Robust Normative Comparisons of Socially Risky Situations

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April 3th 2007

Abstract

In this paper, we characterize and implement robust dominance criteria for comparing societies on the basis of the way they expose their members to various *risks*. Risks are modelled as lotteries on the set of distributions, between a given number of individuals, of pairs made of a pecuniary consequence and a state at which the pecuniary consequence occurs. It is assumed that individuals have individualistic Von Neuman-Morgenstern preferences for these risks. Appealing to Harsanyi's aggregation theorem, we provide empirically implementable criteria that coincide with the unanimity, over all such individual preferences, of anonymous and Von Neuman Morgenstern social rankings of risks that are Pareto-inclusive with respect to individual preferences. The empirically implementable criteria that we identify in this fashion can be interpreted as *sequential expected poverty dominance*. Illustrations of the usefulness of the criteria for comparing the exposure to unemployment risk of different segments of the French and US workforce and for appraising the evolution, over time, of risks of violent crimes in India are also provided.

Keywords: Expected utility, ex-ante welfare, stochastic dominance, multidimensional normative analysis, labour precariousness, risks of crime.

JEL classification numbers: C81, D3, D63, D81, I32, J63, J64

1 Introduction

The protection against various kinds of risks that societies provide to their members is a clearly important ingredient for normative evaluation. For instance, some countries, like US or UK, are commonly depicted as having

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"flexible" labour markets in which most of the work force faces a mild, but significant, risk of unemployment with very little compensation and where the wages of those employed are relatively high. Other countries, like France, are to the contrary portrayed as having "rigid" labour markets in which a fraction of the work force is more or less protected from the risk of being unemployed, even though it enjoys moderate wages, while the remaining part of the work force is exposed to a high risk of unemployment which, if it arises, is the object of significant pecuniary compensation. A natural question to ask from a normative point of view is: what form of organization of the labour market is better ? Analogously, one may be interested in comparing countries - or the same country at different points in time - on the basis of their distributions of income and risk of crime, or risk of infant mortality (see e.g. Gravel *et al.* (2007) or Gravel and Mukhopadhyay (2007) for examples of such comparisons).

In this paper, we theoretically characterize and empirically implement *robust criteria* for comparing societies in terms of their performance in protecting their members against risks. While the analysis has the potential for further generalization, we restrict attention to *binary* risks which, for each individual, can be described by three parameters: A probability of occurrence of a "good" state (say being employed), a pecuniary consequence contingent on the good state (say the income earned if employed), and a pecuniary consequence contingent on the bad state (say the welfare benefit received if unemployed). While we consider risks that have pecuniary consequences, we do allow for the possibility that these risks have also non-pecuniary consequences by allowing that a dollar received in the bad and the good states be valued differently.

Assuming that individuals have Von Neuman-Mongenstern (VNM) preferences over these risks, and acknowledging that a *distribution of* these *individual risks* can be seen as a *socially risky situation*, we derive empirically implementable criteria for comparing socially risky situations that coincide with the unanimity, taken over some (suitably large) class of individual VNM preferences, of all Pareto consistent social rankings that satisfy themselves the VNM properties. It is in this (unanimity) sense that the criteria are considered as "robust". Because of Harsanyi (1955)'s aggregation theorem, we know that a VNM ranking of socially risky situations that respects, in the usual Pareto-sense, individual VNM preferences can be thought of as resulting from the comparisons of the *sum* of the individuals' VNM expected utility functions. Hence, a ranking of socially risky situations agreed upon by all social VNM rankings who respect individuals' VNM rankings is nothing else than a ranking that commands unanimity over all sums of individual VNM expected utility functions taken in some suitable class.

Robust comparisons of distributions of socially risky situations can be seen as particular instances of robust *multidimensional* normative evaluation (see e.g. Atkinson and Bourguignon (1982), Kolm (1977)) in which one is interested in comparing distributions of several attributes by requiring the unanimity over a class of utilitarian rankings. After all, probability of occurrence of a good state, and state-contingent pecuniary consequences can be seen as individual attributes whose distributions can be compared by multidimensional stochastic dominance criteria such as those developed in Atkinson and Bourguignon (1982). Yet, the additional structure imposed to the problem by the fact that the considered attributes are ingredients of risks that are evaluated, both at the social and the individual level, by VNM preferences turns out to be quite significant. As shall be seen, the empirically implementable criteria that are obtained in this context are quite different from the abstract first or second order multidimensional stochastic dominance criteria of the Atkinson and Bourguignon (1982) variety.

We characterize in fact two implementable criteria, each of which corresponding to a specific property of the VNM individual utility functions over which the unanimity of Paretian social ranking is looked for. We first consider VNM utility functions that are increasing with respect to state-contingent income and that are such that marginal utility of income is, weakly, lower in the good state than in the bad one. We find that the empirically implementable criterion that corresponds to the Paretian unanimity over this class of VNM utilities is what we call "Sequential Expected Headcount Poverty" (SEHP) dominance. According to this criterion, distribution of risks A is better than distribution B if, for any poverty line, the *expected number of individuals* in the bad state who fall below the line is no greater in A than in B and if the expected number of individuals who are below the line in either state is also weakly smaller in A than in B. The sequential aspect of this criterion arises from the fact that in order for A to dominate B, one looks first at expected number of poor in the bad state and, in a second step, at the total expected number of poor in either state. Notice that this criterion requires giving a priority to poverty in the bad state as compared to poverty in the good state. This reflects of course the assumption made on the VNM utility function that the marginal utility of income is greater in the bad state than in the good one.

The second, and more restricted, family of VNM utility functions considered in this type of situations satisfy, in addition to the above properties, the requirement that the marginal utility of income is, in every state, decreasing with income and that this decrease in the marginal utility of income is more important in the bad state than in the good one. Put differently, we assume that the state dependant utility function is concave in every state and is "more concave" in the bad state than in the good one. We then show that the implementable criterion that correspond to the Paretian unanimity over this class of VNM utility function, assuming again VNM preference from the part of the social evaluator, is what we call "Sequential Expected Poverty Gap" (SEPG) dominance. This criterion works just like the SEHP one, but with poverty gap, rather than headcount poverty, used as the poverty measure.

That these criteria are helpful for comparing societies on the basis of their performance in protecting their members against risks is illustrated with data on unemployment risk in France and US and on crime risk in India. In the later case, we illustrate that the SEHP criterion is more discriminatory than the corresponding abstract Atkinson and Bourguignon criterion. In the former case, we show that our criteria does not enable one to compare US and France in terms of the exposure to unemployment risk that they provide to their citizens. The empirical illustration also suggests that, contrary to intuition, the exposure to unemployment risks of female is not comparable to that of male in both country. This conclusion is surprising when one knows that the distribution of labour income among male single adults dominates that among female single adults for the generalized Lorenz criterion. The reason why this dominance does not translate into a dominance of male exposure to risk over that of female is that the average probability of being employed is higher among females than above males. The analysis also reveals that young segments of the workforce population have a worse exposure to unemployment risk than older ones.

The plan of the remaining part of the paper is as follows. The next section introduces the normative and empirically implementable criteria and establishes the formal equivalence between them. The third section applies the criteria to the cross-countries comparisons of labour market risk and to the evaluation of the evolution, over time, of the distribution of individual consumption and crime in India. The fourth section concludes.

2 Theory

2.1 Normative criteria

We consider societies made of a given number, n say, of individuals, indexed by i. While the theoretical results of this paper deal with comparisons of societies with a given number of individuals, it is straightforward to extend them to societies with variable number of individuals. Societies expose their members to *binary* risks in which every individual can fall into *two* possible states, "bad" (b) and "good" (g), and receives, as a function of the state, a pecuniary consequence which we shall refer to as "income". As shall be seen, we do not preclude *a priori* the possibility that individuals also value the states in a non-pecuniary fashion. We assume incomes to be non-negative real numbers.[1] We call *socially risky situation* a specific pattern of exposure of the individuals to binary risks. Formally, we model such a socially risky situation as a probability distribution, or a lottery, p on the set $\mathbb{X} = (\{b, g\} \times \mathbb{R}_+)^n$ of all vectors of state-income pairs, one such pair for each individual.

[1] The assumption that income be non-negative is inessential.

A typical element x of $\mathbb{X}$ writes:

$$x = (s_1, y_1, ...s_n, y_n)$$

where, for $i = 1, ..., n$, $s_i \in \{b, g\}$ denote a state in which i falls and y_i denote i's income in that state. We also denote by $(s_i, y_i; s_{-i}, y_{-i})$ the vector of state-income pairs where individual i gets the pair $(s_i, y_i) \in \{b, g\} \times \mathbb{R}_+$ and all individuals other than i get the vector of state-income pairs $(s_{-i}, y_{-i}) \in (\{b, g\} \times \mathbb{R}_+)^{n-1}$. To remain in tune with the binariness of the risk considered herein, we require lotteries to satisfy the following condition:

Condition 1 *For every individual i, if $p(s_i, y_i; s_{-i}, y_{-i}) > 0$ for some $(s_i, y_i) \in \{b, g\} \times \mathbb{R}_+$ and $(s_{-i}, y_{-i}) \in (\{b, g\} \times \mathbb{R})^{n-1}$, then $p(s_i, y'_i; s'_{-i}, y'_{-i}) = 0$ for all $y'_i \in \mathbb{R}_+$ such that $y'_i \neq y_i$ and $(s'_{-i}, y'_{-i}) \in (\{b, g\} \times \mathbb{R})^{n-1}$.*

This condition requires therefore lotteries to have finite support, and to be such that they never assign a positive probability to two different incomes received by an individual in a given state, no matter what can be the state and the incomes of the others. Let $\mathbb{L}$ be the set of all probability distributions on $\mathbb{X}$ that satisfy condition 1. Since a lottery p in $\mathbb{L}$ assigns a positive probability to a unique income for individual i in every state in which i can fall with positive probability, we denote by $y^p_{\sigma i}$ the value of this unique income that is given positive probabilities by p in the state $\sigma \in \{b, g\}$. Moreover, since, for every individual and state achieved with positive probability, there is a unique income received by the individual in that state, we can think of a lottery p in $\mathbb{L}$ as being defined only on the set $\{b, g\}^n$ of combinations of individual states, and we can accordingly denote by $p(s)$ the (joint) probability of the individuals being in the configuration of states $s = (s_1, ..., s_n)$.

Every individual i is assumed to have a VNM preference ordering $\succsim_i$ on $\mathbb{L}$, with asymmetric and symmetric factors $\succ_i$ and $\sim_i$ respectively. This means that there exists a utility function $\Phi_i : (\{b, g\} \times \mathbb{R}_+)^n \to \mathbb{R}$ such that, for every lotteries p and q in $\mathbb{L}$, one has:

$$p \succsim_i q \Leftrightarrow \sum_{s \in \{b,g\}^n} p(s)\Phi_i(s_1, y^p_{s_1 1}, ..., s_n, y^p_{s_n n}) \geq \sum_{s \in \{b,g\}^n} q(s)\Phi_i(s_1, y^q_{s_1 1}, ..., s_n, y^q_{s_n n}) \tag{1}$$

We refer to the numerical representation of $\succsim_i$ provided by (1) as to the expected utility representation. We further assume that individuals are selfish and, therefore, only care about the state in which they fall and the pecuniary consequence they get in that state, and that they have the same selfish preference. In order to write formally this condition, we notice that any (joint) lottery p in $\mathbb{L}$ viewed as a joint lottery on $\{b, g\}^n$ induces an individual i's (marginal) binary lottery p_i on $\{b, g\}$ defined, for $\sigma \in \{b, g\}$, by:

$$p_i(\sigma) = \sum_{\{s \in \{b,g\}^n : s_i = \sigma\}} p(s).$$

For notational convenience, and using the fact that probabilities sum to 1, we write p_i instead of $p_i(g)$ and $1 - p_i$ instead of $p_i(b)$. With this piece of notation, the assumption that individuals have the same selfish VNM preference means that there exists a function $U : \{b,g\} \times \mathbb{R}_+ \to \mathbb{R}$ such that, for every i and every lotteries p and q in $\mathbb{L}$, one has:

$$p \succsim_i q \Leftrightarrow p_i U(g, y^p_{gi}) + (1 - p_i) U(b, y^p_{bi}) \geq q_i U(g, y^q_{gi}) + (1 - q_i) U(b, y^q_{bi}) \quad (2)$$

Also for notational convenience, we write, for every $y \in \mathbb{R}$, and $\sigma \in \{b,g\}$, $U_\sigma(y) = U(\sigma, y)$. By doing so however, we keep in mind that we always view U as a function of two variables, one of which being discrete, having $\{b,g\} \times \mathbb{R}_+$ as a domain.

Lotteries in $\mathbb{L}$ are compared by a social ranking $\succsim$ (with asymmetric and symmetric factors $\succ$ and $\sim$ respectively) that satisfies the VNM properties and respects the weak Pareto principle with respect to individual preferences. In the same fashion as in (1), the first property means that there exists a function $\Phi : (\{b,g\} \times \mathbb{R}_+)^n \to \mathbb{R}$ such that, for every lotteries p and q in $\mathbb{L}$, one has:

$$p \succsim q \Leftrightarrow \sum_{s \in \{b,g\}^n} p(s) \Phi(s_1, y^p_{s_1 1}, ..., s_n, y^p_{s_n n}) \geq \sum_{s \in \{b,g\}^n} q(s) \Phi(s_1, y^q_{s_1 1}, ..., s_n, y^q_{s_n n}) \quad (3)$$

The second property requires $\succsim$ to be such that, for two lotteries p and q in $\mathbb{L}$, if $p \succ_i q$ for all i, then $p \succ q$.

By virtue of a version of Harsanyi (1955)'s aggregation theorem due to Weymark (1993), any VNM social ordering of $\mathbb{L}$ that respects the weak Pareto principle can be written as a (positively) weighted sum of the individual's expected utility representations of their VNM preference. We formally state this fact as follows.

Proposition 1 *Let $(\succsim_1, ..., \succsim_n)$ be a profile of identical selfish VNM individual preference orderings on $\mathbb{L}$ and let $\succsim$ be a VNM social ordering of $\mathbb{L}$ that satisfies the weak Pareto principle. Then, there exists non-negative numbers $\lambda_1, ...\lambda_n$, one of which at least being strictly positive, such that, for every two lotteries, p and q in $\mathbb{L}$, one has:*

$$p \succsim q \Leftrightarrow \sum_{i=1}^{n} \lambda_i [p_i U_g(y^p_{gi}) + (1 - p_i) U_b(y^p_{bi})] \geq \sum_{i=1}^{n} \lambda_i [q_i U_g(y^q_{gi}) + (1 - q_i) U_b(y^q_{bi})] \quad (4)$$

Proof. *The result follows immediately from proposition 4 in Weymark (1993), after noticing that the condition called "independent prospects" in Weymark (1993) is satisfied herein if* $(\succsim_1, ..., \succsim_n)$ *are identical selfish VNM individual preferences, and after applying definition (1) to individual preferences.* ■

If one further assumes that the social ranking is *anonymous* in the sense of being indifferent to a permutation of the individuals' names,[2] then the λ_is of condition (4) must all be identical. Because of this, it follows from proposition 1 that any anonymous VNM social ordering $\succsim$ of $\mathbb{L}$ satisfying the weak Pareto principle with respect to a profile $\langle \succsim_i \rangle_{i=1}^n$ of identical selfish VNM preferences can be written, for lotteries p and q in $\mathbb{L}$, as:

$$p \succsim q \Leftrightarrow \sum_{i=1}^{n}[p_i U_g(y_{gi}^p) + (1-p_i) U_b(y_{bi}^p)] \geq \sum_{i=1}^{n}[q_i U_g(y_{gi}^q) + (1-q_i) U_b(y_{bi}^q)] \quad (5)$$

In this paper, we propose empirically implementable criteria that coincide with the ranking of lotteries in $\mathbb{L}$ that would be agreed upon by *all* social orderings that can be written as per (5) for a large class of individual expected utility functions. To that extent, the considered criteria shall be referred to as "robust".

Yet, it is probably worth noticing that the legitimacy of formula (5) for comparing social risks rides on the acceptance of *both* the "*ex ante*" view that ethical judgements about the goodness of risky social states should result from the aggregation of the individuals' preferences before the resolution of the underlying uncertainty *and* the requirement that the social ranking that results from the aggregation should satisfy the VNM properties. The later requirement may be seen as particularly demanding in view of the well-known criticism that has been launched against it by Diamond (1967). Indeed, if one is indifferent between giving a kidney for sure to Bob and giving a kidney for sure to Ann, then the requirement that the social ranking of risky situations satisfy the VNM properties implies that one must also be indifferent between giving the kidney for sure to either one of the two person and basing the decision of who should get the kidney on the flip a coin. Since spontaneous intuition tends to favour the flip of a coin in a situation like this, some are willing to endorse Roemer (1996)'s view that, with this example, "Diamond has presented a knockdown argument against the ethical attractiveness of" (p. 140) equation (5) as a basis for comparing alternative socially risky situations. Yet others, such as Fleurbaey (2006), are less convinced by the devastating character of Diamond's critique. After

[2] The careful writting of this anonymity condition requires one to impose multi-profile conditions on the social ordering $\succsim$. Details as to how this can be done can be found in Coulhon and Mongin (1989).

all, since in the end only one person will get the kidney anyway, why should flipping a coin be considered better than giving the kidney for sure to either one of the two ? Further discussions of the merit of the ex ante vs the ex post view point in normative analysis can be found in Gadjos and Maurin (2003).

At any rate, we base our notion of normative dominance on formula (5). Specifically, we define as follows the notion of normative dominance with respect to a class of expected utility functions.

Definition 1 (Normative dominance) *Social risk p normatively dominate social risk q for a class $\mathbb{U}$ of utility functions defined on $\{b, g\} \times \mathbb{R}_+$, denoted $p \succsim_{\mathbb{U}} q$, if for all functions U in the class, one has:*

$$\sum_{i=1}^{n}[p_i U_g(y_{gi}^p) + (1 - p_i)U_b(y_{bi}^p)] \geq \sum_{i=1}^{n}[q_i U_g(y_{gi}^q) + (1 - q_i)U_b(y_{bi}^q)]$$

In order to define formally the class of utility functions considered in this paper, we introduce the following notation which eases the statement of the properties of utility function when, as is the case here, they are not assumed to be everywhere differentiable. Specifically, if G is a function from a subset A of $\mathbb{R}$ to $\mathbb{R}$ and α is a number in A, we denote, for every strictly positive real number Δ such that $\alpha + \Delta \in A$ and by $G^{\Delta}(\alpha)$ the discrete first derivative of G evaluated at α by:

$$G^{\Delta}(\alpha) = \frac{G(\alpha + \Delta) - G(\alpha)}{\Delta} \tag{6}$$

This definition can, of course, be applied recursively for different Δ so that, for instance, $G^{\Delta_1\Delta_2}(\alpha)$, interpreted to be the second order discrete derivative of G, is defined by:

$$G^{\Delta_1\Delta_2}(\alpha) = \frac{\frac{G(\alpha+\Delta_2+\Delta_1)-G(\alpha+\Delta_2)}{\Delta_1} - [\frac{G(\alpha+\Delta_1)-G(\alpha)}{\Delta_1}]}{\Delta_2}$$

We now introduce the classes of utility functions considered in this paper.

$\mathbb{U}_1 = \{U : \{b, g\} \times \mathbb{R}_+ : U_b^{\Delta}(\alpha) \geq U_g^{\Delta}(\alpha) \geq 0$ and $U_g(\alpha) \geq U_b(\alpha)$ for all α, $\Delta \in \mathbb{R}_+$ $\}$

$\mathbb{U}_2 = \mathbb{U}_1 \cap \{U : \{b, g\} \times \mathbb{R}_+ : U_b^{\Delta_1\Delta_2}(\alpha) \leq U_g^{\Delta_1\Delta_2}(\alpha) \leq 0$ for all α, Δ_1, $\Delta_2 \in \mathbb{R}_+\}$

The class $\mathbb{U}_1$ contains all utility functions that are, in every state, *increasing with income* and that satisfy the additional requirements that, for a given income level, the utility enjoyed in the good state is no smaller than that in the bad state and the marginal utility of income is weakly larger in the bad state than in the good one. The class $\mathbb{U}_2$ contains all functions that belong to both $\mathbb{U}_1$ which satisfy the additional requirements that:

1) the marginal utility of income is decreasing, in every state, with income and

2) the decrease in the marginal utility of income is weakly more important in the bad state than in the good one.

2.2 Implementable criteria

The first implementable criterion that is introduced is the Sequential Expected Headcount Poverty (SEHP) dominance criterion. It is formally defined as follows.

Definition 2 (Sequential Expected Headcount Poverty dominance) *For every p and $q \in \mathbb{L}$, we say that p SEHP dominates q, denoted $p \succsim_{SEHP} q$ if, for every poverty line t, one has:*

$$\sum_{\{i:y_{bi}^p \leq t\}} (1-p_i) \leq \sum_{\{i:y_{bi}^q \leq t\}} (1-q_i) \tag{7}$$

and:

$$\sum_{\{i:y_{bi}^p \leq t\}} (1-p_i) + \sum_{\{i:y_{gi}^p \leq t\}} p_i \leq \sum_{\{i:y_{bi}^q \leq t\}} (1-q_i) + \sum_{\{i:y_{gi}^q \leq t\}} q_i \tag{8}$$

In words, socially risky situation p dominates socially risky situation q for the SEHP criterion if, no matter how one defines the poverty line, the *expected numbers of individuals who will be in the bad state and poor for this poverty line* is no greater in p than in q (condition (7)) and (condition (8), if the expected number of individuals who are poor in either state is also smaller, weakly, in p than in q. The importance of the requirement, expressed in condition (7), that headcount poverty be calculated, for those who are in the bad state, *in expectation only* is worth noticing. If one uses a poverty line sufficiently large, it is easy to see that condition (7) requires the expected number of individuals who will be in the bad state to be, weakly, smaller in the dominating distribution than in the dominated one. But it requires more than that. To illustrate, consider the following example of two non-pecuniary risks in a two-individual society.

Example 1 *Let p and q be defined by*:

$$\begin{aligned}
p_1 &= 1/3,\ y_{g1}^p = 2 = y_{b1}^p \\
p_2 &= 11/12,\ y_{g2}^p = 3 = y_{b2}^p \\
q_1 &= 1/2,\ y_{g1}^q = 1 = y_{b1}^q \\
q_2 &= 3/4,\ y_{g2}^q = 3 = y_{g2}^q
\end{aligned}$$

The expected headcount poverty in the bad state and in either states are given in the following table, for different choices of poverty lines:

line	E. poverty in bad state in p	E. poverty in either state in p	E. poverty in bad state in q	E. poverty in either state in q
below 1	0	0	0	0
between 1 and 2	0	0	1/2	1
between 2 and 3	2/3	1	1/2	1
above 3	5/4	2	5/4	2

As can be seen, there are no poverty lines for which the expected number of poor in either state is larger in p than in q and there are some poverty line (between 1 and 2) for which the expected number of poor in either state is larger in q than in p. Hence on the sole basis of headcount poverty dominance (condition (8)), p would dominate q. Notice also that the expected number of individuals who will be in the bad state is $3/4$ *in both q and p. Yet, the situation p does not dominate situation q because there are some poverty lines (between 2 and 3) for which the expected number of individuals who are both poor (i.e. strictly below the line) and in the bad state is* $2/3$ *in p while it is only* $1/2$ *in q.*

The previous example depicted a situation where a clear verdict on the comparison of two socially risky situations could have been obtained on the basis of headcount poverty dominance irrespective of the state (condition (8)) but was "vetoed" by the non-satisfaction of condition (7). The following example shows, somewhat conversely, a case where condition (8) is satisfied at equality for all poverty lines and where, for this reason, it can not be used to compare the risks but where condition (7) enables such a comparison to be made.

Example 2 *Let p and q be defined by*:

$$\begin{aligned} p_1 &= 1/3,\ y^p_{b1} = 3 = y^p_{g1} \\ p_2 &= 2/3,\ y^p_{b2} = 1 = y^p_{g2} \\ q_1 &= 1/3,\ y^q_{b1} = 1 = y^q_{g1} \\ q_2 &= 2/3,\ y^q_{b2} = 3 = y^q_{b2} \end{aligned}$$

As can be seen, the two incomes received (irrespective of state) in both situations are the same so that headcount poverty irrespective of the state is the same. Notice also that the expected number of individuals in the bad state (or for that matter in the good one) is 1 in the two situations so that the criterion of the expected number of individuals in the bad state can not be used to rank the two risky situations. Yet, for every poverty line, the expected number of individuals in the bad state who are poor is never greater in p than in q and, for some poverty lines such as 2*, the expected number of individuals who are poor in the bad state is 2/3 in q and 1/3 in p. Hence p dominates q for the SEHP criterion.*

The second empirically implementable dominance criterion is the analogue of SEHP dominance one, but using poverty gap, instead of headcount poverty, as a measure of poverty. We call it, for this reason, the Sequential Expected Poverty Gap (SEPG) criterion. In order to define this criterion, we denote, for any income a and poverty line t, the *poverty gap* $P(t,a)$ of income a for the poverty line t by:

$$P(t,a) = \max(t-a,0] \tag{9}$$

This poverty gap is, as usual, interpreted as the minimal amount of income that is required to get a person with an income of a out of poverty when poverty is defined as falling short of having an income of t. With this notation, we define sequential expected poverty gap as follows.

Definition 3 (Sequential Expected Poverty Gap dominance) *For every p and $q \in \mathbb{L}$, we say that p SEPG dominates q, denoted $p \succsim_{SEPG} q$ if, for every poverty line t, one has:*

$$\sum_{i=1}^{n}(1-p_i)P(t,y_{bi}^{p}) \leq \sum_{i=1}^{n}(1-q_i)P(t,y_{bi}^{q}) \tag{10}$$

$$\sum_{i=1}^{n}[(1-p_i)P(t,y_{bi}^{p}) + p_iP(t,y_{gi}^{p}) \leq \sum_{i=1}^{n}[(1-q_i)P(t,y_{bi}^{q}) + q_iP(t,y_{gi}^{q}) \tag{11}$$

and

$$\sum_{i=1}^{n} p_i \geq \sum_{i=1}^{n} q_i \tag{12}$$

In words, socially risky situation p dominates situation q for the sequential poverty gap dominance criterion if the *expected amount of money* that is required to eliminate poverty *in the bad state* is lower in p than in q for every conceivable poverty line (condition (10)) and if the expected amount of money that is required to eliminate poverty *in either state* is also weakly smaller in p than in q for every poverty line. The remarks made for the SEHP criterion with respect to the independence of the conditions (7) and (8) are obviously valid for the SEPG dominance criterion and conditions (10) and (11). As is well-known from standard one-dimensional dominance analysis (see e.g. Atkinson (1970) or Dasgupta *et al.* (1973)), the criterion of income poverty gap dominance is equivalent to (generalized) Lorenz dominance. Yet, it not difficult to see that expected Lorenz dominance in the bad state is neither necessary nor sufficient for having expected poverty gap dominance in the bad state.

However there is a relation between expected poverty gap dominance in either state and generalized *expected Lorenz dominance* if some extra assumptions can be made on the risky situations. Specifically, assume, plausibly, that all socially risky situations are such that, for all individuals, the income received in the bad state is never greater than that received in the good state. In such a case, it turns out that requiring condition (8) to hold, for two distributions, for all poverty line implies that the distribution of *expected incomes* in the dominating distribution Lorenz dominates that in the dominated income. In order to define this expected Lorenz dominance criterion, consider any socially risky situation p and define the vector $y^{Ep}$of expected incomes in p by:

$$y_i^{Ep} = p_i y_{gi}^p + (1 - p_i) y_{bi}^p$$

for every $i = 1, ..., n$. Denote also, for any vector y in $\mathbb{R}^n$ by $y_{(.)}$ the permutation of y such that $y_{(i)} \leq y_{(i+1)}$ for every $i = 1, ..., n-1$. With this notation, we define the Expected Lorenz (EL) domination ranking of risky situations as follows.

Definition 4 (Expected Lorenz dominance) *Socially risky situation p Expected Lorenz dominates situation q, denoted $p \succsim_{EL} q$ if, for all $m \in \{1, ..., n\}$ one has*

$$\sum_{i=1}^{m} y_{(i)}^{Ep} \geq \sum_{i=1}^{m} y_{(i)}^{Eq}$$

In words, p EL dominates q if the vector of expected incomes associated to p Lorenz dominates, in the general sense, that associated to q. We now notice that, if p and q are two risky situations in which the income received by any individual is weakly lower in the bad state than in the good one, then the requirement for inequality (8) to hold between p and q for all poverty line implies that p EL dominates q but that the converse implication does not hold even under the assumption $p$and q satisfy inequality (7) for all poverty lines.

Proposition 2 *Assume that p and q are two risky situations in $\mathbb{L}$ such that, for every i, $y_{gi}^p \geq y_{bi}^p$ and $y_{gi}^q \geq y_{bi}^q$, and such that the inequality (8) holds for all poverty lines t. Then $p \succsim_{EL} q$. However, for any such p and q for which (7) hold for all poverty lines, $p \succsim_{EL} q$ does not imply that (8) holds for all poverty lines t.*

Proof. *Let p and q be two risky situations in $\mathbb{L}$ such that, for every i, $y_{gi}^p \geq y_{bi}^p$ and $y_{gi}^q \geq y_{bi}^q$, and assume that the inequality:*

$$\sum_{i=1}^{n} [(1 - p_i) P(t, y_{bi}^p) + p_i P(t, y_{gi}^p) \leq \sum_{i=1}^{n} [(1 - q_i) P(t, y_{bi}^q) + q_i P(t, y_{gi}^q)$$

holds for all poverty lines t. Using (9), this inequality writes:

$$\sum_{i=1}^{n}[(1-p_i)\max(t-y_{bi}^{p},0)+p_i\max(t-y_{gi}^{p},0)] \leq$$

$$\sum_{i=1}^{n}[(1-p_i)\max(t-y_{bi}^{p},0)+p_i\max(t-y_{gi}^{p},0)]$$

which is equivalent to:

$$\sum_{i:y_{bi}^{p}\leq t}(1-p_i)(t-y_{bi}^{p})+\sum_{i:y_{gi}^{p}\leq t}p_i(t-y_{gi}^{p}) \leq$$

$$\sum_{i:y_{bi}^{q}\leq t}(1-p_i)(t-y_{bi}^{q})+\sum_{i:y_{gi}^{q}\leq t}p_i(t-y_{gi}^{q}) \qquad (13)$$

Now, since $y_{gi}^{p} \geq y_{bi}^{p}$ and $y_{gi}^{q} \geq y_{bi}^{q}$, one has, for every t:

$$\#\{i : y_{gi}^{p} \leq t\} \subset \#\{i : y_{bi}^{p} \leq t\}$$

and

$$\#\{i : y_{gi}^{p} \leq t\} \subset \#\{i : y_{bi}^{p} \leq t\}$$

Since the inequality (8) holds for all t, it holds in particular for t such that:

$$\#\{i : y_{gi}^{p} \leq t\} = \#\{i : y_{bi}^{p} \leq t\} \qquad (14)$$

and

$$\#\{i : y_{gi}^{q} \leq t\} = \#\{i : y_{bi}^{q} \leq t\}. \qquad (15)$$

Indeed, for any individual i for which either $y_{gi}^{p} > t \geq y_{bi}^{p}$ or $y_{gi}^{q} > t \geq y_{bi}^{q}$, one can raise t by Δ in such a way that $t+\Delta \geq \max\ (y_{gi}^{p},\ y_{gi}^{q}) \geq \max\ (y_{bi}^{p},\ y_{bi}^{q})$. Choose therefore a t such that (14) and (15) hold and write (13) as:

$$\sum_{i:y_{gi}^{q}\leq t}[(1-q_i)(t-y_{bi}^{q})+q_i(t-y_{gi}^{q}) \geq$$

$$\sum_{i:y_{gi}^{p}\leq t}[(1-p_i)(t-y_{bi}^{p})+p_i(t-y_{gi}^{p}) \geq$$

$$\sum_{i:y_{gi}^{q}\leq t}[(1-p_i)(t-y_{bi}^{p})+p_i(t-y_{gi}^{p}) \qquad (16)$$

because of the max condition in inequality (8). Inequality (16) can also be written as:

$$\#\{i : y_{gi}^{q} \leq t\}t - \sum_{i:y_{gi}^{q}\leq t}[(1-p_i)y_{bi}^{p}+p_i y_{gi}^{p}] \leq$$

$$\#\{i : y^q_{gi} \leq t\}t - \sum_{i:y^q_{gi}\leq t} [(1-q_i)y^q_{bi} + q_i y^q_{gi}]$$

$$\iff$$

$$-\sum_{i:y^q_{gi}\leq t} y_i^{Ep} \leq -\sum_{i:y^q_{gi}\leq t} y_i^{Eq}$$

$$\Leftrightarrow$$

$$\sum_{i:y^q_{gi}\leq t} y_i^{Ep} \geq \sum_{i:y^q_{gi}\leq t} y_i^{Eq}$$

which, if required for any t, is clearly equivalent to EL dominance of q by p. To show that El dominance of q by p does not imply that (8) holds for all poverty lines even when (7) does, we consider the following 2-individual example of socially risky situations:

$$\begin{aligned} p_1 &= 1/3,\ y^p_{g1} = 3 = y^p_{b1} \\ p_2 &= 2/3,\ y^p_{g2} = 1+\varepsilon,\ y^p_{b2} = 1 \\ q_1 &= 1/3,\ y^q_{g1} = 1 = y^q_{b1} \\ q_2 &= 2/3,\ y^q_{g2} = 3+\varepsilon,\ y^q_{b2} = 3 \end{aligned}$$

for some $\varepsilon > 0$. Clearly

$$y^{Ep}_{(1)} = 1 + \frac{2}{3}\varepsilon > y^{Eq}_{(1)} = 1$$

and

$$y^{Ep}_{(1)} + y^{Ep}_{(2)} = 1 + \frac{2}{3}\varepsilon + 3 = y^{Eq}_{(1)} = 1 + 3 + \frac{2}{3}\varepsilon$$

so that $p \succ_{EL} q$. Moreover, if we consider the following table:

line t	*E. poverty gap in bad state in p*	*E. poverty gap in either state in p*	*E. poverty gap in bad state in q*	*E. poverty in either state in q*
below 1	*0*	*0*	*0*	*0*
between 1 and 1+ε	*1/3(t-1)*	*1/3(t-1)*	*2/3(t-1)*	*t-1*
between 1+ε and 3	*1/3(t-1)*	*t-1-2/3ε*	*2/3(t-1)*	*t-1*
between 3 and 3+ε	*t-7/3*	*2t-4-2/3ε*	*t-5/3*	*t4/3-4*
above 3	*t-7/3*	*2t-4-2/3ε*	*t-5/3*	*2t-4-2/3ε*

we notice that expected poverty gap in the bad state is lower in p than in q for all poverty lines but that, for poverty lines between 3 and $3+\varepsilon$, the expected poverty gap in either state is $2t-4-2\varepsilon/3$ in p while it is $4t/3-4$ in q and:

$$\begin{aligned} 2t - 4 - 2\varepsilon/3 &> 4t/3 - 4 \\ &\Leftrightarrow \\ 2t/3 &> 2\varepsilon/3 \end{aligned}$$

if ε is sufficiently small.

■

Notice that the criterion of SEPG dominance entails also the requirement, expressed in condition (12), that the expected number of people that will be in the good state be no smaller in the dominating distribution than in the dominated one. As shall be seen below, after the proof of theorem 2, this requirement may be dispensed with if one is willing to assume that there exists sufficiently high income level for which the individual is indifferent between the bad and the good state.

We now provide, in the next subsection, normative foundation for each of these criteria by showing that it coincides with the ranking of socially risky situations that commands unanimity over all anonymous Paretian and VNM social rankings who assume that individual preferences can be represented by expected utility functions in one the two classes defined above.

2.3 Equivalence results

The first theorem establishes an equivalence between normative dominance for the class $\mathbb{U}_1$ and SEHP dominance.

Theorem 1 *Let p and q be two socially risky situations in $\widehat{\mathbb{L}}$. Then $p \succsim_{\mathbb{U}_1} q$ if and only if $p \succsim_{SEHP} q$.*

Proof. *For the first implication, assume that $p \succsim_{\mathbb{U}_1} q$. Then, the inequality:*

$$\sum_{i=1}^{n}[p_i U_g(y_{gi}^p) + (1-p_i)U_b(y_{bi}^p)] \geq \sum_{i=1}^{n}[q_i U_g(y_{gi}^q) + (1-q_i)U_b(y_{bi}^q)] \tag{17}$$

holds for all functions $U : \{b, g\} \times \mathbb{R}_+ \to \mathbb{R}$ in $\mathbb{U}_1$. Consider, for any $t \in \mathbb{R}_+$, the function $V^t : \{b, g\} \times \mathbb{R}_+ \to \mathbb{R}$ defined, for any $a \in \mathbb{R}_+$, by:

$$\begin{aligned} V_b^t(a) &= -1 \text{ if } a \leq t \\ &= 0 \text{ otherwise} \end{aligned}$$

and:

$$V_g^t(a) = 0.$$

It can easily be checked that V^t belongs to $\mathbb{U}_1$. Hence, inequality (17) holds for V^t so that we have:

$$\begin{aligned} \sum_{i=1}^{n}[p_i V_g^t(y_{gi}^p) + (1-p_i)V_b^t(y_{gi}^p)] &\geq \sum_{i=1}^{n}[q_i V_g^t(y_{gi}^q) + (1-q_i)V_b^t(y_{bi}^q)] \\ &\Leftrightarrow \\ \sum_{\{i:y_{bi}^p \leq t\}} -(1-p_i) &\geq \sum_{\{i:y_{bi}^q \leq t\}} -(1-q_i) \\ &\Leftrightarrow \\ \sum_{\{i:y_{bi}^p \leq t\}}^{n} (1-p_i) &\leq \sum_{\{i:y_{bi}^q \leq t\}}^{n} (1-q_i) \end{aligned}$$

as required by condition (7) of the definition of SEHP dominance. For verifying the necessity of condition (8), we can write inequality (17) for the function $\widehat{V}^t : \{b, g\} \times \mathbb{R}_+ \to \mathbb{R}$ *defined, for any* a *and* $t \in \mathbb{R}_+$, *by:*

$$\begin{aligned} \widehat{V}_b^t(a) &= \widehat{V}_g^t(a) = -1 \text{ if } a \leq t \\ &= 0 \text{ otherwise} \end{aligned}$$

after noticing, straightforwardly, that this function also belongs to $\mathbb{U}_1$.
For the other implication, we need to show that the fact of having (7) and (8) satisfied for every non-negative real number t *is sufficient for the inequality (17) to hold for all utility functions in* $\mathbb{U}_1'$. *For this sake, let, for any* $\pi \in [0, 1]$ *and* $(a, b) \in \mathbb{R}_+^2$, $V(\pi, a, b)$ *be defined by:*

$$V(\pi, a, b) = \pi U_g(a) + (1 - \pi) U_b(b) \tag{18}$$

and write (17) as:

$$\sum_{g=1}^{\overline{g}} \sum_{h=1}^{\overline{h}} \sum_{j=1}^{\overline{j}} \Delta f(\pi_g, a_h, b_j) V(\pi_g, a_h, b_j) \geq 0 \tag{19}$$

where:
1) $g = 1, ..., \overline{g}$, $h = 1, ..., \overline{h}$ *and* $j = 1, ..., \overline{j}$ *are the indices of the bounds* π_g, a_h *and* b_j *of a partition of the interval* $[0, 1]$ *(for* g*) and* $[0, \overline{y}]$ *(for* h *and* j*, for some suitably large* $\overline{y} \in \mathbb{R}_+$*) such that, for all* $i = 1, ..., n$, *there exists* g *and* $g' \in \{1, ..., \overline{g}\}$ *and* h, h', j *and* $j' \in \{1, ..., \overline{h}\}$ *for which:*

$$\begin{aligned} p_i &= \pi_g,\ q_i = \pi_{g'}, \\ y_{gi}^p &= a_h \text{ and } y_{gi}^q = a_{h'} \\ y_{bi}^p &= b_j \text{ and } y_{bi}^q = b_{j'} \end{aligned}$$

Moreover the bounds of the intervals' partitions satisfy:

$$\begin{aligned} \pi_1 &= a_1 = b_1 = 0, \\ \overline{h} &= \overline{j} \\ a_h &= b_j \text{ for } h, j \in \{1, ..., \overline{h}\} \text{ such that } h = j. \\ \pi_{\overline{g}} &= 1 \text{ and} \\ a_{\overline{h}} &= b_{\overline{j}} = \overline{y} \end{aligned}$$

and :
2) $\Delta f(\pi_g, a_h, b_j) = \#\{i : p_i = \pi_g,\ y_{gi}^p = a_h \wedge y_{bi}^p = b_j\} - \#\{i : q_i = \pi_g,\ y_{gi}^q = a_h \wedge y_{bi}^q = b_j\}$ *(we of course allow for the possibility that the cardinality of either of the two sets that enters in that difference be zero). We now proceed by decomposing (19) using Abel's identity (see for instance (Fishburn*

and Vickson (1978); eq 2.49)). Doing first the decomposition with respect to the inner (j− indexed) sum operator yields:

$$\sum_{g=1}^{\overline{g}}\sum_{h=1}^{\overline{h}}[\sum_{j=1}^{\overline{j}}\Delta f(\pi_g,a_h,b_j)V(\pi_g,a_h,\overline{y})-\sum_{m=1}^{\overline{j}-1}\Delta_m(\sum_{j=1}^{m}\Delta f(\pi_g,a_h,b_j))\frac{\Delta V_b(\pi_g,a_h,b_m)}{\Delta_m}]\geq 0 \tag{20}$$

where, for every $m = 1,...,\overline{j}-1$, $\Delta V_b(\pi_g,a_h,b_m) = V(\pi_g,a_h,b_{m+1}) - V(\pi_g,a_h,b_m)$ *and* $\Delta_m = b_{m+1}-b_m \geq 0$. *Applying Abel decomposition formula to each term of this expression with respect this time to the h-indexed sum operator yields:*

$$\sum_{g=1}^{\overline{g}}[\sum_{h=1}^{\overline{h}}\sum_{j=1}^{\overline{j}}\Delta f(\pi_g,a_h,b_j)V(\pi_g,\overline{y},\overline{y})-\sum_{l=1}^{\overline{h}-1}\Delta_l(\sum_{h=1}^{l}\sum_{j=1}^{\overline{j}}\Delta f(\pi_g,a_h,b_j))\frac{\Delta V_a(\pi_g,a_l,\overline{y})}{\Delta_l}$$

$$-\sum_{h=1}^{\overline{h}}\sum_{m=1}^{\overline{j}-1}\Delta_m\sum_{j=1}^{m}(\Delta f(\pi_g,a_h,b_j))\frac{\Delta V_b(\pi_g,\overline{y},b_m)}{\Delta_m}$$

$$+\sum_{l=1}^{\overline{h}-1}\sum_{m=1}^{\overline{j}-1}\Delta_m(\sum_{h=1}^{l}\sum_{j=1}^{m}\Delta f(\pi_g,a_h,b_j))(\frac{\Delta V_b(\pi_j,a_{l+1},b_m)}{\Delta_m}-\frac{\Delta V_b(\pi_j,a_l,b_m)}{\Delta_m})]\geq 0 \tag{21}$$

where for every $l = 1,...,\overline{h}-1$, $\Delta V_a(\pi_g,a_l,b_m) = V(\pi_g,a_{l+1},b_m)-V(\pi_g,a_l,b_m)$ *and* $\Delta_l = a_{l+1}-a_l \geq 0$. *Finally, after noticing the obvious fact that:*

$$\sum_{g=1}^{\overline{g}}\sum_{h=1}^{\overline{h}}\sum_{j=1}^{\overline{j}}\Delta f(\pi_g,a_h,b_j) = n-n = 0$$

one can apply Abel decomposition formula to the g-indexed sum operator and obtain, using (18) and the definition (6) of a discrete derivative:

$$-\sum_{g=1}^{\overline{g}-1}[\Delta F(\pi_g,\overline{y},\overline{y})[\pi_{g+1}-\pi_g][U_g(\overline{y})-U_b(\overline{y})]$$

$$-\sum_{l=1}^{\overline{h}-1}\Delta_l[\Delta F(1,a_l,\overline{y})-\sum_{g=1}^{\overline{g}-1}\Delta F(\pi_g,a_l,\overline{y})[\pi_{g+1}-\pi_g]]U_g^{\Delta_l}(a_l)$$

$$-\sum_{m=1}^{\overline{j}-1}\Delta_m[\sum_{g=1}^{\overline{g}-1}\Delta F(\pi_g,\overline{y},b_m)[\pi_{g+1}-\pi_g]]U_b^{\Delta_m}(b_m)\geq 0 \tag{22}$$

where, for every g, h and j, $\Delta F(\pi_g,a_h,b_j)$ *is defined by:*

$$\Delta F(\pi_g,a_h,b_j) = \sum_{e=1}^{g}\sum_{l=1}^{h}\sum_{m=1}^{j}\Delta f(\pi_e,a_l,b_m)$$

Now, since $\Delta_l = \Delta_m$ *and* $a_l = b_m$ *for every* $l,\ m \in \{1, ..., \overline{j} - 1\}$ *such that* $l = m$*, and since:*

$$U_b^{\Delta_m}(a_l) = U_g^{\Delta_l}(a_l) + U_b^{\Delta_m}(a_l) - U_g^{\Delta_l}(a_l)$$

we can write (22) as:

$$-\sum_{g=1}^{\overline{g}-1}[\Delta F(\pi_g, \overline{y}, \overline{y})[\pi_{g+1} - \pi_g][U_g(\overline{y}) - U_b(\overline{y})]$$

$$-\sum_{l=1}^{\overline{h}-1}\Delta_l[\Delta F(1, a_l, \overline{y}) - \sum_{g=1}^{\overline{g}-1}[\pi_{g+1} - \pi_g][\Delta F(\pi_g, a_l, \overline{y}) - \sum_{g=1}^{\overline{g}-1}\Delta F(\pi_g, \overline{y}, a_l)]]U_g^{\Delta_l}(a_l)$$

$$-\sum_{l=1}^{\overline{h}-1}\Delta_l[\sum_{g=1}^{\overline{g}-1}\Delta F(\pi_g, \overline{y}, b_m)[\pi_{g+1} - \pi_g][U_b^{\Delta_l}(a_l) - U_g^{\Delta_l}(a_l)] \geq 0 \tag{23}$$

Define now, for every g*,* h *and* j*,* $\Delta F^{ab}(\pi_g, a_h, b_j)$ *by:*

$$\Delta F^{ab}(\pi_g, a_h, b_j) = \sum_{l=1}^{h}\sum_{m=1}^{j}\Delta f(\pi_e, a_l, b_m) \tag{24}$$

$= \#\{i : p_i = \pi_g, y^p_{gi} \leq a_h \ \wedge\ y^p_{bi} \leq b_j\} - \#\{i : q_i = \pi_g, y^q_{bi} \leq a_h \ \wedge\ y^q_{bi} \leq b_j\}$

With this definition, it can be noticed that, for every h *and* j*, one has:*

$$\begin{aligned}
&\sum_{g=1}^{\overline{g}-1}[\Delta F(\pi_g, a_h, b_j)[\pi_{g+1} - \pi_g] \\
= \ &(\pi_2 - \pi_1)\Delta F^{ab}(\pi_1, a_h, b_j) + \\
&(\pi_3 - \pi_2)[\Delta F^{ab}(\pi_1, a_h, b_j) + \Delta F^{ab}(\pi_2, a_h, b_j)] + \\
&... \\
&(1 - \pi_{\overline{g}-1})[\Delta F^{ab}(\pi_1, a_h, b_j) + ... + \Delta F^{ab}(\pi_{\overline{g}-1}, a_h, b_j)] \\
= \ &\Delta F(\pi_{\overline{g}-1}, a_h, b_j) - \sum_{g=1}^{\overline{g}-1}\pi_g \Delta F^{ab}(\pi_g, a_h, b_j) \\
= \ &\Delta F(1, a_h, b_j) - \sum_{g=1}^{\overline{g}}\pi_g \Delta F^{ab}(\pi_g, a_h, b_j) \\
= \ &\sum_{g=1}^{\overline{g}}\Delta F^{ab}(\pi_g, a_h, b_j) - \sum_{g=1}^{\overline{g}}\pi_g \Delta F^{ab}(\pi_g, a_h, b_j) \\
= \ &\sum_{g=1}^{\overline{g}}(1 - \pi_g)\Delta F^{ab}(\pi_g, a_h, b_j)
\end{aligned} \tag{25}$$

Using (25) and (24), one can write (23) as:

$$\sum_{i=1}^{n}(p_i - q_i)[U_g(\overline{y}) - U_b(\overline{y})]$$

$$-\sum_{l=1}^{\overline{h}-1}\Delta_l[\sum_{i:y_{gi}^p \le a_l} p_i + \sum_{i:y_{bi}^p \le a_l}(1-p_i) - [\sum_{i:y_{gi}^q \le a_l} q_i + \sum_{i:y_{bi}^q \le a_l}(1-q_i)]]U_g^{\Delta_l}(a_l)$$

$$-\sum_{l=1}^{\overline{h}-1}\Delta_l(\sum_{i:y_{bi}^p \le a_l}(1-p_i) - \sum_{i:y_{bi}^q \le a_l}(1-q_i)][U_b^{\Delta_l}(a_l) - U_g^{\Delta_l}(a_l)] \ge 0 \quad (26)$$

Clearly, in order for inequality (26) to hold for all utility functions U in $\mathbb{U}_1$, it is sufficient to have, for every a_l:

$$\sum_{i:y_{gi}^p \le a_l} p_i + \sum_{i:y_{bi}^p \le a_l}(1-p_i) - [\sum_{i:y_{gi}^q \le a_l} q_i + \sum_{i:y_{bi}^q \le a_l}(1-q_i)] \le 0$$

as required by condition (8) and:

$$\sum_{i:y_{bi}^p \le a_l}(1-p_i) - \sum_{i:y_{bi}^q \le a_l}(1-q_i) \le 0$$

which is condition (7), which in turns implies, for a_l sufficiently large:

$$\sum_{i=1}^{n}(p_i - q_i) \ge 0.$$

Hence conditions (7) and (8) are sufficient for the inequality (26) to be valid for every U in $\mathbb{U}_1$. ■

The next theorem establishes the equivalence between normative dominance over the class $\mathbb{U}_2$ and sequential expected poverty gap dominance.

Theorem 2 *Let p and q be two socially risky situations in $\mathbb{L}$. Then $p \succsim_{\mathbb{U}_2} q$ if and only if $p \succsim_{SEPG} q$.*

Proof. *As for the proof of the previous theorem, assume first that $p \succsim_{\mathbb{U}_2} q$ and, accordingly, that inequality (17) holds for all utility functions $U : \{b, g\} \times \mathbb{R}_+ \to \mathbb{R}$ in $\mathbb{U}_2$. Consider, for any $t \in \mathbb{R}_+$, the function $\widetilde{V}^t$: $\{b, g\} \times \mathbb{R}_+ \to \mathbb{R}$ defined, for any $a \in \mathbb{R}_+$, by:*

$$\widetilde{V}_b^t(a) = \min(a - t, 0)$$

and:

$$\widetilde{V}_g^t(a) = 0$$

The function of one variable $\widetilde{V}_b^t$ is the "angle" function used in the classical proof of the Hardy-Littlewood-Polya theorem made by Berge (1959). The reader can verify that $\widetilde{V}^t$ belongs to $\mathbb{U}_2$ (in particular $\widetilde{V}_b^t$ is more concave than $\widetilde{V}_g^t$ and has a larger first discrete derivative than $\widetilde{V}_g^t$) and, therefore, that the inequality:

$$\sum_{i=1}^{n}[p_i\widetilde{V}_g^t(y_{gi}^p)+(1-p_i)\widetilde{V}_b^t(y_{bi}^p)] \geq \sum_{i=1}^{n}[q_i\widetilde{V}_g^t(y_{gi}^q)+(1-q_i)\widetilde{V}_b^t(y_{bi}^q)] \quad (27)$$

holds. Using the definition of $\widetilde{V}^t$, inequality (27) writes:

$$\begin{aligned} \sum_{i=1}^{n}(1-p_i)\min(y_{bi}^p-t,0) &\geq \sum_{i=1}^{n}(1-q_i)\min(y_{bi}^q-t,0) \\ &\Leftrightarrow \\ \sum_{i=1}^{n}(1-p_i)\max(t-y_{bi}^p,0) &\leq \sum_{i=1}^{n}(1-q_i)\max(t-y_{bi}^q,0) \end{aligned}$$

as required by condition (10) of the definition of SEPG dominance. Condition (11) of this definition can be obtained in the same fashion by writing down inequality (17) for the function $\overline{V}^t : \{b,g\}\times\mathbb{R}_+ \to \mathbb{R}$ defined, for any a and $t\in\mathbb{R}_+$, by:

$$\overline{V}_b^t(a)=\overline{V}_g^t(a)=\min(a-t,0)$$

after noticing, straightforwardly, that this "angle" function also belongs to $\mathbb{U}_2$. Finally, condition (12) of SEPG dominance is derived by considering, as a function in $\mathbb{U}_2$, the trivial function $V : \{b,g\}\times\mathbb{R}_+ \to \mathbb{R}$ defined by:

$$\begin{aligned} V_b(a) &= 0 \text{ and} \\ V_g(a) &= 1 \text{ for all } a\in\mathbb{R}_+. \end{aligned}$$

For the other implication, we proceed just as in the proof of theorem 1 by writing inequality (17) in the form of (19) and by doing the Abel decomposition of (19) until we reach condition (23). If we then proceed one step further and Abel decompose each term of (23), we obtain:

$$\sum_{k=1}^{\overline{g}-2}(\sum_{g=1}^{k}(\pi_{g+1}-\pi_g))[\Delta F(\pi_{k+1},\overline{y},\overline{y})-\Delta F(\pi_k,\overline{y},\overline{y})][U_g(\overline{y})-U_b(\overline{y}]$$

$$-\sum_{h=1}^{\overline{h}-1}\Delta_h[\sum_{g=1}^{\overline{g}}\pi_g\Delta F^{ab}(\pi_g,a_h,\overline{y})+\sum_{g=1}^{\overline{g}}(1-\pi_g)\Delta F^{ab}(\pi_g,\overline{y},a_h)]U_g^{\Delta_l}(a_{\overline{h}-1})$$
$$+\sum_{l=1}^{\overline{h}-2}\Delta_l(\sum_{h=1}^{l}\Delta_h[\sum_{g=1}^{\overline{g}}\pi_g\Delta F^{ab}(\pi_g,a_h,\overline{y})+\sum_{g=1}^{\overline{g}}(1-\pi_g)\Delta F^{ab}(\pi_g,\overline{y},a_h)])U_g^{\Delta_{l+1}\Delta_l}(a_l)$$

$$-\sum_{j=1}^{\overline{g}}(1-\pi_j)\sum_{h=1}^{\overline{h}-1}\Delta_h[\sum_{g=1}^{\overline{g}-1}\Delta F^{ab}(\pi_g,\overline{y},b_h)[U_b^{\Delta_l}(a_{\overline{h}-1})-U_g^{\Delta_l}(a_{\overline{h}-1})]$$

$$+\sum_{l=1}^{\overline{h}-2}\Delta_l[\sum_{j=1}^{\overline{g}}(1-\pi_j)\sum_{h=1}^{l}\Delta_h\Delta F^{ab}(\pi_g,\overline{y},b_h)][U_b^{\Delta_{l+1}\Delta_l}(a_l)-U_g^{\Delta_{l+1}\Delta_l}(a_l)]$$

$$\geq 0 \tag{28}$$

after noticing that:

$$\begin{aligned}\sum_{g=1}^{\overline{g}-1}(\pi_{g+1}-\pi_g))[\Delta F(1,\overline{y},\overline{y})][U_g(\overline{y})-U_b(\overline{y}] &= \sum_{g=1}^{\overline{g}}(\pi_{j+1}-\pi_j)(n-n)[U_g(\overline{y})-U_b(\overline{y}]\\ &= 0\end{aligned}$$

Define now, for every g, l *and* m, $\Delta F^b(\pi_j,a_l,b_m)$ *and* $\Delta F^a(\pi_j,a_l,b_m)$ *:*

$$\Delta F^b(\pi_g,a_l,b_m)=\sum_{j=1}^{m}\Delta f(\pi_g,a_l,b_j)$$

and

$$\Delta F^a(\pi_g,a_l,b_m)=\sum_{h=1}^{l}\Delta f(\pi_g,a_h,b_m)$$

With these definitions, we observe that:

$$\begin{aligned}&\sum_{h=1}^{l}\Delta_h\Delta F^{ab}(\pi_g,a_h,\overline{y})\\ =\ &(a_2-a_1)\Delta F^b(\pi_g,a_1,\overline{y})+\\ &(a_3-a_2)[\Delta F^b(\pi_g,a_1,\overline{y})+\Delta F^b(\pi_g,a_2,\overline{y})]+\\ &(a_4-a_3)[\Delta F^b(\pi_g,a_1,\overline{y})+\Delta F^b(\pi_g,a_2,\overline{y})+\Delta F^b(\pi_g,a_3,\overline{y})]+\\ &...\\ &(a_l-a_{l-1})[\Delta F^b(\pi_g,a_1,\overline{y})+...+\Delta F^b(\pi_g,a_{l-1},\overline{y})]\\ =\ &a_l\Delta F^{ab}(\pi_j,a_{l-1},\overline{y})-\sum_{h=1}^{l-1}a_h\Delta F^b(\pi_j,a_h,\overline{y})\end{aligned}$$

$$=a_l[\#\{i:p_i=\pi_g\ \wedge\ y_{gi}^p\leq a_{l-1}\}-\#\{i:q_i=\pi_g\ \wedge\ y_{gi}^q\leq a_{l-1}\}]$$

$$-\sum_{h=1}^{l-1}a_h[\#\{i:p_i=\pi_g\ \wedge\ y_{gi}^p=a_h\}-\#\{i:q_i=\pi_g\ \wedge\ y_{gi}^q=a_h\}]$$

$$= \sum_{\{i:p_i=\pi_g \ \& y^p_{gi}\leq a_{l-1}\}} (a_l - y^p_{gi}) - \sum_{\{i:q_i=\pi_g \ \& y^p_{gi}\leq a_{l-1}\}} (a_l - y^q_{gi})$$

$$= \sum_{\{i:p_i=\pi_g \ \& \ y^p_{gi}\leq a_l\}} (a_l - y^p_{gi}) - \sum_{\{i:q_i=\pi_g \ \& \ y^q_{gi}\leq a_l\}} (a_l - y^q_i)$$

so that :

$$\sum_{g=1}^{\overline{g}} \pi_g \sum_{h=1}^{l} \Delta_h \Delta F^{ab}(\pi_j, a_h, \overline{y}) =$$

$$\sum_{g=1}^{\overline{g}} \pi_g [\sum_{\{i:p_i=\pi_g \ \& \ y^p_{gi}\leq a_l\}} (a_l - y^p_{gi}) - \sum_{\{i:q_i=\pi_g \ \& \ y^q_{gi}\leq a_l\}} (a_h - y^q_{gi})] =$$

$$\sum_{i=1}^{n} p_i \max(a_h - y^p_{gi}, 0) - \sum_{i=1}^{n} q_i \max(a_h - y^q_{gi}, 0) =$$

$$\sum_{i=1}^{n} p_i P(a_h, y^p_{gi}) - \sum_{i=1}^{n} q_i P(a_h, y^q_{gi}) \tag{29}$$

Following just the same reasoning, we can also establish that:

$$\sum_{g=1}^{\overline{g}} (1-\pi_g) \sum_{h=1}^{l} \Delta_h \Delta F^{ab}(\pi_g, \overline{y}, a_h) =$$

$$\sum_{g=1}^{\overline{g}} (1-\pi_g)[\sum_{\{i:p_i=\pi_g \ \& \ y^p_{bi}\leq a_l\}} (a_l - y^p_{gi}) - \sum_{\{i:q_i=\pi_g \ \& \ y^q_{bi}\leq a_h\}} (a_h - y^q_{bi})] =$$

$$\sum_{i=1}^{n} (1-p_i) P(a_l, y^p_{bi}) - \sum_{i=1}^{n} (1-q_i) P(a_l, y^q_{bi}) \tag{30}$$

Finally, we notice that:

$$\sum_{k=1}^{\overline{g}-2} \sum_{g=1}^{k} (\pi_{g+1} - \pi_g))[\Delta F(\pi_{k+1}, \overline{y}, \overline{y}) - \Delta F(\pi_k, \overline{y}, \overline{y})] = \sum_{i=1}^{n} (p_i - q_i) \tag{31}$$

Plugging (29), (30) and (31) into (28) enables one to write this inequality as :

$$\sum_{i=1}^{n} (p_i - q_i)[U_g(\overline{y}) - U_b(\overline{y}]$$

$$- \sum_{i=1}^{n} [p_i P(\overline{y}, y^p_{gi}) - q_i(\overline{y}, y^q_{gi}) + (1-p_i) P(\overline{y}, y^p_{bi}) - (1-q_i) P(\overline{y}, y^q_{bi})] U_g^{\Delta_l}(a_{\overline{h}-1})$$

$$+ \sum_{l=1}^{\overline{h}-2} \Delta_l [\sum_{i=1}^{n} [p_i P(a_l, y^p_{gi}) - q_i P(a_l, y^q_{gi}) + (1-p_i)(a_l, y^p_{bi}) - (1-q_i) P(a_l, y^q_{bi})]] U_g^{\Delta_{l+1}\Delta_l}(a_l)$$

$$-\sum_{i=1}^{n}[(1-p_i)P(\overline{y},y^p_{bi})-(1-q_i)P(\overline{y},y^q_{bi})][U_b^{\Delta_{\overline{h}-1}}(a_{\overline{h}-1})-U_g^{\Delta_l}(a_{\overline{h}-1})]$$

$$+\sum_{l=1}^{\overline{h}-2}\Delta_l[\sum_{i=1}^{n}[(1-p_i)P(a_l,y^p_{bi})-(1-q_i)P(a_l,y^q_{bi})][U_b^{\Delta_{l+1}\Delta_l}(a_l)-U_g^{\Delta_{l+1}\Delta_l}(a_l)]$$

$$\geq 0 \tag{32}$$

Quite clearly, if U belongs to $\mathbb{U}_2$, it is sufficient for (32) to hold to have:

$$\sum_{i=1}^{n}(p_i-q_i)\geq 0,$$

$$\sum_{i=1}^{n}[p_iP(a_l,y^p_{gi})-q_iP(a_l,y^q_{gi})+(1-p_i)(a_l,y^p_{bi})-(1-q_i)P(a_l,y^q_{bi})]\leq 0$$

and

$$\sum_{i=1}^{n}[(1-p_i)P(a_l,y^p_{bi})-(1-q_i)P(a_l,y^q_{bi})]\leq 0$$

for all l. But these conditions are nothing else than the conditions (12), (10) and (11) of the definition of SEPG dominance. ■

As can be seen in the sufficiency proof of this theorem, the condition (12) is sufficient for normative dominance over the class $\mathbb{U}_2$ if one assumes, plausibly, that $U_g(\overline{y})-U_b(\overline{y})\geq 0$ (it is, weakly, better to be in the good state than in the bad state even with a very large income). Yet, if one is willing to assume that, for $\overline{y}$ sufficiently large, $U_g(\overline{y})-U_b(\overline{y})=0$, then there is no need to require condition (12) in verifying SEPG dominance. The fact that the consequences, on the implementable criteria, of making assumptions on the behavior of the utility functions at upper bounds of their domain of definition may be non-trivial is well-known in multi-dimensional analysis (see e.g. Bazen and Moyes (2003), Jenkins and Lambert (1993) or Fleurbaey *et al.* (2003)). Yet the assumption that $U_g(\overline{y})-U_b(\overline{y})=0$ is rather implausible, especially for risk of health or violent criminal act (after all, even very rich people suffer from being raped).

3 Empirical illustrations

We now illustrate how the criteria studied in this paper can be useful to generate interesting empirical normative conclusions. These illustrations are all made using *samples* of individual data on different risks. In order to account for the fact that the compared distributions are samples drawn from a larger population, we perform statistical inference based on the Union-Intersection (UI) method as initiated by Bishop *et al.* (1989)[3]. The UI

[3] See Howes (1994) and Tse and Zhang (2004) for an appraisal of this inference methodology.

method supposes that we accept the hypothesis of dominance of a socially risky situation A over a socially risky situation B if none of the poverty inequalities that define the dominance criterion is significantly positive and if at least one of the inequality is significantly negative. Details of the statistical methodology are provided in appendix. All comparisons that are presented herein are performed at the 95% confidence level.

3.1 Evolution of risk of crime in India

We first illustrate the usefulness of the criteria by applying them to the evolution, over time, of the exposure of Indian citizens to risks of violent crimes measured at the individual's district of residence. It is well-known that India has experienced a period of spectacular economic growth in the last twenty years or so. While much researchers who have performed normative evaluation of the Indian growth experience on the sole basis of pecuniary considerations have concluded that this experience has been beneficial for India, there has been few studies that have looked at the impact of the Indian growth on the distribution of non-pecuniary attributes. In one of these studies, Gravel and Mukhopadhyay (2007) have considered, beside individual expenditures, three other non-pecuniary attributes measured at the level of the district of residence of the individuals: infant mortality, literacy rate, and probability of being the victim of a violent crime. They have conducted the analysis using Atkinson and Bourguignon (1982) multidimensional criteria and have concluded that, if one abstracts from crime, there has been a steady and robust increase in social welfare in India over the whole period 1987-2001 as recorded by the Atkinson and Bourguignon *first order dominance criterion*. However, when the probability of being the victim of a violent criminal act is added to the list of attributes, the conclusion holds only for the more ethically demanding *second order dominance criterion* of Atkinson and Bourguignon, and is not valid for the whole period.

In Gravel and Mukhopadhyay (2007), no use is made of the fact that the distribution, between individuals,of probability of crime and expenditures define a social risk, appraised by individuals using selfish VNM preferences, and appraised socially by a Pareto inclusive and anonymous VNM preference. It is of some interest to see whether using this structure affects the normative conclusion as to whether the exposure of Indians to risk of violent crime has unambiguously improved over the period considered in Gravel and Mukhopadhyay (2007).

For this sake, we use the same data set as Gravel and Mukhopadhyay (2007). Specifically, data on households' consumption have been obtained from the 43rd (1987-1988), 52nd (1995-1996) and 58th rounds (2002) of the consumption expenditure surveys conducted by National Sample Survey Organization (NSSO). Individual consumption expenditures have been derived from household consumption expenditures using the Oxford equivalence scale and are in 2002 Rupees[4]. Consumption data have also been made comparable, to the extent possible, in terms of the reference period over which consumption expenditures are recollected by surveyed households. As is well-known (see e.g. Deaton and Drèze (2005) or Himanshu and Sen (2005)) there has been some time inconsistency as to the recall period used in the NSSO questionnaires to determine the spending on various group of commodities, especially the durables, clothing and footwear. In 1987-88 data on these goods have been collected using both a 30 days recall period and a 365 days recall period while only a 365 days recall period was used for 2002 data. In order to make data on these two periods comparable, we have used calculations based on the 365 recall period. However, for 1995-96, half the sample is at 365 days recall period, while the other half is at 30 days period. Because of the lack of information, we could not correct this subsample for this. At the all India level[5], the analysis is based on 131,511 individuals in 2002, 203,228 individuals in 1995-96 and 563,931 individuals in 1987-88.

As the district of residence of each individual is provided in NSSO data for each period, we have assigned to each individual the violent crime rate of his or her district of residence (there were 527 districts in India in 2002). Due to subdivisions in the district areas that have taken place in India over the 1981-2001 period, there are more districts in 2001 and 1991 than in 1981. In order to make the comparisons consistent, we have aggregated data for 1991 and 2001 to adhere to the original, and coarser,1981 districts partition.

Violent crime rates (number of murders, attempted murders, and rapes per million individuals per district) have been obtained, for the same years as NSSO data from the National Crime Record Bureau. We have restricted our attention to the most violent and extreme form of crime to reduce the risk of trend biases due to the evolution of the reporting behavior of the victims of crimes (or their families). It is indeed well-known that crime reporting tends to grow with education and wealth (wealthier and more educated people are more prone to report crime to the police than deprived or less educated ones). Our assumption is that this bias is less important in the case of violent crimes, who tend to be reported to the police no matter what is the wealth or education level of the family of the victim, than for robberies, burglaries, and other types criminal acts.

Figures 1a and Ib show, respectively, the expected headcount poverty curves in the bad state and the headcount poverty curves irrespective of states (which coincide with standard headcount poverty curves here because

[4]Price deflators are the Urban Non Manual Employees price index for urban data and Agricultural Labourers price index for rural ones. Comparisons or pooling between urban and rural data are performed using Deaton (2005) (table 17;3) ideal Fisher index.

[5]For reasons having to do with the unreliability of data coming from Jammu Kashmir and the North Eastearn States of India, we have excluded these troubled areas from our study.

individual consumption is the same in the two states for all individuals).

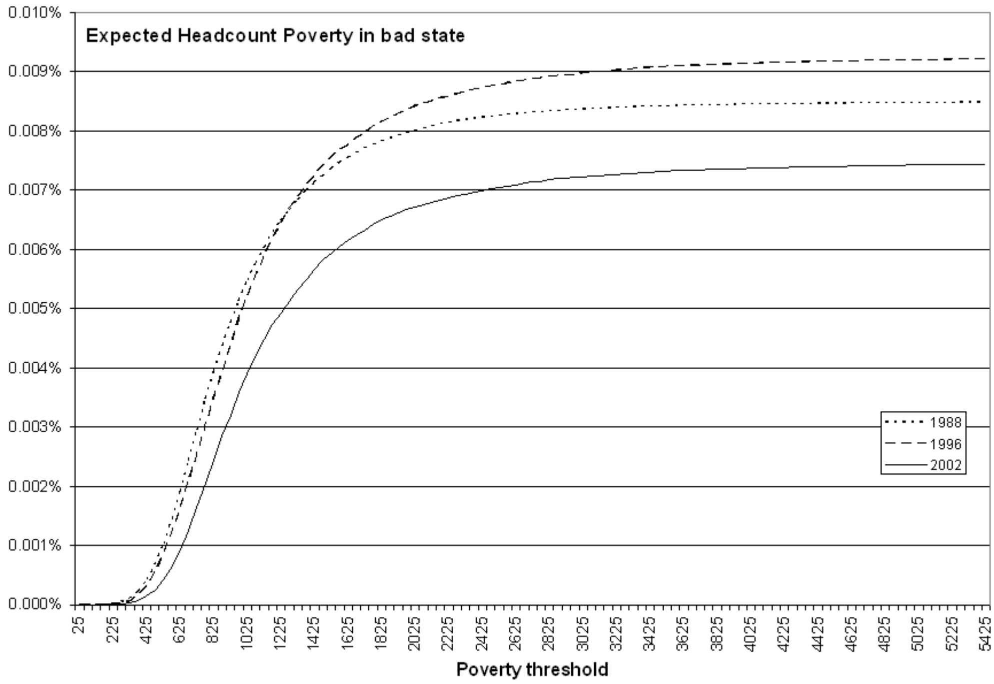

Figure 1a

From these pictures, one can see a clear ranking of the Headcount poverty curves (from 1988 to 2002) and a dominance ranking of the expected head-count poverty curve in the bad state of 2002 over either 1988 or 1996. Yet the expected Headcount poverty curves in the bad state cross between 1996 and 1988, indicating a failure to reach clear cut conclusion as to the ranking of these two years. This impossibility to conclusively rank 1988 and 1996 is clear from the fact the average probability of being the victim of a violent crime in India has increased between 1988 and 1996.

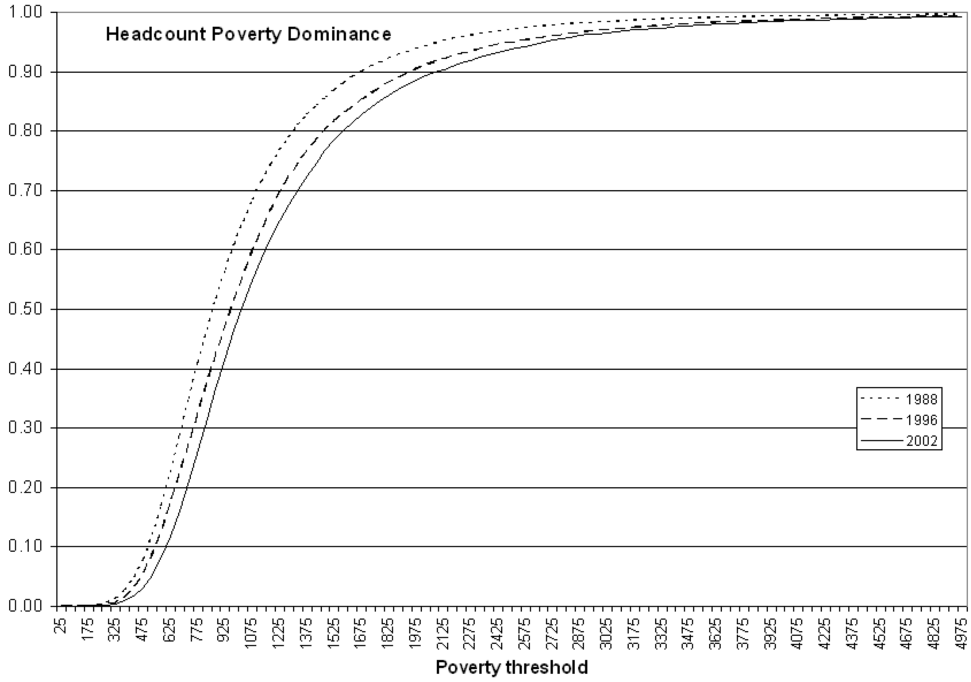

Figure 1b

This visual impression is confirmed by the statistical inference tests whose results are shown in Table 1 below.

	2002 vs 1996		**2002 vs 1988**	
Ranking	2002 $\succsim_{\mathbb{U}'_1}$ 1996		2002 $\succsim_{\mathbb{U}'_1}$ 1988	
T-statistics	Min	Max	Min	Max
SEHP				
condition (7)	-105.36	-2,87	-106.72	-4.51
condition (8)	-47.48	-2.21	-105.75	-2.72
Critical value SMM(92,∞): 3.449				

Table 1: Results of the comparisons with statistical inference

As is clear from the table, the distribution of risk of violent crime in 2002 dominates both that of 1996 or 1988 according to the SEHP dominance criterion. Yet, for the reason indicated above, no conclusion can be obtained for comparison of 1996 and 1988. This illustrates that the criterion proposed in this paper is quite useful to obtain robust normative conclusions. In Gravel and Mukhopadhyay (2007), no first order dominance could be obtained, and one had to resort to second order Atkinson and Bourguignon multidimensional dominance, and the rather demanding properties on the individual utility function that this criterion required, to obtain the verdict. Here, we only need to accept that Indians have VNM preferences for risks that satisfy the rather mild property that the (positive) marginal utility of money is larger in the bad state than in the good one to reach the conclusion, provided of course that we accept to compare social risks by a Pareto inclusive and anonymous VNM social preference.

3.2 Labour market related risks

In the last decade or so, there has been a growing concern for job security and a perception, by many workers in many countries, that their job was becoming increasingly unstable (see e.g. OECD (1997)). These concerns have initiated a literature that attempt at empirically measuring the evolution of job insecurity in various western countries. For the United States, some authors, such as Farber (2005), have suggested that the average job insecurity in the US has increased mildly in the late nineties while others, like Gottschalk and Moffit (1999), have shown no evidence of an increase in the probability of loosing one's job. Similarly, there has been several papers who have examined the evolution of the average risk of involuntary job loss in France. For instance, Givord and Maurin (2004) suggests that the probability of involuntary job loss has increased since the 1980s. It has also been notice by Behagel (2003) and Postel-Vinay (2003) that the increase in the risk of loosing one's job has been larger for low-seniority workers than for high-seniority ones. All of these studies have focused on the average probability of being unemployed and have not make any attempt to derive meaningful normative conclusion out of their conclusion. The criteria examined in this paper are potentially quite useful for appraising the trends in individual exposures to risk of being unemployed.

We illustrate this by comparing the exposure to risks of unemployment of single adult members of the workforce between US and France. We focus on single adults to avoid, at this stage, normatively challenging issues that concern multi-individual households. We use for this purpose the French Labor Force Survey (LFS) and the US Current Population Survey-March Supplement (CPS-MS) for both 2003 and 2004. The LFS contains 50,524 respondents (employees and unemployed) among whom are there 6,953 single individuals without children. In the US CPS, there are 90,314 respondents (employees and unemployed) among whom 7 523 are single without children. In both data sets, the same individuals are observed in 2003 and 2004. The fact that some of them have experienced change in employment status between the two year enable us to assign to each individual in the sample a probability of being unemployed, an income if employed and a (substitution) income if unemployed.

As in Givord and Maurin (2004), we measure the risk of unemployment by the probability of being involuntary unemployed in 2004. This risk means different things for different individual. *For an individual observed unemployed in 2003*, it corresponds to the risk of remaining unemployed in 2004. *For an individual observed employed in 2003*, it corresponds to the risk of loosing his or her job between the 2003 and 2004. We assign probability to every individual by grouping them into homogeneous groups with respect to observable characteristics and by assigning to each individual of the group the same probability of being unemployed. For individual employed in 2003, this probability is nothing else than the fraction of the individuals within the group who became unemployed in 2004. For unemployed individual in 2003, it corresponds to the fraction of individuals in the group that remained unemployed in 2004. There were 38 groups of employed individuals (formed with respect to the level of education, the activity sector, the age and the fact that they work in the private or the public sector) and 10 groups of unemployed (defined on the basis of education, unemployment seniority and gender).

We are interested in "involuntary" unemployment. While the French LFS distinguishes between voluntary and involuntary unemployment, the CPS does not. Hence, we had to adjust our estimated risks by using the Displaced Workers Survey (DWS). The DWS is conducted in the month of January only on the same sample of individuals used in the CPS and asks workers whether or not they were displaced involuntarily from a job at any time in the preceding three-year period. Hence, the DWS enables one to estimate the fraction of unemployed individuals who have been involuntary put into that situation.

In order to compare distributions of risk, we need also to assign a labour income to an employed individual and a replacement income to the unemployed individual. If the individual is observed employed in 2003, his or her monthly activity income is that observed in the data set. If the individual is observed unemployed in 2003, we assign to the individual the monthly labor income that he or she would have earned had he or she been employed. In order to assign this income to an unemployed person, we estimate a wage equation on the sample of employed individuals. We of course account for the possible selection bias that could arise from the fact that we assign to unemployed individuals a wage that has been estimated on a sample of employed households by using Heckman (1979)'s methodology. The independent variables used in the wage equation are seniority (dummy), occupations (6 dummies), industries (5 dummies), city size (10 dummies for France and 8 for United States), education level (6 dummies), age and age squared. We have performed the estimation separately for the samples of female and male singles.

Activity income is then transformed for all individuals into disposable income by subtracting income taxes (net of possible income tax credit) and by adding welfare payments, if any (especially the French "Revenu minimum d'insertion" or RMI). Finally, we assign to each individual a substitution income received in case of unemployment on the basis of the legislation in the two countries. This substitution income is principally made of unemployment benefits and/or social welfare payments. The unemployment benefits are function of the past activity income and the intensity of work (full/part time). Unemployment benefits are more generous in France than in US since the duration of benefit can go up one year in US while it does not go above 26 weeks in the US. The only welfare payment that is considered in France is the RMI ($ 392 US per month). As much of welfare payments in the US are given to family with at least one children, we ignore these benefits in this study devoted to single adults. Housing benefits ("*Allocation Personnalisée au Logement*" and "*Allocation Logement*" in France, Low-rent public housing and Housing choice vouchers in the US) are also ignored since we do not have information on housing prices.

Summary statistics on the probability estimates and the average activity income and substitution income are shown in table 2. All pecuniary figures pertaining to US and France are in US dollars, corrected for Purchasing Power Parity using OECD scales.

	France	United States
Probability of good state (%)	88.67	93.84
	(19.88)	(12.74)
Female	89.66	94.64
	(18.68)	(12.17)
Male	87.90	93.13
	(20.74)	(13.18)
<30 old	87.08	93.66
	(19.32)	(12.33)
>30 old	89.4	93.90
	(20.10)	(12.87)
Probability of remaining employed	95.99	95.51
	(3.97)	(9.45)
Probability of becoming employed	39.93	69.68
	(13.81)	(24.33)
Monthly income (PPP $)		
Mean income in employment	1,284	2,508
	(685.28)	(2428)
Mean replacement income	885	856
	(564.05)	(362.81)

Table 2: summary statistics, France USA

It is clear that the probability of being in the good state (employed) is significantly higher and equally distributed amongst single workers in the

United States than in France. Notice that this seems to be true only for those individuals who were unemployed in 2003 (unemployment inertia is stronger in France than in the US). Among the employed individuals, there does not seem to be much difference between the probability of keeping one's job for one year in France (95.99 %) and in the US (95.51 %). This however seems to be specific to the population of single adults without children. The other estimations that we have done for the two populations suggest that, if we include the other members of the workforce, the probability of keeping one's job is also significantly higher in US than in France. Moreover France seems also to be more "unequal" than in the US in terms of the way it distributes the probability of keeping one's job across its single adults. The gap in average probability of good state is larger between women and men and between "old" (above thirty) and "young" workers in France than in the United States. We can also note that women seem to face, in both countries, a lower probability of being unemployed. This can be explained in part by the fact that there is a larger proportion of women working in the less risky public sector.

Table 3 provides some aggregate information on the distribution of probabilities of being employed according to income in both countries Here again, the differences in these probabilities are smaller in the US than in France, where the probability of being employed appears to be strongly increasing with the disposable income. The risk faced by poor workers is also significantly higher in France than in the United States. The difference between women and men show roughly the same pattern in the two countries. In the bottom and the top of the income distribution, women face to a higher probability of being unemployed while in the middle of the income distribution, their risk is significantly lower.

	France				United States				Diff
Income bracket	Total	Women	Men	t-Test	Total	Women	Men	t-Test	t-Test
$[0, 1000]$	81.51	80.98	82.18	-1.43	95.29	95.05	95.57	-0.34	-10.87
$]1000, 1500]$	89.67	93.74	87.33	11.47	92.49	94.08	90.97	3.80	-3.73
$]1500, 2000]$	94.41	95.18	93.87	1.37	90.83	94.94	87.50	10.98	5.47
$]2000, 2500]$	95.08	96.85	93.91	3.31	93.39	94.39	92.38	2.29	1.94
$]2500, 3000]$	96.56	96.46	96.78	-0.70	94.60	95.19	94.06	1.12	2.27
$]3000, 4000]$	97.51	97.24	97.84	-0.07	95.29	94.45	96.00	-1.10	2.33
$]4000, \infty]$	96.07	96.10	96.06	0.01	95.81	94.14	96.78	-1.30	0.22

Table 3: Aggregate information on the distribution of unemployment risks and income

Can we now make more normatively meaningful comparisons between these two countries based on the criteria developed in this paper ? Figures 2a and 2b show, respectively, expected Headcount poverty curves in the bad state (inequality (7)) and expected headcount poverty curves in either state (inequality (8)) for the total adult population and for the subsamples made of male and female adults in both US and France.

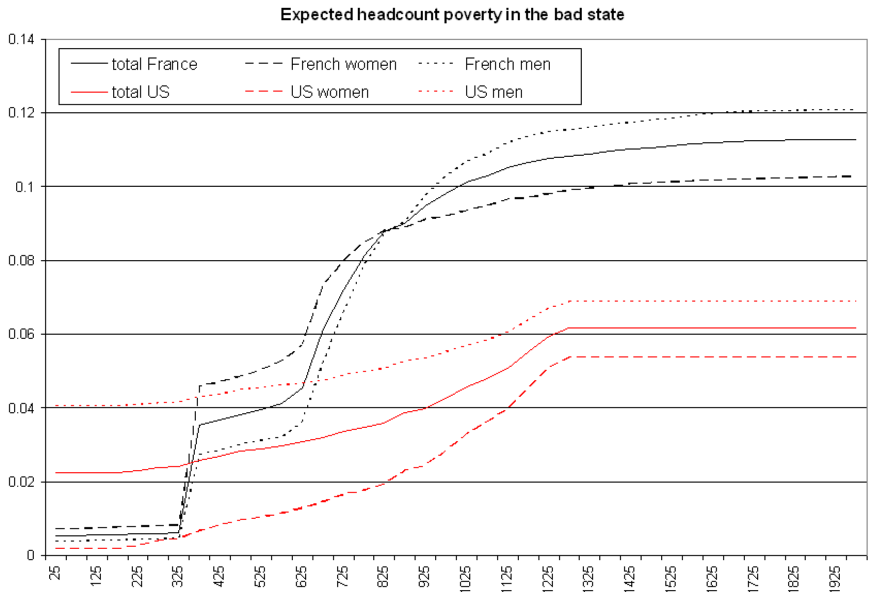

Figure 2a

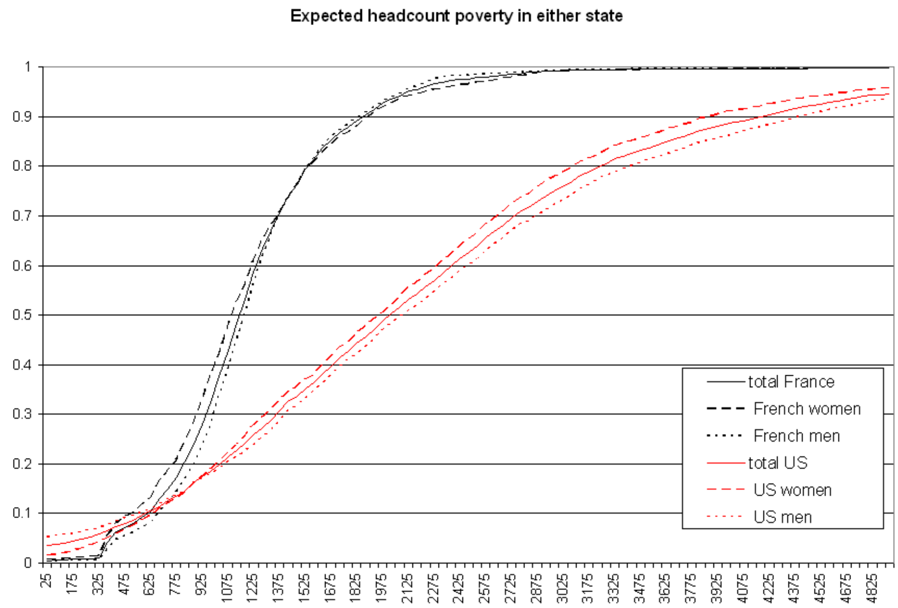

Figure 2b

The French and the US situations are rather different. If we look at the expected headcount poverty in the bad state curves, we notice that the three curves in France cross while they are completely ordered in the US (the expected fraction of the male population being poor in the bad state is higher than that of the female population at every poverty line). Moreover, there is a smaller expected fraction of the women population that is poor in the bad state in US than in France. However all curves cross when one looks (figure 2b) at the expected fraction of the population that is poor in either state. This suggests that no dominance can be obtained between any two of these populations (total US, total France, French women, French men, US women and US men). Because of the social protection provided by the French RMI, the expected fraction of population that is poor in either state is lower in France than in the US for very low poverty lines (the effect of the RMI is quite clear in figure 2b. This suggests that France and US will be difficult to compare in terms of their distribution of labour market related risks.

Table 5 shows the results of the comparisons of all the above populations using both SEHP and SEPG dominance based on statistical inference methods, rather than visual inspection of graphs. It also shows the results of the comparisons, using the same criteria, of old and young segments of the population (using the same cut-off age of 30 years old).

	SEHP Dominance			**SEPG Dominance**		
Comparison	Ranking	Min T	Max T	Ranking	Min T	Max T
France vs USA	?	-12.81	60.90	?	-17.93	53.31
Critical value SMM(392,∞): 4.2						
FRANCE						
Female vs Male	?	-3.85	9.39	?	-3.71	5.30
Young vs Old	**OLD**	0.03	12.94	**OLD**	3.74	12.74
Critical value SMM(197,∞): 3.64						
USA						
Female vs Male	?	-17.42	6.28	?	-16.73	4.84
Young vs Old	**OLD**	-1.93	14.63	**OLD**	-1.89	15.63
Critical value SMM(258,∞): 4.12						

Table 5: French-US Comparisons

While we are unable to compare US with France by any criterion studied herein, we are capable of deriving, within each countries, interesting normative conclusions about the exposure to unemployment risks of various segments of the single households population. For instance, no dominance verdict can be obtained, within each country, between men and women. This conclusion may seem surprising when one knows that, in both countries, the distribution of either activity income or substitution income among men generalized Lorenz dominates that among women. Yet, as mentioned, the exposure to risks of the two subpopulations do not show such a dominance, because women tend to have, at least in some income categories, a better protection against the risk of being involuntarily unemployed than men. However, we can notice that, in both countries, the exposure to risk of single adult members of the workforce above thirty is normatively better than that of the young.

4 Conclusion

This paper characterizes two robust criteria for comparing socially risky situations from a normative point of view. The criteria characterized, SEHP and SEPG dominance, are considered as robust because all VNM rankings of socially risk situations that are anonymous and Pareto inclusive over a very wide class of identical individualistic VNM preferences would agree with their verdict. As briefly illustrated with empirical data, SEHP and SEPG dominance criteria are easy to use and capable of producing interesting conclusions. Among other things, as illustrated in the Indian example, they increase significantly the discriminatory power of more abstract multidimensional dominance criteria à la Atkinson and Bourguignon (1982). They are also potentially useful in comparing different population of individuals in terms of their exposure to risks of unemployment. While the empirical applications provided in this paper have been mainly illustrative, we are confident that much more could be done with the criteria. It is also probably important to generalize the approach to non-binary risks.

5 Appendix: Testing methodology

We briefly recall here the working of the UI inference methodology, advocated by Bishop *et al.* (1989) or Bishop and Formby (1999), that is used in this paper. We illustrate the methodology for the SEHP dominance criterion but the method works just as well for the SEPG dominance one;

Suppose that we want to compare population-wise socially risky situations P and Q, according to the SEHP criterion, based on the comparisons of samples p and q. Denote as $\widehat{ED}_p^b(t)$ and $\widehat{ED}_p(t)$ the estimated expected poverty in the bad state and the estimated expected poverty in either state in p that are defined as follows:

$$\begin{aligned}\widehat{ED}_p^b(t) &= \frac{1}{n_p}\sum_{i=1}^{n_p}(1-p_i)\cdot I(x_i^p \leq t) \\ \widehat{ED}_p(t) &= \frac{1}{n_p}\sum_{i=1}^{n_p} p_i \cdot I(y_i^p \leq t) + \frac{1}{n_p}\sum_{i=1}^{n_p}(1-p_i)\cdot I(y_i^p \leq t)\end{aligned}$$

where n_p is the number of individuals in the sample corresponding to p. One can of course define in an analogous fashion $\widehat{ED}_q^b(t)$ and $\widehat{ED}_p(t)$ for q. We infer that distribution p dominates q if $\widehat{ED}_p^b(t) \leq \widehat{ED}_q^b(t)$ and $\widehat{ED}_p(t) \leq \widehat{ED}_q(t)$ for all possible existing levels of income t. The question is whether we can make this inference in a statistically significant way. Consider a grid of K poverty thresholds, $(t_1, ..., t_K)$ and define statistics:

$$
\begin{aligned}
T_k^b &= \frac{\widehat{ED}_p^b(t_k) - \widehat{ED}_q^b(t_k)}{\left(\widehat{\omega}_p^b(t_k)/n_p + \widehat{\omega}_q^b(t_k)/n_q\right)} \\
T_k &= \frac{\widehat{ED}_p(t_k) - \widehat{ED}_q(t_k)}{\left(\widehat{\omega}_p(t_k)/n_p + \widehat{\omega}_q(t_k)/n_q\right)}
\end{aligned}
$$

where $\widehat{\omega}_p^b(t_k)$ and $\widehat{\omega}_p(t_k)$ are the estimations of the asymptotic variance of $\widehat{ED}_p^b(t_k)$ and $\widehat{ED}_p(t_k)$ respectively. In the same line of reasoning of Davidson and Duclos (2000) and Duclos *et al.* (2006) based on the law of large numbers and the central limit theorem, we can derive these estimations of the asymptotic variance of the estimates of expected poverty as follows:

$$
\begin{aligned}
\widehat{\omega}_p^b(t_k) &= \frac{1}{n_p}\sum_{i=1}^{n_p}\left[(1-p_i)\cdot I(x_i^p \leq t)\right]^2 - \left[\widehat{ED}_p^b(t_k)\right]^2 \\
\widehat{\omega}_p(t_k) &= \frac{1}{n_p}\sum_{i=1}^{n_p}\left[(1-p_i)\cdot I(x_i^p \leq t)\right]^2 + \frac{1}{n_p}\sum_{i=1}^{n_p}\left[p_i \cdot I(y_i^p \leq t)\right]^2 - \left[\widehat{ED}_p(t_k)\right]^2
\end{aligned}
$$

As mentioned in section 3, the UI rule says that we infer a SEHP dominance of P over Q based on the sample estimates of expected poverty if none of the poverty inequalities that define the criterion is statistically positive and at least one is statistically negative. Given our formal definitions, this means that the UI rule says that:

- If $\max\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) < C_\alpha$ and $\min\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) < -C_\alpha$, we infer that p dominates q.
- If $\min\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) > -C_\alpha$ and $\max\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) > C_\alpha$, we infer that q dominates p.
- If $\min\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) > -C_\alpha$ and $\max\left(T_1^b, ..., T_K^b, T_1, ..., T_K\right) < C_\alpha$, we infer that p and q are not different.
- we infer that p and q are not-comparable otherwise.

where C_α is the critical value for a level of significance of α determined from the Studentized Maximum Modulus distribution provided by Stoline and Ury (1979). The degree of freedom used is $2 \cdot K$, that that corresponds the number of equalities that we want to test simultaneously (K inequalities for expected poverty in the bad state, and K inequalities for expected poverty in either state.

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