Problem set in Consumer theory: updated version

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Problem 1 For every preference defined below, indicates if it is convex (weakly or strictly), increasing (weakly or strictly), locally non satiable, continuous, complete and transitive (explains and draw the sets $NW \succeq$)

(i) $(x_1, x_2) \succeq (y_1, y_2) \Leftrightarrow x_1 + x_2 \geq y_1 + y_2$ and max $(x_1, x_2) \geq$ max (y_1, y_2)
(ii) $(x_1, x_2) \succeq (y_1, y_2) \Leftrightarrow x_1 \geq y_1 + 2$ or $x_2 \geq y_2 - 1$
(iii) $(x_1, x_2) \succeq (y_1, y_2) \Leftrightarrow x_1 \geq y_1$ or $x_1 = y_1$ and $x_2 \geq y_2$.
(iv) $(x_1, x_2) \succeq (y_1, y_2) \Leftrightarrow \min(x_1, x_2) \geq \min(y_1, y_2)$ and max $(x_1, x_2) \geq$ max (y_1, y_2)

Problem 2 Shasikanta's preferences for consumption bundles of two goods are as follows. For every pair of bundles A and B, Shasikanta considers that A is strictly better than B if A contains strictly more of at least one good than B. Furthermore, Shasikanta is indifférent between two bundles A et B only if the two bundles are equal (that is they contain the same quantity of the two goods). Are Shasikanta preferences transitive ? Complete ? Continuous ? Locally non-satiable ? Convex ? Justify.

Problem 3 Conrad is an amateur of souvlakis who is endowed with the following preferences. For every pair A and B of bundles of souvlakis and money available to other use than souvlakis, Conrad *weakly prefer* A to B *if and only if*:

1) The quantity of souvlakis available A exceeds the quantity available in B by at least two units and

2) The money available to other use than souvlakis in A is not lower than that available in B by more than 100 euros.

Are Conrad preferences transitive ? Complete ? Continuous ? Locally non-satiable ? Convex ? Justify and draw a representative $NW_{\succsim}(x_1, x_2)$ set.

Problem 4. Kumar has preference for two goods that are numerically represented by the utility function

$$U(x_1, x_2) = \min(x_1, x_2^2)$$

a) Find the Marshallian and Hicsksian demand functions (or correspondances) as well as the indirect utility and the expenditure functions.

b) Is good 1 a luxury or a necessity good ?

Problem 5. The rugby team A has beaten rugby team B. Rugby team B has beaten rugby team C while teams A and C have done a dead-heat. .

a) Is the binary relation "has not lost against" defined on the set of teams $\{A,B,C\}$ complete and transitive ? Justify

b) Is the binary relation “is the brother of" defined on the set of all individuals living in Marseille complete and transitive ?

Problem 6. True or false ? (justify) If leisure is an inferior good for some household, then the number of hours that this household would wish to supply on the market will be an increasing function of the hourly wage.

Problem 7 True of false? (justify) Preeti has a yearly income of ω_1 euros this year. She is certain to receive ω_2 euros next year. If Preeti decices to borrow this year at some interest rate r, she would certainly decide to borrow if the interest rate was lower.

Problem 8 Suppose that at prices $(p_1, p_2) = (5, 10)$, a rational consumer endowed with a wealth of 100 consumes the bundle $(6, 7)$. Suppose that an econometrician has measured the following derivatives:

$$\frac{\partial h_1(5, 10, V(5, 10, 100))}{\partial p_1} = -2$$
$$\frac{\partial h_1(5, 10, V(5, 10, 100))}{\partial p_2} = +1$$
$$\frac{\partial x_1(5, 10, V(5, 10, 100))}{\partial R} = 2/7$$

Estimate the bundle that the consumer would have choosen if he or she had faced prices $(p_1, p_2) = (5, 11)$.

Problem 9 An individual consumes housing and money available for other things than housing. Housing is available in two different cities: city A and city B. The individual can not live in the two cities in the same time. The preferences of the individual for the three goods are measured by the utility function:

$$U(x_1, x_2, x_3) = (x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}})x_3$$

where x_1, x_2 and x_3 denote, respectively, the number of squared meters occuppied in city A, the number of squared meters occuppied in city B and the number of euros available for other things than housing.

a) Define the consumption set of this individual. Is this set convex ?

b) Define the Marshallian demand for each of these three goods. Interpret your results. Are these Marshallian demand continuous ? Why ?

Problem 10 A preference is said to be *homothetic* if it can be measured by a utility function that is homogenous of degree 1. Show that if a preference is homothetic, then the Marshallian cross-price effects should all be equal (that is, $\frac{\partial x_i^M(.)}{\partial p_j} = \frac{\partial x_j^M(.)}{\partial p_i}$ must hold for every pair of goods i and j).

Problem 11 The expenditure function of some consumer is given by:

$$D(p_1, p_2, u) = (\frac{2}{3}p_1 + p_1^{\frac{1}{2}} p_2^{\frac{1}{2}} + \frac{1}{3}p_2)u$$

Find the Marshallian demand of this consumer.

Problem 12 Sylvester has preferences for Pop Corn (good 1) and Pepsi (good 2) that are represented by the utility function $U(x_1, x_2) = -\frac{1}{x_1} - \frac{1}{x_2}$. Find the Marshallian demand correspondence and indicates whether or not they are single-valued (that is to say, whether or not they are functions).

Problem 13 Crocodile Dundee has preferences for hamburgers (good 1) and champagne (good 2) that are represented by the utility function $U(x_1, x_2) = x_1 + x_2^{\frac{1}{2}}$. What conditions (if any) must be imposed on prices and Crocodile's wealth for Croco dile to find optimal to abstain from eating hamburgers?

Problem 14 Find the Marshallian demand behaviour of a consumer's whose preferences for two goods are represented by the utility function

$$U(x_1, x_2) = (1 + x_1)(2 + x_2)$$

(a) Are the Marshallian demand correspondance in fact functions ?

(b) Find the indirect utility function, the expenditure function and the Hicksian demand correspondances.