

Efficiency, Equality and Poverty: Review questions 1

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Question 1: The following table shows the average income (in thousand euros per year) in each of the five quintiles of the population of four countries.

Country	1st quintile	2nd quintile	3rd quintile	4th quintile	5th quintile
1	5	8	9	17	18
2	6	7	9	16	19
3	8	8	8	8	25
4	8	9	10	13	18

(a) Are there rankings of countries that would be unanimously agreed upon by all utilitarian ethics who assume that the average individual in each decile transforms income into happiness with the same increasing and concave function? If so indicate what the rankings are and why? If not, indicate also why?.

(b) When the distributions have the same mean, rank them using the Coefficient of variation as an inequality index. Comment the possible difference between this rankings and those obtained in a) (if any).

(c) Are there rankings of countries that would not be unanimously agreed upon by all utilitarian ethics who assume that the average individual in each decile transforms income into happiness with the same increasing and concave function, but that would be agreed upon by the smaller class of such utilitarian ethics who makes the extra assumption that the decrease in marginal utility of income arises at a decreasing rate? Again, justify.

Question 2 Why is the coefficient of variation of the logarithm of income not a good index of inequality?

Question 3: Consider an individual who cares about two goods, indexed by i . These goods are only available in discrete amounts so that if x_i is the quantity of good i for $i = 1, 2$, then $x_i = 0, 1, 2, \dots$. That is to say $x_i \in \mathbb{N}_+$. The maximal conceivable quantity of good i is assumed to be bounded at level $\bar{x}_i > 1$ for $i = 1, 2$. Suppose one is interested in ranking, with a reflexive and transitive binary relation \succsim having asymmetric and symmetric factors \succ and \sim all non-empty subsets of the set $X = \{(x_1, x_2) \in \mathbb{N}_+ : x_i \leq \bar{x}_i\}$, each of these subsets being interpreted as an opportunity set. Budget sets of the form $B(p_1, p_2, R) = \{(x_1, x_2) \in X : p_1x_1 + p_2x_2 \leq R\}$ for some positive prices $p_i > 0$ ($i = 1, 2$) and income R are of course important such subsets. Suppose that we require \succsim to satisfy the following axioms:

- 1) $\{(x_1, x_2)\} \sim \{(y_1, y_2)\}$ for all (x_1, x_2) and $(y_1, y_2) \in X$
- 2) $\{(x_1, x_2), (y_1, y_2)\} \succ \{(x_1, x_2)\}$ for all (x_1, x_2) and $(y_1, y_2) \in X$
- 3) $A \succsim B \Leftrightarrow A \cup C \succsim B \cup C$ for all subsets A, B and C of X satisfying $A \cap C = B \cap C = \emptyset$.

Show that, for any two budget sets $B(p_1, p_2, R)$ and $B(q_1, q_2, R')$, if R/p_i and R'/q_i are large but lower than \bar{x}_i (for $i = 1, 2$), one has $B(p_1, p_2, R) \succsim B(q_1, q_2, R') \Leftrightarrow \frac{R}{p_1^2 p_2^2} \geq \frac{R'}{q_1^2 q_2^2}$. Justify carefully and comment.

Question 4: Do we need welfarism to give ethical foundations to so-called robust methods for comparing alternative distributions of income. Justify by recalling what is welfarism and what are robust methods for comparing income distributions ?

Question 5: Can we get out of Arrow's impossibility result if we define collective interest to be a function of individual's utility function when these are assumed to be ordinally measurable and interpersonally comparable ? If so, justify by saying what is Arrow's impossibility theorem, what is the meaning of ordinal measurability and interpersonal comparability and what definition of collective interest as a function of individuals utilities can be obtained. If not, then indicate, by providing similar justifications, what minimal extra measurability and/or comparability assumptions are required to escape out of Arrow's impossibility theorem.

Question 6: Show that none of the five Arrows' axiom is implied by the four remaining others.

Question 7: Consider the following ranking \succsim of all finite subsets (opportunity sets) of $\{a, b, c\}$.

$\{a, b, c\} \succ \{b, c\} \sim \{a, c\} \sim \{c\} \sim \{a, b\} \succ \{a\} \sim \{b\}$. Can this ranking be thought of as coming from someone who is unsure about the preference he or she will have when choosing from each opportunity set and who ranks opportunity sets on the basis of the expected maximal utility achieved in each of these set, with expectation taken over all preferences that the individual can have ? If yes, find the preferences and the probabilities attribute to them. If not, say why.

Question 8: Provide an example of a criterion for comparing sets of objects on the basis of their diversity that takes as given a notion of dissimilarity between objects that is only qualitative or ordinal in nature. Comment.

Question 9 True or false ? (justify). Assume that x and y are two distributions of income between n individuals having the same mean and for which there exists a $k \in \{1, \dots, n\}$ such that

$$\sum_{h \leq i} x_{(h)} \geq \sum_{h \leq i} y_{(h)}$$

for all $i \leq k$ with at least one inequality being strict and

$$\sum_{l \leq m} x_{(l)} \geq \sum_{l \leq m} y_{(l)}$$

for all $m > k$ (if any). Assume also that the variance of x is weakly smaller than that of y . Then

$$\sum_{i=1}^n \ln x_i > \sum_{i=1}^n \ln y_i$$

Question 10 True or false (justify and comment). Utilitarianism can justify rape in certain circumstances.

Question 11. True or false (justify) welfarism is not a very helpful paradigm to theories of justice if individual welfare is not interpersonally comparable.