The segregative properties of endogenous formation of jurisdictions with a welfarist central government*

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Abstract

This paper examines the segregative properties of Tiebout processes of jurisdiction formation in the presence of a central government which makes equalization transfers to jurisdictions in such a way as to maximize a welfarist objective. All agents - the households, the local governments and the central government - are assumed to make their choices simultaneously, taking as given the choices of others. The central government is assumed to pursue a generalized utilitarian objective. The main result of the analysis is that, if the utility function used by generalized utilitarian central government is additively separable, the class of preferences that guarantees the wealth segregation of any stable jurisdiction structure is unaffected by the presence of a central government.

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1 Introduction

There is a wide presumption that decentralized processes of jurisdiction formation *à la* Tiebout (1956) lead individuals to self-sort into homogenous communities. Gravel and Thoron (2007) investigate the validity of this intuition within the classical model of jurisdictions formation developed by Westhoff (1977) (see also Greenberg and Weber (1986), Demange (1994) and Kessler and Lulfesmann (2005) among many others). In this model, unequally wealthy households with the same preference for a local public good and a private good choose simultaneously their place of residence from a finite set of locations. Households who choose the same location form a *jurisdiction* and produce a local public good by applying a democratically chosen tax rate to all residents' wealth. Any such simultaneous choice of residence by the households is referred to as a *jurisdiction structure*.

The analysis of this literature concerns *stable* jurisdiction structures, which satisfy the additional property of being *robust* to *individual deviations*. The question raised by Gravel and Thoron (2007) is whether stable jurisdiction structures lead households to self sort, or segregate, themselves by their wealth. The notion of segregation used is that known under the heading of *consecutiveness* in the coalition formation literature (see e.g. Greenberg and Weber (1986)). A jurisdiction structure is segregated in this sense if, for any two jurisdictions with different *per capita* wealth, the richest individual in the poorer jurisdiction is (weakly) poorer than the poorest individual in the richer jurisdiction. Gravel and Thoron (2007) identify a condition on households preferences that is necessary and sufficient for the segregation of any stable jurisdiction structure. The condition requires the public good to *always* be a gross complement to, or *always* be a gross substitute for, the private good. While stringent, and violated by several well-known preferences, including additively separable ones, this Gross Substitutability-Complementarity (GSC) condition is not implausible. For this reason, Gravel and Thoron (2007) seems to provide theoretical ground for the belief that decentralized processes of jurisdiction formation are inherently segregative.

In this paper, we investigate the extent to which this conclusion is affected by the introduction of a *central government*. Introducing a central government in models of endogenous jurisdiction formation strikes us as an important step toward improving the realism of these models. In many countries, one finds indeed a juxtaposition of several levels of government: central and local. It is also commonly observed that the central government puts into place *equalization payment schemes* that redistribute funds across jurisdictions so as to achieve specific normative objectives. It seems, therefore, of some interest to examine the consequence of central government's intervention on the segregative properties of the endogenous formation of local jurisdictions by freely mobile households.[1]

Doing this requires one to specify:

1) the instruments available to the central government,

2) the objective of the central government and

3) the nature of the interaction between the central government, the households and the local governments.

As for the first point, we assume that the central government taxes households at a fixed (possibly negative) rate and redistributes tax revenues between jurisdictions in such a way as to maximize some objective function. Although stylized, this modeling of the redistribution performed by the central government does not provide a bad approximation of many *existing* systems of equalization payments. It is consistent both with the so-called *horizontal* equalization payments scheme of the sort existing in Scandinavian countries and Germany - and the *vertical* schemes observed in several other countries (like Belgium, France, Canada, Australia, Switzerland and India, to mention just a few).[2]

As for the objective of the central government, we assume it to be *welfarist*. We consider more specifically the somewhat large family of *generalized utilitarian* social objectives that compare alternative packages of equalization grants and tax rates on the basis of the sum of some (increasing and concave) transformation of the households' utilities. This family, characterized in classical choice theory by plausible axioms (see e.g. Blackorby, Bossert, and Donaldson (2005), ch. 4), contains the standard utilitarian criterion as well as several others like, for instance, the symmetric mean of order r.

As for the interaction between the households and the governments (central and local), we assume that agents make their decisions *simultaneously*, taking the behavior of others as given. In this setting, we define a *stable jurisdiction structure with a central government* to be an assignment, to every location, of a local tax rate, a central government net transfer, and a set of households that is immune to unilateral deviation from the part of every agent.

The question addressed is whether the GSC condition on households' preferences remains necessary and sufficient for ensuring the segregation of any stable jurisdiction structure with a generalized utilitarian central government We first discuss, by means of examples, how the presence of a

[1]To the best of our knowledge, the only theoretical paper that has introduced a central government in a setting with Tiebout-like processes of local jurisdiction formation is Nechyba (1997). Yet, the setting considered by Nechyba, which involves the purchase of location-specific indivisible house, and where the central government provides central public goods is quite different from the classical Whestoff one examined herein.

[2]See e.g. Gravel and Poitevin (2006) for a theoretical normative analysis of equalization payment in federations and Tarroux (2010) for an empirical investigation in the Canadian context. A broader discussion of equalization in structures with multiple levels of governance is provided by Boadway (2006).

generalized-utilitarian central government may significantly affect the set of stable jurisdiction structures. Yet we show that, if the households' preferences are additively separable, the GSC condition remains necessary and sufficient for the segregation of any stable jurisdiction structure with a generalized utilitarian central government. Hence, it appears that, at least for households with additively separable preference, the presence of a generalized utilitarian central government does not affect the segregative properties of decentralized processes of jurisdiction formation, even though it affects a great deal the set of stable jurisdiction structures.

The rest of the paper is organized as follows. The next section introduces the model and illustrates by some examples how the presence of a central government affects the set of stable jurisdiction structures. Section 3 states and proves the main results and section 4 concludes.

2 The model

As in Gravel and Thoron (2007) and Westhoff (1977), we consider economies with a continuum of households indexed by the interval $[0, 1]$ interval. Any such economy consists of three elements.

First, there is a *wealth distribution* modeled as a Lebesgue measurable, increasing and bounded from above function $\omega : [0, a] \to \mathbb{R}_{++}$ that associates to each household $i \in [0, 1]$ its strictly positive private wealth ω_i.[3] Assuming the function ω to be increasing is a convention according to which households are ordered by their wealth ($i \leq i' \Longrightarrow \omega_i \leq \omega_{i'}$).

The *second* ingredient is a specification of the household's preferences, taken to be the same for all households. We assume that household's preference for the local public good (Z) and the private good (x) is represented by a twice differentiable, strictly increasing and strictly concave utility function $U : \mathbb{R}^2_+ \to \mathbb{R}$ bounded from below.[4] For the result on the necessity of the GSC condition for segregation, we make the additional assumption that the utility function that represents the household's preference and that is used by the central government is *additively separable* so that it can be written, for every $(\overline{Z}, \overline{x}) \in \mathbb{R}^2_+$, as:

$$U(\overline{Z}, \overline{x}) = f(\overline{Z}) + h(\overline{x}) \tag{1}$$

for some twice differentiable increasing and concave real valued functions f and h having both $\mathbb{R}_+$ as domain. Given any bundle of public and private

[3] To alleviate notation, for any $i \in [0, 1]$, any set A and any function $f : [0, 1] \to A$, we write f_i rather than $f(i)$.

[4] The assumption of strict concavity is an inessential simplification that garantees the uniqueness of the solution to the standard consumer program. The assumption that the utility is bounded from below guarantees that the maxmin criterion that is approached in the limit by some members of the generalized utilitarian family of social objectives used by the central government is well-defined.

good $(\overline{Z}, \overline{x}) \in \mathbb{R}_+^2$, we denote by $MRS(\overline{Z}, \overline{x})$ the *marginal rate of substitution of public good to private good* evaluated at $(\overline{Z}, \overline{x})$ defined by:

$$MRS(\overline{Z}, \overline{x}) = \frac{\partial U(\overline{Z}, \overline{x})/\partial Z}{\partial U(\overline{Z}, \overline{x})/\partial x} \tag{2}$$

We also denote by $Z^M(p_Z, p_x, R)$ and $x^M(p_Z, p_x, R)$ the households' Marshallian demands for the public and private good (respectively) when the prices of these goods are p_Z and p_x and the household's income is R. Marshallian demand functions are the (unique under our assumptions) solution of the program:

$$\max_{Z,x} U(Z, x) \text{ subject to } p_Z Z + p_x x \leq R$$

Given again our assumptions, Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of $\mathbb{R}_+^3$). We emphasize that we view Marshallian demands as dual representations of preferences rather than descriptions of actual behavior (after all households rarely if ever purchase local public goods on competitive markets). The indirect utility function corresponding to U is denoted by V and is defined as usual by:

$$V(p_Z, p_x, R) = U(Z^M(p_Z, p_x, R), x^M(p_Z, p_x, R))$$

We further assume that the Marshallian demand for the local public good satisfies the following additional *regularity* condition (introduced and discussed in Gravel and Thoron (2007)).

Condition 1: If there exists a public good price $\overline{p}_Z$, an income level R and a non-degenerate interval I of strictly positive real numbers such that, $Z^M(\overline{p}_Z, p_x, R) = Z^M(\overline{p}_Z, p'_x, R)$ for all prices p'_x and p_x in I, then, for all $(p_Z, p_x, R) \in \mathbb{R}_+^3$, we must have $Z^M(p_Z, p_x, R) = h(p_Z, R)$ for some function $h : \mathbb{R}_{++}^2 \to \mathbb{R}_+$.

We let $\mathcal{U}$ denote the set of all bounded from below utility functions that are twice differentiable, strictly increasing, strictly concave and whose Marshallian demand functions satisfy condition 1 and we let $\mathcal{U}^{\mathcal{A}}$ denote the subset of those functions that are additively separable.

Finally, the *third* element in the description of an economy is a finite set $\mathbb{L} = \{1, ..., L\}$ of possible locations.

The problem considered is that of identifying the properties of the *jurisdiction structures with a central government* that can emerge when households freely choose their location and share, when they locate at the same place, the benefit of a local public good produced by local tax revenues and a central government grant. This intervention of the central government is the main distinctive ingredient of our model as compared to what is done in the literature (e.g. Demange (1994), Gravel and Thoron (2007), Greenberg (1983), Kessler and Lulfesmann (2005),Westhoff (1977), Greenberg and Weber (1986)).

Specifically, we define a *jurisdiction structure with a central government* for the economy $(\omega, U, \mathbb{L})$ to be a Lebesgue measurable *location* function $L : [0,1] \to \mathbb{L}$, a (local) tax vector $t \in \mathbb{R}^L$, a central government grant vector $g \in \mathbb{R}^L$ and a central government wealth tax rate $c \in \mathbb{R}$ that satisfy:

$$\sum_{l \in \mathbb{L}} g_l \leq c \int\limits_{[0,1]} \omega_i d\lambda \tag{3}$$

and, for every $l \in \mathbb{L}$:

$$t_l \varpi_l + g_l \geq 0 \tag{4}$$

and:

$$1 - t_l - c \geq 0 \tag{5}$$

where λ denotes the Lebesgue measure over (Lebesgue measurable) subsets of $[0,1]$ and where, for every location $l \in \mathbb{L}$, $\varpi_l = \int\limits_{L_l^{-1}} \omega_i d\lambda$. The interpretation given to the (Lebesgue measurable) set $L_l^{-1} = \{i \in [0,1] : L_i = l\}$ is that it is the community of households assigned to l by the location function L. This community will form a jurisdiction. We let $\mu_l^L = \lambda(L_l^{-1})$ denote the "measure of households living at l" under the location function L and we denote by $\omega_{L_l^{-1}}$ the restriction of the measurable function ω to the measurable set L_l^{-1}. The possibility that $\mu_l^L = 0$ for some l is, of course, not ruled out. We interpret $\omega_{L_l^{-1}}$ as the distribution of wealth in jurisdiction L_l^{-1}. Condition (3) requires the central government to balance its budget so that that the sum of the (possibly negative) grants given to the jurisdictions does not exceed the revenues obtained from taxing the households at rate c. Conditions 2) and 3) limit the fiscal power of the central and the local governments to raise taxes and to give grants to the extent compatible with non-negative consumption of public (4) and private (5) spending. We emphasize that negative local tax rates are possible in a world with a central government. A household living in a jurisdiction receiving a large positive grant may prefer local tax rates to be negative and, therefore, use part of the central grant in private spending. Similarly, a central government may want to subsidize private consumption and to choose, for this purpose, a negative income tax rate t.

This modeling of the central government covers both the possibility that it transfers money between jurisdictions without taxing households ("horizontal" equalization) and the combination of horizontal and vertical equalization. But our model, by restricting the taxation power of the central government to linear schemes, rules out the possibility of using wealth tax for redistributive purposes (by making it progressive for instance). Of course providing the central government with the *full* power of redistributing the exogenous individual wealth (by choosing the tax paid by each household for instance based on its characteristic) would devoid the problem examined in this paper of much of its interest. For any central government that is averse to wealth inequality would obviously choose, if given such a power, to equalize wealth perfectly within a given jurisdiction. But between the full power given to a central government of taxing individually each household, and the extremely small one considered here of taxing all of them at the same rate, there is a large spectrum of possibilities that deserve, perhaps, a closer analysis.

Denote by $\Phi(\tau, \varpi, \omega_i, \gamma, c) = U(\tau\varpi + \gamma, (1 - \tau - c)\omega_i)$ the utility received by a household with wealth ω_i living in a jurisdiction with local tax rate τ, aggregate wealth ϖ and central government grant γ when the central government household wealth tax rate is c (provided of course that these parameters satisfy conditions (4) and (5)). The function Φ so defined has several properties that we record in the following lemma (whose straightforward proof is omitted).

Lemma 1 *Let U be a utility function in $\mathcal{U}$ and let $(\tau, \varpi, \omega_i, \gamma, c) \in \mathbb{R}^5$ be such that inequalities (4) and (5) hold strictly. Then Φ is a twice differentiable function of its five arguments, is strictly increasing and concave with respect to ω_i, ϖ and γ (taking τ and c as given) and is strictly concave and single peaked*[5] *with respect to τ (taking ω_i, ϖ, γ and c as given).*

One important property of Φ is its strict single peakedness. It implies that a household with wealth ω_i facing a central tax rate of c has a unique favorite local tax rate of $\tau^*(\varpi, \omega_i, \gamma, c)$ in any jurisdiction with tax base ϖ and central government grant γ to which it may belong. This unique favorite local tax rate is the solution of the program:

$$\max_{\tau} \Phi(\tau, \varpi, \omega_i, \gamma, c) \text{ subject to (4) and (5)} \tag{6}$$

and is, for this reason, a continuous function of its four arguments.

As in Gravel and Thoron (2007), the behavior of this favorite local tax rate function is an important ingredient of the analysis. This behavior is very closely related to that of the Marshallian demand functions, as established in the following lemma whose proof, similar to that of lemma 2 in Gravel and Thoron (2007), is omitted.

Lemma 2 *Let $(\varpi, \omega_i, \gamma, c) \in \mathbb{R}^2_{++} \times \mathbb{R} \times [0, 1]$. Then for all U in $\mathcal{U}$, $\frac{1}{\varpi}[Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\varpi}) - \gamma]$ is the solution of (6).*

[5] A function $f : A \to \mathbb{R}$ ($A \subset \mathbb{R}$) is strictly single peaked if, for all a, b and $c \in A$ such that $a < b < c$, $f(c) > f(b) \Rightarrow f(b) > f(a)$ and $f(a) > f(b) \Rightarrow f(b) > f(c)$.

Lemma 2 states that, in a jurisdiction with aggregate wealth ϖ and central government transfer γ, the favorite tax rate of a household with a (net of central government tax) wealth $\omega_i(1-c)$ can be viewed as the *expenditure* that the household would like to devote to local public good in excess of the central government grant if the prices of public and the private goods were $\frac{1}{\varpi}$ and $\frac{1}{\omega_i}$, and if this household had an income of $1-c+\frac{\gamma}{\varpi}$.

We are interested in the possible segregative properties of the likely outcome of a free choice of location by households in the presence of a central government. This likely outcome must be *stable* in the usual sense of being immune to individual deviations from the part of all agents (households, local governments and central government). Providing a precise definition of stability requires one to specify first the objective pursued by each category of agents.

Households' objectives are clear. Each household seeks for the jurisdiction that offers the (utility) best package of tax burden and (local) public good provision.

In so far as the *local governments* are concerned, we adopt the view that each jurisdiction's choice of local tax rate is minimally democratic in the sense that it is the favorite tax rate of some household whose preference are not too distant from those of the jurisdiction's members. The particular rule used for choosing this "dictator" is inconsequential for the results of this paper. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, such as Westhoff (1977), the jurisdiction tax rate would be the one that occupies the *median position* in the jurisdiction's distribution of favorite taxes. While the analysis of this paper applies to this particular positional rule of selection of the dictator, they are valid for other rules as well. Formally, given an economy $(\omega, U, \mathbb{L})$, and a jurisdiction structure $J=(L,t,g,c)$ with a central government for this economy, we let m_J^l be the "dictator" in l for that jurisdiction structure. The only assumption made on this dictator is that his (her) favorite tax rate be contained between the *infimum* and the *supremum* of the favorite tax rates of the jurisdiction's members. Formally, for the jurisdiction structure with a central government (L,t,g,c), we define, for every location $l\in\mathbb{L}$ the infimum and supremum favorite tax rates t_{l*} and t_l^* respectively by:

$$
\begin{aligned}
t_{l*} &= \inf_{i\in L_l^{-1}} \arg\max_{\tau} \Phi(\tau,\varpi_l,\omega_i,g_l,c) \text{ subject to (4) and (5) and} \\
t_l^* &= \sup_{i\in L_l^{-1}} \arg\max_{\tau} \Phi(\tau,\varpi_l,\omega_i,g_l,c) \text{ subject to(4) and (5)}
\end{aligned}
$$

As for the *central government*, we adopt the *welfarist* view point that it compares alternative packages of equalization grants and household wealth tax rate on the basis of the (Lebesgue measurable) distributions of utility levels that they generate. We view any such distribution of utility levels as the graph of a (Lebesgue measurable) bounded function $\mathfrak{u}:[0,1]\to\mathbb{R}$ that maps household $i \in [0,1]$ into utility level $\mathfrak{u}_i$. When convenient, we alternatively denote by $< \mathfrak{u}_i >_{i\in[0,1]}$ such a function. The graph of these functions are compared by a social ordering [6] R with the usual interpretation that $\mathfrak{u}$ R $\mathfrak{u}'$ means "distribution of utilities $\mathfrak{u}$ is socially weakly better than distribution $\mathfrak{u}'$". We shall, somewhat more specifically, assume that the central government uses a Generalized Utilitarian (GU) ordering. There are many justifications that could be given to this choice (see e.g. Blackorby, Bossert, and Donaldson (2005) ch. 4, theorem 4.7) in a classical social choice setting with a finite number of households. But social choice theory is not very developed for populations with a continuum of individuals (see however Candeal, Chichilnisky, and Induràin (1997), Caplin and Nalebuff (1997), Kaneko (1981) and Kaneko (1984)) and we are not aware of axiomatic results that would justify a generalized utilitarian objective in such a setting. We simply observe that the GU family encompasses many welfarist criteria used in the literature. The Generalized Utilitarian ordering R^{GU} is defined by:

$$\mathfrak{u}\ R^U\ \mathfrak{u}' \Longleftrightarrow \int_{[0,1]} \Psi(\mathfrak{u}_i)d\lambda \geq \int_{[0,1]} \Psi(\mathfrak{u}'_i)d\lambda$$

where $\Psi : \mathbb{R} \to \mathbb{R}$ is a function that evaluates individual utility functions from a social view-point. The Pareto principle requires Ψ to be increasing while utility-inequality aversion considerations suggest that Ψ be concave. We assume throughout that Ψ is differentiable. A well-known example of such a generalized utilitarian criterion is the global symmetric mean of order r (for some real number $r \leq 1$) in which Ψ is defined by:

$$\begin{aligned} \Psi(u) &= u^r \text{ if } r \in]0,1] \\ &= \ln u \text{ if } r = 0 \\ &= -u^r \text{ if } r < 0 \end{aligned}$$

This family contains the standard utilitarian criterion (for $r = 1$) and approaches the *maxmin* criterion (as r approaches $-\infty$).

We are now equipped to define formally what is meant for jurisdiction structure with a central government to be stable.

Definition 1 *Given an economy $(\omega, U, \mathbb{L})$, we say that the jurisdiction structure $J = (L, t, g, c)$ with a central government endowed with a social ordering R is stable if*
1) For every l, $l' \in \mathbb{L}$ and all $i \in L_l^{-1}$, $\Phi(t_l, \varpi_l, \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c)$.
2) For all $l \in \mathbb{L}$, $t_l = \tau^*(\varpi_l, \omega_{m^l_J}, g_l, c)$
3) $< U(t_{L_i}\varpi_{L_i} + g_{L_i}, (1 - t_{L_i} - c)\omega_i) >_{i\in[0,1]}$ R $< U(t_{L_i}\varpi_{L_i} + g'_{L_i}, (1 - t_{L_i} - c')\omega_i) >_{i\in[0,1]}$ for all $g' \in \mathbb{R}^l$ and $c' \in \mathbb{R}$ satisfying (3)-(5).

[6] An ordering is a reflexive, complete and transitive binary relation.

This definition of stability rides of course on the assumption that households as well as local and central governments take their decision simultaneously, considering as given the behavior of others. This assumption generalizes naturally the simultaneous Westhoff framework. Yet it may be viewed as restricting unduly the central government's power of shaping the process of jurisdiction formation. An alternative could have been to assume a *two-stage* setting in which the central government would play *before* households and local governments and would choose its redistributive grants and households tax by anticipating the impact of its choice on the stable jurisdiction structure that would emerge in the second stage. While this alternative would certainly be worth exploring in detail, we have chosen to leave it aside in this paper that represents, to the best of our knowledge, the first attempt to introduce a welfarist central government in Tiebout-like models of jurisdiction formation.[7] We simply notice that the two-stage setting for integrating a central government in a Tiebout-like economy involves delicate modeling issues. Let us mention three of them that come to our mind.

First, choosing a set of equalization grants and wealth tax rate before knowing the jurisdiction structure that prevail raises the problem of the financial viability of the equalizations grants. What if the central government chooses, in the first stage, an equalization scheme which imposes a tax burden to a jurisdiction which, in the second stage, will be empty ? Second, and more importantly, as illustrated in the examples below, there may be many stable jurisdiction structures that correspond to a given central government equalization and taxation scheme. If this is the case, how is the central government going to predict which of the stable jurisdiction structures will emerge ? Third, everything else being the same, the central government, at least if it uses a Pareto inclusive social objective, *may* have a tendency to favour the "trivial" grand jurisdiction structure in which all households are put in the same jurisdiction and where, therefore, there is no role for a central government. The reason for this tendency is that, the local public good being non-rival, producing any quantity of it in a larger jurisdiction tend to be preferable from a social welfare view point because its cost can be shared by a larger number of tax payers.[8]

In order to identify the condition on households preferences that is necessary and sufficient for the wealth-segregation of any stable jurisdiction

[7] As mentionned in introduction, Nechyba (1997) also introduces a central government in a Tiebout-like model of local jurisdiction formation. Yet Nechyba's model, that has an (indivisible) land market, is quite different from ours. The central government in Nechyba (1997) does not merely redistribute wealth and local public goods accross jurisdiction but also provides central public good. Moreover it makes its choice through (some form of) majority voting.

[8] This is true at least if the central government has full information on households' wealths. See Gravel and Poitevin (2010) for an analysis of the (difficult) problem of the optimal choice of a jurisdiction structure when the central government is imperfectly informed about households'wealth.

structure, one needs first to define what is meant by a wealth-segregation. We take the definition to be that used in Gravel and Thoron (2007) (see also Westhoff (1977) and Greenberg and Weber (1986)).

Definition 2 *A jurisdiction structure with a central government* $J = (L, t, g, c)$ *for the economy* $(\omega, U, \mathbb{L})$ *is wealth-segregated if, for every locations* l, $l' \in \mathbb{L}$ *for which* $\lambda(L_l^{-1}) \neq 0$ *and* $\lambda(L_l^{-1}) \neq 0$ *and every households* h, i *and* k $\in [0, 1]$ *such that* $\omega_h < \omega_i < \omega_k$, h, $k \in L_l^{-1}$ *and* $i \in L_{l'}^{-1}$ *imply that* $t_l = t_{l'} = \frac{g_{l'} - g_l}{\varpi_l - \varpi_{l'}}$.

In words, a jurisdiction structure is wealth-stratified if, whenever a jurisdiction j contains two households h and k with different levels of wealth, it also contains all households whose wealth levels are strictly between that of h and k (if there exist such households of course) or, if it does not contain those households, it is because they belong to some (non-null) jurisdiction that offers the same tax rate and the same amount of public good as j.

Before establishing, in the next section, that the introduction of a GU central government does not affect the segregative properties of stable jurisdiction structures, it is probably worth discussing a bit how the introduction of a GU central government nonetheless modifies significantly the set of stable jurisdiction structures.

In many economies, the introduction of a central GU government will tend to sharply reduce this set. This can be probably best seen by considering a central government that pursues a *maxmin objective.* While such an objective does not - strictly speaking - belong to the class of GU criteria, it can be approximated with any degree of precision by some member of this class (for instance a symmetric mean of order r that uses a sufficiently negative value of r). The formal definition of the maxmin ordering, denoted $R^{\min}$, defined over any two bounded Lebesgue measurable functions $\mathfrak{u}$ and $\mathfrak{u}'$, is

$$\mathfrak{u}\ R^{\min}\ \mathfrak{u}' \iff \inf_{i \in [0,1]} \mathfrak{u}_i \geq \inf_{i \in [0,1]} \mathfrak{u}'_i$$

If the central government uses such a maxmin objective, then the set of stable jurisdiction structures that can emerge with such a government is rather limited. For, as established in the following proposition, the only stable jurisdiction structure that can be observed with such a central government are those where the jurisdictions' poorest households have the same wealth in all jurisdictions. The trivial "grand jurisdiction" structure in which all households are in the same jurisdiction is, of course, a particular example of such jurisdiction structures.

Proposition 1 *A jurisdiction structure* (L, g, c, t) *with a maxmin central government is stable as per definition 1 only if for all* l *and* l', $\inf_{i \in L_l^{-1}} \omega_i =$

$\inf_{h \in L_{l'}^{-1}} \omega_h$.

Proof. *By contraposition, assume that* (L, g, c, t) *is a stable jurisdiction structure with a welfarist central government in which there are locations* l *and* $l' \in \mathbb{L}$ *for which* $L_l^{-1} \neq \varnothing$ *and* $L_{l'}^{-1} \neq \varnothing$ *such that* $\inf_{i \in L_l^{-1}} \omega_i \neq \inf_{h \in L_{l'}^{-1}} \omega_h$. *We wish to show that the welfarist central government can not be maxmin. Without loss of generality, assume that the set* L_l^{-1} *contains a household who is the worst off in the whole population. Because this household must belong to the set of poorest households in* L_l^{-1}*, one has:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) \leq \Phi(t_k, \varpi_k, \omega_h, g_k, c) \tag{7}$$

for all $k \in \mathbb{L}$ *and* $h \in L_k^{-1}$. *By clause (1) of definition of stability, one has also:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) \geq \Phi(t_{l'}, \varpi_{l'}, \inf_{i \in L_l^{-1}} \omega_i, g_{l'}, c) \tag{8}$$

and:

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h \in L_{l'}^{-1}} \omega_h, g_{l'}, c) \geq \Phi(t_l, \varpi_l, \inf_{h \in L_{l'}^{-1}} \omega_h, g_l, c) \tag{9}$$

Either (i) $\inf_{h \in L_{l'}^{-1}} \omega_h < \inf_{i \in L_l^{-1}} \omega_i$ *or:*

(ii) $\inf_{h \in L_{l'}^{-1}} \omega_h > \inf_{i \in N_l^{--1}} \omega_i$. *Yet assuming (i) would imply, using (8) and the fact that* Φ *is increasing with respect to private wealth, that:*

$$\Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c) > \Phi(t_{l'}, \varpi_{l'}, \inf_{h \in N_{l'}^{-1}} \omega_h, g_{l'}, c)$$

in contradiction with (7). Hence (i) can not hold. If (ii) holds, then one has, because of (9) and the monotonicity of Φ *with respect to private wealth, that:*

$$\Phi(t_{l'}, \varpi_{l'}, \inf_{h \in L_{l'}^{-1}} \omega_h, g_{l'}, c) > \Phi(t_l, \varpi_l, \inf_{i \in L_l^{-1}} \omega_i, g_l, c)$$

so that the household whose income is (arbitrarily close to) $\inf_{i \in L_l^{-1}} \omega_i$ *is strictly worse off than the poorest household in jurisdiction* l'. *In that case, let* W *be defined by:*
$W = \{l'' \in \mathbb{L} : \Phi(t_{l''}, \varpi_{l''}, \inf_{i \in L_{l''}^{-1}} \omega_i, g_{l''}, c) = \Phi(t_l, \varpi_l, \inf_{i \in L_l^{Z}} \omega_i, g_l, c)\}$. *The set* W *is clearly non-empty since* $l \in W$. *Consider then taking away from jurisdiction* l' *some amount of grant* Δ *and dividing it up equally among all jurisdictions in* W *so as to keep constant the central government budget constraint. For a suitably small* Δ, *this change in the central government transfer policy increases the well-being of all households in the jurisdictions*

contained in W (including the worst off) while keeping the well-being of the worst off household in jurisdiction l' above. This shows that the original equalization grant vector g was not maximizing a maxmin ordering whatever the value of c might have been. ■

The intuition behind this result is simple. A maxmin government wants to transfer money to the jurisdiction that contains one of the worst off households (which must clearly be the poorest in its jurisdiction). By stability, any such worst off household prefers staying in its jurisdiction than moving elsewhere while the households in other jurisdictions - including the poorest in them - also prefer staying where they are than moving to the jurisdiction containing the worst off households. Except in the case when the wealth of these poorest households in all jurisdictions happens to be the same, these two conditions for stability imply that worst-off households are strictly worse off than at least one household which is the poorest in its jurisdiction. But if this is the case, then the transfers given by the central government to jurisdictions are not optimal from a maxmin point of view. Notice that the reasoning holds irrespective of the income tax rate chosen by the central government.

Jurisdiction structures in which all non-empty jurisdictions contain some of the poorest households in the population are, admittedly, rather special. For example, if all stable jurisdiction structures are stratified - in the sense of definition 2 above- then the *only* stable jurisdiction structure with a maxmin central government is the trivial "grand" one in which everybody but a set of measure zero lives in one place. By contrast, as illustrated in the following quasi-linear example, the absence of a central government is commonly compatible with *several* stable jurisdiction structures.

Example 1 *Consider the quasi-linear economy $(\omega, U, \mathbb{L})$ defined by:*

$$\begin{aligned} \omega_i &= 1 \text{ if } i \in [0, 4/7[\\ &= 2 \text{ if } i \in [4/7, 6/7] \\ &= 10 \text{ if } i \in]6/7, 1] \end{aligned}$$

$$U(Z, x) = \ln 7Z + x, \tag{10}$$

and $\mathbb{L} = \{1, 2, 3\}$. Suppose there is no central government. Because the preferences represented by U satisfy what will be called below the GSC condition (see also Gravel and Thoron (2007)), we know that all stable jurisdiction structures will be segregated as per definition 2. Up to irrelevant permutations of locations, there are therefore only four such segregated jurisdiction structure:
1) The perfectly segregated structure in which each of the three intervals $[0, 4/7[$, $[4/7, 6/7]$ and $]6/7, 1]$ form a jurisdiction,
2) the "poor-mixed" structure in which the two poor households categories

belonging to the interval $[0, 6/7[$ *form a jurisdiction and the rich households in the interval* $]6/7, 1]$ *form another,*
3) the somewhat opposite "rich-mixed" structure where the two "rich" categories of households in $[4/7, 1]$ *form a jurisdiction and where the poor households in* $[0, 4/7[$ *form another and*
4) the "grand jurisdiction" structure in which the all households form a unique jurisdiction.
Assume for the sake of this example that majority voting in each jurisdiction so that the dictator in each jurisdiction is the household whose favorite tax rate is the median of the favorite tax rates of the jurisdiction's members. Solving program (6) for $\gamma = c = 0$ *(no central government) and for* U *defined by (10), one obtains easily that this favorite tax rate is* $1/\omega_i$. *This enables one to conclude that the perfectly segregated jurisdiction structure 1) is stable in this example. Indeed if one denotes by 1, 2, and 3 the jurisdictions in which households in the intervals* $[0, 4/7[$, $[4/7, 6/7]$ *and* $]6/7, 1]$ *respectively live, one obtains from clause (2) of the definition of stability that* $t_1 = 1$, $t_2 = 1/2$ *and* $t_3 = 1/10$. *This leads to a provision of public good of* $Z_1 = t_1 \int_{[0,4/7]} \omega_i d\lambda = 4/7$, $Z_2 = t_2 \int_{[4/7,6/7]} \omega_i d\lambda = 2/7$ *and* $Z_3 = t_3 \int_{[0,4/7]} \omega_i d\lambda = 1/7$. *We remark that this structure satisfies clause (1) of the definition of stability. Indeed for any household* $i \in [0, 4/7[$, *one has* $U(Z_1, 0) = \ln 4 > U(Z_2, (1 - t_2)) = \ln 2 + 1/2$ *and* $U(Z_1, 0) = \ln 4 > U(Z_3, (1 - t_3)) = 9/10$. *Similarly a household belonging to* $[4/7, 6/7]$ *and living in jurisdiction 2 has a utility of* $U(Z_2, 2(1 - t_2)) = \ln 2 + 1$ *at its place of residence while it would obtain a utility of* $U(Z_1, 0) = \ln 4$ *if it were to move to jurisdiction 1 and a utility of* $U(Z_3, 2(1 - t_3)) = 9/5$ *if it were to move to the rich jurisdiction 3. Finally, the utility of* $U(Z_3, (1 - t_3)10) = 9$ *achieved by any rich household in jurisdiction 3 is larger than the utility of* $U(Z_2, (1 - t_2)10) = \ln 2 + 5$ *it would get if it were to move to jurisdiction 2, and the utility of* $U(Z_1, 0) = \ln 4$ *it would get if it were to move to jurisdiction 1. The "poor-mixed" structure is also stable in this economy. Indeed, in this structure, the (median) tax rate at jurisdiction 1 (inhabited by households in* $[0, 6/7]$*) is* $t_1 = 1$ *while the tax rate that will prevail at the (rich) jurisdiction 2 inhabited by households in* $[6/7, 1]$ *is* $t_2 = 1/10$. *We have here* $Z_1 = t_1[\int_{[0,4/7]} \omega_i + \int_{[4/7,6/7]} \omega_i] d\lambda = 8/7$. *Clearly:*

$$U(Z_1, 0) = \ln 8 > U(Z_2, 2(1 - t_2)) = 9/5$$

so that no household in $[4/7, 6/7]$ *wants to move to jurisdiction 2. Similarly:*

$$U(Z_1, 0) = \ln 8 > U(Z_2, 1 - t_2) = 9/10$$

so that no household in $[0, 4/7[$ *wants to move to jurisdiction. Lastly:*

$$U(Z_2, (1 - t_2)10) = 9 > U(Z_1, 0) = \ln 8$$

so that the rich prefer staying in jurisdiction 2. It can be checked however that the "rich-mixed" jurisdiction structure is not stable no matter how the local tax rate in the "rich-mixed" jurisdiction is set (provided of course that this tax rate lies in the interval $[1/10, 1/2]$ *as it should if the structure satisfy clause 2) of the definition of stability. Of course the grand jurisdiction structure is stable (because no household with preferences as in this example would choose to move to a desert jurisdiction with zero public good provision. Hence, there are three stable jurisdiction structures in this example without a central government. Thanks to proposition 1, we know that only the grand jurisdiction structure can be stable with a maxmin central government.*

While this example and proposition 1 both suggest that the introduction of a GU central government in a Westhoff model tends to reduce the number of stable jurisdiction structures, it is not difficult to think of economies where the opposite is true. Indeed, in the following example, we have an economy where the introduction of a utilitarian central government increases the number of stable jurisdictions.

Example 2 *Consider the Cobb-Douglas economy* $(\omega, U, \mathbb{L})$ *defined by:*

$$\begin{aligned} \omega_i &= 1 \text{ if } i \in [0, 1/4[\\ &= 3 \text{ if } i \in [1/4, 1] \end{aligned}$$

$$U(Z, x) = \ln Z + \ln x, \tag{11}$$

and $\mathbb{L} = \{1, 2\}$*. Hence we consider here a very simple economy with only two possible place of residence and two types of households: poor ones (in proportion 1/4) and rich ones (in proportion 3/4, admittedly not a very realistic distribution). Solving program (6) for* U *defined by (11), one obtains:*

$$\frac{1}{\varpi}[Z^M(\frac{1}{\varpi}, \frac{1}{\omega_i}, 1 - c + \frac{\gamma}{\varpi}) - \gamma] = \frac{1 - c}{2} - \frac{\gamma}{2\varpi} \tag{12}$$

Suppose first there is no central government so that $c = \gamma = 0$*. Then, any household has a favorite tax rate of* $1/2$ *no matter where it lives. In this (very simple) world without government, the grand jurisdiction is stable. But the (segregated) jurisdiction structure where poor households live in one jurisdiction and rich ones live in another is not stable. Indeed, since the two jurisdiction structure must offer the same tax rate of 1/2 for being stable as per clause (2) of the definition of stability, any poor household will have incentive to leave its jurisdiction and to move to the rich one where, for a same tax payment, it will get a larger public good provision. However,* *the segregated structure may become stable if one introduces a GU central government. Indeed, consider specifically a utilitarian government and the jurisdiction structure (L, t, g, c) defined by:*

$$\begin{aligned} L(i) &= 1 \text{ if } i \in [0, 1/4[\text{ and} \\ &= 2 \text{ if } i \in [1/4, 1] \end{aligned}$$

$$\begin{aligned} t_1 &= 5/4 \\ t_2 &= 25/12 \end{aligned}$$

$$\begin{aligned} \gamma_1 &= 0 \\ \gamma_2 &= -15/4 \\ c &= -3/2 \end{aligned}$$

This jurisdiction structure, in which the central government subsides everybody's income at a rate of 50% and finances this subsidy by collecting a grant of 15/4 from the rich jurisdiction, satisfies clearly conditions (3)-(5). Notice also that that each t_i satisfies condition (12) for its g_i (for $i = 1, 2$) and that:

$$\begin{aligned} \Phi(t_1, \varpi_1, \omega_i, g_1, c) &= \Phi(\frac{5}{4}, \frac{1}{4}, 1, 0, -3/2) \\ &= \ln\frac{5}{16} + \ln\frac{5}{4} = 2\ln\frac{5}{4} - \ln 4 \\ &= \Phi(\frac{25}{12}, \frac{3}{4}, 1, \frac{-15}{4}, -3/2) \\ &= \Phi(t_2, \varpi_2, \omega_i, g_2, c) \end{aligned}$$

and,

$$\begin{aligned} \Phi(t_2, \varpi_2, \omega_i, g_2, c) &= \Phi(\frac{25}{12}, \frac{3}{4}, 3, \frac{-15}{4}, -\frac{3}{2}) \\ &= \ln\frac{15}{16} + \ln\frac{5}{4} = \ln\frac{3}{4} + 2\ln\frac{5}{4} \\ &= \Phi(\frac{5}{4}, \frac{1}{4}, 3, 0, -3/2) \\ &= \Phi(t_1, \varpi_1, \omega_i, g_1, c). \end{aligned}$$

Hence every household prefers (weakly) to live in its jurisdiction then to move. We finally invite the reader to verify that g_1, g_2 and c satisfy the 1st order conditions for the following program solved by the central government (after substituting the budget constraint $c \int_{[0,1]} \omega_i = \frac{5c}{2} = g_1 + g_2$ into the objective):.

$$\max_{g_1, g_2} \frac{1}{4}[\ln(\frac{5}{16} + g_1) + \ln(1 - \frac{2(g_1 + g_2)}{5} - \frac{5}{4})] + \frac{3}{4}[\ln(\frac{75}{48} + g_2) + \ln((1 - \frac{2(g_1 + g_2)}{5} - \frac{25}{12})3)]$$

Given the strict concavity of the objective function, this is sufficient to conclude that this jurisdiction structure with a central government is stable.

These two examples make clear that the introduction of a GU central government may affect substantially the set of stable jurisdiction structures. This set may be shrunk, as in the first example, or enlarged, as in the second.

We now show that, if households preferences are additively separable, the effect of that the central government can have on the endogenous formation of jurisdiction process, as important as it may be, does not affect the segregative properties of the process.

3 RESULTS

As in Gravel and Thoron (2007), the monotonicity of τ^* with respect to household's wealth (given jurisdiction's wealth and central government grant) is a key element for guaranteeing the wealth segregation of stable jurisdiction structures. By lemma 2, this monotonicity of the household's favorite tax rate with respect to private wealth is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if Z^M is monotonically *decreasing* with respect to p_x) or always a gross *substitute* for (if Z^M is monotonically *increasing* with respect to p_x) the private good. We state formally this state of affairs, proved in Gravel and Thoron (2007), using the regularity condition 1 as follows.

Lemma 3 *For every $U \in \mathcal{U}$, the function τ^* that solves (6) is monotonic with respect to ω_i for any given jurisdiction wealth level ϖ and per capita wealth if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

We refer to this property according to which the substitutability/ complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable, the GSC condition is nonetheless a significant restriction that, as discussed in Gravel and Thoron (2007), can be violated even by additively separable preferences.

An information used in Gravel and Thoron (2007) to show that the GSC condition is necessary and sufficient for guaranteeing the segregation of any stable jurisdiction structure is the structure of households' indifference curves in the tax-jurisdiction's wealth space. While this information is also useful in the present context, we need to account for the fact that the relevant space of location characteristics is now *three*, rather than *two*, dimensional and must include central government grant as well as local tax rate and jurisdiction's aggregate wealth. Specifically, the indifference surface of a household with (net of central government) wealth $(1-c)\omega_i$

passing through some point $(\overline{\tau}, \overline{\omega}, \overline{\gamma}) \in \mathbb{R}^3$ such that $\Phi(\overline{\tau}, \overline{\omega}, \omega_i, \overline{\gamma}, c) = \overline{\Phi}$ is the graph of the implicit function $f^{\overline{\Phi}} : [\frac{-\overline{\gamma}}{\overline{\omega}}, 1] \times \mathbb{R} \times \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ defined by $\Phi(\tau, f^{\overline{\Phi}}(\tau, \gamma, c; \omega_i), \omega_i, \gamma, c) \equiv \overline{\Phi}$. The assumption imposed on U guarantees that the function $f^{\overline{\Phi}}$ exists and is derivable everywhere. Its partial derivative $f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$ with respect to τ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_\tau^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{1}{\overline{\tau}}[\frac{\omega_i}{MRS(\overline{\tau}\varpi + \overline{\gamma}, (1 - c - \overline{\tau})\omega_i)} - \varpi] \tag{13}$$

where $\varpi = f^{\Phi}(\overline{\tau}, \overline{\gamma}, c; \omega_i)$. Figure 1 below illustrates the shape of these indifference curves in the (τ, ϖ) plane for given values of $\overline{\gamma}$ and c. Specifically, indifference curves of a household with private wealth $(1-c)\omega_i$ are U-shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen indeed that, at the minimum of an indifference curve, the term within the bracket of (13) is zero thanks to the first order conditions of (6)). As in Gravel and Thoron (2007), and despite what figure 1 suggests, indifference curves need *not* be globally convex. The only property that indifference curves possess is that of being "single-through" (monotonically decreasing at the left of the minimum and monotonically increasing at the right).

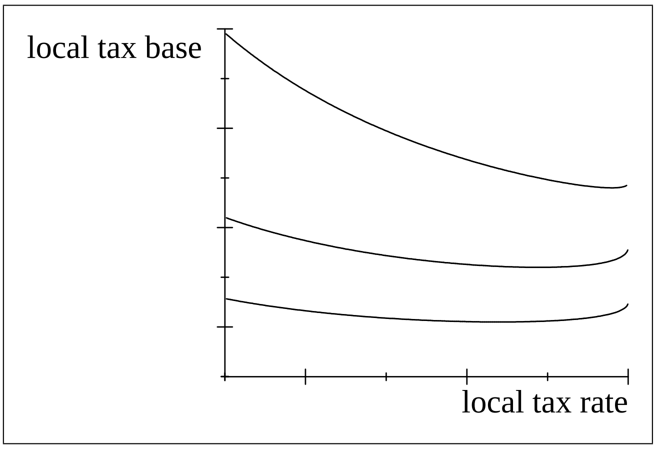

Figure 1

Analogously, one can fix local tax rate at $\overline{\tau}$ and examine the property of the derivative of $f^{\overline{\Phi}}$ with respect to the central government's grant. This partial derivative $f_\gamma^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i,)$ with respect to γ evaluated at $(\overline{\tau}, \overline{\gamma}, c, \omega_i)$ is given by:

$$f_\gamma^{\overline{\Phi}}(\overline{\tau}, \overline{\gamma}, c; \omega_i) = \frac{-1}{\overline{\tau}}$$

Hence, when looked in the $(\gamma, \overline{\omega})$ space, indifference surfaces are straight line with negative slope (if at least $\overline{\tau}$ is positive). There is therefore a constant marginal trade off between tax base and central government grant as envisaged by a mobile household. This is of course not surprising since both central government grant and local tax base are perfectly substitutable ways of getting public expenditure in a given jurisdiction. The rate at which the household is willing to sacrifice local tax base in order to get more central government transfer depends obviously upon the local tax rate that converts tax base into public spending. Figure 2 shows a typical indifference surface in the tax rate, tax base and central government space in which location choice by households is made.

We first establish, in the following lemma, that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space (for a given level of central government grant) coincides with the ordering of the households' wealth when preferences for the public and the private good satisfy the GSC condition. The proof of this lemma, which mimics very closely that of lemma 5 in Gravel and Thoron (2007), is omitted.

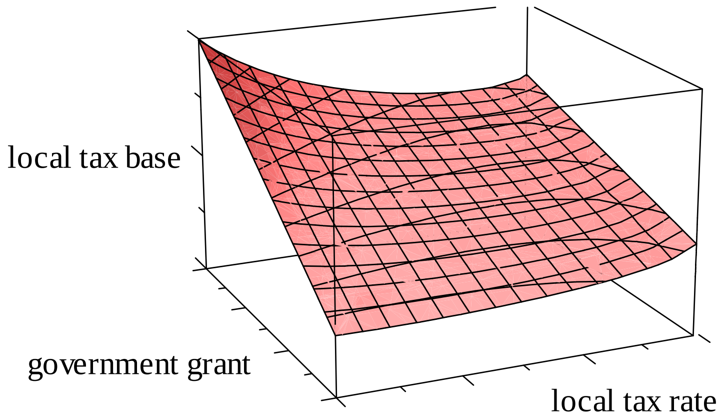

Figure 2

Lemma 4 *Assume that households preferences are represented by a utility function in $\mathcal{U}$. Then, Z^M is everywhere a gross substitute (resp. complement) to the private good if and only if one has, at any $(\overline{\tau}, \varpi, \overline{\gamma}, c) \in \mathbb{R}^4$ satisfying $\overline{\gamma} + \overline{\tau}\varpi \geq 0$ and $1 - \overline{\tau} - c \geq 0$, $f_{\tau}^{\overline{\Phi}_j}(\overline{\tau}, \overline{\gamma}; \omega_h) \leq$ (resp. $\geq$) $f_{\tau}^{\overline{\Phi}_k}(\overline{\tau}, \overline{\gamma}, \omega_k)$ for every h, k such that $\omega_h < \omega_k$ where, for every i, $\overline{\Phi}_i = \Phi(\overline{\tau}, \varpi, \overline{\gamma}, c; \omega_i)$.*

In plain English, this lemma says that the GSC condition is equivalent to the requirement that, for any central government grant, the slope of the indifference surfaces in the local tax rate and jurisdiction wealth space be *monotonic* with respect to private wealth at any point of that space. In proposition 3 below, we shall show that this monotonicity of the slopes in the two-dimensional space of tax rate and aggregate wealth for a given government grant holds true as well in the three dimensional space of government grants, local tax and jurisdiction's wealth.

We now establish that, if preferences are additively separable, and if the utility function aggregated by the generalized-utilitarian central government is the additively separable numerical representation of those preferences, then the GSC condition is *necessary* for the wealth segregation of any stable jurisdiction structure for the two definition of stability.

Proposition 2 *Assume that households preferences are represented by a utility function belonging to $\mathcal{U}^{\mathcal{A}}$. Then, a stable - as per definition 1 or* **??***- jurisdiction structure with a generalized-utilitarian central government that uses an additively separable numerical representation of these preferences as utility function is segregated only if the preferences satisfies the GSC condition.*

Proof. *Assume that the GSC condition is violated. Then there are private good prices p_x^0, p_x^1 and p_x^2 satisfying $0 < p_x^0 < p_x^1 < p_x^2$ and public good price $p_Z > 0$ such that:*

$$\begin{aligned} Z^M(p_Z, p_x^0, 1) &= Z^M(p_Z, p_x^2, 1) \\ &> Z^M(p_Z, p_x^1, 1) \text{ or} \\ &< Z^M(p_Z, p_x^1, 1) \end{aligned}$$

Our objective is to show the existence of an economy where a non-segregated stable jurisdiction structure can be constructed. We provide the construction by assuming the first of these two inequalities, the argument being symmetric for the second. Consider an economy where a mass μ_0 of households have wealth $1/p_x^0$, a mass μ_1 of households have wealth $1/p_x^1$ and a mass μ_2 of households have wealth $1/p_x^2$ and let $\mathbb{L} = \{1, 2\}$. In order to construct a stable jurisdiction structure that is not segregated, we are going to locate households with wealth $1/p_x^0$ and $1/p_x^2$ in location 1 and households with wealth $1/p_x^1$ in location 2. For this purpose we are going to choose positive numbers of households μ_0, μ_1 and μ_2 so that:

$$\frac{\mu_0}{p_x^0} + \frac{\mu_2}{p_x^2} = \frac{\mu_1}{p_x^1} = \frac{1}{p_Z} \tag{14}$$

There is clearly no difficulty, given any p_x^0, p_x^1, p_x^2 and p_Z satisfying the properties above, of finding positive numbers μ_i (for $i = 0, 1, 2$) satisfying (14). Without a central government, such a jurisdiction structure, that is clearly non-segregated, would be stable because the two jurisdictions created would have the same tax base $\frac{1}{p_Z}$ and each household would get its favorite local tax given this tax base in its jurisdiction of residence. With a central *government however, this jurisdiction structure can be stable only if the central government finds optimal, given the two jurisdictions and local tax rates, to perform no equalization grants and to raise no taxes. Hence, we must prove that positive numbers μ_0, μ_1 and μ_2 can be found in such a way that a generalized-utilitarian central government that uses the additively separable numerical representation of households' preferences provided by (1) would find it optimal to give no equalization grants to jurisdictions and to collect no taxes as a result. We must prove that positive numbers μ_0, μ_1 and μ_2 such that:*

$$\begin{aligned}(0,0) \quad \in \quad & \arg\max_{(\gamma,c)\in\mathbb{R}\times[0,1]} \mu_0\Psi(f(Z^M(p_Z,p_x^0,1)+\gamma)+h(x^M(p_Z,p_x^0,1)-\frac{c}{p_x^0})) \\ & +\mu_1\Psi(f(Z^M(p_Z,p_x^1,1)+\frac{2c}{p_Z}-\gamma)+h(x^M(p_Z,p_x^1,1)-\frac{c}{p_x^1})) \\ & +\mu_2\Psi(f(Z^M(p_Z,p_x^2,1)+\gamma)+h(x^M(p_Z,p_x^2,1)-\frac{c}{p_x^2})) \qquad (15)\end{aligned}$$

can be found. Since the objective function of the program (15) is concave, any combination of values of jurisdiction 1's grant γ^* and wealth tax c^* that satisfy the first order conditions of this program is a solution to it. Using the (assumed) fact that $Z^M(p_Z,p_x^2,1)=Z^M(p_Z,p_x^0,1)$, the 1st order conditions of (15) for $\gamma^*=0$ and $c^*=0$ write:

$$(\mu_0\Psi^{0\prime}+\mu_2\Psi^{2\prime})\frac{\partial f(Z^M(p_Z,p_x^2,1))}{\partial Z}=\mu_1\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z} \qquad (16)$$

and:

$$\begin{aligned}&\frac{\mu_0}{p_x^0}\Psi^{0\prime}\frac{\partial h(x^M(p_Z,p_x^0,1))}{\partial x}+\frac{\mu_1}{p_x^1}\Psi^{1\prime}\frac{\partial h(x^M(p_Z,p_x^1,1))}{\partial x}+\frac{\mu_2}{p_x^2}\Psi^{2\prime}\frac{\partial h(x^M(p_Z,p_x^2,1))}{\partial x} \\ &\qquad =\frac{2\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z} \qquad (17)\end{aligned}$$

where, for $k=0,\ 1,\ 2$, $\Psi^{k\prime}>0$ denotes the derivative of Ψ evaluated at $f(Z^M(p_Z,p_x^k,1))+h(x^M(p_Z,p_x^k,1))$. We notice that, thanks to the decreasing monotonicity of the indirect utility function with respect to prices $f(Z^M(p_Z,p_x^2,1)+h(x^M(p_Z,p_x^2,1))<f(Z^M(p_Z,p_x^1,1)+h(x^M(p_Z,p_x^1,1))<f(Z^M(p_Z,p_x^0,1)+h(x^M(p_Z,p_x^0,1))$. Since Ψ is concave, one has therefore:

$$\Psi^{2\prime}>\Psi^{1\prime}>\Psi^{0\prime}>0 \qquad (18)$$

By definition of the Marshallian demands, condition (17) writes (thanks to additive separability):

$$\frac{\mu_0}{p_Z}\Psi^{0\prime}\frac{\partial f(Z^M(p_Z,p_x^0,1))}{\partial Z}+\frac{\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z,p_x^1,1))}{\partial Z}+\frac{\mu_2}{p_Z}\Psi^{2\prime}\frac{\partial f(Z^M(p_Z,p_x^2,1))}{\partial Z}$$

$$= \frac{2\mu_1}{p_Z}\Psi^{1\prime}\frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z}$$

or (since $Z^M(p_Z, p_x^2, 1) = Z^M(p_Z, p_x^0, 1)$):

$$(\mu_0\Psi^{0\prime} + \mu_2\Psi^{2\prime})\frac{\partial f(Z^M(p_Z, p_x^2, 1))}{\partial Z} = \mu_1\Psi^{1\prime}\frac{\partial f(Z^M(p_Z, p_x^1, 1))}{\partial Z}$$

which is nothing else than condition (16). Hence, the only other condition that the numbers μ_0, μ_1 and μ_2 must satisfy beside (14) is condition (16). Since p_x^0, p_x^1, p_x^2 and p_Z are given positive numbers, we have that $\mu_1 = p_x^1/p_Z > 0$. Hence, in order for the proposed jurisdiction structure to be stable with a generalized utilitarian government who chooses (optimally) not to intervene, we only need to find positive numbers μ_0 and μ_2 such that equalities:

$$\mu_2 = \frac{p_x^2}{p_Z} - \frac{p_x^2}{p_x^0}\mu_0 \tag{19}$$

and:

$$\mu_2 = \frac{p_x^1\Psi^{1\prime}\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{p_Z\Psi^{2\prime}\partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}}\mu_0$$

hold. From the first order condition that defines Marshallian demands, we can write the later equality (using additive separability) as: :

$$\mu_2 = \frac{p_x^2\Psi^{1\prime}\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{p_Z\Psi^{2\prime}\partial h(x^M(p_Z, p_x^2, 1))/\partial x} - \frac{\Psi^{0\prime}}{\Psi^{2\prime}}\mu_0 \tag{20}$$

Since the preferences represented by the utility function are additively separable, no good can be inferior and, as a result, the private good is not a Giffen good. Hence x^M is decreasing with respect to its own price so that, since $p_x^1 < p_x^2$, $x^M(p_Z, p_x^1, 1) > x^M(p_Z, p_x^2, 1)$ and, since h is concave, $\partial h(x^M(p_Z, p_x^1, 1))/\partial x < \partial h(x^M(p_Z, p_x^2, 1))/\partial x$. Since by (18), $\Psi^{1\prime} < \Psi^{2\prime}$, we conclude that the intercept of the linear equation (20) is smaller than that of equation (19). Moreover, the abscissa at the origin of the linear equation (19) is p_x^0/p_Z while the abscissa at the origin of equation (20), denoted $\mu^0(2)$ is:

$$\begin{aligned}
\mu^0(2) &= \frac{\Psi^{1\prime}p_x^2\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}p_Z\partial h(x^M(p_Z, p_x^2, 1))/\partial x} \\
&= \frac{\Psi^{1\prime}p_x^1\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}p_Z\partial f(Z^M(p_Z, p_x^2, 1))/\partial Z} \quad \textit{(by separability and definition of Marshallian demands)} \\
&= \frac{\Psi^{1\prime}p_x^1\partial f(Z^M(p_Z, p_x^1, 1))/\partial Z}{\Psi^{0\prime}p_Z\partial f(Z^M(p_Z, p_x^0, 1))/\partial Z} \quad \textit{(since } Z^M(p_Z, p_x^0, 1) = Z^M(p_Z, p_x^2, 1)\textit{)} \\
&= \frac{\Psi^{1\prime}p_x^0\partial h(x^M(p_Z, p_x^1, 1))/\partial x}{\Psi^{0\prime}p_Z\partial h(x^M(p_Z, p_x^0, 1))/\partial x} \quad \textit{(by separability and definition of Marshallian demands)}
\end{aligned}$$

Now, again, since the private good is not inferior (and therefore not Giffen) we obtain, using concavity of h and inequality (18), that $\mu^0(2) > p_x^0/p_Z > 0$. Hence the abscissa at the origin of the linear equation (20) is larger than that of equation (19) so that the two straight lines represented by those two equations are just as in figure 3) and cross in the strictly positive orthant. This shows the existence of positive numbers μ_0 and μ_2 satisfying equations (19) and (20) and this completes the proof.

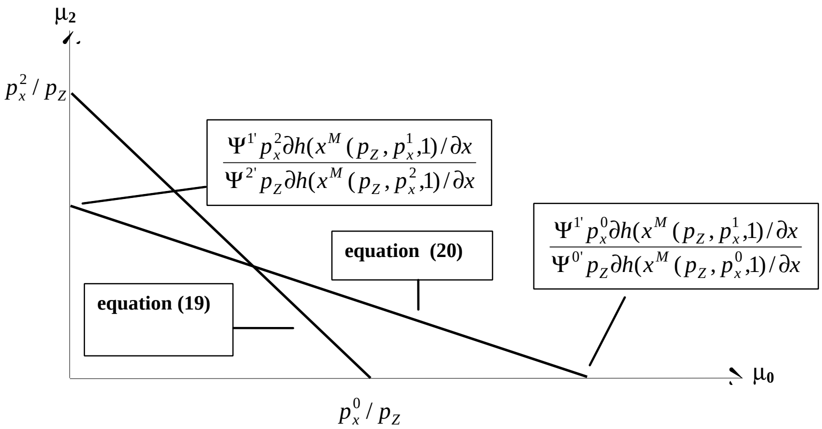

Figure 3.

■

The difference between the proof of this proposition and the corresponding one in Gravel and Thoron (2007) is worth mentioning. The reason for the difference is the additional constraint imposed by the optimality of the central government (non) intervention in the construction of the stable but non-segregated jurisdiction structure for any violation of the GSC condition. This constraint obviously increases the difficulty of the construction of the economy giving rise to such a stable non-segregated jurisdiction structure. The additive separability condition plays a key role in this construction and we do not know whether we could obtain the construction without such an assumption. We emphasize, however, that proposition 2 proves in fact something slightly stronger than what is required. Indeed, what is established in proposition 2 is that, for any violation of the GSC condition obtained with additively separable preferences, one can find a non-segregated stable jurisdiction structure in which a generalized utilitarian government finds it optimal to perform zero equalization (and accordingly to levy no wealth taxes). The possibility of proving, less demandingly, that any violation of the GSC condition can give rise to a non-segregated stable jurisdiction structure in which the utilitarian central government performs non-zero equalization without assuming additive separability of households' utility function remains an open, if not difficult, question.

We notice also that the assumption that there is a continuum of households plays a role in this proof. Specifically, the proof establishes, for any violation of the GSC condition, the existence of positive *real* numbers of households that can be put in a non-segregated but yet stable jurisdiction structure. Yet, we are not capable of proving, for any violation of the GSC condition, the existence of *integer* numbers of households that can be put in a stable but non-segregated jurisdiction structure.

We now establish, without any further condition on household's preferences, the converse proposition that the GSC condition is sufficient for the wealth segregation of any stable jurisdiction structure. No additive separability is needed for this proposition.

Proposition 3 *Assume that households' preferences satisfy the GSC condition and are represented by a utility function in $\mathcal{U}$. Then, any stable jurisdiction structure with a GU central government as per either definition 1 is wealth segregated.*

Proof. *We sketch the argument for the case where the local public good is everywhere a gross complement to the private good. Assume therefore that Z^M is decreasing with respect to p_x and, by contradiction, let (L, g, c, t) be a jurisdiction structure that is not wealth-stratified. Hence, there are locations l and $l' \in \mathbb{L}$ (with $l \neq l'$), and households h, i and $k \in [0, 1]$ with $h < i < k$ for which one has h and $k \in L_l^{-1}$, $i \in L_{l''}^{-1}$ with $\lambda(L_l^{-1}) > 0$ and, $\lambda(L_{l''}^{-1}) > 0$ and either $t_l \neq t_{l'}$ or $t_l\varpi_l + g_l \neq t_{l'}\varpi_{l'} + g_{l'}$. It is clear that if only one of the two inequalities $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\varpi_l + g_{l'}$ holds, then the jurisdiction structure can not be stable because there would be unanimity of the inhabitants of one of the jurisdictions l and l' to go to the jurisdiction with the low tax rate (if $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l = t_{l'}\omega_{l'} + g_{l'}$) or to the jurisdiction with the largest public good provision (if $t_l = t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\omega_l + g_{l'}$). Hence one can assume that both $t_l \neq t_{l'}$ and $t_l\varpi_l + g_l \neq t_{l'}\varpi_l + g_{l'}$ hold. Define now ϖ by:*

$$\begin{aligned} t_l\varpi + g_{l'} &= t_l\varpi_l + g_l \\ &\Leftrightarrow \\ \varpi &= \varpi_l + \frac{g_l - g_{l'}}{t_l} \end{aligned}$$

Clearly one has:

$$\begin{aligned} U(t_l\varpi + g_{l'}, (1 - c - t_l)\omega_m) &= \Phi(t_l, \varpi, \omega_m, g_{l'}) \\ &= U(t_l\varpi_1 + g_l, (1 - c - t_l)\omega_m) \\ &= \Phi(t_l, \varpi_l, \omega_m, g_l, c) \end{aligned} \tag{21}$$

for every household m. For this non-stratified jurisdiction structure to be *stable, one must have:*

$$\begin{aligned}\Phi(t_l, \varpi_l, \omega_h, g_l, c) &\geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \\ \Phi(t_l, \varpi_l, \omega_i, g_l, c) &\leq \Phi(t_{l'}, \varpi_{l'}, \omega_i, g_{l'}, c)\end{aligned}$$

and

$$\Phi(t_l, \varpi_l, \omega_k, g_l, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c)$$

or, using (21):

$$\Phi(t_l, \varpi, \omega_h, g_{l'}, c) \geq \Phi(t_{l'}, \varpi_{l'}, \omega_h, g_{l'}, c) \quad (22)$$

$$\Phi(t_l, \varpi, \omega_i, g_{l'}, c) \leq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_i, g_{l'}, c) \quad (23)$$

and

$$\Phi(t_l, \varpi, \omega_k, g_{l'}, c) \geq \Phi(t_{l'}, \overline{\omega}_{l'}, \omega_k, g_{l'}, c) \quad (24)$$

By lemma 4, the slopes of indifference curves in the space of all combinations of local tax rate and jurisdiction aggregate wealth are ordered by individual wealth for any level of central government grant and, therefore, for the level $g_{l'}$*. Hence indifference curves of households h, i and k at* (t_l, ϖ) *must be as they are depicted in figure 4. Clearly from this figure, unless indifference curves of households k and i or h and i cross in the "wrong" order at a point such as* (τ, ϖ'')*, the set of combinations of local tax rates and jurisdiction aggregate wealth that i considers as weakly worse (given central government grant* $g_{l'}$*) than* (t_l, ϖ) *is contained in the set of such combinations that either household h or household k considers as strictly worse than* (t_l, ϖ)*. Hence, unless indifference curves of two households cross in the wrong order at* (τ, ϖ'')*, inequalities (22)-(24) can not simultaneously hold for distinct combinations* (t_l, ϖ) *and* $(t_{l'}, \varpi_{l'})$ *of local tax rates and jurisdiction tax rates. This prove that unless the GSC condition is violated, the*

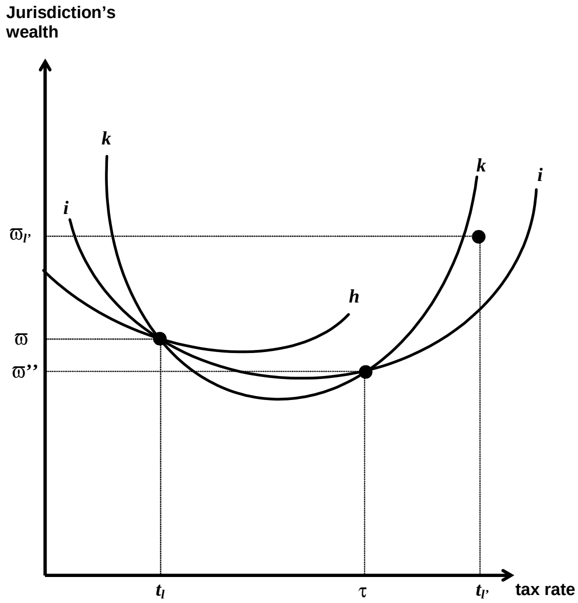

Figure 4.

jurisdiction structure can not be stable. ■

4 Conclusion

The main conclusion of this paper is that the welfarist - in fact generalized-utilitarian - intervention of a central government does not alter the segregative properties of endogenous jurisdiction formation, at least when this jurisdiction formation is modelled within a framework *à la* Westhoff. Specifically, the GSC condition that it is necessary and sufficient to impose on household preferences for guaranteeing the wealth segregation of any stable jurisdiction is not affected by the presence of a central government as modelled in this paper. This of course does not mean that central government intervention does not affect jurisdiction formation. As examples 1 and 2 discussed in section 2 reveal, the redistributive behavior of the GU central government affects quite sharply the set of stable jurisdiction structures. Yet the jurisdictions structures that remain stable under a generalized utilitarian central government are segregated under exactly the same conditions on household's preferences than would be the case without a central government.

While we believe that the message according to which central government intervention does not modify the segregative forces underlying Tiebout-like processes of jurisdiction formation is of some interest, it is worth recalling the limitations of the analysis on which it stands. For one thing, the result is obtained, at least for its necessity part, under the assumption that preferences are additively separable. It would be nice if this assumption could be relaxed. Another limitation of the analysis lies, perhaps, in the simultaneous setting in which the decisions by households, central and local government are considered. A third limitation of the analysis is the rather limited power given to the central government in our model to redistribute private wealth. Extending the analysis of this paper over these limitations, as well as many others, seems to us a worthy objective for future research.

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