

# Freedom-based Measurement of Living Standard

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This paper contrasts conventional real GDP and price indicators of living standard, interpreted as revealing information on *welfare*, with others, that aim at reflecting individual *freedom*. It is argued that freedom-based indices are easier to use and interpret than traditional real GDP ones. Illustrations of the differences between the two classes of indices are provided for international comparisons, as well as for the evaluation of growth and inflation in France.\*

## I. Introduction

Does the average French citizen have a better standard of living now than 10 years ago? Is the standard of living of an average (South) Korean citizen lower than that of a Portuguese? What impact on the standard of living of an average person with a given monetary income does changes in prices have in the last year? Those are examples of questions asked – and answered – very commonly. Yet they are not easy to answer. There are at least two reasons to this difficulty.

The *first* one is that the very notion of an “average” individual is abstract. Actual individuals are all different and experience various changes in living standard over time and/or across countries or states. What happens to the living standard of an “average person” is very likely to be unrepresentative of the reality experienced concretely by actual individuals. For the most part, we will not make any serious attempt in addressing this difficulty in this paper and will stick to the common usage of reasoning about an “average person”, being of course perfectly aware of the abstractness of this notion. We say “for the most part” because we shall, on one occasion, discuss the possibility of using conventional indices with the more ambitious objective of evaluating *distributions* of living standards between the citizens. As we will see however, it is rather difficult to do this in a satisfactory way.

The *second* difficulty arises from the very definition of the concept of “living standard” itself, as noticed among many others, by SEN [1987]. Virtually all concrete answers given to the questions raised above are expressed in terms of two categories of variables: *income* and *prices*. This is not surprising. Most individuals covered by the notion of living standard evolve in market economies where they freely convert their monetary income into goods that satisfy their needs and desires. They do this conversion by paying, for each unit of every good, its market price. Hence the mere knowledge of someone’s monetary income as well as the prices of all goods is enough for knowing the person’s *purchasing power* or, in the economists’ jargon, the person’s *budget set*. How can this purchasing power be measured in a simple and ethically

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meaningful way? This is the main question to which the various indicators used in practice to measure economic growth, standard of living, or inflation provide answer.

For many economists raised in the so-called *welfarist* tradition, the importance of the budget set lies, ultimately, in its ability to provide the person who faces it with a bundle of goods that is the source of a high *welfare* level. It is beyond the scope of this paper to examine in detail the meaning of the notion of welfare that is at stake here (see e.g. GRIFFIN [1986], SEN [1987, 1991] and SUMMER [1996]). It will suffice for our purpose to view, rather vaguely, welfare as an index of the extent to which the individual's *preferences* are satisfied. This view is vague because it does not specify what lies behind the person's preferences (i.e. happiness, desire, needs, etc.).

How can a budget set, which comprises *many* bundles that the individual can afford, be converted into a *single* welfare level? The answer usually provided to this question lies in the assumption of "rational choice" by individuals. Given their purchasing power, individuals are indeed assumed to choose a bundle of goods that provide them with the largest possible welfare. This rationality assumption thus connects any budget set to its unique maximal welfare level. Of course making this connection operational in terms of usable indices of living standard requires additional information on the bundle of goods chosen – by assumption rationally – by an "average person". As mentioned above, and as will be described below, most methods used in practice to evaluate living standards require, in addition of data on prices and income, information on a "representative basket" of goods. Such a representative basket is used to determine the weighted average of prices or to establish a *Purchasing Power Parity* (PPP) conversion of currencies in international comparisons.

This welfarist credo has fueled a growing skepticism in the last thirty years, thanks notably to the work of SEN [1979] (see also SEN [1985;1991] and RAWLS [1971]). It is, again, beyond the scope of this paper to discuss and appraise the reasons underlying this state of affairs. It will suffice for our purpose to recognize that welfare has been considered to be an inadequate, or at least incomplete, ingredient for describing an individual's situation, and that the rationality assumption according to which individuals make welfare maximizing choices has been questioned even by welfarists themselves (see for instance BLACKORBY, BOSSERT and DONALDSON [2005]).

As an alternative to welfare, it has been suggested by some to base the appraisal of an individual situation on *freedom of choice*. There is indeed a long tradition in economics and elsewhere for viewing individual freedom as an important thing, if not *the* most important thing, in normative appraisal. Members of this tradition in economics include KNIGHT [1947], HAYEK [1960], BUCHANAN [1975] and SEN [1988], among many others. When envisaged from this perspective, the importance of the individual purchasing power lies not so much in the welfare that this purchasing power enables the individual to achieve but, rather, in the freedom of choice that it provides. Making this alternative workable requires of course a definition of freedom of choice. The literature, illustrated by the contributions of JONES and SUGDEN [1982], PATTANAIK and XU [1990;1998] and XU [2004] (among others, see the surveys of GRAVEL [2006;2009]) has made efforts to providing plausible and workable such definitions. In a nutshell, freedom of choice tends to be defined by the *size of the opportunity set* faced by an individual. In our context, this amounts to defining freedom as the *number of bundles* of goods that the individual can afford, given income and prevailing prices.

The object of this paper is to illustrate the potential usefulness of empirically implementable summary measures of standard of living that reflect the average individual's freedom of choice, rather than well-being. As we hope to convince the reader, freedom-based measures of standard of living are simpler to use and to construct than welfare-based ones, be it simply because, contrary to the latter, they do not require data on a (more than often arbitrary) "reference basket". Moreover, the use of these measures can be justified by precise axioms that identify their meaning in a crisp way. As we shall recall, few of the most commonly used welfare-based indices are so crisply interpretable in terms of ethical principles. Finally, we will show, using empirical data, that freedom-based indices tend to provide conclusions that are not qualitatively different from those achieved out of welfare-based indices concerning the evaluation of economic growth, international comparisons of living standard, and measurement of inflation.

The remaining part of this paper is organized as follows. The next section recalls the main welfarist framework in which budget sets are evaluated and discusses the justification given to conventional exercises of measuring growth and inflation, and of performing international comparisons of living standards. The third section discusses how budget sets can be evaluated on the basis of the freedom to consume that they provide. It also derives a simple method for comparing opportunity sets in this perspective. The fourth section illustrates the difference between welfarist and freedom based indicators of standard of living and section five concludes.

## II. Welfare-based Indices of Living Standard

### II.1 Preliminaries

Consider an individual living in a particular community (for example a country, a province or a region, possibly distinguished by the time period at which it is examined). The individual has an income of  $I$  units of *numéraire* that he or she can freely spend on a number,  $l$  say, of goods and services available on markets at strictly positive prices  $p_1, \dots, p_l$ .<sup>1</sup> The  $l$  prices  $(p_1, p_2, \dots, p_l)$  and the individual income  $I$  define the **budget set**  $B(p_1, p_2, \dots, p_l, I)$  defined by:

$$B(p_1, p_2, \dots, p_l, I) = \{(x_1, \dots, x_l) \in \mathbb{R}_+^l : \sum_{j=1}^l p_j x_j \leq I\}.$$

This set, which contains all bundles of goods that the individual can afford, given prices and income, provides therefore a complete description of the individual's *purchasing power*. While the budget set is completely defined by prices and income, it is obviously unaffected by the choice of the numéraire used to measure those. Doubling all prices and income for instance – or converting euros into rupees – does not affect the set of bundles of goods that are available

1. We keep throughout the assumption that the number of goods is fixed across the compared communities. In practice however, and especially in the case of long term intertemporal comparisons, this assumption is somewhat unsatisfactory. After all, what were the price of cellular phones in the fifties?. There are various solutions that can be proposed to this well-known "new goods problem". The easier, but probably unsatisfactory, would be to consider that the price of cellular phones in the fifties were infinite. An alternative would be to adopt an approach *à la* LANCASTER [1971] and to view goods as locations in a more fundamental space of characteristics that are priced by some hedonic function (see e.g. ROSEN [1974] and TRIPLET [1994]). A recent discussion of the new goods problem in the context of conventional price indices is provided by HAUSMAN [2003].

for choice to the individual. On the other hand the budget set expands if income increases and prices do not change and it shrinks if some prices increase and nothing else happens. Things are not so clear when, as is typically the case in practice, prices and income change in various directions simultaneously. It is therefore of some importance to obtain a summary measure of the individual purchasing power.

Welfarist economists believe that individuals obtain welfare out of their consumption of goods. They also very often believe that individuals choose from their budget set a bundle of goods that provides them with the highest possible welfare level. This double belief serves as justification for most commonly used indices of individual living standard. If an individual chooses a bundle of goods in a budget set, one concludes that this chosen bundle offers – at least weakly – more welfare than any other affordable bundle. Hence, under the assumption that individuals make “rational” choices, knowing the chosen bundle as well as the prices and income parameters that define this set enables one to perform meaningful comparisons of budget sets. The concept of *per capita* Gross Domestic Product (GDP) is often used for that purpose.

## II.2 *Real Per Capita GDP*

A common definition of GDP is of being the value, at market prices, of all final goods and services sold in the community. Formally, the GDP of a community  $i$ , denoted  $GDP^i$ , is defined by:

$$GDP^i = \sum_{j=1}^l p_j^i X_j^i \quad (1)$$

where  $p_j^i$  is the price of the good  $j$  ( $j = 1, \dots, l$ ) observed in community  $i$  (and labelled in the currency used in that community) and  $X_j^i$  is the *total quantity* of good  $j$  sold in that community. When used to appraise *individual* living standard, the GDP is often divided by the number of individuals in community  $i$ . This *per capita* GDP can be interpreted as the cost, at community  $i$ 's prices, of the bundle of goods chosen by an average individual in the community. Let  $x_j^i = X_j^i/n$  denote the *per capita* – or average – consumption of good  $j$  in community  $i$  where  $n$  denotes the number of individuals living in  $i$  (taken to be the same in every community to simplify notation). What meaning can be given to per capita GDP comparisons across communities?

Not much if one limits oneself to so-called *nominal* comparisons of GDP that do not account for price changes. Indeed, suppose that we compare two values of per capita GDP. The larger of the two values may be larger because prices are higher and goods consumption is lower, or because prices are lower and good consumption is higher, or because of a combination of the two possibilities. Insofar as it is the bundle of goods chosen by the average individual that matters for appraising, in the welfarist perspective, this individual's living standard, it is important to disentangle somehow changes in per capita GDP that are due to price changes from those that are due to quantity changes. The typical way to do this disentanglement is to compare the two GDP levels at a *common set of prices*. When one does this, one is comparing what are called *real per capita GDP* levels.

### II.3 Welfare Meaning of Real per Capita GDP Comparisons

To be specific, consider two communities  $h$  and  $i$ . Suppose we are given, for each of these two communities, the prices and the per capita consumption of every good. On the basis of this information, when can we say unambiguously that the average individual in one community has higher welfare than that of the other? Here is a test that answers this question:

#### Real GDP comparison test:

If  $\sum_{j=1}^l p_j^i x_j^i \geq \sum_{j=1}^l p_j^i x_j^h$  (per capita GDP in community  $i$  is weakly higher than the cost, at community  $i$ 's prices, of the average community  $h$ 's chosen bundle), then community  $i$ 's average individual is weakly better off than he or she would be with the average community  $h$  bundle.

The logic underlying this test is clear. If inequality  $\sum_{j=1}^l p_j^i x_j^i \geq \sum_{j=1}^l p_j^i x_j^h$  is observed, it means that, at prices  $(p_1^i, \dots, p_l^i)$  where the bundle  $(x_1^i, \dots, x_l^i)$  was chosen by the average individual in  $i$ , the bundle  $(x_1^h, \dots, x_l^h)$  chosen in community  $h$  was also *affordable* to this individual. Hence, if this individual is rational, the welfare achieved with the chosen bundle can not be smaller than what it would have been with the affordable bundle  $(x_1^h, \dots, x_l^h)$ . Moreover, if individuals are never satiated, one can derive the *stronger* conclusion that, if the *strict* inequality  $\sum_{j=1}^l p_j^i x_j^i > \sum_{j=1}^l p_j^i x_j^h$  is observed (per capita GDP in  $i$  is strictly larger than the cost, at  $i$ 's prices, of community  $h$ 's bundle), the average individual in  $i$  achieves a *strictly higher* welfare than he or she would get with the bundle chosen in  $h$ . Indeed, the strict inequality means that, at  $i$ 's prices, the average individual in  $i$  would have spent less money on the bundle chosen in  $h$  than on his or her chosen bundle. If this individual can not be satiated, and is rational, then the only reason why he or she would accept to spend strictly more money on a bundle is if the bundle provides strictly more welfare.

While this test provides a welfare-based justification for making real GDP comparisons across communities, it is not free from interpretational difficulties. Indeed, suppose first that the two inequalities:

$$\sum_{j=1}^l p_j^i x_j^i \geq \sum_{j=1}^l p_j^i x_j^h \tag{2}$$

and

$$\sum_{j=1}^l p_j^h x_j^h \geq \sum_{j=1}^l p_j^h x_j^i \tag{3}$$

hold simultaneously and that at least one of the two inequalities is strict. This means that the average individual in community  $i$  is better off than he or she would be with the bundle chosen by the average individual in community  $h$  and, conversely, that the later individual is better off with his or her choice than with community  $i$ 's bundle, with at least one of the two "better off" statement being strict. This situation is clearly problematic. Perhaps it comes from the fact that the average individuals in the two communities differ (for instance the average Korean individual prefers his or her average bundle to that chosen by the average Portuguese and the Portuguese one has converse preferences). But this renders difficult the ranking of Portugal and Korea on the basis of welfare. If Koreans are happier in Korea than they would be in

Portugal while Portuguese are happier in Portugal than they would be in Korea, which of the two countries provides the best standard of living?

On the other hand, if the representative individuals in the two communities have the same preferences, then observing simultaneously the inequalities (2) and (3), with at least one strict is even more problematic. Indeed, in that case, inequality (2) indicates that our (unique) individual is better off in  $i$  than in  $h$  while inequality (3) indicates that the same individual has opposite preference. In the economic jargon, the simultaneous occurrence of these two inequalities represents a violation of the (weak) *axiom of revealed preferences* (see e.g. SAMUELSON [1948] for an early statement and VARIAN [2006] for a more recent survey). This violation is plainly incompatible with the rationality postulate that individuals choose in their budget set a preferred basket of goods. But if individuals do not make rational choices, it is difficult to interpret real per capita GDP figures from a welfarist point of view. In any case therefore, observing simultaneously (2) and (3) raises serious problems of interpretation.

Another difficulty arises if none of the two inequalities (2) and (3) is observed. In that case, it can *neither* be said that the average individual in community  $h$  is better off than what he or she would be with the basket consumed by the average community  $i$  individual *nor* that the average community  $i$  individual is better off with his or her chosen basket than with the basket chosen by the average individual of community  $h$ . The simultaneous violation of inequalities (2) and (3) is simply *uninformative* on the welfare levels achieved by the average individuals in the two communities. This is so irrespective of whether or not the individuals in the two communities can be assumed to have the same preference.

The following table summarizes the normative conclusions that can be obtained from per capita real GDP comparisons on the basis of the GDP test in a welfarist perspective, when representative individuals in the compared communities have identical or different preferences.

inequalities	same preference	different preferences
(2) is true and (3) is false	individual is better off in community $i$ than in community $h$	individual in community $i$ prefers his/her bundle to that of community $h$
(2) is false and (3) is true	individual is better off in community $h$ than in community $i$	individual in community $h$ prefers his/her bundle to that of community $i$
(2) and (3) are true	inconsistent with rational choice	every individual prefers his/her bundle to others'
(2) and (3) are false	uninformative	uninformative

#### II.4 Welfare Meaning of Real PER CAPITA GDP Comparisons Between “Similar” Communities

There are *three* obvious limitations to the welfarist interpretation of real per capita GDP comparisons discussed in the preceding subsection. *First*, as mentioned in the introduction, the interpretation concerns the welfare of an abstract “average individual” who may not represent very well the unequally distributed well-being of the many individuals that are involved. *Second*, comparisons of real *per capita* GDP may be uninformative if none of the inequalities (2) and (3)

are true. *Third*, comparisons of per capita GDP may be difficult to interpret if both inequalities (2) and (3) are true.

For certain types of comparisons of communities, it is possible to provide a somewhat different welfarist interpretation of GDP test that avoids the first two limitations. On the other hand, there is no easy way out of the third limitation. The kinds of comparisons that enable such an alternative interpretation concern situations involving what could be called “*small*” changes in prices, income and quantities. Some comparisons of GDP made in practice concern, indeed, situations that are not very distant from each other. This is clearly the case when one is interested in comparing France in 2007 with France in 2008, or when one compares countries with similar consumption patterns and price levels.

Specifically, take communities  $h$  and  $i$  again and assume that the numbers

$$dI = \sum_{j=1}^l p_j^i x_j^i - \sum_{j=1}^l p_j^h x_j^h$$

and  $dp_j = p_j^i - p_j^h$  that measure differences, respectively, in *per capita* GDP and prices between the two communities are “small” relative to the observed levels of income and prices. There is no precise definition of what “small” means. Let us simply say that the results that we are about to describe will be all the more accurate as the changes in prices and income are small. These results will enable one to provide a welfare interpretation of GDP comparisons that is more conclusive than those underlying the two tests discussed previously. Moreover, the conclusion obtained will not concern a single average individual only but, instead, the whole community of individuals considered.

This requires of course that something be said on the criterion used to normatively evaluate a community as a whole. All individuals living in such a community – call it  $h$  – are assumed to face common prices  $(p_1^h, \dots, p_l^h)$ . Yet these individuals can differ by their income and by the function they use to convert income and prices into welfare, after making rational choices from their budget set. Specifically, for an individual  $k$  (with  $k = 1, \dots, n$ ) living in  $h$ , we denote by  $I_k^h$  his or her income and by  $V_k(p_1, \dots, p_l, I)$  the maximal welfare that this individual can achieve, when endowed with an income of  $I$  and facing prices  $(p_1, \dots, p_l)$ . Welfarist economists evaluate a community on the sole basis of its distribution of welfare levels  $(v_1, \dots, v_n)$ . This evaluation is made by assigning a number  $W(V_1, \dots, V_n)$  to any such distribution of welfare levels. Such a number – interpreted as an index of *social* welfare – is assigned to each distribution of welfare level by the *social welfare function*  $W$ . Such a function summarizes the welfarist ethics used to make normative evaluation in the community. Philosophers and economists have examined various forms that such a social welfare function can take. It is for instance commonly held that  $W$  should be increasing with respect to individual welfare so that the ethics is not of a sadistic variety. It is also typically assumed that  $W$  is quasi-concave so that the ethics has a weak preference for welfare equality, everything else being the same.

The function  $W$  we shall specifically consider is the one according to which the observed distribution of income in that community is optimal. That is, we are going to consider the function  $W$  for which the observed distribution of incomes in community  $h$ , denoted  $(I_1^h, \dots, I_n^h)$  is the solution of the program:

$$\max_{I_1, \dots, I_n} W(V_1(p_1^h, \dots, p_l^h, I_1), \dots, V_n(p_1^h, \dots, p_l^h, I_n)) \text{ s.t. } \sum_{i=1}^n I_i = GDP^h$$

If this is the case, and if the income distribution provides everyone with a positive amount of income, it satisfies – under the usual differentiability assumption – the necessary first order conditions:

$$\frac{\partial W}{\partial V_k} \frac{\partial V_k}{\partial I} = \frac{\partial W}{\partial V_m} \frac{\partial V_m}{\partial I} = a \text{ for all individuals } k \text{ and } m \quad (4)$$

and for some strictly positive number  $a$ . These conditions simply say that, if income is optimally – from the view point of the ethics represented by  $W$  – distributed in community  $h$ , then the ethical benefit – again as evaluated by  $W$  – of giving an additional euro to someone must be equalized – to some number  $a$  – across individuals.

Given (4), consider the impact, on the distribution of welfare in  $h$ , of bringing prices and individual incomes to what they are in community  $i$ . If the prices and income changes required by this move are small, their impact on social welfare, denoted  $dW$ , can be approximated by:

$$dW = \sum_{k=1}^n \frac{\partial W}{\partial V_k} \left( \frac{\partial V_k}{\partial I} dI_k + \sum_{j=1}^l \frac{\partial V_k}{\partial p_j} dp_j \right) \quad (5)$$

where, for every individual  $k$ ,  $dI_k = I_k^i - I_k^h$  and, for community  $j = h, i$ ,  $I_k^j$  is the income achieved by individual  $k$  in community  $j$ . Of course, by definition of GDP one has:

$$\sum_{k=1}^n dI_k = ndI \quad (6)$$

Using equation (4), condition (5) can also be written as:

$$dW = a \left[ \sum_{k=1}^n [dI_k + \sum_{j=1}^l \frac{\partial V_k / \partial p_j}{\partial V_k / \partial I} dp_j] \right] \quad (7)$$

Using Roy's identity, we can alternatively write equation (7) as:

$$dW = a \left[ \sum_{k=1}^n dI_k - \sum_{j=1}^l X_j^h dp_j \right]$$

or, using (6) and the definition of GDP provided by (1):

$$\begin{aligned} dW &= a \left[ \sum_{j=1}^l [p_j^i X_j^i - p_j^h X_j^h - X_j^h dp_j] \right] \\ &= a \left[ \sum_{j=1}^l [p_j^i X_j^i - p_j^h X_j^h - X_j^h (p_j^i - p_j^h)] \right] \\ &= a \left[ \sum_{j=1}^l p_j^i X_j^i - \sum_{j=1}^l p_j^h X_j^h \right] \end{aligned} \quad (8)$$

Since  $a$  is a positive number if income is optimally distributed in  $h$ , the sign of the right hand side of this expression is determined exclusively by the sign of  $\sum_{i=1}^l p_j^i X_j^i - \sum_{i=1}^l p_j^h X_j^h$ . Hence, equality (8) provides the following alternative real GDP test:

1. *Alternative real GDP test 1*: Distribution of well-being is ethically better in  $i$  than in  $h$  for the ethics that considers optimal the income distribution in  $h$  if and only if GDP is higher in  $i$  than in  $h$  when the two GDPs are compared at community  $i$ 's prices.

Using an analogous reasoning, but looking at things from the view point of the (possibly different) community  $i$ 's ethics, one can obtain an analogous alternative real GDP test 2.

2. *Alternative real GDP test 2*: Distribution of well-being is ethically better in  $i$  than in  $h$  for the ethics that considers optimal the income distribution in  $i$  if and only if GDP is higher in  $i$  than in  $h$  when the two GDPs are compared at community  $h$ 's prices.

An important difference between these alternative tests and the test described previously is that the alternative tests provide **definite** answers in terms of welfare comparisons. Indeed, *per capita* GDP evaluated at community  $i$ 's prices is higher in  $i$  than in  $h$  **if and only if** the distribution of welfare is better in  $i$  than in  $h$  for the ethics that considers optimal the distribution of income in  $h$ . Hence, with the alternative tests, we can **never** be in the inconclusive fourth line of the table above. Of course it can happen that real per capita GDP is higher in  $i$  than in  $h$  when  $i$ 's prices are used but higher in  $h$  than in  $i$  when the two are compared at  $h$  prices. In that case, one must conclude *either* that the social welfare function that judges optimal the income distribution in  $i$  is different than that which judges optimal the income distribution in  $h$ , *or* that individuals do not make rational choices and, therefore, that Roy's identity does not apply. This problem is similar to the one encountered for the first two GDP tests concerning opposing individual preferences with respect to the two average bundles.

While the possibility provided by these two alternative tests of drawing ethical conclusion on the distribution of welfare, rather than on the welfare of an abstract average individual, may be seen as an advantage, the fragility of the ethical conclusion thus obtained is worth stressing. The conclusion is based, indeed, on the ethics that considers optimal the actual distribution of income observed in one of the communities under comparison. Suppose that the communities are France in 2007 and France in 2008. It is not clear to us that the ethics that considers optimal the distribution of income in France in either of the two years would command widespread support.

## II.5 *GDP-Based Indices of Living Standards in Practice*

When using GDP-based indices to evaluate standards of living in applied work, few practitioners connect these to welfarist theory in the way just described. Rather, they simply compare GDP, either across time or across countries, using a common set of prices and do not interpret their comparisons in terms of well-being. The meaning of these comparisons is therefore doubtful. An exception to this is the vast literature, nicely surveyed in DIEWERT [2007], that deals with the economic approach to index numbers. While this literature bases index numbers on explicit utility maximization the connections with welfarism are not explicitly spelled out and the arguments are not presented in terms of revealed preferences. Yet, the interpretation of the economic approach to index numbers is formally similar to that discussed above.

In applied work, there are at least three main instances where GDP-based comparisons of living standard are performed:

- 1) the evolution over time of the GDP for a given community (the issue of economic growth).
- 2) the international comparisons of per capita GDP
- 3) The evolution over time of prices, and the measurement of inflation.

### II.5.1 Measurement of Economic Growth

Economic growth is commonly defined to be the rate of growth of real GDP, or sometimes per capita GDP, of a community between two points of time. In principle, this measurement of economic growth is nothing else than a particular instance of the comparison discussed above, where communities  $h$  and  $i$  are taken to be the same community observed at two points of time (say that  $i$  is current time and  $h$  is an early period). In practice however, growth measurement proceeds by dividing GDP of the periods by a *price index* and by calling the result of this division a real GDP. The *growth rate* is then defined to be the rate of variation of this real GDP. A price index for a community  $i$  is the ratio of the cost of a *reference bundle* at prices prevailing in that community to the cost of the reference bundle at some reference price. Formally, let  $(x_1^r, \dots, x_l^r)$  be some reference bundle of the  $l$  goods and let  $p_1^r, \dots, p_l^r$  be the reference prices. Then, the price index of community  $i$  for these reference prices and bundles, denoted  $P_r^i$ , is defined by:

$$P_r^i = \frac{\sum_{j=1}^l p_j^i x_j^r}{\sum_{j=1}^l p_j^r x_j^r} \quad (9)$$

and the so-called real GDP of community  $i$ , denoted  $\widehat{GDP}_i$ , is given by:

$$\widehat{GDP}_i = \frac{GDP_i}{P_r^i} \quad (10)$$

One then defines real growth rate  $\rho$  between periods  $h$  and  $i$  to be:

$$\rho = 100 \times \left( \frac{\widehat{GDP}_i}{\widehat{GDP}_h} - 1 \right) \quad (11)$$

What meaning can be given to this growth rate or, equivalently, to the ratio  $\widehat{GDP}_i/\widehat{GDP}_h$ ? Using (9) and (10), this ratio writes:

$$\frac{\widehat{GDP}_i}{\widehat{GDP}_h} = \frac{\sum_{j=1}^l p_j^i x_j^i \sum_{j=1}^l p_j^h x_j^r}{\sum_{j=1}^l p_j^i x_j^r \sum_{j=1}^l p_j^h x_j^i}$$

It is clearly difficult to interpret this expression further if nothing is said on the reference bundle of goods. Very commonly, one assumes that the reference bundle is the one consumed in one of

the two periods under consideration. If for instance we assume that the bundle consumed in  $h$  is used as reference, then the ratio can be written as:

$$\frac{\widehat{GDP}_i}{\widehat{GDP}_h} = \frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^i x_j^h}$$

In this case, having the ratio greater than one and, therefore, growth positive is equivalent to requiring community  $i$  to be better off than community  $h$  as per the GDP test above (equation (2)). A similar, but reverse, conclusion could be obtained if one were using community  $i$ 's prices as references. Of course, when measuring growth, it is important to check that growth is positive for both sets of prices (community  $h$  and  $i$ ). If growth is positive for one set of price and negative for another, one would be in the interpretatively difficult situation analogous to that of the third line of the table above.

It is also worth recalling that, if a positive growth rate is indicative of an improvement in social – under alternative GDP tests – or average – if the two first GDP test are used – welfare in a community observed at two points of time, the comparison of growth rates between different communities is difficult to interpret if the GDP of the two communities are evaluated at different sets of prices.

### II.5.2 International Comparisons of per capita GDP

When international comparisons of GDP are involved, one faces the issue of expressing prices and incomes into a common numéraire. Since the budget set, and therefore the rational choice made from this set, is unaffected by the choice of the numéraire, there is no problem in doing this. More than often, it is the US dollar that is used as numéraire so that all prices and incomes are expressed in that currency, using exchange rates prevailing at the time where the currency conversions are performed. Of course, comparisons of nominal *per capita* GDP expressed in US dollars have no meaning if different prices prevail in the compared communities. It is to address this issue that many *per capita* GDP comparisons use so-called **Purchasing Power Parities** (PPP) conversion factors to correct for price differences between communities. A PPP conversion factor is a rate of exchange of the numéraire in terms of some international standard, typically taken to be the value, at some reference prices ( $p_1^r, \dots, p^r$ ) expressed in US dollars, of a reference bundle of goods ( $x_1^r, \dots, x_j^r$ ). In that context, the ratio:

$$\frac{\sum_{j=1}^l p_j^r x_j^r}{\sum_{j=1}^l p_j^i x_j^r}$$

is defined to be the PPP exchange rate of community  $i$ . Notice that this ratio is nothing else than the inverse of the price index defined previously. In the current context, this ratio is interpreted to be the amount of community  $i$ 's currency that is required per US dollar spent on the reference bundle. In that context, the PPP adjusted per capita GDP of community  $i$ , noted  $\overline{gd}p^i$  is defined to be:

$$\begin{aligned}
 \overline{gdp}^i &= \frac{\sum_{j=1}^l p_j^r x_j^r}{\sum_{j=1}^l p_j^i x_j^r} \overline{gdp}^i \\
 &= \frac{\sum_{j=1}^l p_j^r x_j^r}{\sum_{j=1}^l p_j^i x_j^r} \sum_{j=1}^l p_j^i x_j^i \tag{12}
 \end{aligned}$$

What can be the meaning of PPP adjusted per capita comparisons ? Specifically, assume that PPP adjusted per capita GDP is higher in community  $i$  than in community  $h$  so that inequality:

$$\overline{gdp}^i > \overline{gdp}^h \tag{13}$$

Given (12), this inequality can equivalently be written as:

$$\frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^i x_j^r} > \frac{\sum_{j=1}^l p_j^h x_j^h}{\sum_{j=1}^l p_j^h x_j^r}$$

What can be inferred from this information? Again, the answer is: not much without further assumption on the choice of the reference bundle that enters into the definition of PPP. Suppose for instance that one uses the bundle consumed by the average country  $h$ 's individual as reference so that  $x_j^r = x_j^h$  for any good  $j$ . In that case, the above inequality writes:

$$\begin{aligned}
 \frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^i x_j^h} &> \frac{\sum_{j=1}^l p_j^h x_j^h}{\sum_{j=1}^l p_j^h x_j^h} \\
 &\Leftrightarrow \\
 \sum_{j=1}^l p_j^i x_j^i &> \sum_{j=1}^l p_j^i x_j^h
 \end{aligned}$$

so that it implies (GDP test 1) that the average individual in community  $i$  prefers his or her bundle to that consumed in  $h$ . Yet in order to make per capita GDP comparisons comparable between more than two countries, the reference bundle used in PPP conversion factor is seldom, if ever, the bundle chosen by one of the two compared countries. One finds, instead, alternative more or less “objective” reference bundles, ranging from the Big Mac sandwich advocated

by the magazine *The Economist* - to more sophisticated bundles that aim at representing the average “world citizen” choice. Yet, for an arbitrary reference bundle that is unconnected with the two compared communities, inequality (13) carries no meaning whatsoever in terms of welfare. Hence no ethically meaningful conclusions can be derived from PPP adjusted per capita GDP comparisons the way they are usually conducted.

### II.5.3 Measurement of the Cost of Living

As performed by several statistical agencies – including the French INSEE – the *inflation rate* is defined to be the rate of increase – in percentage – of *average prices* between two periods (say  $h$  and  $i$ , with  $i$  being the most recent period). Average prices are, again, calculated as the price of a reference bundle of goods  $(x_1^r, \dots, x_l^r)$  or, alternatively, as a weighted average of the prices, with the weights given by the quantitative importance of the corresponding good in the reference bundle. With such a reference bundle, the inflation rate between period  $h$  and  $i$ , denoted  $\Delta P$ , is defined by:

$$\Delta P = 100 \left( \frac{\sum_{j=1}^l p_j^i x_j^r}{\sum_{j=1}^l p_j^h x_j^r} - 1 \right)$$

There are two main approaches for selecting the reference bundle used in the definition of the inflation rate. One is the so-called Laspeyres approach. It uses the bundle consumed by the average individual in the earlier period  $h$  as reference. The other is the Paasche approach that takes as reference the bundle consumed in the most recent  $i$  period.

What is the meaning of the rate of inflation? Not much in itself. Indeed, if nothing is known about the evolution of the income of the individual who is facing the prices described in this rate, it is difficult to say anything about the welfare impact of prices changes. In common discussions, it is usually assumed that inflation measures a depreciation of purchasing power of someone who has a fixed income. Is this assumption correct? Specifically can we infer something on the change in welfare experienced by an individual with a given income between two periods from the inflation rate prevailing between the two periods? The answer to this question is positive if either the Paasche or the Laspeyres approach is used to define inflation.

Notice first that we can define the income of the average individual in any period to be the *per capita* GDP of the community in that period (under the assumption that the individual consumes all his or her income, or, alternatively, that saving is included in the list of goods). If we do that, we can conclude from the observation that inflation – as measured the Paasche approach – is higher than the rate of growth of the individual income that individual welfare has decreased between the two period. Indeed having a larger inflation rate, in the Paasche sense, than the rate of growth in income means that inequality:

$$\frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^h x_j^i} \geq \frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^h x_j^h} \tag{14}$$

holds or, equivalently, that:

$$\sum_{j=1}^l p_j^h x_j^h \geq \sum_{j=1}^l p_j^h x_j^i$$

holds. Hence, following the logic of the above GDP test, one can conclude that the average individual in  $h$  prefers his or her bundle of goods to that consumed by the (possibly different) average individual in  $i$ . A special case of this rule applies if there are no income changes. In that case, a positive inflation rate in the Paasche sense implies a welfare loss for the average individual in community  $h$ .

In a somewhat analogous line, suppose that inflation, as measured by the Laspeyres' approach, is lower than the rate of income growth so that the inequality:

$$\frac{\sum_{j=1}^l p_j^i x_j^h}{\sum_{j=1}^l p_j^h x_j^h} \leq \frac{\sum_{j=1}^l p_j^i x_j^i}{\sum_{j=1}^l p_j^h x_j^i} \tag{15}$$

holds or, equivalently, that:

$$\sum_{j=1}^l p_j^i x_j^h \leq \sum_{j=1}^l p_j^i x_j^i$$

holds. Then, following again the reasoning of the earlier GDP test, one concludes that the average individual in  $i$  prefers his or her bundle to that consumed in  $h$ . Applying this result to the particular case where income does not change between the two periods, we conclude from this that a **negative inflation**, in the Laspeyres sense, is indicative of a welfare gain of the average consumer in period  $i$ . Of course, if Laspeyres' inflation is positive and Paasche inflation is negative, then we are faced with an indeterminacy if we follow the logic of the first GDP test. Likewise, if Laspeyres inflation is negative and Paasche inflation is positive, we conclude that there are either conflicting views of welfare changes between the two communities or a violation of the individual rationality.

Using equation (8), and assuming zero changes in income, we can equivalently conclude, more strongly, that Laspeyres and Paasche inflation are equivalent to ethical worsening in the sense of the two alternative tests if price changes are small.

## II.6 Limitations of Welfare-based Measures of Living Standards

Let us summarize the limitation of the welfare-based measures of standard of living described so far.

1. In order to make meaningful welfare-based comparisons of living standard, information must be available on good prices, average income (or GDP) *and* a reference bundle.
2. Even when all this information is available, there are instances where comparisons of living standards do not carry any ethical meaning

3. When ethical meaning can be assigned to comparisons of living standard, this ethical meaning always rides on the strong assumption of rational individual behavior.

As will now be seen, these three limitations can be overcome by using different indicators of living standards based on individual freedom of choice, rather than welfare. Freedom-based indicators require information on prices and income but not on a reference bundle of goods. Moreover meaningful comparisons of freedom-based indicators can be obtained in all circumstances without any additional assumption on the individual behavior or on the fact that differences between communities are not small. The price to pay for this benefit is to accept the ethical idea to replace welfare by freedom of choice as the proper aspect of an individual situation to be looked at. Another limitation of freedom-based indicators is that they do not lead to meaningful conclusions about the distribution of individual freedoms even when the differences between communities are small.

### III. Freedom-based Indices of Standard of Living

The freedom that an individual has to make the choices he or she may want to make, irrespective of whether or not he or she will actually make them, is clearly an important dimension of the individual situation. According to some philosophers, it could also be one of the most important such dimension. Alexis de Toqueville for instance wrote some time ago that freedom is the most important aspect of an individual situation and adds that “the man who asks of freedom anything other than itself is born to be a slave”.

Yet agreement with Alexis de Toqueville is not necessary for recognizing the importance of freedom of choice for human beings. It is quite possible to argue that freedom is important not only for itself – as de Toqueville claimed – but, also, for the benefit that it provides in terms, say, of a higher welfare. After all, an individual who cares about welfare and makes choices with the objective of achieving as much welfare as possible will benefit from having his or her freedom enlarged. It is therefore quite possible to hold the view that freedom is important even in an instrumental perspective of enabling individuals to achieve better – welfare-wise say – outcomes. Can we design numerical indicators of individual freedom that could lead to intertemporal and international comparisons ?

A relatively recent literature in economics has made some progress toward answering this question. The notion of freedom captured by this literature is that of *opportunity* or *power* to do things. Individuals are described as facing opportunity sets and as making, in a further stage, a choice from these sets. Freedom is envisaged, in this perspective, *before* the choices are made and the question addressed is that of measuring the freedom offered by an opportunity set. Measurement of freedom is achieved very often through *axiomatic* reasoning. That is, simple properties, or axioms, that a plausible definition of the statement “offers more freedom than” could verify are specified and a characterization of the ranking of opportunity sets that satisfy these properties, if any, is obtained. A ranking of sets that has appeared often in this axiomatic work is the *number* of available options.

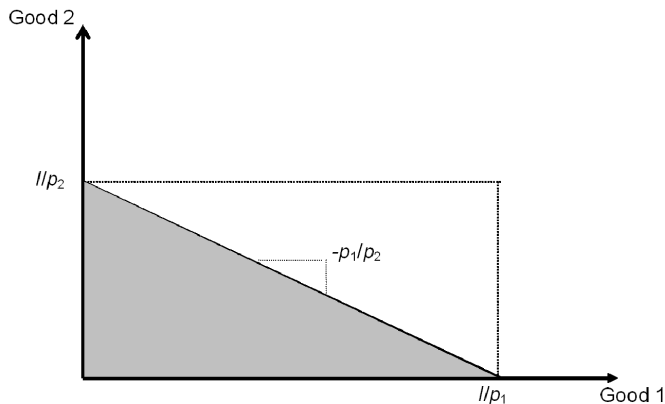
We are in this paper interested in economic opportunities as these are specified by the individual’s budget set. XU [2004] has provided an axiomatic justification for using a specific

measure of the “size” of the budget set: its *volume*. The volume of the budget set can be seen as the *number of bundles* of goods available to the individual facing the set. Ranking opportunity sets on the basis of their number of available options has been suggested by many (see e.g. HAYEK [1960], JONES and SUGDEN [1982], SUPPES [1987], PATTANAİK and XU [1990], CARTER [1999] and VAN HEES [2000]) as a plausible definition of freedom of choice. The connection between the volume of a budget set and the number of bundles that the set offers is easily seen if we assume that goods are available in discrete amounts: 0, 1, 2, etc. units. The volume of the budget set is nothing else than a continuous approximation of the number of distinct bundles of goods that the individual can afford, given his or her budget set.<sup>2</sup>

From a mathematical point of view, the volume of the budget set  $B(p_1, \dots, p_l, I)$ , denoted  $v(p_1, \dots, p_l, I)$ , is defined by:

$$v(p_1, \dots, p_l, I) = \frac{I^l}{l! \prod_{j=1}^l p_j}$$

A geometrical illustration of this formula as a measure of the volume of the budget set is provided in the FIGURE 1 below for the case where there are only two goods. In that two-dimensional setting the volume – in fact the area – of the rectangular triangle that describes the budget set  $B(p_1, p_2, I)$  is indeed simply half the area of the rectangle of height  $I/p_2$  and basis  $I/p_1$ . The generalization of this formula to the arbitrary  $l$ -goods case is straightforward.



2. It could be argued that not every bundle of the budget set is equally valuable from a freedom-based point of view, and, in particular, that bundles whose cost is strictly less, at the prevailing price, than the consumer’s income do not contribute at all to freedom. Such a concern would naturally lead one to consider the size of the budget *line* – rather than the budget *set* – as an appropriate measure of freedom. In the abstract literature on ranking opportunity sets, there has been some papers, like PATTANAİK and XU [1998], who have characterized rankings of opportunity sets on the basis of the number of elements of their sets of essential alternatives, rather than on their total number of elements. We are not aware of any axiomatic characterization of such rankings in the more structured contexts of budget sets considered herein, where the essential alternatives of a budget set could be those who belong to the boundary of the set. We believe that obtaining such a characterization would be of definite interest.

Since the ranking of budget sets induced by the comparisons of their volume is unaffected if a monotonically increasing transformation is applied to  $v$ , we shall measure the freedom offered by the budget set  $B(p_1, \dots, p_l, I)$  by the number  $\widehat{v}(p_1, \dots, p_l, I)$  defined by:

$$\begin{aligned} \widehat{v}(p_1, \dots, p_l, I) &= (I!)^{1/l} [v(p_1, \dots, p_l, I)]^{1/l} \\ &= \frac{I}{\prod_{j=1}^l p_j^{1/l}} \end{aligned} \tag{16}$$

Hence, with this specification, the freedom of choice offered by a budget set is measured by the ratio of the income over the *geometric mean* of the prices. Notice that no reference bundle is required for defining freedom in this fashion.

Is this measure of freedom plausible? Yes if one believes that the following three axioms should be satisfied by any freedom-based ranking of budget sets.

**Axiom 1.** (Monotonicity) If the budget set  $B$  contains the budget set  $B'$  as a subset, then  $B$  offers no less freedom than  $B'$  and if  $B$  contains  $B'$  as a proper subset, then  $B$  offers strictly more freedom than  $B'$ .

**Axiom 2.** (Invariance with respect to units of measurement) Suppose that budget  $B(p_1, \dots, p_l, I)$  offers weakly more freedom of choice than  $B(p'_1, \dots, p'_l, I')$ . Then for any good  $j$  that has a strictly positive price in the two budgets, we must also have that budget  $B(p_1, \dots, p_{j-1}, \lambda p_j, p_{j+1}, \dots, p_l, I)$  offers weakly more freedom than budget  $B(p'_1, \dots, p'_{j-1}, \lambda p'_j, p'_{j+1}, \dots, p'_l, I')$  for any strictly positive real number  $\lambda$ .

**Axiom 3.** (symmetry between goods) Suppose that budget  $B(p_1, \dots, p_l, I)$  offers weakly more freedom than budget set  $B(p'_1, \dots, p'_l, I')$ . Then, if the prices  $(\pi_1, \dots, \pi_l)$  result from a permutation of the prices  $(p_1, \dots, p_l)$ , it must be that  $B(\pi_1, \dots, \pi_l, I)$  offers also weakly more freedom than  $B(p'_1, \dots, p'_l, I')$ .

The *monotonicity* axiom is natural. It says that changes in prices and income that do not reduce the affordability of any bundle can not reduce freedom. It says also that changes in prices and income that leave affordable all bundles to which the consumer had previously access *and* that make affordable bundles that were not so before strictly increase freedom. It seems quite difficult to think of a conception of freedom of choice that would violate this axiom and that would, therefore, consider that enlarging the set of available options could, in some circumstances, reduce freedom.

The axiom of *Invariance with respect to units of measurement* is also somewhat natural. It says that changing the unit of measurement of any good should not change the ranking of budget sets in terms of their freedom. For instance, suppose that good  $j$  refers to a particular brand of beer and that the quantity of beer is measured in bottles of half a litre. Hence for that unit of measurement, the price of beer is the amount of numéraire that the individual must pay to get one bottle of half a litre of beer. Suppose it is considered, for this method of measuring units of beer, that budget set  $B(p_1, \dots, p_{j-1}, p_j, p_{j+1}, \dots, p_l, I)$  offers more freedom than budget set  $B(p'_1, \dots, p'_{j-1}, p'_j, p'_{j+1}, \dots, p'_l, I')$ . What the axiom says is that this verdict should *not* be affected by a change in the unit of measurement of beer. Suppose for instance that we decide to

measure beer in litres rather than in bottles of half a litre. This means that the price of beer in the first budget, which was  $p_j$  units of numéraire for half a liter, will now become  $2p_j$  units of numéraire per liter. Similarly, in the second budget the price of beer will move from  $p'_j$  units of currency per bottle of half a litre to  $2p'_j$  units of currency per litre. The axiom says that changing the unit of measurement in this fashion does not affect the ranking of the two budget sets.<sup>3</sup>

The axiom of *symmetry between goods* is clearly more disputable. It says that the nature of the goods involved does not matter for freedom appraisal. Specifically, consider a budget set  $B(p_1, \dots, p_l, I)$  and assume that it offers more freedom than the budget set  $B(p'_1, \dots, p'_l, I')$ . Consider now *permuting* the prices of some (or all) of the goods in such a way that, say, the price of butter becomes the price of housing and the price of housing becomes the price of butter. Let us denote as  $(\pi_1, \dots, \pi_l)$  the result of this price permutation. The axiom requires that the freedom standing of the budget set  $B(\pi_1, \dots, \pi_l, I)$  with the prices permuted in this fashion *vis-à-vis*  $B(p'_1, \dots, p'_l, I')$  should be the same as the standing of the original (unpermuted) budget set  $B(p_1, \dots, p_l, I)$ . Should it really? Suppose that budget sets  $B(p_1, \dots, p_l, I)$  and  $B(p'_1, \dots, p'_l, I')$  differ only by the fact that, in the first budget, housing price is significantly lower while wine price is mildly higher. In particular, all other prices and income are the same in the two budget sets. It is plausible that, in such a situation, the first budget set could be considered as offering more freedom of choice than the second. It is plausible that a sharp drop in housing price significantly enlarge freedom of choice even when accompanied by a small increase in the wine price. Yet this does not need to entail that a symmetric sharp drop in wine price accompanied by a mild increase in housing price also enlarge freedom of choice. This is clear if housing is perceived as a “more important” good, than wine. Yet the symmetry axiom rules out the possibility for some good to be *a priori* more important than another. This axiom imposes on all goods to have the same importance in terms of their contribution to freedom.

Comparing budget sets on the basis of their volume as per formula (16) is tightly connected to these three axioms. Specifically, XU [2004] has shown that the only reflexive, complete and transitive ranking of budget sets that satisfies the three axioms is the ranking according to their volume, as defined by formula (16). Hence, accepting monotonicity, invariance with respect to the unit of measurement of the goods and symmetry between goods forces one to accepting to measure freedom by formula (16) and, conversely, endorsing formula (16) as a good measure of freedom entails accepting the three axioms.

Yet, as argued above, the symmetry axiom that forces all goods to have the same importance may be considered suspicious. Can we obtain implementable indices for comparing budget sets on the basis of freedom *without* imposing symmetry? It turns out that we can, at least if we accept to restrict attention to *continuous* rankings of budget sets. The mathematical property of continuity appears, we believe, quite acceptable in our context. It says that small changes in prices and/or income should not lead to big changes in the ranking of two budget sets. More precisely, if one set is ranked strictly above another, then the verdict should be robust to arbitrarily small perturbations in prices and income. Let us state this property formally.

3. Invariance with respect to units of measurement is the name given by Xu (2004) to this axiom. It is fair to say that this axiom implies in fact much more than a mere invariance with respect to units of measurement. Indeed, this axiom is akin to a (strong) forme of scale invariance, which may not be appealing in all circumstances.

**Axiom 4.** (Continuity) For any two budgets  $B(p_1, \dots, p_l, I)$  and  $B(q_1, \dots, q_l, I')$  such that  $B(p_1, \dots, p_l, I)$  offers strictly more freedom than  $B(q_1, \dots, q_l, I')$ , there exists a number  $\varepsilon > 0$  such that  $B(p_1^\varepsilon, \dots, p_l^\varepsilon, I^\varepsilon)$  offers strictly more freedom than  $B(q_1, \dots, q_l, I')$  and that  $B(p_1, \dots, p_l, I)$  offers strictly more freedom than  $B(q_1^\varepsilon, \dots, q_l^\varepsilon, I^{\varepsilon'})$  for all combinations of prices and income  $(p_1^\varepsilon, \dots, p_l^\varepsilon, I^\varepsilon)$  and  $(q_1^\varepsilon, \dots, q_l^\varepsilon, I^{\varepsilon'})$  such that  $|p_j^\varepsilon - p_j| \leq \varepsilon$ ,  $|q_j^\varepsilon - q_j| \leq \varepsilon$ ,  $|I^\varepsilon - I| \leq \varepsilon$  and  $|I^{\varepsilon'} - I'| \leq \varepsilon$  for  $j = 1, \dots, l$ .

If we restrict attention to continuous rankings of budget sets in this sense, then we can justify measuring freedom by a formula similar to (16), but that allows prices at the denominator to be raised to any exponent whatsoever (and not only to  $\frac{1}{l}$ ). Specifically, we can justify measuring the freedom offered by the budget set  $B(p_1, \dots, p_l, I)$  by a function  $f$  that writes:

$$f(p_1, \dots, p_l, I) = \frac{I}{p_1^{\alpha_1} p_2^{\alpha_2} \dots p_l^{\alpha_l}} \tag{17}$$

for some strictly positive numbers  $(\alpha_1, \dots, \alpha_l)$  that can be chosen, without loss of generality, in such a way that they sum to one. Formula (16) is clearly a particular case of such a function that imposes the additional assumption that  $\alpha_1 = \alpha_2 = \dots = \alpha_l = \frac{1}{l}$ . While measuring freedom of choice by a function  $f$  that can be written as per (17) has the advantage of allowing the prices of the various goods to be treated differently (e.g. housing price may have a bigger exponent than wine price), it does not provide any indication as to the choice of the appropriate exponent.

Let us now establish, in the following proposition, that comparing budget sets on the basis of a function  $f$  that can be written as per (17) is the only continuous way to rank budget sets that satisfy monotonicity and invariance with respect to units of measurement. This simple proposition rides heavily on theorem 4.7 of CANDEAL and INDURAIN [1995].

**Proposition 1.** Assume that income and prices are strictly positive. Then, a continuous, reflexive, complete and transitive ranking of budget sets satisfies monotonicity and invariance from units of measurement if and only if it is based on a function  $f$  which can be written as per (17) for some numbers  $\alpha_1, \dots, \alpha_l$  satisfying  $\alpha_j \in [0, 1]$  and  $\sum_{j=1}^l \alpha_j = 1$ .

**Proof.** If income is strictly positive, a budget set  $B(p_1, \dots, p_l, I)$  is defined by the  $l$  “normalized” prices  $(\rho_1, \dots, \rho_l)$  defined by  $\rho_j = p_j/I$  for  $j = 1, \dots, l$ . Hence, ranking budget sets amounts to ranking vectors of such normalized prices. If the ranking is reflexive, complete, transitive and continuous, and if all normalized prices can be considered, then by DEBREU [1954] theorem, there exists a function  $\Phi : \mathbb{R}_+^l \rightarrow \mathbb{R}$  that numerically represents it. That is to say budget  $B(p_1, \dots, p_l, I)$  offers weakly more freedom than budget  $B(q_1, \dots, q_l, I')$  if and only if  $\Phi(p_1/I, \dots, p_l/I) \geq \Phi(q_1/I', \dots, q_l/I')$ . Clearly, the ranking is monotonic with respect to set inclusion if and only if  $\Phi$  is decreasing with respect to each of its  $l$  argument. It can also be checked that if the ranking satisfies invariance from units of measurement, it satisfies the property of multiplicative invariance of CANDEAL and INDURAIN [1995; def. 4.6]. For this reason, theorem 4.7 of CANDEAL and INDURAIN [1995] (which requires  $\Phi$  to be defined on  $\mathbb{R}_{++}^l$ ) applies so that  $\Phi$  can be written as

$$\Phi(\rho_1, \dots, \rho_l) = \rho_1^{\alpha_1} \dots \rho_l^{\alpha_l}$$

for some real numbers  $a_1, \dots, a_l$ . Any such number must be negative if  $\Phi$  is decreasing in its argument. Hence, defining  $b_l = -a_l > 0$  we can write  $\Phi$  as:

$$\Phi(p_1/I, \dots, p_l/I) = \frac{I^{b_1+\dots+b_l}}{p_1^{b_1} p_2^{b_2} \dots p_l^{b_l}}$$

for any strictly positive prices  $p_1, \dots, p_l$  and income  $I$ . Acknowledging the ordinality of measurement provided by  $\Phi$  and defining  $f(p_1, \dots, p_l, I)$  by

$$f(p_1, \dots, p_l, I) = [\Phi(p_1/I, \dots, p_l/I)]^{\frac{1}{b_1+\dots+b_l}}$$

gives the result (by posing  $\alpha_j = b_j/(b_1 + \dots + b_l)$ ). □

Basing comparisons of living standard on either formula (16) or (17) is easy. One only needs to know the (average) income in a community and the  $l$  prices that prevail, an information that is required anyway to perform conventional GDP-based comparisons. Yet, contrary to conventional GDP-comparisons, one does not need to assume anything on a representative bundle of goods. Moreover, as was said, all comparisons of communities on the basis of either formulas (16) or (17) can be interpreted in terms of freedom of choice, and the interpretation is crisply expressed in terms of the axioms used in the characterization of each of these two formulas. However, measuring standard of living by formula (17) requires one to specify the exponents  $\alpha_j$  which weight the prices of the various goods. Theory does not provide any guidance for making this choice. A possibility, explored in the next section, is to weight each price according to the budget share of the good in some “reference” budget. This however reintroduces the need of choosing the reference budget. Moreover, it is noteworthy that if one weights prices as per the good’s budget share in some reference budget, then, the ranking of budget sets based on formula (17) coincide with that obtained with the so-called Törnqvist-Theil family of index numbers (see e.g. DIEWERT [1976] and the discussion in DIEWERT [2007]). Of course the interpretation of the two formulas – freedom-based here and welfarist-based in DIEWERT [1976] – is quite different. Moreover, it is not clear that weighting prices as per budget shares in some actual budget is the natural thing to do under the freedom-based interpretation of formula (17). If the prices of some of the goods – like education or health – are subsidized, then weighting these prices as per the budget share devoted to the spending on these goods may underestimate the contribution of these goods to freedom.

In the next section, we illustrate how freedom-based comparisons of living standards can be made and compared with conventional GDP-welfare based ones.

## IV. Freedom vs Welfare-based Indices: an Empirical Appraisal

### IV.1 Measurement of Growth in France

We first show how conventional evaluation of growth rate of real per capita GDP, as per formula (11), compares with that of the growth rate of freedom as measured either by the symmetric formula (16), or by some non-symmetric specification of (17). Data used in these calculations come from the National Account provided by INSEE (*Comptes nationaux - Base 2000, Insee*).

The data provides information on GDP at 2000 prices. GDP at current prices for different years are then calculated using inflation rates provided by INSEE (and calculated as per the Laspeyres method). Determining the value of the product of prices raised at specific powers – as per the denominators of expressions (16) and (17) – requires of course data on the price of *each* good. Yet we did not have access to data on individual prices. Hence we had to resort to the imperfect alternative of using average price indices for 82 *categories* of goods and treating each of these price index as if it was the price of a single good. The price index for each group of good is calculated by INSEE using the Laspeyres method. To that extent, our calculation of the product of prices raised at specific exponents is slightly inconsistent because it takes the product of price indices which are themselves weighted sums of more disaggregated prices. This inconsistency must be kept in mind when comparing the measurement of growth provided by the different methods. The definition of the goods categories is provided in appendix A.

We have considered several weighting scheme of prices in the definition of freedom as per (17). The first is the symmetric one provided by (16). For the others, we have raised the price of each good at an exponent given by the good's *budget share* – defined as the ratio of the expenditure on the good over total income – in various reference situations. Two reference situations are the expenditure pattern for France in 1997 and in 2007. The exact values of the budget shares of the various goods in France for each of the two years are also given in Appendix A. As can be seen from this appendix, budget shares calculated by INSEE are somewhat surprising for certain categories of goods. For instance, according to INSEE, an average French individual devoted only 6.88% of his or her income to housing in 2007. This comes from the fact that INSEE attributes zero housing expenses to all households who own their housing. This is clearly problematic. Because of these limitations, we have also present measures of growth in freedom in which good prices are raised at exponents as per the these goods budget shares in the average Congo and US budgets. As these two weightings are based on the World Bank data used for our international comparisons (see next section), they use a coarser aggregation of the goods into categories than that provided by INSEE.

FIGURE 1 compares welfare-based growth rates of real GDP and growth rate of freedom under the five specifications of weighing of prices. Growth rate of GDP are calculated, for each year, using both the price of the previous year and the price of the current year as the reference price.

As can be seen, growth measurement appears to be somewhat sensitive to the choice of the indicator. Freedom-based indicators who use the weighting of prices as per French budget shares tend to provide an evaluation of real growth that is quite in tune with what is provided by usual real per capita GDP methods. This is of course not surprising given what has been said above with respect to the possible interpretation of freedom-based indicators as Törnqvist-Theil indices. To a lower extent, this congruence between freedom-based and welfare-based evaluation of growth is also observed when freedom is evaluated by a specification of formula (17) that uses US shares as price exponents. On the other hand the appraisal of growth is quite different if symmetric weighting of all prices (as per formula (16)) is used or if the weighting of prices is done on the basis of the Congo's budget shares. In the first case, freedom-based evaluation tends to exceed welfare-based one while yielding the same qualitative verdict of a steady growth in France over the period. When Congo's budget shares are used, the appraisal

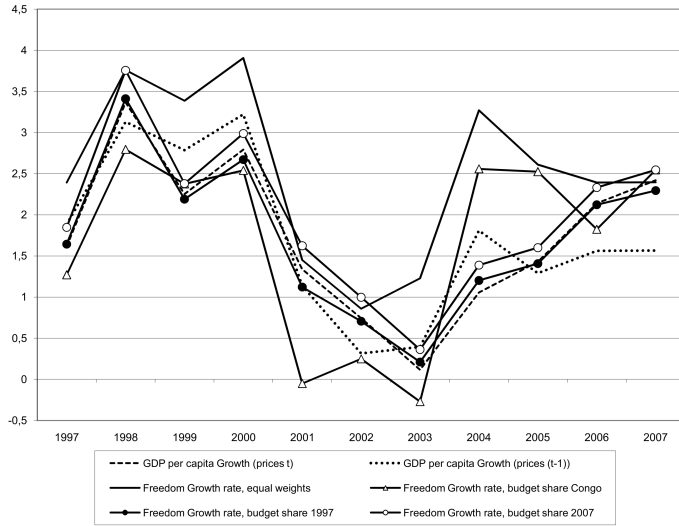


FIGURE 1 – Freedom vs welfare based growth

of growth is not only quantitatively, but also qualitatively different from those obtained on the basis of conventional real per capita GDP. For instance, for 2001 and 2003 – two years where growth was considered to be small but positive by welfare-based conventional indicators – the freedom-based measure who uses a weighting of prices as per Congo’s budget shares concludes in a reduction of freedom. The reason for this is that, in these two years, the increase in the prices of food products (who represent a large share of Congo’s budget and whose prices are weighted more heavily in formula (17)) was sufficiently large to outweigh the small increase in nominal income. On the other hand, for the most recent years, the Congo-based freedom evaluation makes one more optimistic about France’s performance than what could be inferred from looking at conventional real per capita GDP.

As argued above, the relative congruence between welfare-based and freedom-based measures of living standard should not come up as a big surprise. Even if one adopts the welfarist point of view that budget sets are only important insofar as they provide welfare to the individual who faces them, it is clear that this individual, at least if he or she chooses in a welfare maximizing fashion, will benefit – welfare-wise – from facing “larger” sets. Freedom can be instrumental to welfare as well as being, possibly, an issue of intrinsic importance. Yet, what is suggested by this figure is that, while similar, the welfare and freedom based evaluations of growth do lead sometimes to different qualitative conclusions as to whether or not growth was positive for certain years. This figure also suggests that the freedom based evaluation of growth is likely to be sensitive to the weighting of the prices used in formula (17). Obtaining a plausible weighting of prices is therefore an important priority in the agenda of implementation of freedom-based indicators of living standards.

## IV.2 *International Comparisons*

Our international comparisons of living standards are based on data provided by the 2005 International Comparison Program (ICP) coordinated by the World Bank. We take from ICP data on per capita GDP and on *price level indices* for a collection of goods' categories, listed in the appendix. It should be noticed that the 12 group categories used in the ICP are much broader, and therefore less homogenous, than those used by INSEE. Prices indices are used to determine PPP conversion factor of each currency into US dollars, as described by formula (13), so as to obtain standard PPP adjusted comparisons of per capita GDP. It is important to note that the (complex) method used by the World Bank to construct the reference bundle in the formula (13) does not enable one to assign a clear welfare meaning to international comparisons of PPP adjusted real per capita GDP obtained out of this reference bundle.

Just as for the measurement of growth in France, we have treated price indices as if they were the single prices that appear in formulas (16) or (17). TABLES 1, 2 and 3 provide the ranking of 138 countries based on freedom using, respectively, symmetric treatment of all goods (formula (16)) and exponents defined as the average budget share in US (one of the richest country) and Congo (the poorest). The precise value of each budget share is provided in appendix B. TABLE 4 shows the ranking of countries based on conventional welfare-based PPP adjusted per capita GDP comparisons.

TABLE 5 provides a rank correlation matrix for the four rankings and FIGURE 2 plots the countries rank according to the the three freedom ranking against their standard PPP per capita GDP. As can be seen, the rankings of countries induced by the four indices are highly correlated. Yet the correlation between the different freedom based rankings is slightly larger than between each of the freedom ranking and the conventional PPP adjusted GDP-based comparisons. Despite of this, several countries see their rank affected by the switch to freedom as compared to traditional PPP adjusted GDP indicator. For instance, changes in position arise for 111 countries if US budget shares are used and for 117 if the budget shares are those of Congo. However, it is true that for a large number of these countries, the changes in ranking that take place are modest and concern, for the most part, one, two or three positions in the overall scale. Countries in that case include China, India, Pakistan, US, UK Germany and Japan. France standing in the ranking happens to be somewhat more sensitive to the choice of a freedom-based measure of living standard rather than a PPP adjusted per capita one. Its relative position increases indeed from the 24th (if PPP adjusted per capita is used) up to the 18th (with a freedom based assessment using Congo budget shares). It is worth noticing that the ranking of countries based on Congo's budget shares is the freedom-based ranking that differs the most from the PPP adjusted per capita GDP one.

TABLE 1 – Freedom ranking with equal weighting of prices

Luxembourg 1	Finland 24	Latvia 47	Macedonia 70	Bolivia 93	Lesotho 116
Qatar 2	Spain 25	Croatia 48	Peru 71	Iraq 94	Côte d'Ivoire 117
Norway 3	Italy 26	Mexico 49	Columbia 72	Congo 95	Cambodia 118
Brunei 4	Taiwan 27	Chile 50	Ukraine 73	Philippines 96	Sao Tomé 119
US 5	Bahrein 28	Argentina 51	Bosnia 74	Cape Verde 97	Kenya 120
Kuwait 6	N. Zeland 29	Malaysia 52	Ecuador 75	Morocco 98	Bengladesh 121
Ireland 7	Greece 30	Belarus 53	Azerbaijan 76	Moldova 99	Benin 122
Switzerland 8	Cyprus 31	Kazakhstan 54	Tunisia 77	Pakistan 100	Ghana 123
Netherlands 9	Israel 32	Venezuela 55	Albania 78	Mongolia 101	Guinea 124
Austria 10	Slovenia 33	Mauritius 56	Fiji 79	Angola 102	Burkina Faso 125
Denmark 11	Korea 34	Eq. Guinea 57	Maldives 80	India 103	Nepal 126
Macao 12	Czechia 35	Botswana 58	Egypt 81	Sudan 104	Mali 127
Iceland 13	Portugal 36	Romania 59	Armenia 82	Yemen 105	Madagascar 128
Singapore 14	Saudi Arabia 37	Uruguay 60	Namibia 83	Viet Nam 106	Togo 129
Canada 15	Oman 38	Gabon 61	Syria 84	Kyrgyzstan 107	Sierra Leone 130
Sweden 16	Malta 39	Bulgaria 62	China 85	Nigeria 108	Mozambique 131
Belgium 17	Hungary 40	Brazil 63	Jordan 86	Cameroon 109	Centre Afr. 132
Australia 18	Estonia 41	Lebanon 64	Georgia 87	Djibouti 110	Rwanda 133
Hong-Kong 19	Slovakia 42	Serbia 65	Swaziland 88	Chad 111	Niger 134
UK 20	Lithuania 43	S. Africa 66	Paraguay 89	Laos 112	Malawi. 135
Germany 21	Russia 44	Turkey 67	Bhutan 90	Senegal 113	Guinea Biss. 136
France 22	Iran 45	Montenegro 68	Sri Lanka 91	Tajikistan 114	Liberia 137
Japan 23	Poland 46	Thailand 69	Indonesia 92	Mauritania 115	Congo Dem. 138

TABLE 2 – Freedom ranking, US weighting of prices

Luxembourg 1	Taiwan 24	Croatia 47	Montenegro 70	Congo 93	Senegal 116
Qatar 2	Finland 25	Iran 48	Ukraine 71	Indonesia 94	Sao Tome 117
Norway 3	Spain 26	Malaysia 49	Azerbaijan 72	Iraq 95	Cambodia 118
Brunei 4	Italy 27	Kazakhstan 50	Peru 73	Moldova 96	Cote d'Ivoire 119
US 5	Greece 28	Chile 51	Bosnia 74	Cape Verde 97	Kenya 120
Ireland 6	N. Zeland 29	Mexico 52	Columbia 75	Morocco 98	Benin 121
Kuwait 7	Bahrein 30	Belarus 53	Ecuador 76	Philippines 99	Ghana 122
Macao 8	Cyprus 31	Argentina 54	Albania 77	Pakistan 100	Bengladesh 123
Singapore 9	Israel 32	Eq. Guinea 55	Tunisia 78	Mongolia 101	Burkina Faso 124
Netherlands 10	Slovenia 33	Venezuela 56	Armenia 79	Angola 102	Nepal 125
Canada 11	Czechia 34	Mauritius 57	Egypt 80	Kyrgyzstan 103	Guinea 126
Switzerland 12	Korea 35	Gabon 58	Fiji 81	India 104	Mali 127
Austria 13	Malta 36	Botswana 59	Georgia 82	Chad 105	Madagascar 128
Iceland 14	Saudi Arabia 37	Romania 60	Swaziland 83	Sudan 106	Togo 129
Hong Kong 15	Portugal 38	Bulgaria 61	China 84	Yemen 107	Sierra Leone 130
Australia 16	Oman 39	Lebanon 62	Maldives 85	Viet Nam 108	Mozambique 131
Denmark 17	Hungary 40	Uruguay 63	Namibia 86	Tajikistan 109	Centre Afr. 132
UK 18	Estonia 41	Serbia 64	Syria 87	Nigeria 110	Rwanda 133
Belgium 19	Slovakia 42	Brazil 65	Sri Lanka 88	Laos 111	Niger 134
Sweden 20	Lithuania 43	S. Africa 66	Paraguay 89	Cameroon 112	Malawi. 135
Japan 21	Russia 44	Thailand 67	Bhutan 90	Djibouti 113	Guinea Biss. 136
Germany 22	Latvia 45	Turkey 68	Jordan 91	Mauritania 114	Liberia 137
France 23	Poland 46	Macedonia 69	Bolivia 92	Lesotho 115	Congo Dem. 138

TABLE 3 – Freedom ranking, Congo weighting of prices

Luxembourg 1	Hong Kong 24	Croatia 47	Thailand 70	Morocco 93	Senegal 116
Qatar 2	Italy 25	Mexico 48	Montenegro 71	Indonesia 94	Chad 117
US 3	Bahrein 26	Chile 49	Peru 72	Iraq 95	Sao Tomé 118
Norway 4	Greece 27	Argentina 50	Ecuador 73	Cape Verde 96	Kenya 119
Kuwait 5	N. Zeland 28	Belarus 51	Bosnia 74	Philippines 97	Cambodia 120
Netherland 6	Israel 29	Kazakhstan 52	Colombia 75	Congo 98	Bengladesh 121
Ireland 7	Cyprus 30	Malaysia 53	Tunisia 76	Moldova 99	Benin 122
Brunei 8	Japans 31	Lebanon 54	Azerbaijan 77	Mongolia 100	Nepal 123
Austria 9	Slovenia 32	Mauritius 55	Albania 78	Pakistan 101	Ghana 124
Switzerland 10	Portugal 33	Uruguay 56	Fiji 79	Angola 102	Burkina Faso 125
Denmark 11	Taiwan 34	Venezuela 57	Armenia 80	India 103	Mali 126
Canada 12	Czechia 35	Brazil 58	Egypt 81	Yemen 104	Madagascar 127
Sweden 13	Malta 36	Iran 59	Paraguay 82	Sudan 105	Togo 128
UK 14	Saudi Arabia 37	Romania 60	Namibia 83	Kyrgyzstan 106	Guinea 129
Iceland 15	Oman 38	Botswana 61	Jordan 84	Vietnam 107	Mozambique 130
Belgium 16	Hungary 39	Bulgaria 62	Goergia 85	Cameroon 108	Rwanda 131
Australia 17	Korea 40	S. Africa 63	Syria 86	Lao 109	Sierra Leone 132
France 18	Estonia 41	Eq. Guinea 64	Maldives 87	Djibouti 110	Centr.e Afr. 133
Germany 19	Slovakia 42	Turkey 65	Swaziland 88	Lesotho 111	Niger 134
Finland 20	Lithuania 43	Gabon 66	China 89	Tajikistan 112	Malawi. 135
Singapore 21	Poland 44	Serbia 67	Bhutan 90	Nigeria 113	Guinea Biss. 136
Macao 22	Russia 45	Macedonia 68	Bolivia 91	Cote d'Ivoire 114	Liberia 137
Spain 23	Latvia 46	Ukraine 69	Sri Lanka 92	Mauritania 115	Congo Dem. 138

TABLE 4 – PPP adjusted per capita GDP ranking of countries

Luxembourg 1	France 24	Gabon 47	Thailand 70	Angola 93	Sao Tomé 116
Qatar 2	Italy 25	Chili 48	Ecuador 71	Georgia 94	Cambodia 117
Norway 3	Spain 26	Botswana 49	Bosnia 72	Sri Lanka 95	Lesotho 118
Brunei 4	Bahrein 27	Eq. Guinea 50	Peru 73	Indonesia 96	Sao Tomé 119
Kuwait 5	Taiwan 28	Russia 51	Tunisia 74	Iraq 97	Benin 120
US 6	Greece 29	Malaysia 52	Columbia 75	Philippines 98	Kenya 121
Singapore 7	N. Zelan 30	Mexico 53	Ukraine 76	Cape Verde 99	Bengladesh 122
Ireland 8	Cyprus 31	Argentina 54	Albania 77	Mongolia 100	Ghana 123
Macao 9	Israel 32	Iran 55	Albania 78	Pakistan 101	Burkina Faso 124
Hong Kong 10	Slovenia 33	Lebanon 56	Egypt 79	Moldova 102	Nepal 125
Iceland 11	Korea 34	Mauritius 57	Namibia 80	Yemen 103	Mali 126
Switzerland 12	Saudi Arabia 35	Venezuela 58	Swaziland 81	Sudan 104	Madagascar 127
Canada 13	Maltal 36	Romania 59	Jordan 82	Viet Nam 105	Guinea 128
Netherland 14	Oman 37	Bulgaria 60	Fiji 83	India 106	Togo 129
Austria 15	Czechia 38	Uruguay 61	China 84	Cameroon 107	Rwanda 130
Denmark 16	Portugal 39	Kazakhstan 62	Syria 85	Djibouti 108	Sierra Leone 131
Australia 17	Hungary 40	Serbia 63	Maldiv 86	Nigeria 109	Mozambique. 132
Belgium 18	Estonia 41	Brazil 64	Armenia 87	Laos 110	Malawi 133
Sweden 19	Slovakia 42	Belarus 65	Paraguay 88	Chad 111	Centre Afr. 134
UK 20	Lithuania 43	S. Africa 66	Bhutan 89	Kirghyzistan 112	Niger 135
Germany 21	Poland 44	Montenegro 67	Congo 90	Mauritania 113	Guinea Biss. 136
Finland 22	Croatia 45	Turkey 68	Bolivia 91	Senegal 114	Liberia 137
Japan 23	Latvia 46	Macedonia 69	Morocco 92	Côte d'Ivoire 115	Congo Dem. 138

TABLE 5 – Rank correlation matrix

	Freedom US	Freedom Congo	Freedom symmetric
Freedom Congo	0.99521	1	
Freedom symmetric	0.99848	0.99688	1
GDP	0.99558	0.99297	0.99603

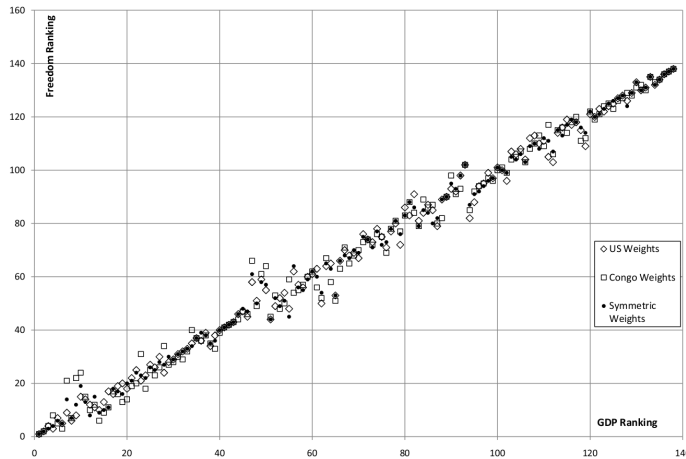


FIGURE 2 – Freedom vs GDP rankings of countries

It is interesting to see that some countries can be quite substantially affected by using freedom, rather than per capita GDP, as the metric of living standard. Belarus for instance jumps from the 65th to the 51th position (with Congo’s budget share). Other relatively big gainers from the move to freedom are Russia (climbs from the 51th to the 44th position) and Netherland (from the 14th to the 7th position). Untypical is the standing of Iran who sometimes benefits from basing international comparisons of living standard on freedom (by moving from the 55th to the 45th position if equal weighting of prices is assumed) and sometimes loses (its position falls to the 59th if freedom assessment uses Congo’s budget shares). Big losers from the switch to freedom as a metric of standard of living are Singapore (who slips from the 7th position up to the 21st), Hong Kong (from 10th to 24th), Gabon (47th to 69th) and Botswana (49th to the 61st).

It is also important to notice that, while there are differences between the three freedom-based rankings of countries that reflect the different weighting of the prices in formula (17), the differences are not astonishing and tend to be smaller than those that distinguish either of these three rankings from the conventional PPP adjusted per capita GDP one. This suggests that freedom based rankings of countries are not too sensitive to the choice of the prices weighting factors.

### IV.3 Measurement of Prices in France

We now evaluate the evolution of prices in France according to seven methods: standard Laspeyres, standard Paasche and the five harmonic means of prices raised at identical, or different, exponents, as required by the freedom-based approach. As for the measurement of economic growth, our empirical analysis is based on INSEE Harmonized Consumption Price Index and uses, therefore, the same grouping of prices into the 85 categories as in the measurement of growth in France. To make the measurement of inflation ethically meaningful in every year, we perform it as in the formula (14) and (15) using, for every year, the bundle of the previous year as the reference bundle in the Laspeyres approach and the bundle of the current year in the Paasche approach. It is important to notice that, by so doing, we differ from INSEE who keeps constant the bundles of reference over all years. As discussed earlier, the INSEE method enables one to compare – welfare wise – any year with the reference year but does not enable ethically meaningful year-to-year comparisons.

FIGURE 3 shows inflation rates according to Laspeyres, Paasche and the five geometric means of prices using, as for growth, symmetric weighting of prices and weighting as per the budget share of the corresponding good in France in 1997 and 2007, as well as in Congo and in US.

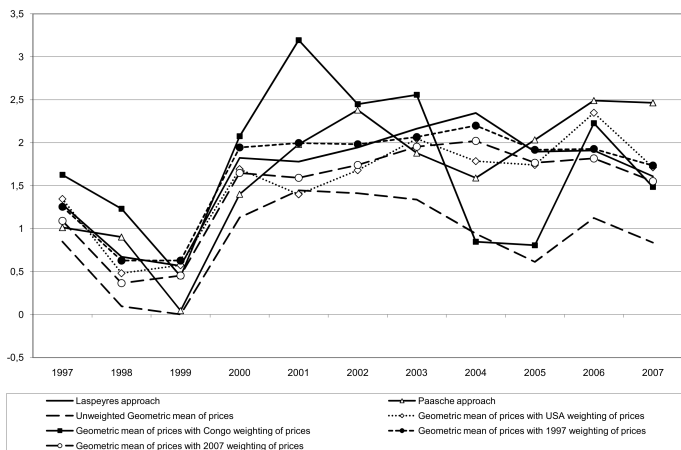


FIGURE 3 – Inflation rates

This figures parallels of course closely what was observed for growth in FIGURE 1. This is of course not surprising since real growth is, after all, nothing else than nominal growth minus the rate of inflation and that nominal income growth is the same no matter whether it is appraised by a welfare-based or a freedom-based indicator. Here again, one finds that welfare-based and freedom-based measures of inflation are closer when the weighting of prices in the freedom measures is done according to French budget than with either of the three other weighting methods. Depreciation in purchasing power appears lower when appraised by a freedom-based method that assigns equal weight to all prices as compared to a conventional Laspeyres or Paasche-based evaluation. To the contrary, weighting prices as per the corresponding shares of

the goods in the Congo budget leads to an greater evaluation of the depreciation of purchasing power, especially because of the high increase in food prices that have taken place in the second half of the period, an increases that was largely echoed in the popular press. Notice that the verdict as to whether or not inflation was positive may be affected by the choice of the indicator. In 1999 for instance, inflation measured by the freedom-based symmetric geometric mean was mildly negative, while it was positive when measured by the Laspeyres and the Paasche approach or by other freedom-based measures. But except for 1999, the qualitative verdict of a yearly depreciation of purchasing power in France – at least for someone with a given income – is obtained for all seven indicators.

## V. Conclusion

We have illustrated in this paper the usefulness of measuring standard of living in a way that reflects the average individual's freedom of choice, rather than his or her welfare as commonly done by usual measures of standard of living. Freedom-based measures have the merit of being clearly interpretable in terms of ethical meaning, a property that is not shared by commonly used indices of living standard. Freedom-based measures have also the advantage of requiring less information than welfare-based ones since they avoid the need of specifying a reference bundle for calculating average prices. Yet, some freedom-based measures require a weighting of prices which can be done according the good budget shares in some representative budget. Doing this however reintroduces the need of specifying a reference budget. As suggested by the illustration, the appraisal of living standard happens to be somewhat sensitive – albeit not outlandishly so – to the choice of the weightings.

All in all, the normative conclusion obtained out of freedom-based measures of living standards, no matter how the weighting is performed, are not radically different from those obtained by conventional welfare-based indicators. There are differences, especially in the ranking of countries, or, for some specification of the weightings, for the evaluation of growth in France. But the differences are not huge.

As mentioned in the text, it is not totally surprising that welfare-based and freedom-based measures of living standard give comparable results. Even if one adopts a welfarist point of view, it is clear that having more freedom in the sense of a budget set with a larger “volume” increases the well-being that this individual can obtain out of the set. Because of this, it is all the more normal that the ordinal ranking obtained from comparing budget sets on the basis of their volume is somewhat similar to the one obtained from performing revealed preference comparisons based on an observed bundle of good that has been chosen from them. In our view, this congruence between easy-to-use and informationally parcimonious freedom-based indicators on the one hand and difficult-to- interpret and informationally demanding welfare-based ones on the other should be seen as an additional argument in favour of the formers, rather than an argument against them.

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## Appendix

### A. Good Categories for the INSEE Harmonized Price Index and Budget Shares

1. Bread and cereals (1997 = 0.0272, 2007 = 0.0229)
2. Meat (1997 = 0.0624, 2007 = 0.045)
3. Fish and seafoods (1997 = 0.0130, 2007 = 0.0133)
4. Milk, cheese and eggs (1997 = 0.0273, 2007 = 0.0234)
5. Oils and fats (1997 = 0.0065, 2007 = 0.0043)
6. Fruits (1997 = 0.0130, 2007 = 0.0109)
7. Vegetables (1997 = 0.0149, 2007 = 0.0152)
8. Sugar, jam, honey, chocolate and confectionery (1997 = 0.0127, 2007 = 0.0106)
9. Other Food products (1997 = 0.0029, 2007 = 0.0062)
10. Coffee, tea and cocoa (1997 = 0.0051, 2007 = 0.0047)
11. Mineral waters, soft drinks, fruit and vegetable juices (1997 = 0.0078, 2007 = 0.0092)
12. Spirits (1997 = 0.0052, 2007 = 0.0050)
13. Wine (1997 = 0.0173, 2007 = 0.0103)
14. Beer (1997 = 0.0020, 2007 = 0.0021)
15. Tobacco (1997 = 0.0211, 2007 = 0.0195)
16. Clothing materials (1997 = 0.0005, 2007 = 0.0001)
17. Garments (1997 = 0.0511, 2007 = 0.0405)
18. Other clothing articles and accessories (1997 = 0.0051, 2007 = 0.0042)
19. Cleaning, repair and hire of clothing (1997 = 0.0017, 2007 = 0.0015)
20. Footwear including repair (1997 = 0.0143, 2007 = 0.0108)
21. Housing rents (1997 = 0.0639, 2007 = 0.0688)

22. Materials for maintenance and repair of the dwelling (1997 = 0.0028, 2007 = 0.0029)
23. Services for maintenance and repair of the dwelling (1997 = 0.0157, 2007 = 0.0129)
24. Water supply and miscellaneous services relating to the dwelling (1997 = 0.0134, 2007 = 0.0189)
25. Electricity (1997 = 0.0272, 2007 = 0.0222)
26. Gas (1997 = 0.0115, 2007 = 0.0140)
27. Liquid fuels (1997 = 0.0076, 2007 = 0.0085)
28. Solid fuels (1997 = 0.0007, 2007 = 0.0005)
29. Heat energy (1997 = 0.0020, 2007 = 0.0016)
30. Furniture and furnishings (1997 = 0.0219, 2007 = 0.0157)
31. Carpets and other floor coverings (1997 = 0.0016, 2007 = 0.0007)
32. Repair of furniture, furnishings and floor coverings (1997 = 0.0008, 2007 = 0.0012)
33. Cleaning textiles (1997 = 0.0064, 2007 = 0.0050)
34. Household electrical appliances (1997 = 0.0120, 2007 = 0.0094)
35. Repair of household appliances (1997 = 0.0016, 2007 = 0.0007)
36. Glassware, tableware and household utensils (1997 = 0.0087, 2007 = 0.0076)
37. Tools and equipment for house and garden (1997 = 0.0034, 2007 = 0.0053)
38. Non-durable domestic goods (1997 = 0.0115, 2007 = 0.0104)
39. Domestic services and household services (1997 = 0.0056, 2007 = 0.0102)
40. Health (1997 = 0.0510, 2007 = 0.0430)
41. Motor cars (1997 = 0.0404, 2007 = 0.045)
42. Motor cycles, bicycles and other vehicles (1997 = 0.0020, 2007 = 0.0035)
43. Spares parts and accessories for personal transport equipment (1997 = 0.0350, 2007 = 0.0326)
44. Fuels and lubricants for personal transport equipment (1997 = 0.0489, 2007 = 0.0421)
45. Maintenance and repair of personal transport equipment (1997 = 0.0275, 2007 = 0.02312)
46. Other services with respect to personal transportation (1997 = 0.0095, 2007 = 0.0100)
47. Passenger transport by rail (1997 = 0.0069, 2007 = 0.0052)
48. Passenger transport by road (1997 = 0.0067, 2007 = 0.0060)
49. Passenger transport by air (1997 = 0.0056, 2007 = 0.0086)
50. Passenger transport by sea and inland waterway (1997 = 0.0003, 2007 = 0.0004)
51. Combined passenger transport (1997 = 0.0062, 2007 = 0.0024)
52. Other transport services (1997 = 0.0008, 2007 = 0.0008)
53. Postal services (1997 = 0.0029, 2007 = 0.0024)
54. Telephone and telefax equipment (1997 = 0.0003, 2007 = 0.0016)
55. Telephone and telefax services (1997 = 0.0169, 2007 = 0.0285)
56. Equipment for reception, recording and reproduction of sounds and pictures (1997 = 0.0075, 2007 = 0.0070)
57. Photographic and cinematographic equipment and optical instruments (1997 = 0.0009, 2007 = 0.0030)
58. Computers equipment (1997 = 0.0003, 2007 = 0.0081)
59. Recording media (1997 = 0.0091, 2007 = 0.0050)
60. Repair of audio-visual, photographic and information processing equipment (1997 = 0.0019, 2007 = 0.0015)

61. Other major durables for recreation and culture (1997 = 0.0031, 2007 = 0.0022)
62. Games, toys and hobbies (1997 = 0.0060, 2007 = 0.0051)
63. Equipment for sport, camping and open-air recreation (1997 = 0.0024, 2007 = 0.0051)
64. Gardens, plants and flowers (1997 = 0.0065, 2007 = 0.0070)
65. Pets and related products; veterinary and other services for pets (1997 = 0.0032, 2007 = 0.0067)
66. Recreational and sporting services (1997 = 0.0080, 2007 = 0.0122)
67. Cultural services (1997 = 0.0154, 2007 = 0.0167)
68. Books (1997 = 0.0050, 2007 = 0.0045)
69. Newspapers and periodicals (1997 = 0.0104, 2007 = 0.0088)
70. Miscellaneous printed matter; stationery and drawing materials (1997 = 0.0057, 2007 = 0.0038)
71. Package holidays (1997 = 0.0014, 2007 = 0.0029)
72. Education (1997 = 0.0037, 2007 = 0.0057)
73. Restaurants and cafés (1997 = 0.0546, 2007 = 0.0450)
74. Cafeterias (1997 = 0.0163, 2007 = 0.0141)
75. Accommodation services (1997 = 0.0195, 2007 = 0.0132)
76. Hairdressing salons and personal grooming (1997 = 0.0111, 2007 = 0.0112)
77. Appliances and products for personal care (1997 = 0.0198, 2007 = 0.0218)
78. Jewellery, clocks and watches (1997 = 0.0076, 2007 = 0.0076)
79. Other personal belonging (1997 = 0.0066, 2007 = 0.0059)
80. Insurance (1997 = 0.0110, 2007 = 0.0245)
81. Financial services (1997 = 0.0041, 2007 = 0.0064).
82. Other services (1997 = 0.0145, 2007 = 0.0137)

## **B. Goods' categories for the ICP Price Index and budget shares**

1. Food and non-alcoholic beverages (Congo = 0.62, US = 0.06)
2. Alcoholic beverages and tobacco (Congo = 0.02, US = 0.02)
3. Clothing and footwear (Congo = 0.05, US = 0.04)
4. Housing, water, electricity, gas and other fuels (Congo = 0.12, US=0.16)
5. Furnishings, household equipment and household maintenance (Congo = 0.03, US = 0.04)
6. Health (Congo = 0.04, US=0.18)
7. Transport (Congo = 0.03, US=0.11)
8. Communication (Congo = 0.01, US=0.02)
9. Recreation and culture (Congo = 0.01, US=0.09)
10. Education (Congo = 0.03, US=0.09)
11. Restaurants and hotels (Congo = 0.01, US=0.06)
12. Miscellaneous goods and services (Congo = 0.03, US=0.13)

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