Comparer les inégalités entre pays occidentaux : Le revenu disponible est-elle la seule variable pertinente ?

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1. Introduction

La réapparition de préoccupations relatives aux inégalités dans le débat social a accru le besoin de disposer de comparaisons internationales crédibles en la matière. De telles comparaisons sont destinées à fournir des indications de nature *normative* sur la manière avec laquelle les différents systèmes sociaux ou les différentes politiques en vigueur dans différents pays allouent les ressources rares entre leurs citoyens. C'est dans cette perspective qu'un certain nombre d'études récentes (voir par exemple Gottschalk and Smeeding (1997)) ont révélé l'existence de différences substantielles entre pays occidentaux en terme d'égalité de revenus disponibles en montrant, en accord avec ce que suggèrent nombre de débats populaires, que les pays anglo-saxons comme les Etats-Unis et le Royaume Uni sont plus inégaux que les pays d'Europe continentale du centre et du sud qui paraissent eux-mêmes plus inégaux que les pays d'Europe du Nord. Si ces constatations sont utiles, elles ne concernent que la seule distribution des revenus disponibles. Or cette dernière ne représente évidemment pas la seule caractéristique des sociétés qui soit pertinente pour les fins de l'évaluation normative.

L'insuffisance du revenu disponible est manifeste si l'on adopte le point de vue éthique "bien êtriste" suivant lequel la seule information pertinente pour l'évaluation normative est la distribution des niveaux de bien être individuels.[1] Il paraît en effet clair que le revenu disponible ne constitue pas le seul facteur influençant le bien être individuel. Ce dernier dépend également de la santé, de l'éducation (ou de l'information) ainsi que de l'accès à différents services publics (routes, moyens de transports publics, qualité de l'environnement naturel, sécurité publique, etc.) d'usage souvent gratuit ou subventionnés. De ce seul point de vue, une simple comparaison des distributions de revenu disponibles a fort peu de chance de nous fournir un apperçu adéquat de la distribution des bien êtres au sein de la population.

Mais le caractère insuffisant de la distribution des revenus disponibles pour les fins de l'évaluation normative apparaît également à la lunette d'un point de vue éthique alternatif dit de "l'*égalité des opportunités*". A en croire ce point de vue, développé par un certain nombre de philosophes et d'économistes, (voir par exemple Dworkin (1981), Arneson (1989), Cohen (1990) ou Roemer (1998)), les inégalités qui importent pour les fins de l'appréciation normative sont celles qui résultent de facteurs qui, comme le milieu social d'origine, échappent à la responsabilité des individus. En revanche cette approche

[1] Héritier moderne de l'utilitarisme classique de Bentham et Becaria, le "bien êtrisme" est une théorie de la justice distributive qui est largement utilisé en économie (voir par exemple Blackorby et al. (2001)) et qui est également défendu par un grand nombre de philosophes anglo-saxons (voir par exemple instance Griffin (1986) and Summer Summer (1996)).

ne considère pas comme étant problématique d'un point de vue éthique les différences de revenu attribuables à des facteurs comme le niveau d'effort résultant du libre exercice de la responsabilité individuelle. Cette approche recommande donc des politiques qui réduisent les inégalités résultant de facteurs, appelés *circonstances* par Roemer (1998), pour lesquels les individus ne peuvent être tenus responsables tout en laissant ces derniers assumer les pleines conséquences de l'exercice de leur responsabilité. Cette approche suggère donc qu'une attention particulière soit accordée aux contributions respectives des circonstances et de la responsabilité individuelle dans la détermination des inégalités individuelles de revenu.

Dans ce rapport, nous mobilisons successivement ces deux théories éthiques pour comparer plusieurs grands pays occidentaux sur le plan des inégalités en ne limitant pas les comparaisons au seul revenu disponible.

A l'intérieur du cadre bien êtriste, la première partie du rapport présente des comparaisons de treize pays occidentaux (Allemagne, Australie, Autriche, Canada, Espagne, Etats-Unis, France, Italie, Portugal, Suède, Suisse et Royaume Uni sur la base de l'inégalité de leurs citoyens en terme de revenu disponible et d'accès à certains *services publics*. Il nous apparaît en effet important d'introduire les biens publics dans la liste des attributs individuels pertinents pour l'évaluation des inégalités. Les pays occidentaux présentent en effet des différences substantielles dans la manière avec laquelle ils fournissent des services publics à leurs citoyens. Dans des pays unitaires et centralisés comme la France, le Royaume Uni et le Portugal, la fourniture de services publics est décidée dans une large mesure au niveau de l'Etat central et est considérée, pour cette raison, comme fournissant un accès à une quantité et une qualité de service public relativement égale sur l'ensemble du territoire national. Dans des pays à structure fédérale comme l'Allemagne, le Canada ou les Etats-Unis, ainsi que dans des pays unitaires décentralisés comme l'Italie ou l'Espagne, la fourniture de plusieurs services publiques est décidée dans une certaine mesure à des niveaux infra-nationaux (province, région ou état) et peut, pour cette raison, varier de manière significative d'une région à l'autre. Même si plusieurs pays fédéraux s'efforcent de corriger ces possibles inégalités interrégionales d'accès aux services publics au moyen de mécanismes de péréquation fiscale,[2] il existe une croyance répandue au sein des pays centralisés à structure unitaire que les pays à structure fédérale présentent une moins bonne performance en matière d'égalité dans l'accès de leurs citoyens aux services publics. Ce type de croyance a certainement nourri, en France, une partie de l'opposition à la politique de décentralisation amorcée par le gouvernement Raffarin. Il apparaît donc sur plan intéressant d'examiner la performance comparative des pays fédéraux/décentralisés et des pays unitaires/centralisés en matière d'inégalité à la fois en termes d'accès au service

[2]La exemple, l'article 36.2 the la constitution canadienne énonce que :

> "le parlement et le gouvernement du Canada s'engagent sur le principe de mettre en place des paiements de péréquation pour s'assurer que les gouvernements provinciaux ont des recettes suffisantes pour fournir des niveaux de service public raisonnablement comparables à des niveaux de prélèvement fiscaux raisonnablement comparables".

Des mécanismes analogues sont enchassés dans la constitution de la République Fédérale Allemande.

public et de revenu disponible.

L'introduction, en plus du revenu disponible, d'un indicateur d'accès aux services et aux biens publics est également utile pour éclairer les comparaisons de distributions de revenus disponibles entre des pays présentant des différences substantielles dans la taille de leur secteur public. Par exemple, une comparaison naïve de la distribution de revenu disponible entre les Etats-Unis et l'Italie au moyen d'un critère standard comme la courbe de Lorenz généralisée va révéler une dominance de l'Italie par les Etats-Unis. Une bonne part de cette dominance provient du fait que le revenu disponible *per capita* des Etats-Unis est plus du double de celui de l'Italie. Dans la mesure où le Produit Intérieur Brut (PIB) per capita de l'Italie représente environ 70% du PIB américain, la grande différence de revenu disponible par tête entre les deux pays provient du fait que l'Italie prélève en impôts et en taxes une plus grande fraction de son revenu brut moyen que les Etats-Unis. Une comparaison plus approfondie des Etats-Unis et de l'Italie sur la base des inégalités et du bien être devrait donc tenir compte de l'utilisation faite par les deux pays de leurs recettes fiscales. Si ces dernières sont uniquement utilisées pour financer des paiements de transferts, une analyse limitée aux distributions de revenu disponible paraît adéquate. Mais si les recettes fiscales sont utilisées en partie pour financer la production de biens et de services publics (comme cela semble être le cas pour la comparaison des Etats-Unis et de l'Italie) d'une manière qui est susceptible de varier d'une région à l'autre, il apparaît important de comparer la distribution jointe de ces biens publics et des revenus disponibles.

La première partie du rapport s'intéresse de manière spécifique à la distribution de deux biens publics mesurés à un niveau infra-national : *Le taux de mortalité infantile* et *le nombre moyen d'élèves par classe dans les écoles publiques.* Comme indicateur de bien public, le taux régional de mortalité infantile est interprété comme la probabilité qu'une décision, prise par un ménage moyen dans la région, d'avoir un enfant conduise à la mort de cet enfant avant l'âge d'un an. Cette probabilité est interprétée comme étant un output brut du système de santé de la région du pays où il est mesuré, output qui dépend à la fois de l'information disponible aux parents en matière de prévention de la mortalité infantile (par exemple en subissant des examens médicaux réguliers durant et après la grossesse) et de la qualité des hopitaux et du personnel médical. Même si le niveau et la répartition interrégionale de cet output brut ne peut pas être uniquement attribué à la politique publique, il paraît incontestable qu'il en dépend dans une large mesure. Le nombre moyen d'élèves par classe dans les écoles publiques peut s'interpréter, pour sa part, comme le résultat quasi exclusif de la dépense publique. Par contre, et contrairement à la prévention du risque de mortalité infantile dont on peut penser qu'il est une source intrinsèque de contribution au bien être individuel (au moins pour des adultes qui envisagent de se reproduire), la taille moyenne des classes dans les écoles publiques ne semble pas affecter *directement* le bien être des individus (les parents ou les enfants ne tirent aucun bien être du fait intrinsèque de recevoir un enseignement dans une classe de petite taille). C'est uniquement l'effet, assez méconnu du reste, des petites classes sur l'accumulation du capital humain qui semble être une source de concernement pour les individus concernés.

Une critique fréquente adressée à l'approche bien êtriste en tant que théorie de la justice concerne la difficulté qu'il peut y avoir, pour une agence gouvernementale ou un planificateur social, à rassembler l'information nécessaire pour procéder aux *comparaisons interpersonnelles* de bien être que cette approche requiert. De fait, l'emploi opératinel du bien êtrisme exige que les niveaux de bien être atteints par les individus en fonctions des ressources externes et internes dont ils disposent soit comparables d'un individu à l'autre. Si cette hypothèse ne pose pas de difficultés insurmontables au niveau du théorique - on peut très bien accepter l'idée que les niveaux de bien être, convenablement mesurés, des différents individus soient comparables entre eux - elle soulève la difficulté pratique de savoir par quelle fonction précise on peut mesurer le bien être individuel. Nous prenons dans ce rapport ce problème au sérieux en utilisant une approche en termes de *dominance* qui reste agnostique sur la manière précise avec laquelle le bien être est supposé mesuré. L'approche en terme de dominance procède en supposant simplement de la fonction qui associe un niveau de bien être à chaque configuration d'attributs possibles qu'elle satisfait un certain nombre de propriétés minimales et en exigeant l'*unanimité* de tous les classements bien êtriste qui supposent du bien être individuel qu'il dépend des ressources externes par une fonction satisfaisant ces propriétés. [3] Cette approche en terme de dominancee souffre du défaut de ne fournir qu'un classement *incomplet* des distributions d'attributs individuels (un pays peut en dominer un autre pour une spécification particulière de la fonction de mesure du bien être individuel mais le classement inverse peut prévaloir pour une autre spécification de cette fonction). Par contre, lorsque cette approche conduit à un classement, celui-ci s'avère particulièrement robuste sur le plan éthique. De fait, l'approche en terme de dominance que nous mobilisons dans ce rapport parvient à classer de manière robuste environ 30% des paires de pays concernés. Nous pouvons, en spécifiant la fonction de bien être et en l'agrégeant par un indice d'inégalités multidimensionnelles, compléter ce classement, tout en étant conscient du caractère contestable et arbitraire des comparaisons de pays non-classés par l'approche en termes de dominance.

Les grands résultats que nous tirons de ces classements sont les suivants. D'abord, l'élargissement multidimensionnel des comparaisons de pays en termes d'inégalité conduit à une réduction significative du classement des pays "anglo-saxons" (Australie, Etats-Unis et Royaume Uni) et une amélioration notable du classement d'un pays comme le portugal par rapport à ce qui résulterait d'une analyse unidimensionnelle basée sur la seule distribution des revenus disponibles. Par ailleurs, les résultats ne révèlent aucune tendance claire, pour les pays fédéraux, à exhiber des performances moindres en matière d'égalité que leurs homologues unitaires. Finalement, les résultats révèlent une corrélation entre les indices d'inégalités unidimensionnelles fondés sur la seule distribution des revenus disponibles et leurs généralisations multidimensionnels. Ce dernier état de faits suggère au moins que la pratique répandue de limiter les comparaisons internationales à la seule distribution du

[3] Une liste de références classiques à l'approche en termes de dominance dans la littérature économique inclurait, dans le cas unidimensionnel, Atkinson (1970), Kolm (1969) Sen (1973). Comme nous l'expliquons dans la première partie du rapport, l'approche en termes de dominance est beaucoup moins développée dans le contexte multidimensionnel.

revenu disponible ne fournit pas une apperçu grossièrement mauvais de la performance relative des principaux pays occidentaux en matière d'inégalités multidimensionnelles.

La seconde partie du rapport propose pour sa part une comparaison internationale des inégalités qui se fait l'écho de l'approche en termes des inégalités des opportunités. De manière spécifique, cette partie du rapport s'emploie à mesurer et à comparer l'égalité des opportunités d'acquisition du revenu disponible dans quelques pays de l'OCDE. Ici encore, les résultats sont destinés à compléter les comparaisons plus traditionnelles d'inégalités en termes de revenu disponible. Pour cette raison, nous nous intéressons à la manière avec laquelle les performance relatives des pays en matière d'égalité des opportunités sont reliées à celles qui concernent l'égalité des revenus disponibles. De fait, si certains pays adoptent des politiques qui favorisent l'égalité des opportunités plutôt que celle des revenus disponibles finaux, on pourrait s'attendre à observer des différences dans le classements des pays sur la base de ces deux critères. Mais il est également possible que les pays qui favorisent l'égalité des revenus disponibles soient aussi ceux qui favorisent l'égalité des opportunités.

La mesure du niveau d'atteinte d'un objectif d'égalisation des opportunités d'acquisition du revenu disponible est une tâche difficile qui n'a été entreprise que dans de très rares travaux (voir par exemple Roemer et al. (2003)). En particulier, la partition des attributs individuels entre les "circonstances", dont l'individu ne peut pas être tenu pour responsable, et les éléments qui appartiennent à sa sphère de responsabilité ne va pas de soi. Egalement difficile est la mesure de la contribution des circonstances à l'inégalité totale observée. Differentes méthodes ont été proposées dans la littérature pour résoudre ces problèmes. Dans ce rapport, nous assimilons le *milieu social d'origine* de chaque individu à ses "circonstances". De fait, il semble y avoir unanimité chez les tenants de l'approche en terme d'inégalité des opportunités pour voir dans le milieu social d'origine de l'individu un facteur déterminant de sa réussite sociale - au moins mesurée à l'aulne du revenu disponible - pour lequel l'individu ne saurait être tenu pour responsable. La définition de l'égalité des opportunités qui est proposée dans ce rapport, et qui est développée en détail dans la seconde partie, est basée sur la notion d'inégalité *conditionelle*. Nous adoptons en effet le point de vue suivant lequel une comparaison d'inégalités d'opportunités peut s'assimiler à une comparaison de distributions de revenus disponibles conditionnelles au milieu social d'origine des individus. Reste à savoir comment mesurer de manière synthétique le milieu social d'origine de l'individu. Une étude conduite en Inde serait, à l'évidence, encline à s'intéresser à la caste des parents. Dans le cadre des quelques pays de l'OCDE considérés ici, nous retiendrons comme variable de "conditionnement" une mesure discrète du niveau d'éducation des parents qui est reconnu par de nombreuses études comme exerçant un effet déterminant sur la réussite sociale des enfants.

Les données que nous utilisons dans ce rapport ont été collectées par Roemer et al. (2003) avec pour objectif spécifique de véhiculer de l'information sur les revenus disponibles et les milieux sociaux d'origines d'échantillons représentatifs d'individus dans le cadre de l'approche en termes d'égalité des opportunités. Cette base de donnée a été construite à partir d'échantillons d'individus collectés par les agences statistiques de neuf pays : Allemagne, Belgique, Etats-Unis, France, Italie, Pays Bas, Norvège, Royaume Uni et

Suède. Ces bases de données contiennent de l'information détaillée sur différentes sources de revenus disponibles ainsi que, de manière moins complète, sur l'éducation du père de chaque individu échantilloné.

L'analyse révèle, là aussi, un certain nombre de faits intéressants. Le plus important d'entre eux est le constat que, à l'instar de ce que suggére l'analyse multidimensionnelle de la première section du rapport, il existe une corréelation significative entre le classement des pays en termes d'égalité de revenus disponibles et celui en termes d'égalité des opportunités. Ici aussi donc, un classement des pays standard en termes d'égalités des revenus ne fournit pas une mauvaise approximation du classement de ces pays en termes d'égalité des opportunités. Ceci étant, la corrélation n'est pas parfaite, et les imperfections semblent concerner les pays qui apparaissent dans la tranche intermédiaire des classements. De fait, il apparaît que des pays comme la France et l'Italie ont une moins bonne performance en matière d'égalité des opportunités que le laisserait suggérer leur classement en termes d'égalité des revenus alors qu'un verdict quelque peu opposé semble s'appliquer à des pays comme l'Allemagne ou les Pays Bas.

Le reste du rapport est organisé de la manière suivante. Le premier chapitre, rédigé par N. Gravel et B. Tarroux, décrit la méthodologie et les résultas obtenus dans le contexte des comparaisons internationales de distributions de revenus disponibles et d'accès à des biens publics en terme de dominance bien êtriste. Le second chapitre, écrit par A. Lefranc, N. Pistolesi et A. Trannoy est consacré pour sa part à l'approche en termes d'égalité des opportunités. Les deux chapitres sont rédigés de manière à servir de support à des publications ultérieures dans des revues académiques. Cet objectif explique le choix linguistique qui a été fait pour leur rédaction. Ils peuvent, par ailleurs, être lus de manière séparés.

2. MULTIDIMENSIONAL INEQUALITY IN DISPOSABLE INCOME AND ACCESS TO PUBLIC GOODS

1. Introduction

It has long been recognized (see for instance Kolm (1976a), Kolm (1976b), Kolm (1977), Atkinson and Bourguignon (1982), Atkinson and Bourguignon (1987), Rawls (1971), Sen (1987) and Sen (1992)) that monetary income is not the only individual attribute that is relevant for distributive justice. Yet this widely, and increasingly, held belief in the truly multidimensional nature of human beings has failed so far to give rise to empirical implementations. Much applied comparisons of communities (countries, provinces, etc.) on the basis of their performance in achieving better redistributive objectives continue to be performed solely in terms of a single numerical indicator, very often identified with monetary income.[1]. It is only recently that a timorous alteration of this unidimensional trend, noticeably through the (yet unpublished) work of Crawford (2005) and Duclos et al. (2003),has appeared.

The insufficient development of the theory of multidimensional inequality measurement, as compared to its unidimensional cousin, contributes certainly for no small part to this state of affairs. Unidimensional measurement of inequality theory has reached for quite a long time its full fruition. It rides on an equivalence, first established by Hardy et al. (1952), and popularized, among economists, by Sen (1973) (see also Kolm (1969) and Dasgupta et al. (1973)) between four plausible answers to the basic question : When can we say that a distribution A of one attribute between n individuals is unambiguously more equal than another distribution B ? Assuming that the total amount of the attribute to be distributed is the same in both distributions, the four equivalent answers to this question are :

(1) When A could be obtained from B by a finite sequence of bilateral transfers between a richer individual to a poorer one.

(2) When A would be ranked above B by all utilitarian social planners who assume that individuals convert income into utility by the same *increasing* and *concave* utility function.

(3) When poverty, as measured by the poverty gap, is lower in A than in B no matter

[1] Another unidimensional indicator that is the object of a growing literature is health (see e.g. Allison and Foster (1999)).

what is the poverty line.

(4) When the Lorenz curve associated to A lies everywhere above that corresponding to B.

These equivalences, which have been generalized to distributions involving different total quantities of the attribute and/or different numbers of individuals,[2] are the theoretical foundation of unidimensional inequality measurement. By themselves, they enable an unambiguous ranking of unidimensional distributions with non-crossing Lorenz curves. As clearly illustrated in many applied studies, non-intersecting Lorenz curves can be observed in quite a significant fraction of cases. And, when more ethically demanding inequality indices are used by researchers to compare all distributions, the compatibility of these indices with the incomplete ranking provided by any of the four answers is usually considered to be a minimal requirement.[3]

While we do not have at our disposal such a theoretical foundation for measuring multidimensional inequalities, the (slow) progresses that have been made in the last twenty years on this question do not make us completely deprived on that matter. Following the important contribution by Atkinson and Bourguignon (1982)) and the less noticed one of Bourguignon (1989), as well as the more recent work by Fleurbaey et al. (2003) or Gravel and Moyes (2004), we dispose of a few operational dominance criteria that rank alternative distributions of *two* attributes in the same way as would all utilitarian social planners believing that individuals convert attributes into utility by the same function satisfying specific properties.[4] The main property considered in this paper is *decreasing increasingness* with respect to all attributes. By this, we mean the requirement for the marginal utility of every attribute to be positive and decreasing with respect to the two attributes. This property implies therefore the requirement of the utility function to be *concave with respect to every attribute*, everything else being the same. It also implies the decreasingness of marginal utility of one attribute with respect to other attributes, reflecting a *substitutability* between attributes as means of achieving a given level of well-being. Translated into distributional or statistical language, this property captures the idea that, at given marginal distributions of the attribute, reducing the correlation

[2] Distributions with different numbers of individuals can be made comparable by applying the so-called Dalton population principle according to which the replication of the same distribution an arbitrary number of time does not change its distributional characteristics. Distributions with different total amount of the attribute can be unambigously compared just as those with the same total amount if increments of attribute are added to the bilateral transfers in the answer (1) and the Lorenz curve is replaced by the generalized Lorenz one (see e.g. Shorrocks (1983)) in the answer (4).

[3] For instance, the failure of the variance of the logarithm to satisfy this requirement explains the drastic reduction in the use of this particular inequality index in applied studies observed in the last 30 years.

[4] While the multidimensional approach used in this report follows the route opened up by Atkinson and Bourguignon (1982), this is not the only way to proceed. See Kolm (1976a), Kolm (1976b), Kolm (1977), Koshevoy (1995) and Koshevoy (1998) for alternative approaches and Trannoy (2004) and Weymark (2004) for recent surveys. The translation of the Atkinson and Bourguignon criteria into rankings that are agreed upon by all utilitarian judgements based on "plausible" properties on the individual utility function is not easy if the number of attributes is increased. See Hadar and Russell (1974) (theorem 5.8) for an overview of the difficulties.

between the two attributes reduces inequalities. It is, of course, an unavoidable feature of multidimensional comparisons that they involve, along with an examination of the distribution of every attribute in isolation, an investigation of the way by which different attributes are *co (r) related*.

Of course requiring unanimity of all utilitarian social planners when individual utility depends on two attributes is likely to be more demanding than when individual utility depends on one attribute. When two attributes are considered, conditions on each attribute must be checked along with conditions on the correlation between the attributes. It is therefore to be expected that the fraction of conclusive rankings that can be agreed upon by all utilitarian planners who assume that the individual utility function satisfy the aforementioned property will be lower in the multidimensional case than in the unidimensional one. Yet, thanks to the work of Bourguignon (1999), Maasoumi (1986) and Tsui (n.d.), we have also have at our disposal a few multidimensional inequality indices that enable us to rank any pair of two-dimensional distributions in a way that is compatible with the incomplete ranking provided by utilitarian unanimity.

This chapter is therefore an attempt to depart from the unidimensional practice of inequality appraisal and to compare inequalities across twelve OECD countries (Australia, Austria, Canada,France, Germany, Italy, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States) in terms of both private disposable income and access to *public goods*. It is organized as follows. In the next section, we present the theoretical criteria and indices used to perform the comparisons. Section 3 discusses the data and the methodology used to perform the tests as well as the results of the comparisons and section 4 provides some conclusion.

2. Presentation of the criteria and indices

2.1. Unidimensional analysis

We first recall the well-known criteria used to compare distributions of one attribute (income) assumed to be cardinally measurable and transferable across n households.[5] This later assumption is quite crucial for the interpretation of the methodology adopted here (see e.g. Allison and Foster (1999) for a unidimensional comparisons of the distributions of health indicators not assumed to be cardinally measurable).

Let x and $y \in \mathbb{R}_+^n$[6] be two distributions of the attribute between the n households indexed by i $(i = 1, ..., n)$. We interpret x_i and y_i as the amount of the attribute received by household i in distributions x and y respectively. Let $\mu(x) = \sum_{i=1}^{n} \frac{x_i}{n}$ and $\mu(y) = \sum_{i=1}^{n} \frac{x_i}{n}$ denote

[5]We focus the discussion on the case where the number of individuals is the same. As is well-known, cases where the number of individuals differ between distributions can be transformed into cases with the same number of individuals after appropriate replications of the distributions.

[6]The assumption for the attribute to be measured by a non-negative number is not essential for most of the results but is nonetheless adopted to enhanced intuition (negative incomes are rather rare, even though they arise sometimes in data set on disposible income).

the mean of x and y respectively. For every vector z in $\mathbb{R}^n$, we denote by $z_{(.)} = (z_{(1)}, ..., z_{(n)})$ the ordered permutation of z such that, for all $i = 1, ..., n-1$, $z_{(i)} \leq z_{(i+1)}$.

Much of the comparisons performed in this chapter are based on the symmetric utilitarian criterion. Let $U : \mathbb{R}_+ \to \mathbb{R}$ be a particular utility function that transforms the attribute into individual well-being. For the utility function U, the symmetric utilitarian criterion ranks x above y if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$. As shown in D'Aspremont and Gevers (1977) (see also Denicolò (1999) for a recent and concise proof of this), utilitarianism is the only Pareto inclusive and anonymous way to aggregate individual utilities into a social ranking when individual utility is assumed to be cardinally measurable and when utility differences are assumed to be interpersonally comparable. The symmetric requirement that all individuals use the same function to convert the attribute into well-being seems genuine to the unidimensional nature of the analysis. If two individuals where different in their ability to convert the attribute into well-being, this difference should be accounted for and included in the analysis, which would then become multidimensional.

Obviously, the assumption that the social planner has all the required information to measure utility cardinally and to perform interpersonal comparison of utility differences that justifies the use of utilitarianism is a strong one. A more acceptable assumption, which lies behind the dominance approach, is to assume that the social planner is willing to measure utility cardinally and to perform interpersonal comparisons of utility differences, but does not know which exact function to use. It only knows that the function satisfies some basic properties and, being careful, it only accepts to make a definite ranking of two distributions when the symmetric utilitarian criterion ranks them in the same fashion for all the utility functions satisfying the properties.

We define formally the concept of utilitarian dominance for a class of utility functions $\mathbb{U}$ satisfying a specific set of properties as follows.

Definition 1. ***(Utilitarian dominance)****. We say that x utilitarian dominates y for the class of functions $\mathbb{U}$, denoted $x \succeq_{\mathbb{U}} y$, if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$ for all utility functions U in $\mathbb{U}$.*

The following two classes of utility functions are typically considered in unidimensional analysis :

$\mathbb{U}^{U1} = \{U : \mathbb{R}_+ \to \mathbb{R}$ such that U is increasing$\}$ and

$\mathbb{U}^{U2} := \{U : \mathbb{R}_+ \to \mathbb{R}$ such that U is increasing and concave$\}$[7]

As recalled in introduction, a well-known accomplishment of unidimensional inequality measurement theory has been to provide easy-to-check operational statistical criteria that are equivalent to alternative notions of utilitarian dominance. For the sake of completeness, we recall what are these criteria and the equivalences.

Definition 2. ***(Quantile Dominance)****. We say that x quantile dominates y, denoted $x \succeq_Q y$, if $x_{(i)} \geq y_{(i)}$ for all i.*

[7] A function $g : A \to \mathbb{R}$ where $A \subseteq \mathbb{R}^k$ for $k = 1, 2, ..$ is increasing if $a \geq b \Rightarrow g(a) \geq g(b)$ and is concave if $g(\lambda a + (1-\lambda)b) \geq \lambda g(a) + (1-\lambda)g(b)$ for every a and b in A and any $\lambda \in [0, 1]$.

In words, x quantile dominates y if the ith poorest individual in x has at least as much of the attribute as the ith poorest individual in y. Clearly, such a dominance can never be observed between two distributions of the same total quantity of the attribute.

Definition 3. ***(Headcount poverty dominance)***. *We say that x headcount poverty dominates y, denoted $x \succeq_{Hp} y$, if $\#\{i : x_i \leq t\} \leq \#\{i : y_i \leq t\}$ for every possible poverty threshold $t \in \mathbb{R}_+$.*

In words, x headcount poverty dominates y if the *number* of individuals whose income are below some poverty line is lower in x than in y *no matter what is the poverty line*. This criterion is nothing else than a discrete version of the first order stochastic dominance one (see e.g. Hadar and Russell (1974) for a classical statement).

Definition 4. ***(Generalized Lorenz dominance)***. *We say that x generalized Lorenz dominates y, denoted $x \succeq_{GL} y$ if, for every $k = 1, ..., n$, it is the case that $\sum_{i=1}^{k} x_{(i)} \geq \sum_{i=1}^{k} x_{(i)}$.*

In words, x generalized Lorenz dominates y if the total quantity of the attribute possessed by the k poorest individuals in x is at least as large as the corresponding quantity possessed by k poorest individuals in y. The numbers $\frac{\sum_{i=1}^{k} x_{(i)}}{k}$ and $\frac{\sum_{i=1}^{k} y_{(i)}}{k}$ are the values of the ordinates of the Generalized Lorenz curves (see Shorrocks (1983)) for x and y, respectively, that connect the points associating, to every rank k in the (ascending ordered) income distribution ($k = 1, ..., n$), the average income held by all individuals with rank lower than k.

Definition 5. ***(Poverty gap dominance)***. *We say that x poverty gap dominates y, denoted $x \succeq_{PG} y$ if, for every poverty threshold t, it is the case that $\sum_{i=1}^{n} \max(t - x_i, 0) \leq \sum_{i=1}^{n} \max(t - .y_i, 0)$*

In words, x poverty gap dominates y if, no matter what is the poverty threshold, a lower quantity of the attribute is needed in x than in y to eliminate totally the poverty defined by the threshold. This criterion is a discrete version of second order stochastic dominance.

In the following proposition, we summarize the well-known equivalences that exist between some of these criteria and utilitarian dominance.

Proposition 1. *$x \succeq_{\mathbb{U}^{U1}} y \Leftrightarrow x \succeq_{Q} y \Leftrightarrow x \succeq_{Hp} y$ and $x \succeq_{\mathbb{U}^{U2}} y \Leftrightarrow x \succeq_{GL} y \Leftrightarrow x \succeq_{PG} y$*

When the total amount of the attribute differs across distributions, the generalized Lorenz criterion is sensitive to both the overall available quantity and its dispersion. For this reason, some economists assert that the generalized Lorenz criterion is not a "pure inequality" criterion and prefer referring to it as a "welfare criterion". It is widely believed that a "pure inequality" criterion should, ideally, be independent from the "size of the cake". There are two standard ways to think about this independence : A *relative* and an *absolute* one. In the relative conception of inequality, it is the distribution of the *ratio* of the individual income over the mean income (or the total income) that is deemed important. In the absolute conception, the proper *equalisandum* is assumed to be the *difference* between the individual income and the mean (or the total income). Despite arguments given in favour of an absolute notion of inequality by Kolm (1976a) and Moyes (1987), it is the relative notion which has received the most attention in the applied literature.

We follow this tradition by using, on occasion, the relative Lorenz domination criterion defined as follows.

Definition 6. ***(Relative Lorenz dominance)****. We say that x relatively Lorenz dominates y, denoted $x \succeq_{RL} y$ if, for every $k = 1, ..., n$, it is the case that $\sum_{i=1}^{k} \frac{x_{(i)}}{\mu(x)} \geq \sum_{i=1}^{k} \frac{y_{(i)}}{\mu(y)}$.*

It is, probably, worth saying that the "pure inequality" considerations which underlie the Relative Lorenz criterion when applied to distributions with different amounts of the attribute are not easy to rationalize in a utilitarian or, for that matter, welfarist framework where individual are selfish. Applying the result of proposition 1, the only way to do this rationalization is to assume that individuals derive utility from the *ratio* of their income over the mean. This implies, in particular, that the well being of every individual is decreasing with respect to the amount of attribute held by others, everything else being the same. Hence, rationalizing pure inequality attitudes in a utilitarian setting is akin to assuming that individuals are envious or jealous.

In cases where relative and/or generalized Lorenz criteria are inconclusive, we resort to the strategy of ranking income distribution by the utilitarian utility function underlying the Atkinson-Kolm inequality index. This amounts to comparing, across distributions, the number $W_{AK}(z;\varepsilon)$ defined, for any distribution z of one attribute, and every number $\varepsilon > 0$, by

$$I_{AK}(z;\varepsilon) = \sum_{i=1}^{n} \frac{1}{1-\varepsilon} z_i^{1-\varepsilon} \tag{2.1}$$

if $\varepsilon \neq 1$ and by

$$I_{AK}(z;1) = \sum_{i=1}^{n} \ln z_i$$

otherwise. This particular choice of a family of complete rankings of income distributions is largely motivated by our desire to make comparisons with their bidimensional counterparts.

2.2. Multidimensional analysis

Assume now that there are two attributes. A distribution z of the two attributes is described as a $2 \times n$ matrix of non-negative numbers[8] which we write as

$$z = \begin{bmatrix} z_{11} & z_{21} & ... & z_{n1} \\ z_{12} & z_{22} & ... & z_{n2} \end{bmatrix}$$

where, for every $i = 1, ..., n$ and $j = 1, 2$, z_{ij} represents the amount of attribute j received by individual i in the distribution z.

Using, for the reasons sketched in the previous section, utilitarian dominance as the basic normative criterion for comparing alternative distributions of the two attributes, our task is to propose plausible properties that individual utility could satisfy when it is assumed to be a function of two attributes. To define these properties, it is convenient, but not necessary, to assume that the utility function is differentiable with respect to its 2 argument to the required degree. For every function G of k variables ($k \geq 2$), we denote by $G_j(z)$ its jth partial derivative evaluated at the 2dimensional vector z. The class of utility functions that are considered are the following.

$\mathbb{U}^{M1} = \{U : \mathbb{R}^2_+ \to \mathbb{R}$ such that $U_j(z) \geq 0$ and $U_{12}(z) \leq 0$ for every $z \in \mathbb{R}^2_+$ and $j = 1, 2\}$

$\mathbb{U}^{M2} = \{U : \mathbb{R}^2_+ \to \mathbb{R}$ such that $U_j(z) \geq 0$ for $j = 1, 2$, $U_{12}(z) \leq 0$ and either $U_{11}(z)$ or $U_{22}(z)$ for every $z \in \mathbb{R}^2_+\}$

$\mathbb{U}^{M3} = \{U : \mathbb{R}^2_+ \to \mathbb{R}$ such that $U_j(z) \geq 0$, $U_{jk}(z) \leq 0$, $U_{112}(z) \geq 0$, $U_{122}(z) \geq 0$ and $U_{1122}(z) \leq 0$ for every $z \in \mathbb{R}^k_+$, $j = 1, 2$ and $k = 1, 2\}$

Functions in $\mathbb{U}^{M1}$ have the property of being *increasing* with respect to every attribute, and of having a marginal utility of every attribute that is *decreasing* with respect to the other attribute. Functions in $\mathbb{U}^{M2}$ satisfy, in addition to the properties of the functions in $\mathbb{U}^{M1}$, the intuitively plausible requirement that the marginal utility of at least one attribute is decreasing with respect to the level of that attribute. Notice however that the functions in the class $\mathbb{U}^{M2}$ are *not* required to satisfy the property that the marginal utility of each attribute be decreasing with respect to the same level of attribute. Functions in $\mathbb{U}^{M3}$ satisfy, in addition to the properties satisfy by the functions in $\mathbb{U}^{M2}$, the (less intuitive) requirements that the rate of decrease in the marginal utility of any attribute should be decreasing with respect to the other attribute, and that the decrease in the rate of decrease should it self be increasing with respect to the attribute!!!.

Atkinson and Bourguignon (1982) have proposed two operational criteria that are equivalent to the rankings provided by all utilitarian planners who assume that individual utility functions are in $\mathbb{U}^{M1}$ and $\mathbb{U}^{M3}$ respectively. For the sake of completeness, we recall the definition of these criteria, which we reframe in the discrete setting considered here.

Definition 7. ***(Bidimensional headcount poverty dominance)*** *Distribution x bidimensionally headcount poverty dominates distribution y, denoted $x \succeq_{Hp2} y$ if, for every pair of poverty thresholds t_1 and t_2, $\#\{i : (x_{i1}, x_{i2}) \leq (t_1, t_2)\} \leq \#\{i : (y_{i1}, y_{i2}) \leq (t_1, t_2)\}$.*

[8] We maintain the assumption that the quantities of each attribute is non-negative even though, here again, it is not essential.

In words, x bidimensionally headcount poverty dominates y if, for every two poverty thresholds (one such threshold for every attribute), the number of individuals that are considered poor with respect to the two attributes is lower in x than in y. This criterion is a straightforward generalization of the unidimensional poverty head count dominance one where people can be poor with respect to either of the attribute. It can be noted that if x bidimensionally headcount poverty dominates y, then x headcount poverty dominates y for every attribute but that the converse does not hold.

The second criterion, first introduced by Atkinson and Bourguignon (1982), is a two-dimensional generalization of the poverty gap dominance criterion presented above.

Definition 8. *(**Bidimensional poverty gap dominance**) Distribution x bidimensionally poverty gap dominates y, denoted $x \succeq_{BPG} y$, if, for all poverty lines t_1 and t_2 one has $\sum_{i=1}^{n} \max(t_1 - x_{i1}, 0) \max(t_2 - x_{i2}) \leq \sum_{i=1}^{n} \max(t_1 - y_{i1}, 0) \max(t_2 - y_{i2}, 0)$*

In words, x bidimensionally poverty gap dominates y if, for all pairs of poverty lines (one such line for every attribute), the product of the amounts of the two attributes that will be necessary to eliminate all poverty defined by the poverty lines is lower in x than in y.

The third operational criterion considered in this paper is a generalization of the Bourguignon (1989) criterion due to Gravel and Moyes (2004). It is defined formally as follows.

Definition 9. *(**Bidimensional ordered poverty gap dominance**) Distribution x dominates distribution y by the bidimensional ordered poverty gap criterion, denoted $x \succeq_{OPG} y$ if, for all attributes j ($j = 1, 2$), and all sequences $\{t_1^h, t_2^h\}_{h=1}^k$ of k pairs of poverty lines ordered in such a way that $0 = t_1^1 \leq t_1^2 \leq ... \leq t_1^k$ and $t_2^1 \geq t_2^2 \geq ... \geq t_2^k = 0$, one has :*

$$\sum_{h=1}^{k-1} \sum_{\{i:x_{i1} \in [t_1^h, t_1^{h+1}]} \max(t_2^h - x_{i2}, 0) \leq \sum_{h=1}^{k-1} \sum_{\{i:y_{i1} \in [t_1^h, t_1^{h+1}]} \max(t_2^h - y_{i2}, 0)$$

and

$$\sum_{h=1}^{k-1} \sum_{\{i:x_{i2} \in [t_2^{k+1-h}, t_2^{k-h}]} \max(t_1^{k+1-h} - x_{i1}, 0) \leq \sum_{h=1}^{k-1} \sum_{\{i:y_{i2} \in [t_2^{k+1-h}, t_2^{k-h}]} \max(t_1^{k+1-h} - y_{i2}, 0)$$

In words, x dominates y by the bidimensional ordered poverty gap criterion if, for every negatively co-monotone sequence of pairs of poverty lines (one such pair for each attribute) the minimal amount of each attribute that is necessary to eliminate the poverty defined by these lines is lower in x than in y. The basic innovation of this criterion, as compared to its unidimensional poverty gap dominance cousin, is to admit the idea, underlying the property of negative co-monotonicity, that the *attribute-specific poverty lines* used to measure poverty in a distribution of two attributes must be *decreasing with respect to the other attribute*. At a given level of wealth, an individual who benefits from good public services should be less likely to be considered poor than her co-citizen who suffers from a low level of public service.

This criterion is a generalization of the one proposed by Bourguignon (1989) for comparing distributions of a transferable attribute between households differing in terms of their need, in the specific case where the distribution of households into need categories (different values of the other variable) is assumed to be fixed. We note that this criterion is quite demanding as it requires, for each negatively co-monotone sequence of pairs of poverty lines, that the dominating distribution be less poor, as per the poverty gap criterion, than the dominated one for both attributes. At first blush, the Bidimensional ordered poverty gap dominance criterion may be considered difficult to implement, because of the need to consider all logically possible decreasing poverty line function for every attribute. Yet, it is easy to see that the only poverty lines that matter for the criterion are those which correspond to the observed levels of the attribute in the two distributions under consideration. As there are only a finite number of those poverty lines, the criterion is quite easy, if not computer time intensive, to implement. Furthermore, as will be discussed in section 3, it is possible to express this criterion in terms of an algorithm that reduces this computer time intensivity to a significant extent.

We now provide formally the statement of the equivalence between each of these three operational criteria and its utilitarian dominance counterpart.

Proposition 2. *(Atkinson and Bourguignon (1982)) For every two distributions of two attributes x and y, $x \succeq_{\mathbb{U}^{M1}} y \Leftrightarrow x \succeq_{Hp2} y$ and $x \succeq_{\mathbb{U}^{M3}} y \Leftrightarrow x \succeq_{BPG} y$.*

Proposition 3. *(Gravel and Moyes (2004)) For every two distributions of two attributes x and y, $x \succeq_{\mathbb{U}^{M2}} y \Leftrightarrow x \succeq_{OPG} y$.*

In view of these two propositions, as well as of the definitions of $\mathbb{U}^{M1}$, $\mathbb{U}^{M3}$ and $\mathbb{U}^{M3}$, it is clear that $x \succeq_{Hp2} y$ implies $x \succeq_{OPG} y$ and that $x \succeq_{OPG} y$ implies $x \succeq_{SL} y$ but that the reverse implications do not hold. Hence the Sequential Ordered Lorenz criterion is more discriminant than the Ordered Poverty Gap one who is in turn more discriminant than the Bidimensional Headcount Poverty criterion. As usual with dominance analysis, the increase in discriminatory power gained from switching from one criterion must be balanced against the decreasing plausibility of the properties of the individual utility function assumed in the corresponding utilitarian dominance criterion. The class $\mathbb{U}^{M3}$ may seem particularly exhausting in this respect.

We remark also that, among other things, both Bidimensional Ordered Poverty Gap dominance and Bidimensional poverty gap imply unidimensional Generalized Lorenz dominance dimension by dimension. Because of this, it is very likely that the requirement of dominance for either of the three bidimensional criteria will be more demanding than the corresponding unidimensional one. In view of this possible lack of discriminatory power of the dominance criteria in the bidimensional case, it may be of interest to perform the analysis by means of bidimensional inequality indices.

As in the case of unidimensional comparisons, we do this by specifying a particular individual utility function of the two attribute and by comparing alternative distributions of two attributes between n individuals on the basis of the sum of the individual utilities defined by this particular utility function. The sum of individual utilities can therefore be interpreted as an index of multidimensional inequality.[9]

In order to be consistent with the dominance criteria defined above, the individual utility function used to define I^U needs to belong to the class $\mathbb{U}^{M3}$ (and therefore, to $\mathbb{U}^{M2}$ and $\mathbb{U}^{M1}$). We consider two classes of such utilitarian indices.

The first of these two classes has been proposed by Bourguignon (1999). For every real numbers α, β and strictly positive real numbers w_1 and w_2, the index $I_B^U(\alpha, \beta, w_1, w_2; z)$ of this class is defined by

$$I_B^U(\alpha, \beta, w_1, w_2; z) = \sum_{i=1}^{n} \left[w_1 z_{i1}^{\beta} + w_2 z_{i2}^{\beta} \right]^{\frac{\alpha}{\beta}} \tag{2.2}$$

It can be checked that if $0 \leq \alpha \leq \beta \leq 0$, the utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$ defined by $U(x_1, x_2) = \left[w_1 x_1^{\beta} + w_2 x_2^{\beta} \right]^{\frac{\alpha}{\beta}}$ belongs to $\mathbb{U}^{M3}$. For this reason The Bourguignon index I_B^U is consistent with bidimensional headcount dominance, bidimensional ordered poverty gap dominance and sequential Lorenz dominance.

The other class of utility index that is considered herein is the *Cobb Douglas index* I_{CD}^U which, for any real numbers γ, r_1 and r_2, and any distribution z of two attributes between n individuals is defined by :

$$I_{CD}^U(r_1, r_2; z) = \sum_{i=1}^{n} \gamma z_{i1}^{r_j} z_{i2}^{r_2} \tag{2.3}$$

In order for the utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$ defined by $U(x_1, x_2) = \gamma x_1^{r_1} x_2^{r_2}$ to belong to $\mathbb{U}^{M3}$, it is necessary and sufficient that the numbers γ, r_1 and r_2 be negative. As the utilitarian criterion requires cardinal unit interpersonal comparisons of utility, there is no loss of generality in assuming $\gamma = -1$. Notwithstanding this restriction on the parameters γ, r_1 and r_2, this inequality index has been characterized by Tsui (n.d.) (theorem 4) for the case where the mean of each attribute is constant across distributions.

It can be noticed that when applied to distributions of one attribute between n individuals, the two classes of indices become members of the Atkinson-Kolm family defined by equation (2.1). Cobb-Douglas indices (for negative γ and r_1) cover the cases where $\varepsilon > 1$ while Bourguignon indices covers the case where $\varepsilon \in]0, 1[$.

As in the unidimensional case, one can apply any of the criteria or indices defined in this section to distributions of *individual shares* of the total of each attribute even though it is not easy to justify this practice by utilitarian arguments. Formally, defining $\mu(z_j) = \sum_{i=1}^{n} \frac{z_{ij}}{n}$ to be the symmetric mean of the attribute j (for $j = 1, 2$) in the bidimensional distribution z, this amounts to applying the aforementioned criteria and indices to the

[9] See Weymark (2004) for a recent survey of the so-called "ethical approach" to inequality indices applied to multidimensional inequality appraisal.

comparisons of the *normalized distributions* $\widetilde{z}$ defined, for every distribution $z \in \mathbb{R}^{2n}$ by :

$$\widetilde{z} = \begin{bmatrix} \frac{z_{11}}{n\mu(z_1)} & \frac{z_{21}}{n\mu(z_1)} & \cdots & \frac{z_{n1}}{n\mu(z_1)} \\ \frac{z_{12}}{n\mu(z_2)} & \frac{z_{22}}{n\mu(z_2)} & \cdots & \frac{z_{n2}}{n\mu(z_2)} \end{bmatrix} \tag{2.4}$$

3. Empirical implementation

3.1. Data

As mentioned in introduction, the individual attributes considered are disposable income, regional infant mortality rate and regional average pupils/teacher ratio in public schools.

Table 1 reports the countries studied, their income surveys and their year and sample size. The income concept used is household's disposable income which *includes* labour income (wages and self-employment earnings such as small business or farm incomes), capital income (dividend, rents and interest) and transfers payments *but is net* of any income tax and social security contributions. As data sets provide information on household composition, we account for the heterogeneity in households sizes by transforming households disposable incomes into households equivalent disposable income using the OECD equivalence scales. Finally, we adjust incomes to account for differences in the purchasing power of the different currencies by using purchasing power parities supplied by the OECD. In table 2, we report sample weighted average of equivalent disposable income for each country as well as the *per capita* GDP for 2002 in US dollars based on purchasing power parities.

TABLE 1 :OVERVIEW OF DATA SETS

Country	Data-set	Sample Size
Australia	Household, Income and Labour Dynamics in Australia (HILDA) - 2000	19 914
Austria	European Community Household Panel (ECHP) - 1998	8 109
Canada	Survey of Consumer Finances 1998	91 254
France	ECHP - 1998	14 563
Germany	ECHP - 1998	15 118
Italy	ECHP - 1998	19 000
Portugal	ECHP - 1998	9 735
Spain	ECHP - 1998	16 486
Sweden	ECHP - 1998	12 451
Switzerland	Survey of Income and Consumption - 2001	9 220
United Kingdom	ECHP - 1998	12 307
United States	Current Population Survey - 1998	131 617

The public goods variables are infant mortality and pupils/teacher ratio in public schools (elementary and secondary). Both of these variables are collected at a regional level which depends upon the availability of the information in the various data sets. The levels where the NUTS1 for the European Union countries, the states for Australia, the divisions for United States and the regions for Switzerland (see the appendix for more detailed information). Table 2 also reports the weighted average of infant mortality and pupils/teacher ratio at public school for each country.

Overall tables 1 and 2 reveal well-known facts, among which that United States has the largest average disposable income and the largest rate of infant mortality. United Kingdom is the country which has the worst performance in terms of pupils/teacher ratio (followed by Canada and UK) while Portugal (closely followed by Austria) does rather well on that matter.

Table 2 : MAIN CHARACTERISTICS OF DATA SET

Country	Mean disposable income (PPP$)	GDP per capita (PPP$)	Mean infant mortality	Mean number of pupils per teacher
Australia	21,990	28,100	5.12	14.94
Austria	19,150	28,900	4.92	9.92
Canada	19,960	30,330	5.20	16.05
France	18,150	27,200	4.50	14.10
Germany	17,650	25,900	4.67	14.25
Italy	13,490	25,600	4.91	11.75
Portugal	11,485	18,400	6.17	9.16
Spain	12,995	22,400	4.76	12.96
Sweden	15,165	27,200	2.95	12.66
Switzerland	26,680	30,500	4.65	14.72
United Kingdom	19,810	28,000	5.50	24.17
United States	28,265	36,100	8.63	15.94

3.2. Statistical methodology

All comparisons performed in this papers can be expressed in terms of poverty comparisons using either headcount and/or poverty gap criteria.

For comparisons involving disposable income, we have proceed in grouping individual incomes into several (typically twenty) quantiles. We have assigned to every quantile the average disposable income of the individual in that quantile. There is obviously some arbitrariness in this quantile division. Yet, the sensitivity tests that we have performed suggest that the rankings are relatively robust to it (we have done the tests going from 10 to 100 quantiles categories).We of course abstain from doing this aggregation of individual values into quantiles for the comparison of distributions of public goods (infant mortality and pupils/teacher ratio) because the umber of distinct values taken by these variables is rather small.

In order to account for the fact that the compared distributions of disposable income are samples drawn for a larger population, we perform statistical inference using two different methods : the intersection-union (IU), initiated by Howes (1994) and Kaur et al. (1994) and the union-intersection (UI) advocated among others by Bishop and Formby (1999). A comparison of the two rules is performed by Howes (1994). The common principle underlying these methods consists in doing statistical inference at each point of a grid in a sequential manner based on a given number, K say, of subhypothesis.

To be specific, define, for each procedure :

$$\begin{aligned} H_0^i &: \gamma_i^A \geqslant \gamma_i^B \\ H_A^i &: \gamma_i^A < \gamma_i^B \\ \overline{H}_0^i &: \gamma_i^B \geqslant \gamma_i^A \\ \overline{H}_A^i &: \gamma_i^B < \gamma_i^A \\ \text{for } i &= 1, ..., K \end{aligned}$$

where γ_i^j can be either the headcount poverty or the poverty gap for the distribution j ($j = A, B$) for the poverty limit i. According to the UI rule $\succeq^{UI}$, the rejection region of the null hypothesis is the intersection of the rejection of K subhypothesis and the non-rejection region of the null hypothesis is the union of the non-rejection regions of the K subhypothesis. In other words, we accept dominance of A over B (resp. of B over A) if we fail to reject one of the K null subhypothesis $\overline{H}_0^i$ (resp.H_0^i) and we reject dominance if we reject all of the K null subhypothesis.

By contrast, the IU rule $\succeq^{IU}$says that the rejection region of the null hypothesis is the union of K subhypothesis and the non-rejection region of the null hypothesis is the intersection of the non-rejection regions of the K subhypothesis. In other words, we accept dominance of A over B (resp. of B over A) if we fail to reject all of the K null subhypothesis $\overline{H}_0^i$ (resp.H_0^i) and we reject dominance if we reject any one of the K null subhypothesis.

Let T_i be the test statistic of H_0^i defined by :

$$T_i = \frac{\widehat{\gamma}_i^A - \widehat{\gamma}_i^B}{\left(\frac{\widehat{\omega}_{ii}^A}{N^A} + \frac{\widehat{\omega}_{ii}^B}{N^B} \right)^{\frac{1}{2}}}$$

where $\widehat{\gamma}_i^j$ is the sample estimate of γ_i^j ($i = 1, ..., K$; $j = A, B$), $\widehat{\omega}_{ii}^A$ the variance estimates of $\widehat{\gamma}_i^j$ and N^j the size of the sample drawn from population j, $j = A, B$. The asymptotic variance estimators are derived in Davidson and Duclos (2000) for headcount poverty and the poverty gap and in Duclos et al. (2003) for the bidimensional poverty gap[10] according

[10]Beach and Davidson (1983) provides asymptotic variance for the relative and generalized Lorenz curves (see Bishop et al. (1989) for generalized Lorenz dominance, Bishop and Formby (1999) for relative Lorenz dominance and Bishop et al. (1994) for decile means dominance).

to :

$$\widehat{\omega}_{ii} = \frac{1}{N}\sum_{i:y_i<t}\left[(t-y_i)^{s-1}\right]^2 I(y_i<t)-(\widehat{\gamma}_i)^2 \text{ for unidimensional poverty}$$

$$\widehat{\omega}_{ii} = \frac{1}{N}\sum_{i:y_{i1}<t_1,y_{i2}<t_2}\left[(t_1-y_{i1})^{s-1}(t_2-y_{i2})^{s-1}\right]^2 I(y_i<t)-(\widehat{\gamma}_i)^2 \text{ for bidimensional poverty}$$

where s denote the order of dominance ($s=1$ for headcount poverty and $s=2$ for poverty gap).

The IU rule is defined by :

$$\begin{aligned} A &\succeq {}^{IU}B \Leftrightarrow \max(T_1,...,T_K) < -Z_\alpha \\ B &\succeq {}^{IU}A \Leftrightarrow \min(T_1,...,T_K) > Z_\alpha \end{aligned}$$

while the UI rule is :

$$\begin{aligned} A &\succeq {}^{UI}B \Leftrightarrow \min(T_1,...,T_K) < -C_\alpha \text{ and } \max(T_1,...,T_K) < C_\alpha \\ B &\succeq {}^{UI}B \Leftrightarrow \max(T_1,...,T_K) > C_\alpha \text{ and } \min(T_1,...,T_K) > -C_\alpha \end{aligned}$$

where Z_α is the critical value for a significance level of α (α is the probability of reject H_0 when H_0 is true) derived from the t distribution and C_α is the critical value for α significance level determined from the Student Maximum Modulus (SMM) distribution provided by Stoline and Ury (1979). Clearly, the $\succeq^{UI}$test is more prone to recognize dominance than $\succeq^{IU}$. Put differently, a conclusion that a A dominates B obtained by $\succeq^{IU}$is more robust than an analogous conclusion obtained by $\succeq^{UI}$.

For the ordered bidimensional poverty gap dominance criterion proposed by Gravel and Moyes (2004), which generalizes the Bourguignon (1989) criterion, we adopt the following approach..

Recall from definition 9 that checking for bidimensional ordered poverty gap dominance amount to verifying that the following inequalities :

$$\sum_{h=1}^{k-1}\sum_{\{i:x_{i1}\in[t_1^h,t_1^{h+1}]}\max(t_2^h-x_{i2},0)\leq\sum_{h=1}^{k-1}\sum_{\{i:y_{i1}\in[t_1^h,t_1^{h+1}]}\max(t_2^h-y_{i2},0)$$

$$\sum_{h=1}^{k-1}\sum_{\{i:x_{i2}\in[t_2^{k+1-h},t_2^{k-h}]}\max(t_1^{k+1-h}-x_{i1},0)\leq\sum_{h=1}^{k-1}\sum_{\{i:y_{i2}\in[t_2^{k+1-h},t_2^{k-h}]}\max(t_1^{k+1-h}-y_{i2},0)$$

hold for every sequence of k pairs of poverty lines satisfying $0=t_1^1\leq t_1^2\leq...\leq t_1^k$ and $t_2^1\geq t_2^2\geq...\geq t_2^k=0$, $t_{1j}\geq t_{2j}\geq...\geq t_{nj}$.

A nice way to verify in a systematic manner these inequalities is to apply the algorithm suggested by Bourguignon (1989) in the proof of his main theorem. In order to define this algorithm, denote by $S_j(\mathbf{x},\mathbf{y})$ the support of attribute j (for $j=1,2$) in distributions $\mathbf{x}$

and $\mathbf{y}$ defined by $S_j(\mathbf{x},\mathbf{y}) = \{a \in \mathbb{R}_+ : \exists\ i \in \{1,...,n\}$ such that $x_{ij} = a$ or $y_{ij} = a\}$. Hence $S_j(\mathbf{x},\mathbf{y})$ is the set of all distinct values taken by the attribute j in distributions $\mathbf{x}$ and $\mathbf{y}$. Without loss of generality, denoting by q_j the number of elements in $S_j(\mathbf{x},\mathbf{y})$ (of course $q_j \leq 2n$), we write $S_j(\mathbf{x},\mathbf{y}) = \{z_1^j, ..., z_{q_j}^j\}$ where $z_1^j \leq .z_2^j \leq .. \leq z_{q_j}^j$.

With this notation, we define recursively the functions :

$$\triangle G_k^j(t) = \max_{z \geq t} \left\{ \sum_{\{i:x_{ih}=z_k^h\}} \max(z - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_k^h\}} \max(z - y_{ij}; 0) + \triangle G_{k-1}^j(t) \right\}$$

starting with :

$$\triangle G_1^j(t) = \max_{z \geq t} \left\{ \sum_{\{i:x_{ih}=z_1^h\}} \max(z - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_1^h\}} \max(z - y_{ij}; 0) \right\}$$

for $k = 1, ..., q_{q_h}^h$, $j = 1, 2$, $h = 1, 2$ and $h \neq j$. Hence this algorithm starts by defining $\triangle G_1^j(t)$ to be the biggest poverty gap excedent in attribute j of $\mathbf{x}$ over $\mathbf{y}$ among individuals endowed with the smallest observed quantity of attribute h in either distribution for all poverty lines weakly larger than t. One can then define recursively $\triangle G_k^j(t)$ (for $k = 2, ..., b_h$) to be the biggest cumulative poverty gap excedent in attribute j of $\mathbf{x}$ over $\mathbf{y}$ calculated over all individuals who have, weakly, less than z_k^h units of of attribute h when considering all poverty lines above t that are increasing as the individual's endowment of attribute h decreases. It is easy to verify that distribution $\mathbf{x}$ dominates distribution $\mathbf{y}$ by the ordered poverty gap dominance criterion if and only if

$$\triangle G_k^j(t) \leq 0$$

holds for all $k = 1, ..., q_{q_h}^h$, $j = 1, 2$, $h = 1, 2$ and $h \neq j$.

This algorithm leads itself naturally to statistical testing. For $\triangle G_1^j(t)$, we calculate, for every observed value t [11] of attribute j, the maximal poverty gap excedent of $\mathbf{x}$ over $\mathbf{y}$ for individuals who have precisely z_1^h units of attribute h by considering all observed values of attribute j that are weakly larger than t. Let $z_1^{j*}(t)$ be the poverty line above t such that this poverty gap excedent is maximal. We have thus :

$$\begin{aligned} \triangle G_1^j(t) &= \sum_{\{i:x_{ih}=z_1^h\}} \max(z_1^{j*}(t) - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_1^h\}} \max(z_1^{j*}(t) - y_{ij}; 0) \\ &= P_1^j(\mathbf{x}, z_1^{j*}(t)) - P_1^j(\mathbf{y}, z_1^{j*}(t)) \end{aligned}$$

where $P_k^j(\mathbf{z}, t)$ denote the poverty gap in attribute j in distribution $\mathbf{z}$ for the poverty line t restricted to the individuals who are in some category k.

[11] We actually need to consider also one value of the attribute j that is strictly larger than $z_{q_j}^j$.

From these expressions, we can calculate, following the Davidson and Duclos (2000) methodology, the estimated variance of $\triangle G_1^j(t)$ by the formula :

$$var(\triangle G_1^j(t)) = var(P_1^j(\mathbf{x}, z_1^{j*}(t)) - var(P_1^j(\mathbf{y}, z_1^{j*}(t)))$$

with the estimation $\widehat{var}(P_1^j(\mathbf{x}, z_1^{j*}(t))$ of the variance of $P_1^j(\mathbf{x}, z_1^{j*}(t)$ being given by :

$$\widehat{var}(P_1^j(\mathbf{x}, z_1^{j*}(t)) = \sum_{\{i:x_{ih}=z_1^h\}} [\max(z_1^{j*}(t) - x_{ij}; 0))]^2 - [P_1^j(\mathbf{x}, z_1^{j*}(t)]^2$$

and similarly for the variance of $P_1^j(\mathbf{y}, z_1^{j*}(t)$. From these estimated variances, we calculate the statistical test expression :

$$T = \frac{n^{1/2}[P_1^j(\mathbf{x}, z_1^{j*}(t) - P_1^j(\mathbf{y}, z_1^{j*}(t)]}{\left(\widehat{var}(P_1^j(\mathbf{x}, z_1^{j*}(t)) + \widehat{var}(P_1^j(\mathbf{y}, z_1^{j*}(t))\right)^{0.5}} \tag{3.1}$$

Now, for $\triangle G_2^j(t)$, we similarly calculate, for each observed value t of attribute j :

$$\triangle G_2^j(t) = \max_{z\geq t} \sum_{\{i:x_{ih}=z_1^h\}} \max(z - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_1^h\}} \max(z - y_{ij}; 0) + \triangle G_1^j(z)$$

$$= \sum_{\{i:x_{ih}=z_1^h\}} \max(z_2^{j*}(t) - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_1^h\}} \max(z_2^{j*}(t) - y_{ij}; 0) + \triangle G_1^j(z_2^{j*}(t)$$

$$= P_2^j(\mathbf{x}, z_2^{j*}(t)) - P_2^j(\mathbf{y}, z_2^{j*}(t)) + P_1^j(\mathbf{x}, z_1^{j*}(z_2^{j*}(t))) - P_1^j(\mathbf{y}, z_1^{j*}(z_2^{j*}(t)))$$

where, here again, $z_2^{j*}(t)$ is the poverty line that maximizes the expression :

$$\sum_{\{i:x_{ih}=z_1^h\}} \max(z - x_{ij}; 0) - \sum_{\{i:y_{ih}=z_1^h\}} \max(z - y_{ij}; 0) + \triangle G_1^j(z)$$

If we define now the *cumulative poverty gap* expressions $\overline{P}_2^j(\mathbf{x}, t)$ and $\overline{P}_2^j(\mathbf{y}, t)$ by

$$\begin{aligned}\overline{P}_2^j(\mathbf{x}, t) &= P_2^j(\mathbf{x}, z_2^{j*}(t)) + P_1^j(\mathbf{x}, z_1^{j*}(z_2^{j*}(t))) \text{ and} \\ \overline{P}_2^j(\mathbf{y}, t) &= P_2^j(\mathbf{y}, z_2^{j*}(t)) + P_1^j(\mathbf{y}, z_1^{j*}(z_2^{j*}(t)))\end{aligned}$$

we can obtain an estimate $\widehat{var}(\overline{P}_2^j(\mathbf{z}, t))$ of the variance of $\overline{P}_2^j(\mathbf{z}, t)$ (for $\mathbf{z} = \mathbf{x}$, $\mathbf{y}$) by :

$$\widehat{var}(\overline{P}_2^j(\mathbf{z}, t)) = \sum_{\{i:z_{ih}=z_2^h\}} [\max(z_2^{j*}(t) - z_{ij}; 0))]^2 + \sum_{\{i:z_{ih}=z_1^h\}} [\max(z_1^{j*}(z_2^{j*}(t)) - z_{ij}; 0))]^2 - \overline{P}_2^j(\mathbf{z}, t)^2$$

From this expression, one can construct a statistical test analogous to expression (3.1). Repeating this procedure enables us therefore to test for the statistical significance of any of the expression $\triangle G_k^j(t) \leq 0$ following either the UI or IU methodology.

In our empirical implementation, we perform inference test for poverty gap and for the three univariate distributions and the two bivariate distributions at 95% confidence.

4. One-dimensional comparisons

4.1. Distributions of disposable income

Table 3 shows the differences in the number of comparisons obtained with UI rule. For headcount poverty dominance, the UI rule is able to rank conclusively 32% of all the possible pairwise comparisons. For poverty gap (or Lorenz) dominance, the percentage of conclusive rankings jump to some 74%. This indicates how significant is the discriminatory power of the dominance approach in the one-dimensional cases : All utilitarian social planners who assume that the individual transform income into well being by the same increasing and concave utility function would agree on the ranking of 75% of OCDE countries. There is therefore not much of a political debate to be performed on these rankings.

Figure 1 shows the results of pairwise statistical Lorenz comparisons performed at 5 per cent confidence level by the UI methodology. The reading of the figure is simple : If a country A dominated a country B if and only if it is related to it by a connected set of downward and horizontal segments. For quantile and generalized Lorenz dominance, Switzerland appears at the top of the ranking : It is never dominated while it dominates ten of the eleven other countries in terms of the quantile criterion. The hierarchy exhibits a second group of countries, made of Australia, Canada, France, Germany, Sweden and United Kingdom, that is only dominated by Switzerland and who dominates Italy, Spain and Portugal. United States occupies a special position in that it is never dominated but dominates far less countries than Switzerland.

Table 3 : Summary of simple and statistical comparisons of income distributions

Table 3 : Summary of simple and statistical comparisons of income distributions

	No	%
Quantile Dominance		
Dominance	21	32
Crossings	45	68
Equivalence		
Generalized Lorenz Dominance		
Dominance	42	74
Crossings	24	36
Equivalence		
Relative Lorenz Dominance		
Dominance	39	59
Crossings	27	41
Equivalence		

As mentioned in the previous section, the Lorenz criterion applied to distributions of actual income is sensitive to both *per capita* disposable income (the size of the cake) and its dispersion. Researchers are often interested in comparing countries on the basis of the distributions of the shares of the total disposable income. The results of the comparisons of the 12 countries on the basis of their relative Lorenz curves are presented in Figure 2. We complete this ranking by providing complete ranking of both actual (Table 4) and normalized (Table 5) income distributions based on the comparisons of the AK welfare index for alternative values of the parameter ε. The most equal country in relative term appears to be Sweden while United States is the more unequal country. It is interesting to note the special position of Spain which turns out to be significantly affected by the size of the quantiles used. If we use 10 equal quantiles, Spain appears to be less equal than almost all other countries according to the Lorenz criterion applied to normalized distributions. Yet this conclusion is not valid if the incomes are grouped into 20 quantiles because the 5% richest individuals own in Spain a lower fraction of total income than in countries like Sweden or Germany. The only country that dominates Spain in term of distributions of income share is Austria at a 1% confidence level.

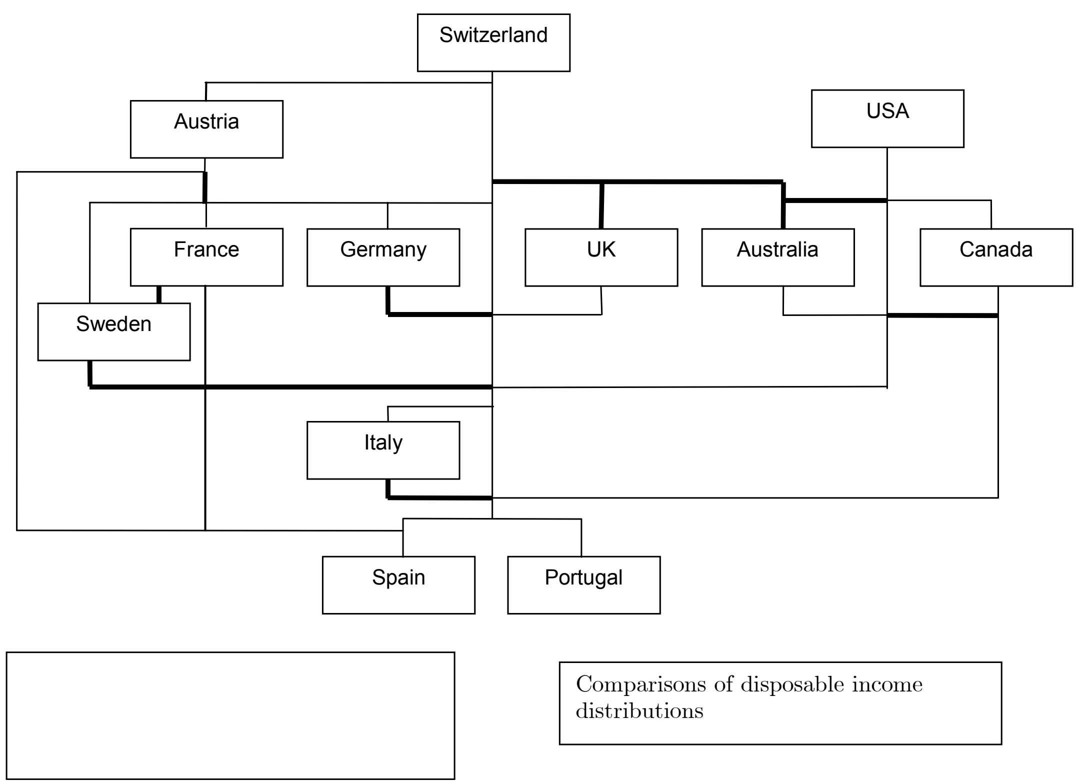

Figure 1

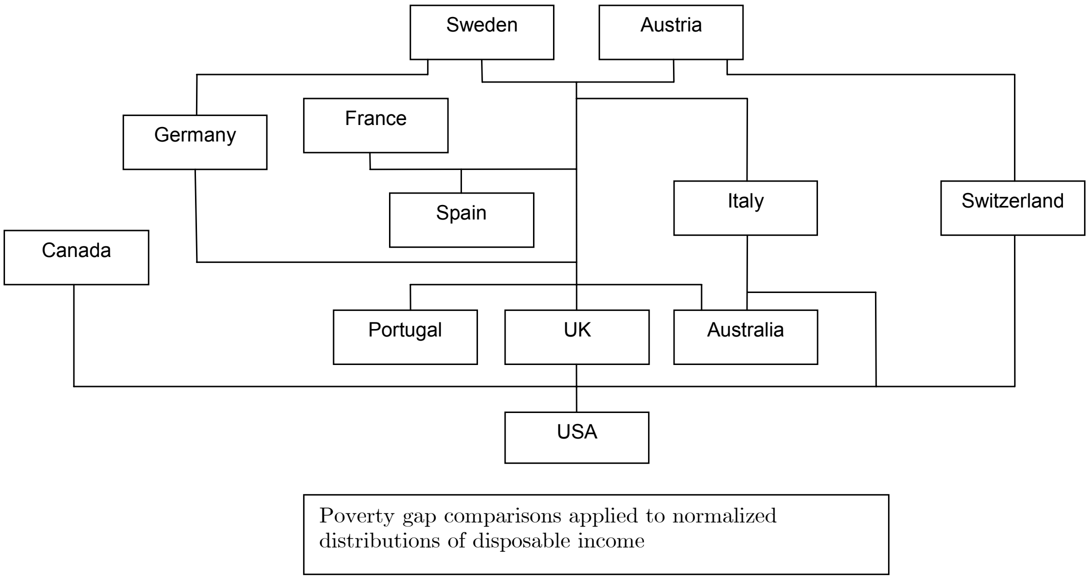

Figure 2

Table 4 : Country ranks based on AK indices (actual distributions)

country	**rank** ($\varepsilon = 0.5$)	**rank**($\varepsilon = 1$)	**rank** ($\varepsilon = 1.5$)	**rank** ($\varepsilon = 2$)
Australia	3	3	4	7
Austria	5	4	3	3
Canada	4	5	7	8
France	7	7	5	4
Germany	8	8	8	5
Italy	10	10	10	10
Portugal	12	12	12	12
Spain	11	11	11	11
Sweden	9	9	9	9
Switzerland	2	1	1	1
UK	6	6	6	6
US	1	2	2	2

Table 5 : Country ranks based on AK indices (normalized distributions)

country	**rank** ($\varepsilon = 0.5$)	**rank**($\varepsilon = 1$)	**rank** ($\varepsilon = 1.5$)	**rank** ($\varepsilon = 2$)
Australia	10	10	11	11
Austria	2	2	2	2
Canada	7	7	8	8
France	4	4	4	3
Germany	3	3	3	4
Italy	6	6	6	6
Portugal	11	11	10	9
Spain	9	9	9	10
Sweden	1	1	1	1
Switzerland	5	5	5	5
UK	8	8	7	7
US	12	12	12	12

4.2. Distributions of public goods

Table 6 presents a summary of comparisons for public goods distributions (again using the UI rule). The headcount poverty dominance criterion allows us to rank a bit less than half of the possible pairwise comparisons of distributions of infant mortality rates and about two third of the comparisons in the case of Pupils/teacher ratio. The extra conclusive rankings that are obtained from moving to the generalized Lorenz criterion represent 27 point of percentage for infant mortality and 17 points of percentage in the case of Pupils/teacher ratios. Figures 3 and 4 show the rankings generated by the Lorenz criteria applied to the actual and normalized distributions of regional infant mortality rates.

The hierarchy of countries in terms of their performance in providing and distributing protection against the risk of infant mortality is quite clear. Because of its excellent performance in terms of overall infant mortality prevention, Sweden appears at the top of the hierarchy and dominates all other countries on the sole basis of the quantile dominance criterion. Below Sweden, a second group of countries composed of France, Germany and Switzerland have also the properties of dominating all countries outside the group (Sweden excepted of course) by the Generalized Lorenz criterion. Third in the hierarchy is the group made of Spain and Austria, while Canada, U. K. and Italy would form a fourth group.

Table 6 : Summary of comparisons of public goods distributions

	Health comparisons		Education comparisons	
	No	%	No	%
Quantile Dominance				
Dominance	31	47	44	66
Crossings	35	53	22	33
Equivalence				
Generalized Lorenz Dominance				
Dominance	49	74	55	83
Crossings	17	26	11	17
Equivalence				
Relative Lorenz Dominance				
Dominance	25	38	46	70
Crossings	41	62	20	30
Equivalence				

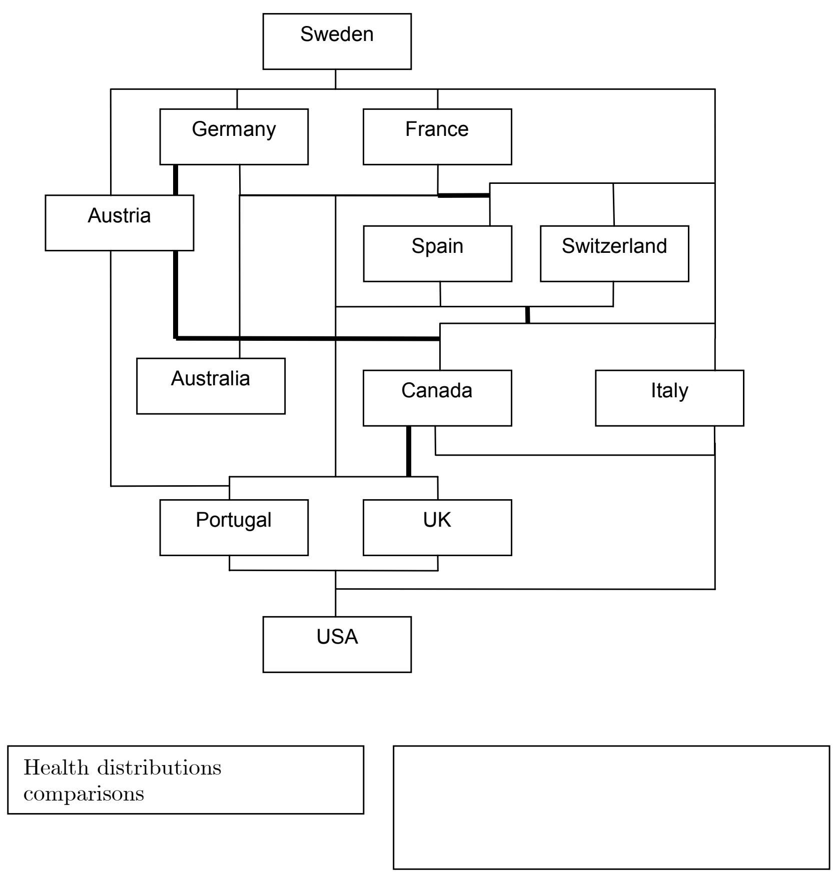

Figure 3

As before, US, dominated by all countries but Australia, appear at the bottom of the hierarchy.

Things are significantly different when one looks at the distributions of the *shares* of total infant mortality owned by the different regions of a country according to the Lorenz, or poverty gap, criterion. The ranking here puts Germany at the top of the hierarchy, dominating 8 of the 10 countries (only Austria and Portugal avoid the German domination). Austria comes second while Sweden and France would probably come third. The bottom of the hierarchy is here occupied by Italy (dominated by all countries but Portugal, Australia, US and Canada). Portugal has the rather peculiar feature of being undominated, even though the only country it dominates is Australia.

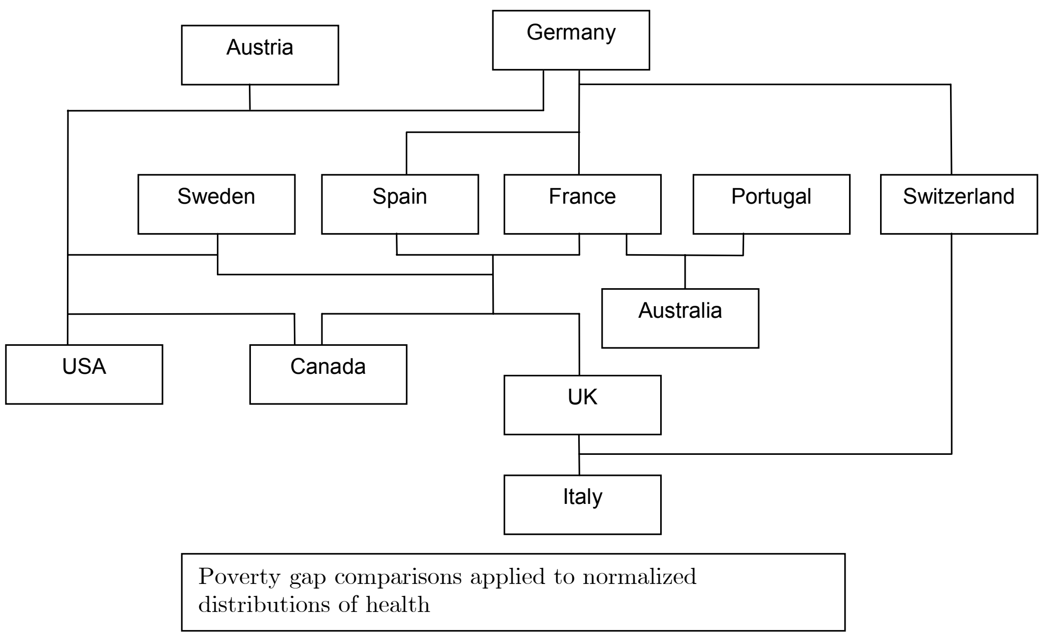

Figure 4

It should be noticed that this comparison of interregional inequalities in the risk of infant mortality show no clear advantage in favour of unitary/centralized countries as opposed to federal/decentralized ones. It is a federal country, Germany which appears at the top of the hierarchy while other federal countries such as US and Canada can be found at the bottom. While unitary countries like France do relatively well, the performance of other unitary countries such as United Kingdom is rather poor.

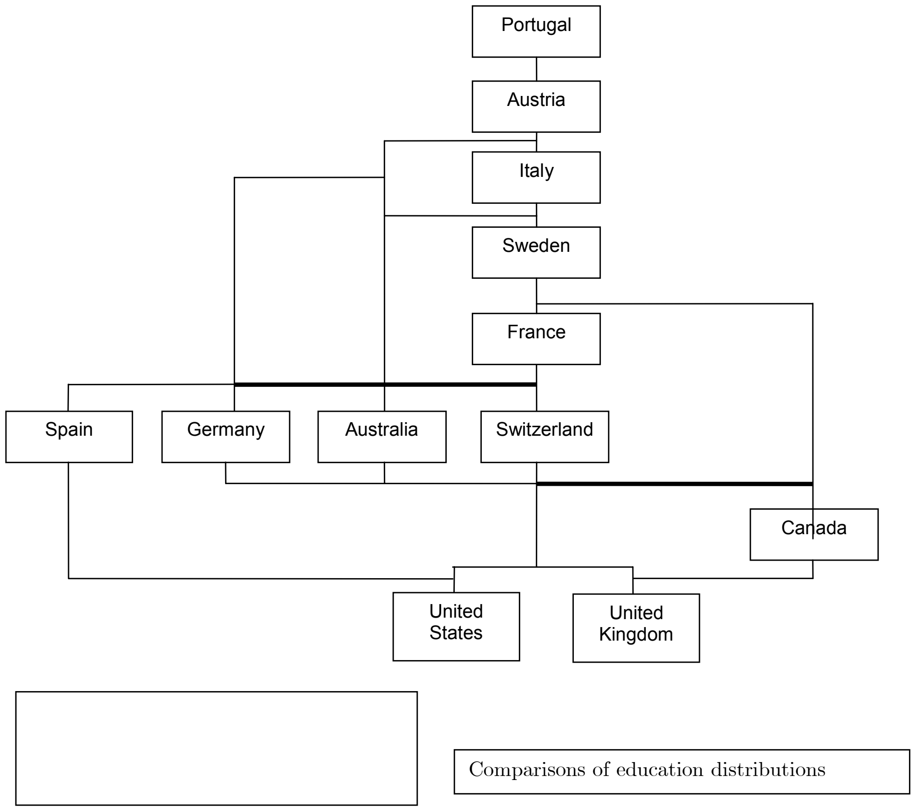

Figure 5

The comparisons, shown in figure 5, of countries in terms of their inequality in giving access to public schools with small classes show a clear domination of Portugal over all other countries (obtained on the sole basis of the quantile dominance criterion) and an abysmal performance of United Kingdom and United States. Between these two extreme cases, one finds three groups of countries which are composed starting from the bottom to the top of Austria, Italy and Sweden (first group)), France, Spain and Germany (second group) and Switzerland and Australia (third group).

The conclusions achieved by looking at the distributions of shares of pupils/teacher ratios, shown in figure 6, are, again, quite different. According to this criteria, the three most equal countries are Austria and Australia. The standing of Canada and UK appears also to be high, even though these two countries perform badly on the basis of the generalized Lorenz criterion. This means that British or Canadian citizens have equal relatively equal access to public schools with large class sizes. A somewhat converse conclusion holds for Germany or Portugal where the average size of classes is low, but the dispersion of the sizes across regions is high. While it is, here again, difficult to derive from these figures firm conclusions about the egalitarian performance of unitary/centralized countries as compared to federal/decentralized one. The federal Australia does quite well in terms of equalizing class sizes among its region while the federal Germany does poorly on that matter. But, at the other extreme, Unitary France does well on that matter while unitary UK does rather poorly.

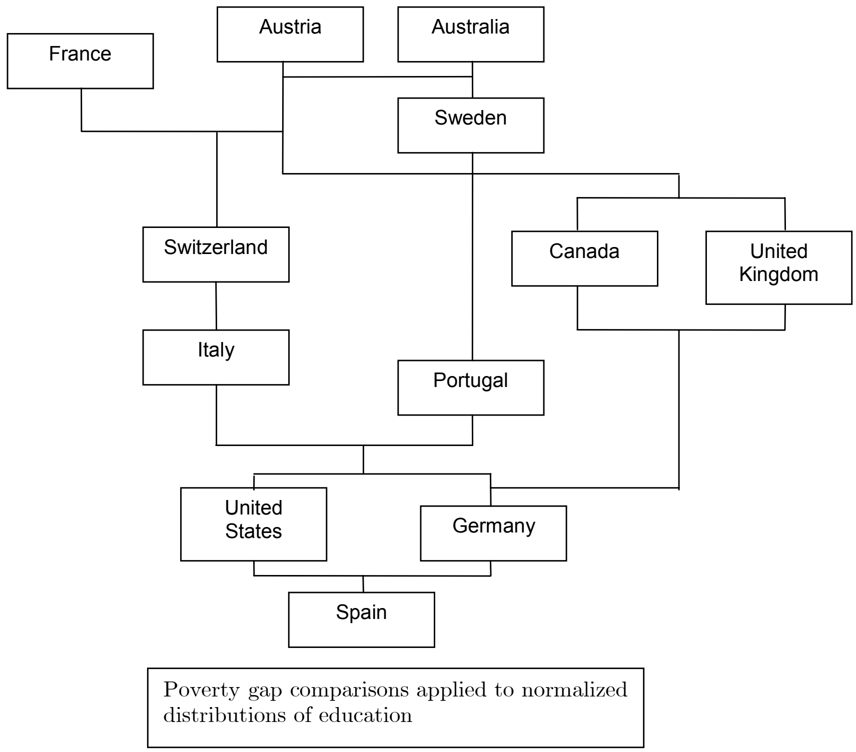

Figure 6

5. Bidimensional Inequality Comparisons

5.1. Comparisons of health and disposable income distributions

The summary of the implementation of the bidimensional comparisons of health and disposable income distributions is given in table 7.

As could be expected, a significantly lower fraction of all possible pairwise comparisons can be ranked conclusively by the bidimensional dominance criteria than by their unidimensional cousins. For instance, the bidimensional headcount poverty dominance criterion leads to a conclusive ranking in only 11 per cent of all possible pairwise comparisons. Switching to the bidimensional ordered poverty gap dominance criterion merely increase the fraction of conclusive comparison to 12% while the move to the sequential Lorenz criterion pushes the fraction up to 24% (using UI rule). Hence, it appears that the switch from headcount poverty dominance criterion to bidimensional ordered poverty gap has less impact on the fraction of conclusive rankings obtained than does the switch from the bidimensional ordered poverty gap criterion to the bidimensional poverty gap criterion. This later criterion is also the only one that is able to provide us some discriminatory power in the ranking of normalized distributions of both health and disposable income.

Table 7 : Summary of multidimensional comparisons of health and disposable income

	actual distributions		normalized distributions	
	No	%	No	%
Bidimensional headcount poverty				
Dominance	7	11		
Crossings	59	89		
Generalized Bourguignon				
Dominance	8	12		
Crossings	58	88		
Bidimensional poverty gap				
Dominance	16	24	4	6
Crossings	50	76	62	94

Figures 7 and 8 show the rankings that emerge from the comparisons of both actual and normalized distributions on the basis of the bidimensional poverty gap criterion. As can be seen, the two rankings differ markedly. With actual distributions, Switzerland appears at the top of the ranking and dominates five countries. Next in the order is a group formed by France, Germany and Sweden who are never dominated and dominate three other countries : Italy, Portugal and Spain. Portugal appears at the very bottom of the hierarchy, dominated by six countries : the four countries cited above plus United Kingdom and Austria. Non surprisingly, United States are never dominated and never dominate. This can be explained by the fact that United States have a significantly higher average disposable income and infant mortality than the other countries.

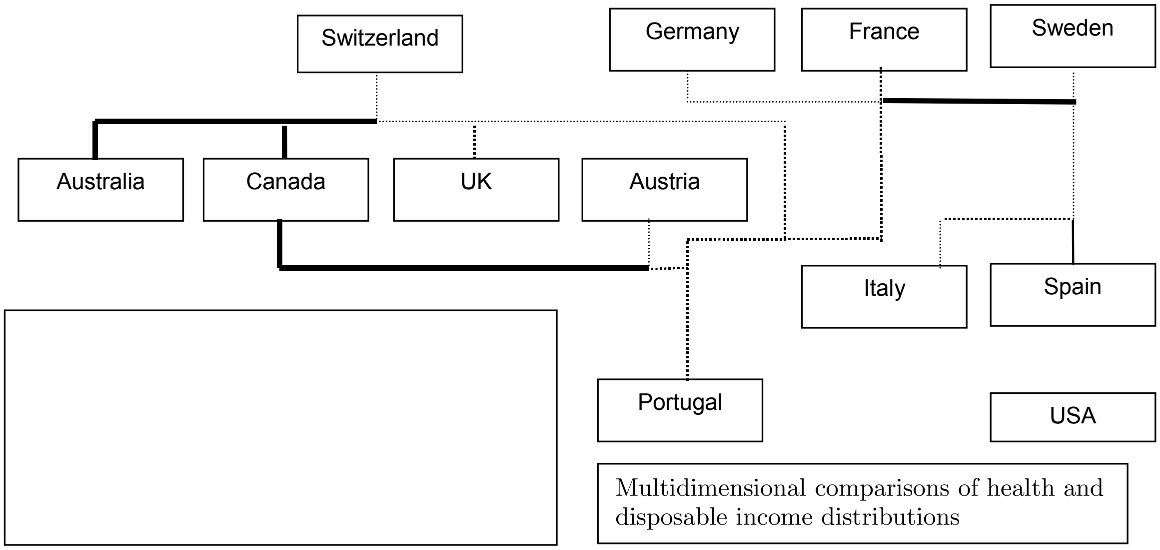

Figure 7

Yet US does not perform very well when accounting for the different sizes of the two cakes (disposable income, higher in the US and prevention from the risk of infant mortality, lower in the US) that it achieves. United States is, indeed, dominated by three countries (Austria, Germany and Sweden) while it dominates no country. In relative terms, the most egalitarian country appears to be Sweden, which dominates two countries. Here again, there is no evidence that federal/decentralized countries perform worse than unitary/centralized one in terms of their performance in allocating the risk of infant mortality and disposable income across their citizens.

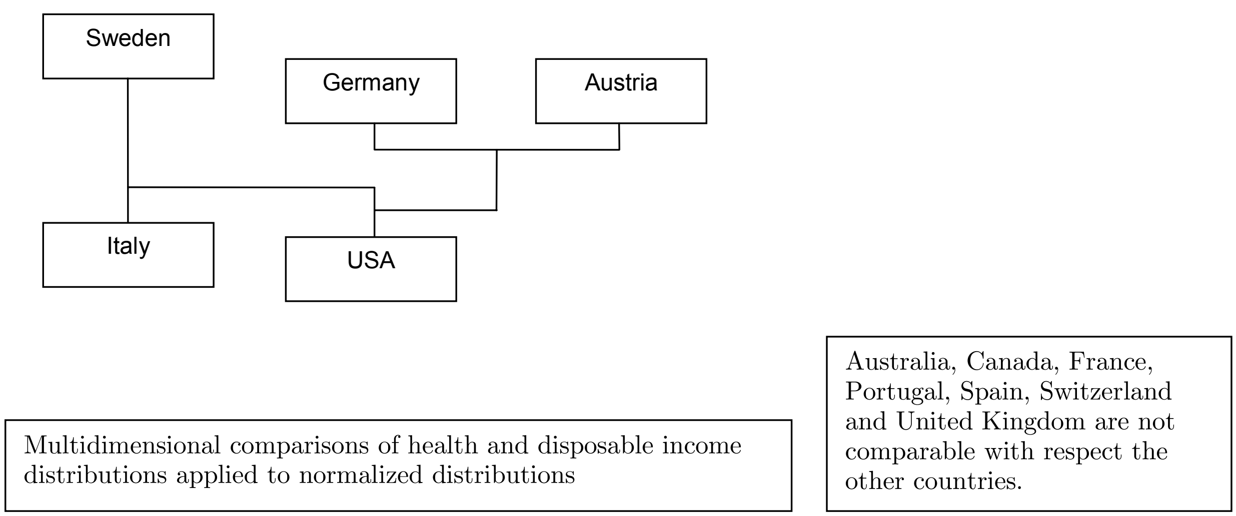

Figure 8

We now provide complete rankings of normalized distributions of infant mortality and disposable income by means of the Bourguignon index for different values of the parameters α and β (assuming w_1 and w_2 to be 1). The first thing that can be noticed is the relative stability of the complete rankings of the countries provided by this index, which are obviously consistent with figure 8. Sweden is the most egalitarian country and the United States is the less egalitarian. There are only two pairwise comparisons that do not resist to the change in the parameters value of the Bourguignon index : Australia and Portugal and Italy and Switzerland. In figure 9, we illustrate these two ambiguous pairwise comparisons and show the values of the parameters where the ranking of the two countries is reversed. As we require $\alpha \leq \beta$, we only focus on the north-west part of the map. We see that Italy dominate Switzerland only on the north-east part of map. Italy is preferred if the social planner chooses a high degree of substitution between income and health (a high β) and a low level of concavity of the utility function (represented by the value of α). As for Australia and Portugal, it is when the utility function is very concave that Portugal is more egalitarian than Australia ($\alpha \leq 0.4$).

TABLE 9 : RELATIVE BOURGUIGNON INDEX FOR SOME VALUE OF α AND β

Country	$\alpha = 0.1$ $\beta = 0.1$	Rank	$\alpha = 0.9$ $\beta = 0.9$	Rank	$\alpha = 0.1$ $\beta = 1$	Rank
Australia	1.9800	10	1.9813	10	1.0671	9
Austria	1.9899	2	1.9900	2	1.0692	2
Canada	1.9834	7	1.9853	7	1.0680	6
France	1.9880	4	1.9878	4	1.0686	4
Germany	1.9882	3	1.9879	3	1.0686	3
Italy	1.9846	6	1.9855	5	1.0680	6
Portugal	1.9790	11	1.9792	11	1.0664	11
Spain	1.9802	9	1.9815	9	1.0670	10
Sweden	1.9907	1	1.9906	1	1.0693	1
Switzerland	1.9867	5	1.9854	6	1.0681	5
United Kingdom	1.9822	8	1.9822	8	1.0672	8
United States	1.9774	12	1.9779	12	1.0662	12

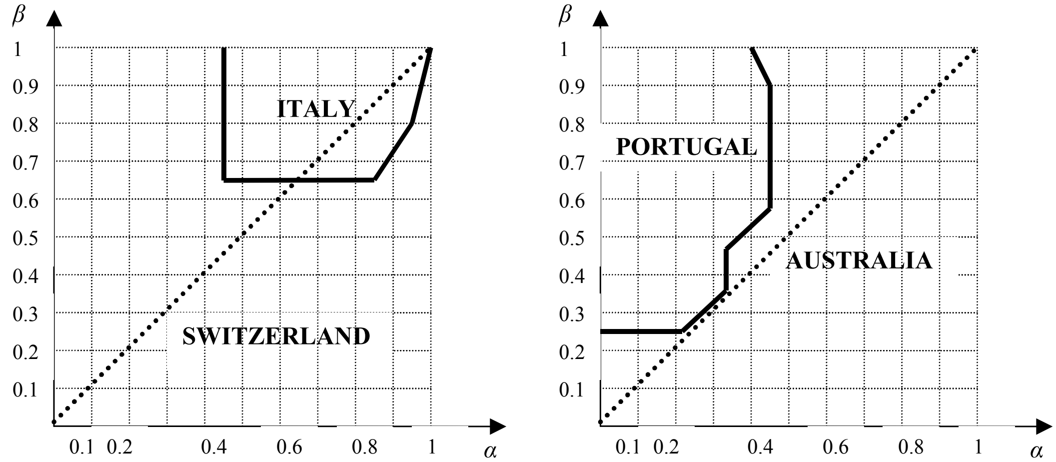

Figure 9

Table 10 presents the ranking of countries resulting from the comparisons of the value taken by the relative Cobb-Douglas index for different negative values of r_1 and r_2. For the selected combination of two parameters, the ranking generated is very similar than that generated by the Bourguignon index and 7 countries (Austria, France, Germany, Italy, Sweden, Switzerland and the United Kingdom, the most egalitarian ones) have the same ranking . While the comparison between Italy and Switzerland is ambiguous for Bourguignon index, the relative Cobb-Douglas index generate a clear conclusion : Switzerland is more egalitarian than Italy. On the other hand, as for the Bourguignon index, the ranking of Australia and Portugal is versatile with the change in the parameter values.

Table 10 : Relative cobb-douglas Index for some value of r_1 and r_2

Country	$r_1 = -0.1$ $r_2 = -0.1$	Rk	$r_1 = -0.6$ $r_2 = -0.1$	Rk	$r_1 = -0.1$ $r_2 = -1$	Rk	$r_1 = -0.4$ $r_2 = -0.4$
Australia	-1.02579	10	-1.27697	12	-1.02579	10	-1.14669
Austria	-1.01263	2	-1.11853	2	-1.01262	2	-1.06686
Canada	-1.02016	7	-1.20866	7	-1.02016	7	-1.11275
France	-1.01484	4	-1.13838	4	-1.01484	4	-1.07822
Germany	-1.01458	3	-1.13657	3	-1.01458	3	-1.07697
Italy	-1.01949	6	-1.19543	6	-1.01951	6	-1.10699
Portugal	-1.02639	11	-1.26120	9	-1.02639	11	-1.14356
Spain	-1.02531	9	-1.26128	10	-1.02532	9	-1.14107
Sweden	-1.01154	1	-1.10713	1	-1.01154	1	-1.06069
Switzerland	-1.01641	5	-1.15366	5	-1.01641	5	-1.08645
United Kingdom	-1.02230	8	-1.21829	8	-1.02230	8	-1.12064
United States	-1.02731	12	-1.27407	11	-1.02731	12	-1.14966

We can of course use either family of indices to compare the actual distributions of the two attributes.. We do this in table 11 for the Bourguignon index and in table 12 for the Cobb-Douglas. We note here again the relative stability of the rankings to the choices of the parameters for the Bourguignon index as compared to the Cobb-Douglas index. In both rankings, US and Switzerland appears at the top while Portugal and Spain appears at the very bottom, reflecting essentially the respective standing of the countries in terms of per capita disposable income.

TABLE 11 : GENERALIZED BOURGUIGNON INDEX FOR SOME VALUE OF α AND β

Country	$\alpha = 0.1$ $\beta = 0.1$	Rank	$\alpha = 0.9$ $\beta = 0.9$	Rank	$\alpha = 0.1$ $\beta = 1$	Rank
Australia	4.6577	3	8439	3	2.6818	3
Austria	4.6477	4	7574	6	2.6699	4
Canada	4.641	5	7806	4	2.6656	5
France	4.6286	7	7223	7	2.6520	7
Germany	4.6218	8	7056	8	2.6455	8
Italy	4.5428	10	5635	10	2.5732	10
Portugal	4.4875	12	4914	12	2.5242	12
Spain	4.5218	11	5445	11	2.5553	11
Sweden	4.5889	9	6236	9	2.6141	9
Switzerland	4.7284	1	9987	2	2.7470	1
United Kingdom	4.6360	6	7732	5	2.6602	6
United States	4.7173	2	10415	1	2.7389	2

While the Cobb-Douglas ranking of these same country gives the same figures for the very top (it however always put Switzerland above US) and the very bottom, it appears to be very versatile with respect to middle range countries. France for instance may be at the fourth or the seven rank depending upon the choice of the parameter.

Country	$r_1 = -0.1$ $r_2 = -0.1$	Rk	$r_1 = -0.6$ $r_2 = -0.1$	Rk	$r_1 = -0.1$ $r_2 = -1$	Rk	$r_1 = -0.4$ $r_2 = -0.4$
Australia	-0.1893	3	-0.00159	4	-0.000040	8	-0.00133
Austria	-0.1894	4	-0.00151	3	-0.000033	3	-0.00131
France	-0.1909	7	-0.00159	5	-0.000036	4	-0.00135
Canada	-0.19005	5	-0.00159	6	-0.000038	6	-0.00134
Germany	-0.1913	8	-0.00161	7	-0.000037	5	-0.00136
Italy	-0.1975	10	-0.00199	8	-0.000055	10	-0.00156
Portugal	-0.2021	12	-0.00232	12	-0.000072	12	-0.00172
Spain	-0.1994	11	-0.00215	11	-0.000064	11	-0.00163
Sweden	-0.1937	9	-0.00172	9	-0.000041	9	-0.00142
Switzerland	-0.1839	1	-0.00128	1	-0.000025	1	-0.00117
United Kingdom	-0.1906	6	-0.00161	8	-0.000039	7	-0.00135
United States	-0.1849	2	-0.00136	2	-0.000030	2	-0.00121

Table 12 : Generalized cobb-douglas Index for some value of r_1 and r_2

5.2. Comparisons of education and disposable income distributions

We finally present the results of the implementation of dominance bidimensional comparisons of education and disposable income distributions in table 12 while the rankings generated by the dominance criteria applied to both the actual and the normalized distributions are depicted in figures 10 and 11 respectively.

Just like in the comparisons of distributions of infant mortality and disposable income, the fraction of conclusive ranking is higher when the dominance criteria are applied to relative distributions then when they are applied to actual ones.

For actual comparisons, the bidimensional headcount poverty dominance criterion enables us to conclude only in 3 percent of the cases. The percentage of conclusive rankings is increased to 11 if one uses the bidimensional ordered poverty gap criterion while the move to bidimensional poverty gap increases this percentage to 18. Hence, bidimensional dominance criteria appear less discriminant for comparing distributions of disposable income and education than in the case of income and health. On the other hand, and contrary to what was the case with the joint distribution of infant mortality and disposable income, the extra discriminatory power obtained by switching from bidimensional headcount poverty to Generalized Bourguignon is larger than that brought about by the move from the Generalized Bourguignon criterion to the bidimensional poverty gap. Again, only the bidimensional poverty gap criterion is conclusive when applied to normalized distributions.

Table 13 : Summary of multidimensional comparisons of education and disposable income

	actual distributions		normalized distributions	
	No	%	No	%
Bidimensional headcount poverty				
Dominance	2	3		
Crossings	64	97		
Generalized Bourguignon				
Dominance	7	11		
Crossings	59	89		
Bidimensional poverty gap				
Dominance	9	14	17	26
Crossings	57	86	49	74

If we now turn to Figures 10 and 11, we can see the ranking of countries produced by the criteria (using again the IU rule) applied to actual and normalized distributions respectively. In the former case, Austria, who dominates five countries : Germany, France, Sweden, Italy and Spain and who is never dominated appears clearly at the top. Three countries (Portugal and the United States) are never dominated and never dominate. Finally, one notes the bad performance of Spain which does not dominate any country but who is dominated by three countries, followed tightly by Canada and UK.

Switching now to comparisons of normalized distributions, we notice, the clear domination of Austria and Sweden (each over 5 countries) and the significant domination of France (over four countries). Spain and UK (who dominates no countries and who are dominated by 5 of them) are clearly at the bottom of the hierarchy.

The complete rankings of countries provided by the Bourguignon and the Cobb-Douglas indexes applied to normalized distributions are presented in table 15 and 16 respectively.

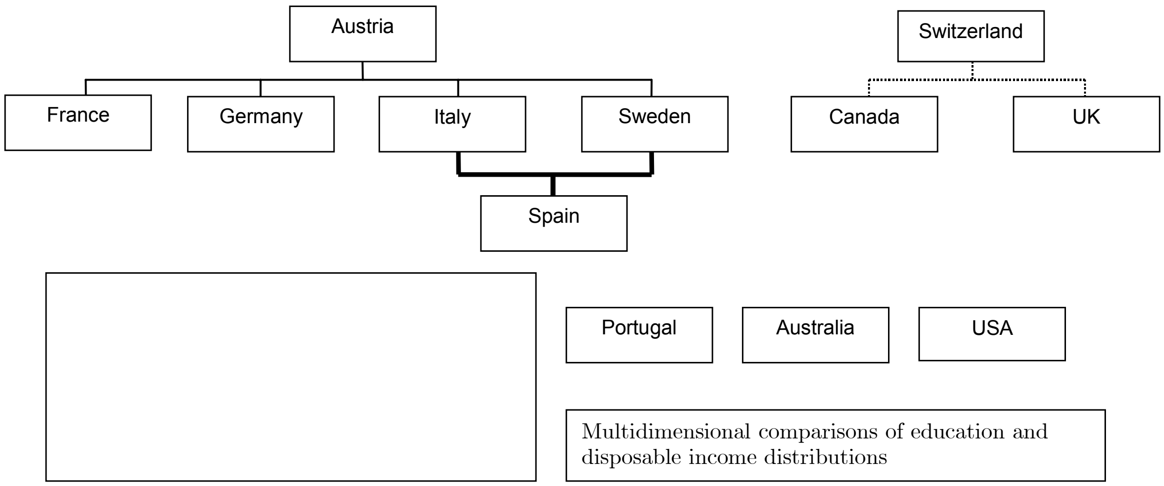

Figure 10

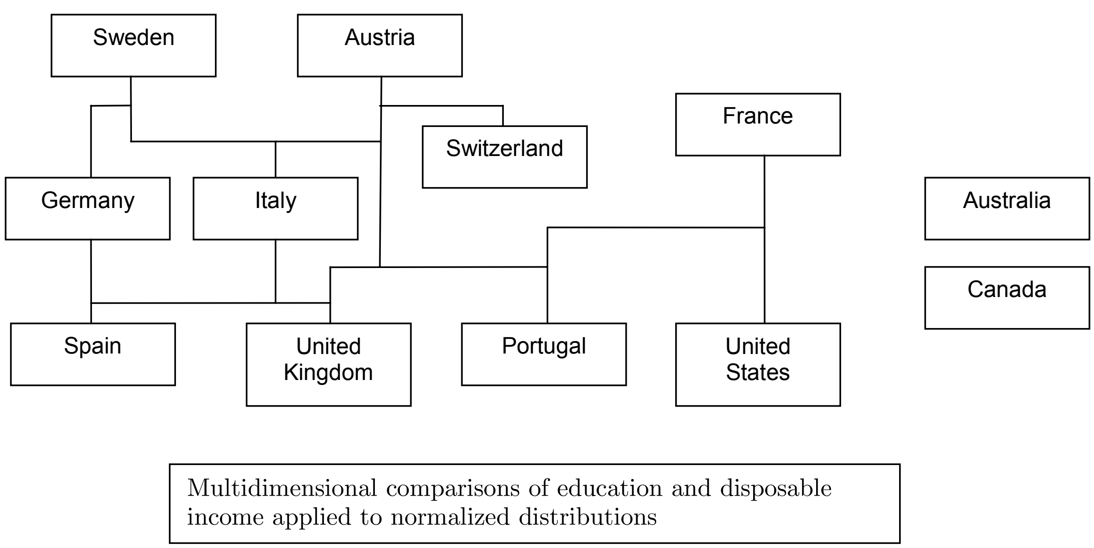

Figure 11

Table 15 : Relative Bourguignon Index for some value of α and β						
Country	$\alpha = 0.1$ $\beta = 0.1$		$\alpha = 0.9$ $\beta = 0.9$		$\alpha = 0.1$ $\beta = 1$	
Australia	1.9800	9	1.9813	7	1.0671	9
Austria	1.9898	2	1.9900	2	1.0692	2
Canada	1.9833	7	1.9852	7	1.0680	7
France	1.9880	3	1.9878	3	1.0687	3
Germany	1.9878	4	1.9875	4	1.0686	4
Italy	1.9845	6	1.9854	5	1.0681	6
Portugal	1.9789	10	1.9791	11	1.0665	10
Spain	1.9781	11	1.9795	10	1.0655	12
Sweden	1.9906	1	1.9905	1	1.0693	1
Switzerland	1.9866	5	1.9853	6	1.0681	5
United Kingdom	1.9821	8	1.9821	8	1.0672	8
United States	1.9767	12	1.9773	12	1.0659	11

As compared with the Bourguignon index, the ranking generated by the Cobb-Douglas appears more sensitive to the particular specification of the parameters values. The only countries whose ranking is unaffected by the change in the parameters are Sweden (1st rank), Austria (2nd rank) and Portugal (9th rank). For a given weight ($|r_1|$) assigned to disposable income, increasing the weight $|r_2|$ given to education reduces the egalitarian standing of Germany as compared to France, Italy and Switzerland whereas a increase in the value of $|r_1|$ for a constant value of $|r_2|$ yields the opposite conclusion. These observations are obviously consistent with one-dimensional conclusions : Germany has a more equal distribution of income than those countries but a less equal distribution of average class sizes across its regions.

Table 16 : Relative cobb-douglas Index for some value of r_1 and r_2								
Country	$r_1 = -0.1$ $r_2 = -0.1$	Rk	$r_1 = -0.6$ $r_2 = -0.1$	Rk	$r_1 = -0.1$ $r_2 = -1$	Rk	$r_1 = -0.4$ $r_2 = -0.4$	Rk
Australia	-1.02302	9	-1.23304	9	-1.02338	8	-1.12689	9
Austria	-1.01196	2	-1.11104	2	-1.01245	2	-1.06311	2
Canada	-1.02028	7	-1.20875	8	-1.02238	6	-1.11326	7
France	-1.01404	3	-1.12931	4	-1.01442	3	-1.07341	3
Germany	-1.01422	4	-1.12742	3	-1.02240	7	-1.07418	4
Italy	-1.01838	6	-1.17905	6	-1.02047	5	-1.09937	6
Portugal	-1.02546	10	-1.24618	10	-1.02786	10	-1.13690	10
Spain	-1.02707	11	-1.24833	11	-1.07837	12	-1.15487	12
Sweden	-1.01080	1	-1.09892	1	-1.01147	1	-1.05648	1
Switzerland	-1.01473	5	-1.13322	5	-1.01614	4	-1.07640	5
United Kingdom	-1.02161	8	-1.20674	7	-1.02498	9	-1.11600	8
United States	-1.02811	12	-1.27511	12	-1.04235	11	-1.15435	11

Figure 12 illustrates these observations for Germany as compared with France and with Switzerland.

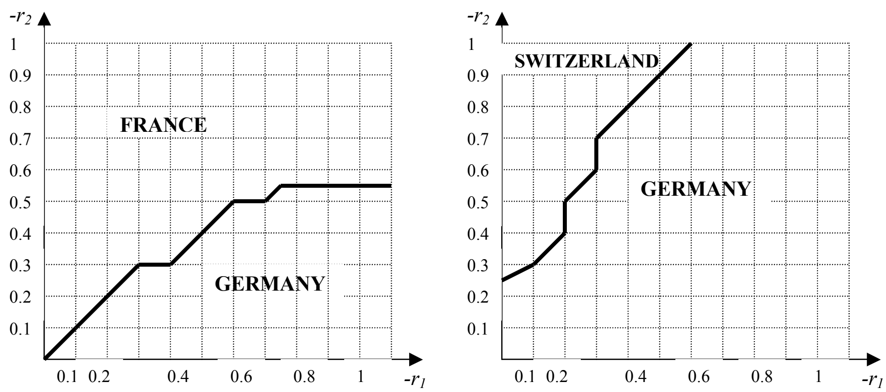

Figure 12

The ranking of actual distributions provided by the Cobb-Douglas index (provided in table 17) is a also more sensitive to the choice of the parameters values than the Bourguignon one (table 18) The versatility is here even greater than with income and infant mortality because it affects also the tails of the rankings. As a matter of fact, France is the only country whose ranking (5th) is preserved in the change of the parameters. We can make the same type of observations as for the Cobb-Douglas index applied to normalized distributions. For a given weight $|r_1|$, the increase in the weight of education ($|r_2|$) yields a different ordering. The most obvious example of this is Portugal, which becomes the most egalitarian country when $r_1 = -0.1$ and $r_2 = -1$, and Switzerland, which is illustrated in figure 12. For a given $|r_1|$ constant, the highest is $|r_2|$, the more Austria is likely to be preferred to Switzerland. We notice however that when we attach the same importance to the two attributes, Austria is always preferred to Switzerland. For a constant social weight on education ($|r_1|$), the increase of the social weight on disposable income ($|r_2|$) change the hierarchy. Austria is not the most egalitarian country, Sweden become less egalitarian than Germany and the United Kingdom and Portugal become the less preferred country. On the other hand, Switzerland becomes the most egalitarian country and the United Kingdom is less unequal.

Country	$r_1 = -0.1$ $r_2 = -0.1$	Rk	$r_1 = -0.6$ $r_2 = -0.1$	Rk	$r_1 = -0.1$ $r_2 = -1$	Rk	$r_1 = -0.4$ $r_2 = -0.4$
Australia	-0.49329	4	-0.00401	4	-5.62667	9	-0.06092
Austria	-0.47486	1	-0.00377	3	-3.74517	2	-0.05154
Canada	-0.50052	8	-0.00420	7	-6.12099	11	-0.06448
France	-0.49569	5	-0.00410	5	-5.37928	5	-0.06129
Germany	-0.49870	7	-0.00417	6	-5.60623	8	-0.06279
Italy	-0.50314	10	-0.00502	10	-4.60877	3	-0.06550
Portugal	-0.50243	9	-0.00570	12	-3.69491	1	-0.06552
Spain	-0.51462	12	-0.00549	11	-5.41961	7	-0.07279
Sweden	-0.49753	6	-0.00439	9	-4.88848	4	-0.06201
Switzerland	-0.47920	2	-0.00328	1	-5.39703	6	-0.05353
United Kingdom	-0.50594	11	-0.00425	8	-6.69779	11	-0.06713
United States	-0.48660	3	-0.00359	2	-5.96262	10	-0.05792

TABLE 17 : GENERALIZED COBB-DOUGLAS INDEX FOR SOME VALUE OF r_1 AND r_2

An interesting question that can be asked from all of this is whether the conclusions obtained from a unidimensional analysis based on disposable income alone are significantly altered by the introduction of the other attribute and the resort to a bidimensional dominance analysis. Clearly the dominance rankings of the countries seems to be somewhat affected by the introduction of multidimensional considerations, as a comparison of figures 1 and 5 reveals. Yet, albeit significant, the difference in the ranking is not astonishing and, from looking at both figures 1 and 2 on the one hand and figure 5 at the other, one could get the impression that bidimensional dominance comparisons are just blurred mixtures of the two unidimensional dominances ones.

TABLE 18 : GENERALIZED BOURGUIGNON INDEX FOR SOME VALUE OF α AND β

Country	$\alpha = 0.1$ $\beta = 0.1$	Rank	$\alpha = 0.9$ $\beta = 0.9$	Rank	$\alpha = 0.1$ $\beta = 1$	Rank
Australia	3.4265	4	7941	3	2,6635	3
Austria	3.4484	3	7075	6	2,6534	6
Canada	3.4042	5	7307	4	2,6469	5
France	3.4015	6	6724	7	2,6342	7
Germany	3.3920	8	6557	8	2,6274	8
Italy	3.3304	10	5137	10	2,5485	10
Portugal	3.2948	12	4416	12	2,4935	12
Spain	3.2999	11	4946	11	2,5275	11
Sweden	3.3701	9	5736	9	2,5943	9
Switzerland	3.4982	1	9488	2	2,7340	1
United Kingdom	3.3925	7	7233	5	2,6418	5
United States	3.4815	2	9918	1	2,7238	2

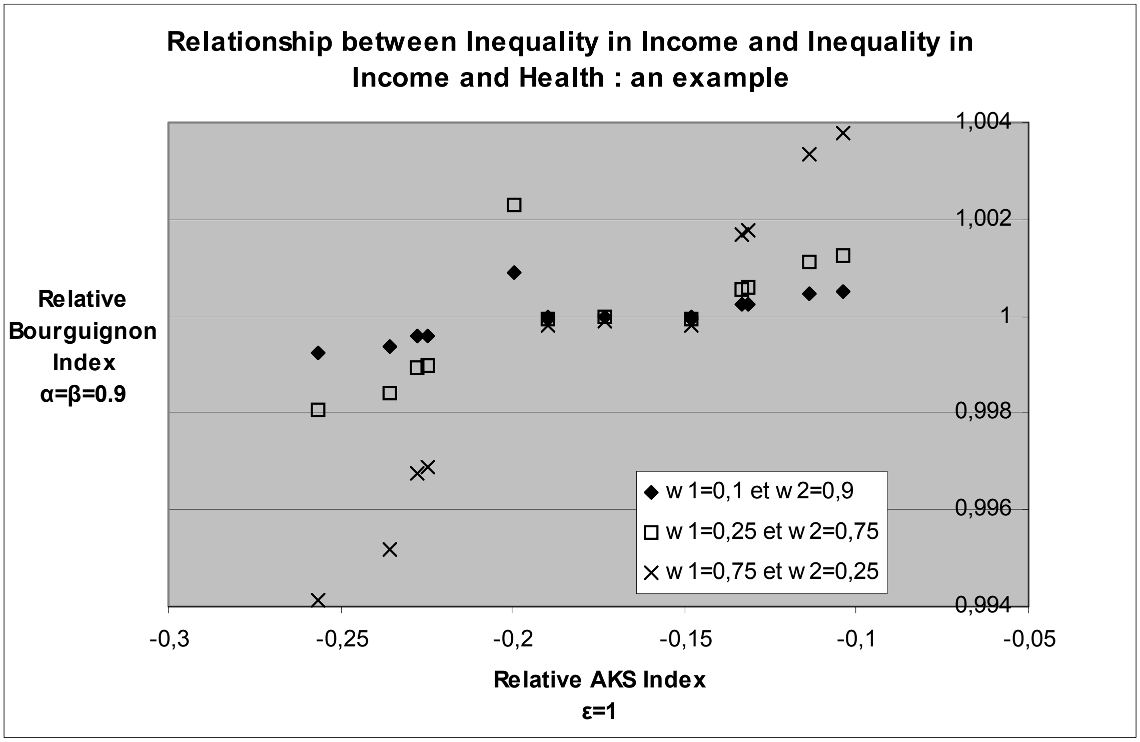

Figure 13

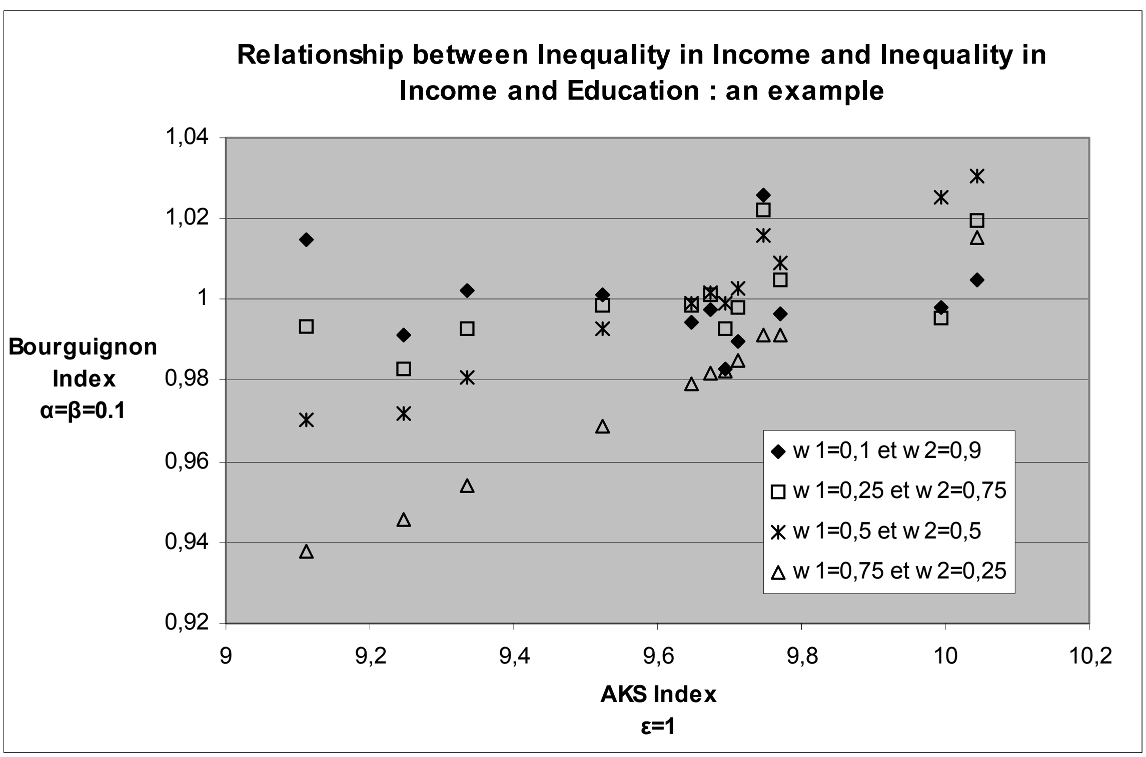

Figure 14
A way to compare differences between multidimensional and unidimensional analysis

is to compare the behavior of unidimensional and multidimensional indices. Figures 13 and 14 show the relationship between Bourguignon multidimensional indices (calculated, for alternative specification of the parameters) and Atkinson-Kolm unidimensional index of income inequality in the case of health and disposable income and education and disposable income respectively. As can be seen, albeit correlated, the behavior of the two types of indices is quite different. The difference obviously depend upon the weight given to disposable income in the calculation of the Bourguignon multidimensional index (after all, the Bourguignon index becomes just a one dimensional income inequality index of the Atkinson and Kolm family if the weight attached to the public good goes to zero). One should also notice that the difference between the two indices is more striking for the distribution of education and income than for the distribution of health and income.

6. Conclusion

The main object of this chapter was to provide the first (to our knowledge) international comparison of OECD countries in terms of their normative performance in allocating disposable income and access to public service to their citizens based on robust multidimensional dominance methodologies. The methodologies used were in part the two multidimensional stochastic dominance criteria proposed by Atkinson and Bourguignon Atkinson and Bourguignon (1982) and a new dominance criterion proposed by Gravel and Moyes Gravel and Moyes (2004) whose discriminating power lies between that of the two Atkinson and Bourguignon Atkinson and Bourguignon (1982) criteria.

The exercise proved to be useful on two fronts : Methodological and empirical. On the methodological front, this paper has illustrated the workability of multidimensional dominance methodology and its ability to rank conclusively a significant fraction (36%) of countries, at least when applied to actual distributions. While this is much lower than the 78% of conclusive comparisons that can be done using unidimensional dominance analysis, the robustness of the multidimensional comparisons from an ethical standpoint is worth keeping in mind : Any conclusive ranking emerging from a multidimensional analysis must entail a dominance in every dimension but not vice versa. For this reason, the core of 36% of the conclusive comparisons that survive the test of multidimensional dominance comparisons can be considered as particularly robust. The analysis has also suggested that, at least for the distribution of education and disposable income, the gain in discriminatory power allowed by a move from Atkinson and Bourguignon first order criterion (the multidimensional headcount poverty dominance criterion) to the ordered poverty gap one one was larger than the gain than the corresponding gain of moving from the ordered poverty gap to Atkinson and Bourguignon second criterion. The somewhat opposite conclusion holds for distribution of health and disposable income.

On the empirical front, the results have suggested that the comparisons of the performance of the OECD countries in terms of their ability to fairly allocate access to public goods and disposable income to their citizens does not indicate any clear advantage in favour of centralized or unitary country on that matter. We believe results of this kind to be of interest to policy and, more to the point, constitution makers, especially perhaps to those who advocate centralized models of governance on the basis of the better distributive performance of centralized countries with respect to access to public goods.

3. Equality of opportunities vs. Equality of outcomes : Are Western societies all alike ?

1. Introduction

The purpose of this chapter is to offer an international comparison of inequality of opportunity, a concept which, in our view, echoes more closely the views held in contemporary societies about inequality. Specifically, we attempt to measure and compare the extent of equality of opportunity for income acquisition in several western societies. This analysis is intended to complement the more conventional one-dimensional comparisons of countries on the basis of disposable income inequality. There is in fact no reason to believe *a priori* the degree of equality of opportunity to be correlated with the degree of equality of outcome (or income). For this reason, we also examine how countries' performance in terms of equality of opportunity relates to their degree of inequality of outcome. Indeed, if some countries favour the concept of equality of opportunity over equality of outcomes, one may observe a somewhat different ranking of countries according to the two criteria. Or it may be that countries that effectively promote equality of outcome are also those which are the most effective in achieving equality of opportunity.

As discussed in introduction, it is not at all an easy matter to analyse the extent of equality of opportunity for income acquisition. Particularly difficult is the definition of the relevant set of circumstances and the measurement of the contribution of these circumstances to the final income earned by a given individual. In this chapter, we focus on individual socioeconomic family-background, a variable that is widely agreed to be an essential part of the set of circumstances that determine one's future. The definition of inequality of opportunity used here borrows from a companion work (Lefranc et al. (2004a)) and it rides on the notion of conditional (in)equality. We take the view that the study of inequality of opportunity can be reduced to a comparison of the distributions of income, *conditional on individual social and economic background*.

The analysis conducted in the chapter is based on a data set gathered by Roemer et al. (2003) and which has been specifically designed to convey information on individual income and socio-economic background. This data set is built from national household surveys for nine countries : Belgium, France, Germany, Great-Britain, Italy, Netherlands, Norway, Sweden and United-States. It contains detailed information on most sources of individual income, as well as information, albeit more limited, on the education of the father of the individual respondent.

Since our data differ from those commonly used in international comparisons of income inequality, we first check that they deliver results on inequality of outcome that are comparable to those found in the literature, before turning to the analysis of equality of opportunity. With respect to inequality of disposable income, we also rank countries according to the criterion of Lorenz dominance which, as recalled in the previous chapter, is known to be a more robust procedure than ranking by the ordering of inequality indexes. As in the previous chapter, a particular attention is paid to the issue of statistical inference, in contrast to many empirical analyses. To this end, we implement a robust non-parametric tests of stochastic dominance that have been developed recently (see e.g. Davidson and Duclos (2000)).

The chapter is organized as follows. The next section presents the definition of equality of opportunity for income acquisition, the statistical procedure and an index of inequality of opportunity. The data are presented in Section 3.3. Section 3.4 includes results about outcome inequality. Section 3.5 draws inequality-of-opportunity comparisons among the nine countries and offers a picture of each country from both perspectives and the last section concludes.

2. From inequality of outcome to equality of opportunity : definition and measurement

When measuring inequality of outcome in empirical work, a wealth of different approaches and indexes can be used. To the contrary, when departing from the analysis of outcome to examine opportunity, one first requires to provide a definition of equality of opportunity that can be implemented empirically.

In this section we offer a formal definition of equality of opportunity and show how it can be used to assess whether equality of opportunity holds. We also develop a cardinal measure of the extent of opportunity inequality that makes cross-country comparisons possible.

2.1. Definition

Equal-opportunity theories differentiate two fundamental sources of inequality among individuals : on the one hand, factors outside the realm of individual choice, usually referred to as *circumstances* ; on the other hand, factors that individuals can be judged responsible for and that can be generically referred to as *effort*. One important principle emphasized by equal-opportunity theories is that differences in circumstances are not a morally acceptable source of inequality. On the other hand, inequality arising from differences in effort need not be corrected. As a consequence, any level of inequality of outcome can be compatible with equality of opportunity. However, when equality of opportunity prevails, no set of circumstances should provide individuals with an advantage over any other set of circumstances. This allows us to derive a condition for equality of opportunity that can be implemented empirically.[1]

In order to derive this condition, one first needs to be more specific about the definition of the advantage that some circumstances s may provide over some others s'. Consider the situation where individuals would be allowed to choose their circumstances (before knowing the level of effort they will exert). In this context, we say that s provides some advantage over s', if all individuals prefer the opportunity set associated with s to the one associated with s'. Now if $S = \{1, \ldots, \overline{s}\}$ denotes the set of all possible circumstances, we say that equality of opportunity prevails if s is not preferred to s' by all individuals.

In the case of income acquisition, the opportunity offered to an individual with circumstances s can be summarized by the conditional income distribution x conditional on s, denoted $F(x \mid s)$. Choosing among elements of S amounts to choosing among income lotteries summarized by their conditional distribution $F(x \mid s)$. Obviously, the definition of equality of opportunity outlined in the previous paragraph is contingent upon the preferences used to rank the opportunity sets offered by different circumstances. We would like the proposed criterion to hold for a sufficiently broad class of preferences. In this paper we use stochastic dominance theory to rank the opportunity offered by different circumstances. In fact, second-order stochastic dominance is equivalent to the Generalized Lorenz (GL) dominance criterion discussed in the previous chapter (see also Shorrocks (1983)). It offers a powerful and quite general criterion for comparing income lotteries.[2] For example, consider two lotteries $F(x \mid s)$ and $F(x \mid s')$. If $F(x \mid s)$ dominates $F(x \mid s')$ according to second-order stochastic dominance, then the lottery associated with s is preferred to the one associated with s' by any individual (a) whose preferences satisfy the axioms of expected utility theory and (b) whose Von-Neuman Morgenstern utility function is increasing and concave.[3]

To summarize, we will say that equality of opportunity is satisfied for the set of circumstances S if and only if :

$$\nexists (s, s') \in S^2 \text{ such that } F(x \mid s) \succ_{SD_2} F(x \mid s')$$

where $\succ_{SD_2}$ denotes second-order stochastic dominance. Defining equality of opportunity as non-dominance with a second order stochastic dominance criterion is equivalent to saying that an individual choosing among these circumstances is unable to rank them.

[1] This characterization of equality of opportunity is developed with greater details in Lefranc et al. (2004b).

[2] $F(x \mid s)$ dominates $F(x \mid s')$ according to second-order stochastic dominance, denoted $F(x \mid s) \succ_{SD_2} F(x \mid s')$, iff $\forall x$, $\int_0^x F(y \mid s)dy < \int_0^x F(y \mid s')dy$.

[3] First-order stochastic dominance would provide an even more robust characterization of equality of opportunity, given our definition, since it only requires the Von-Neuman Morgenstern utility function to be increasing.

2.2. Measurement

Stochastic dominance tests

The condition developed in the previous paragraph suggests a natural empirical test to assess whether equality of opportunity prevails : first, estimate the conditional income distributions associated with some given circumstances and then compare these distributions using second-order stochastic dominance tools. When drawing the GL curves of two conditional income distributions, three situations can occur : (a) one curve lies above the other, (b) the two curves intersect, (c) the two curves are identical. Our definition implies that equality of opportunity prevail in case (b) or in case (c). It is violated in case (a). Case (c) is an interesting particular case of equality of opportunity.[4] Equality of the conditional distributions may be referred to as strong equality of opportunity and can even be detected with a first-order stochastic dominance test.[5] That is why in practice, we estimate the conditional income distributions and we perform non-parametric stochastic dominance tests at the first and second order. The methodology has been developed in Davidson and Duclos (2000). The details of the test statistics and procedure are presented in the appendix.

We implement the following sequence of tests. Comparing two distributions for sub-populations A and B, we test the null that the two distributions are equal. If we fail to reject the null we conclude to strong equality of opportunity between A and B. Note that rejection of the null can occur either because the two distributions A and B intersect or because of dominance at the first order. Consequently, if we reject the null of equality, we go on testing first-order dominance : we test for the dominance of distribution A over B and vice-versa. If the two tests reject dominance, we go on at the second order by drawing Generalized Lorenz curves. We conclude to equality of opportunity in case of a two-way rejection of dominance.

Inequality of opportunity index

One drawback of the characterization of equality of opportunity with an ordinal approach is that it does not allow us to rank different situations in which we would reject equality of opportunity. At the cost of a loss of generality, it is also possible to build an index allowing to measure the degree of inequality of opportunity.

Before proceeding further, it is useful to wonder what kind of minimal properties such an index must satisfy. Borrowing from the literature on inequality indexes (see for instance Sen (1973)), it seems reasonable to require the following properties.

1) *Within-type Anonymity.* The measure must be invariant to any permutation of two individuals of the same type.

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[4] One can notice that it corresponds to the definition of equality of opportunity developed in Roemer (1998).

[5] $F(x\,|\,s)$ dominates $F(x\,|\,s')$ according to first-order stochastic dominance iff $\forall x\ F(x \mid s) < F(x \mid s')$. First-order dominance implies second-order dominance.

2) *Between-type Principle of Transfers of Pigou-Dalton.* Consider two types such that the first one dominates the second one according to the GL test. The measure must decrease if we perform any transfer from some first-type individual to some other second-type individual such that (a) in the ex-ante distribution, the first-type individual is richer than the second-type individual and (b) in the ex-post distribution, the first-type individual is not poorer than the second-type individual, others things being equal. From the Hardy-Littlewood-Polya theorem, it seems clear that the equality of the two distributions may be obtained through a finite sequence of such transfers.

3) *Normalization.* If the CDFs corresponding to all types are identical, then the index must be equal to 0.

4) *Principle of Population.* The measure is invariant to a replication of the population.

5) *Homogeneity of Degree zero.* The measure is invariant to a scale factor applied to all incomes.

This list of properties defines a class of indexes of equality of opportunity. Among it, we favor an index that sounds familiar since it resorts to the most popular index of inequality, the Gini index.

First we ought to agree on a measure of the opportunities offered to individuals of a given type in the space of lotteries. Here we borrow some ideas from the literature about measuring opportunity sets (see Peragine (1999) for a survey). It is natural to see the area under the Generalized Lorenz curve of a given lottery as the feasible opportunity set. Indeed, any lottery dominated according to the GL test belongs to this set. In an influential contribution to the measurement of opportunity sets, Pattanaik and Xu. (1990) axiomatized the cardinal of a discrete set as a measure of opportunity. Among the axioms introduced by the authors, the following monotonicity property reads as follows. Given an opportunity set A and an opportunity y which does not belong to A, $A \cup y$ offers more opportunity than A. When the opportunity set is continuous, counting elements of the opportunity set does not make sense any more. A natural extension is to consider the area below the opportunity set as a cardinal measure of opportunity and, for instance, Bensaid and Fleurbaey (2003) already proposed such a measure when the opportunity set is a budget set. Hence, we use the area under the GL curve of a type as a quantitative assessment of the opportunity of this type.

Let us rank the types according to twice the area under the GL curve, starting from the smallest one. This area for the worst type is equal to $\mu_1(1 - G_1)$ with μ the average and the G the Gini coefficient Yitzhaki (1979) already proposes $\mu(1 - G)$ as a measure of satisfaction of the society, here of the society made of the individuals of the same type. The Gini-opportunity index obeys to the following formula

$$GO(x) = \frac{1}{\mu} \sum_{i=1}^{k} \sum_{j>i} p_i p_j (\mu_j(1 - G_j) - \mu_i(1 - G_i)). \tag{2.1}$$

It computes the weighted sum of all the differences between areas of opportunity sets. Dividing by the average income of the population μ allows to get an index which does not depend on the wealth of the society. This index can be viewed as an extension of the

Gini coefficient since, when there are as many types as individuals, we are back to it,

$$G(x) = \frac{1}{n^2\mu} \sum_{i=1}^{n} \sum_{j>i} (x_j - x_i) \quad (2.2)$$

and therefore the Gini-opportunity index is comprised between 0 and 1. Comparison of formula (2.1) and (2.2) allows to establish $GO(x) \leq G(x)$ and that the Gini-opportunity index increases with the number of types.

Even if it is easily established that the Gini-opportunity index satisfies the above properties, distinctive properties of this index deserves more investigation. Here we do not propose an axiomatization of the index, which will be the subject of further research.

3. Data description

Data requirements for comparing inequality of opportunity for income acquisition across countries turn out to be even more stringent than for comparing inequality of outcome. Indeed, the reliability of the empirical analysis calls not only for comparable measures of individual disposable income. It also requires that individual background be measured in a comparable and homogeneous way across countries.

3.1. Data sets and sample selection

The data used in the empirical analysis come from household surveys and micro-economic administrative data from nine different countries : Belgium, France, West-Germany[6], Great-Britain, Italy, Netherlands, Norway, Sweden, and the United States. All data were collected during the first half of the nineties. For each country, the data sets include information on individual and household income, both pre- and post-fisc, as well as information on individual *circumstances*.

Table 3.1 summarizes the main characteristics of the data sets used for each country. The data used are sub-samples from the original surveys. They were put together by national experts within the context of a previous international comparison of income inequality and equality of opportunity, whose results were presented in Roemer et al. (2003).[7] Although these national data sets were collected independently, much effort was expended to ensure the greatest degree of *ex post* comparability across countries of the different variables used in the analysis.

Needless to say, this represents a challenging task, given the number of countries involved in the present analysis. The comparability of the data across countries remains imperfect and needs to be carefully examined. In fact, one may question the usefulness

[6] East-Germany has been discarded on the ground that, from an economic point of view, it remained a distinct society from the West-Germany in the mid-nineties.

[7] For providing access to the data, we are grateful to Marx (Belgium data), Wagner (German data), Jenkins (British data) Colombino (Italian data), Pommer (Dutch data), Aaberge (Norwegian data), Fritzell (Swedish data), Page and Roemer (US data).

TAB. 3.1 – Data bases

			Year	Obs.
Belgium	PSBH	Panel survey of Belgian households	1992	933
France	BdF	French Household Survey	1994	2 769
West-Germany	GSOEP	German socio-economic panel	1994	1 143
Great-Britain	BHPS	British household panel survey	1991	991
Italy	SHIW	Italian survey of household income and wealth	1993	1 392
Netherlands	AVO	Dutch facilities use survey	1995	1 758
Norway	SLL	Norwegian survey of level of living	1995	576
Sweden	LNU	Swedish level of living survey	1991	1 469
USA	PSID	Panel study of income dynamics	1991	1 119

of such an endeavor given that harmonized micro-economic income data sets, such as the Luxembourg Income Study, the OECD , the ECHP or the World Bank data sets have already been built[8]. However, one unique feature of the data used in this article is that it provides, beside information on individual income, information on individual circumstances (see below). Hence, being able to relate individual income to individual circumstances in nine developed economies makes the data set used here extremely valuable. One further advantage of these data is that these data include information on Sweden and the Netherlands, two countries that are often absent form international comparisons of income inequality.

Samples used in the rest of the paper are restricted to households whose head is a man, aged 25 to 40 at the time of the survey (25 to 50 in Germany).

3.2. Main variables

Individual circumstances

Defining the exact set of individual circumstances is a deep and debatable question. Besides, in empirical work, observing this entire set is clearly out of reach. In this paper, we examine the dependence of individual opportunity on a narrower set of circumstances, namely circumstances relating to individual social background.[9]

For most countries in our data, individual social background is measured by the level of

[8]See Gottschalk and Smeeding (2000) for an analysis of income inequality from the LIS data, OCDE (1998) for OECD data, and Deininger and Squire (1996) for a presentation of the World Bank data. However, these normalized data sets are not immune to statistical problems. See Atkinson and Brandolini (2001) for an assessment of these.

[9]Of course, social origin may influence individual success through a variety of channels such as economic or genetic inheritance. Our interest solely lies in determining the extent to which circumstances influences individual opportunity sets. Identifying these different channels is not the topic of this chapter.

TAB. 3.2 – Samples description

	Observations			Years of education		
	ED_1	ED_2	ED_3	ED_1	ED_2	ED_3
Belgium	425	341	167	< 10	$10-12$	>12
France	1274	703	792	—	—	—
G-Britain	402	307	282	—	—	—
W-Germany	857	142	144	< 10	$10-13$	>13
Italy	245	706	441	< 5	$5-7$	> 7
Netherlands	479	788	491	< 6	$6-9$	> 9
Norway	247	170	159	< 9	$9-11$	> 11
Sweden	825	414	230	< 8	$8-11$	> 11
USA	390	354	375	< 12	12	> 12

Number of observations and number of years of education of the parents for the different sub-samples. — : information about the occupational group of the parents have been used.

education of the father. The only two exceptions are France and Great-Britain for which we only observe the occupational group of the father. For each country, we partition our sample in three categories, Ed_1 to Ed_3, where Ed_3 denotes the most advantaged social background. When using father's education, we account for specificities of national educational systems. When using information on father's social group the classification is as follows : for France, (1) farmers and manual workers, (2) clerks and (3) professionals and self-employed workers ; for Great-Britain, (1) farmers and unskilled manual workers, (2) clerks and skilled manual workers (3) professionals and self-employed workers[10].

Table 3.2 provides details about the classification of social background in each country, as well as the number of observations in each category. In the partitioning of our samples, two constraints had to be taken into account. First the need for sub-samples for each type of social background that would be large enough to allow for the estimation of conditional income distributions. Second the requirement of a meaningful partitioning, with respect to each country's educational and social structure. As a consequence of these two constraints, the comparability of our classification across countries remains imperfect. In particular, one should be aware of differences in the relative size of each group across countries. In France, Great-Britain, the Netherlands, Norway and the US, each group represents between 1/4 and 1/2 of the overall population. This does not hold for Belgium, Germany, Italy and Sweden where one group represents less than 1/6 of the overall population. This should be kept in mind when analyzing the extent of equality of opportunity in section 3.5.

[10]For the French sample it is the occupational group when then individual was 16, In Great-Britain it is the occupational group when he was 14.

Income

We focus on two measures of individual income : gross pre-fisc annual household income and net disposable annual household income.[11] Analyzing both income measures allows to examine the impact of fiscal redistribution on inequality of outcomes and opportunity.

Since household income (both pre- and post-fisc) incorporates a variety of different income sources, similar sources should be taken into account for each country in order to ensure cross-country data comparability. Gross pre-fisc income includes labor income (from both salaried and self-employed workers) and asset income. The only exception is Belgium for which neither self-employment nor capital income is available. This could lead to underestimate inequality in this latter country. Labor income is measured gross of any employee share of social security contributions. Taxes taken into account are income tax as well as housing and property taxes. Transfers include unemployment benefits, all social security benefits (related to sickness, disability, maternity, poverty ...), pensions, child or family allowances and means-tested benefits. Details of income sources taken into account, for each country are provided in table 7.1. To account for differences in household size, income is normalized using the OECD equivalence scale. It amounts to divide household income by the square root of the number of household members.

4. Inequality of outcome

While several papers have already compared the extent of income inequality across countries, using harmonized data, the analysis undertaken here is interesting for two reasons. First, it can be seen as a test of the validity of the data used in this paper. In fact, our results broadly concur with those of previous analysis, which can be interpreted as an evidence of the validity of our data set for performing cross-country comparisons of inequality. Second, while most comparative papers have concentrated on the analysis of inequality indexes, we also compare relative inequality across countries by using Lorenz dominance criterion. The interest of this latter criterion lies in its greater generality, since, as shown by Atkinson (1970), Lorenz dominance among two distributions implies that all relative inequality indexes will consistently provide the same ranking of these distributions. We also pay particular attention to issues of statistical inference and implement Lorenz Dominance tests.[12]

We first discuss the ranking of countries which emerges from these tests before performing a comparison with the results of other studies based on inequality indexes.

[11] In most countries, taxes and employee social security contributions are simulated. Differences across countries regarding the share of social security spending financed by means of employer contribution, employee contribution or income tax is likely to reduce the comparability across countries of gross pre-fisc income levels. Comparison of disposable income distributions across countries does not raise similar concerns.

[12] The methodology of these tests are presented in the appendix.

4.1. Lorenz Dominance tests

One useful way to get a first picture of income inequality in the nine countries is to compare the shape of the income densities. The densities are estimated in logarithm using kernel estimation[13]. Figure 3.1 gives the densities of the distribution of disposable income centered around their mean. The American distribution is reproduced on each graph to make comparisons easier.

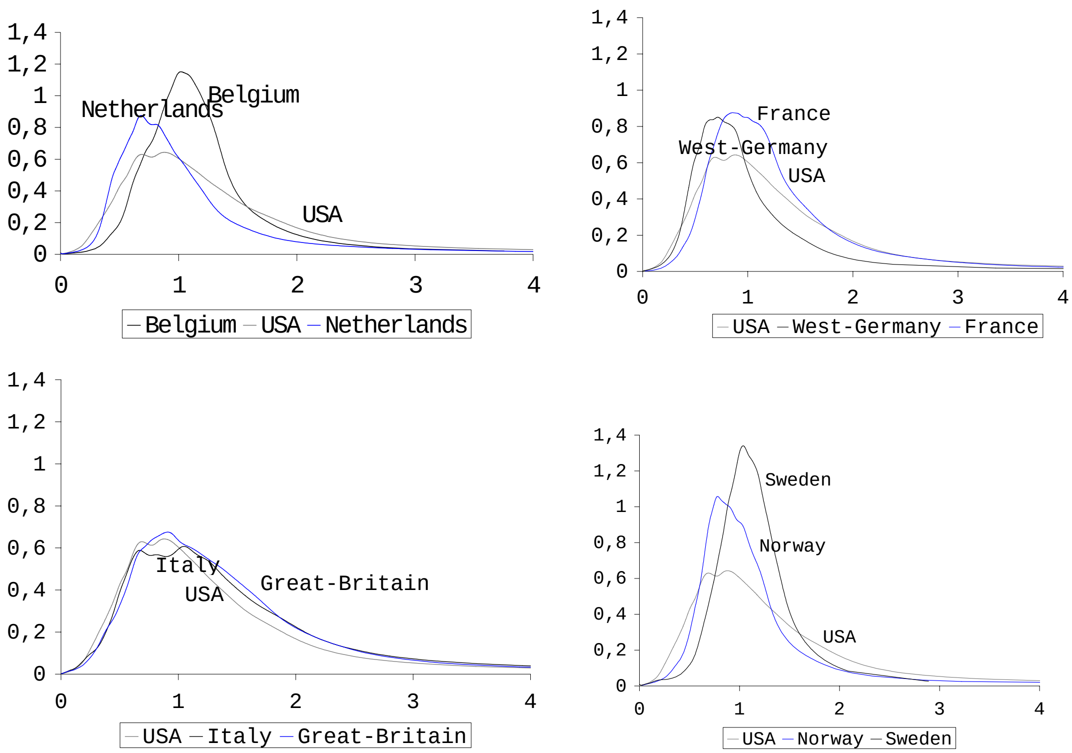

Figure 3.1

The comparison of these densities reveals important differences across countries in the distribution of income. The contrast between Sweden and the US is striking with a fairly symmetric distribution concentrated around its mean for the first one and a strong right skew for the second one. The differences between other European distributions and America's one are less sharp. Norway shares with Sweden a significant polarization around the mode but it is less concentrated than the Swedish distribution. The case of Belgium seems to be fairly similar to these two Nordic countries. The shape of the distribution in the Netherlands, France and Germany is comparable and lies in an intermediate position between Sweden and the US. The British density is closer to the American one than to the distribution in continental Europe, with the exception of Italy. This latter country displays a distribution fairly close to the American and British ones.

To obtain a more precise picture of inequality we consider Lorenz curves. Figure 3.2 shows Lorenz curves for disposable income in each country. As for income densities, the

[13] A Gaussian adaptative bandwidth kernel estimator has been used.

American curve is represented on each graph. Their analysis corroborates our previous comments. On the top-left panel, it is apparent that the Belgian Lorenz curve is above the Dutch curve, which itself dominates the US one. On the top-right panel, one can notice that France and West-Germany have a similar level of inequality. The bottom-left panel confirms that inequality is pretty much the same in GB, the US and Italy. Finally, on the bottom-right panel, one can notice the significant gap between Scandinavian countries and the United-States.

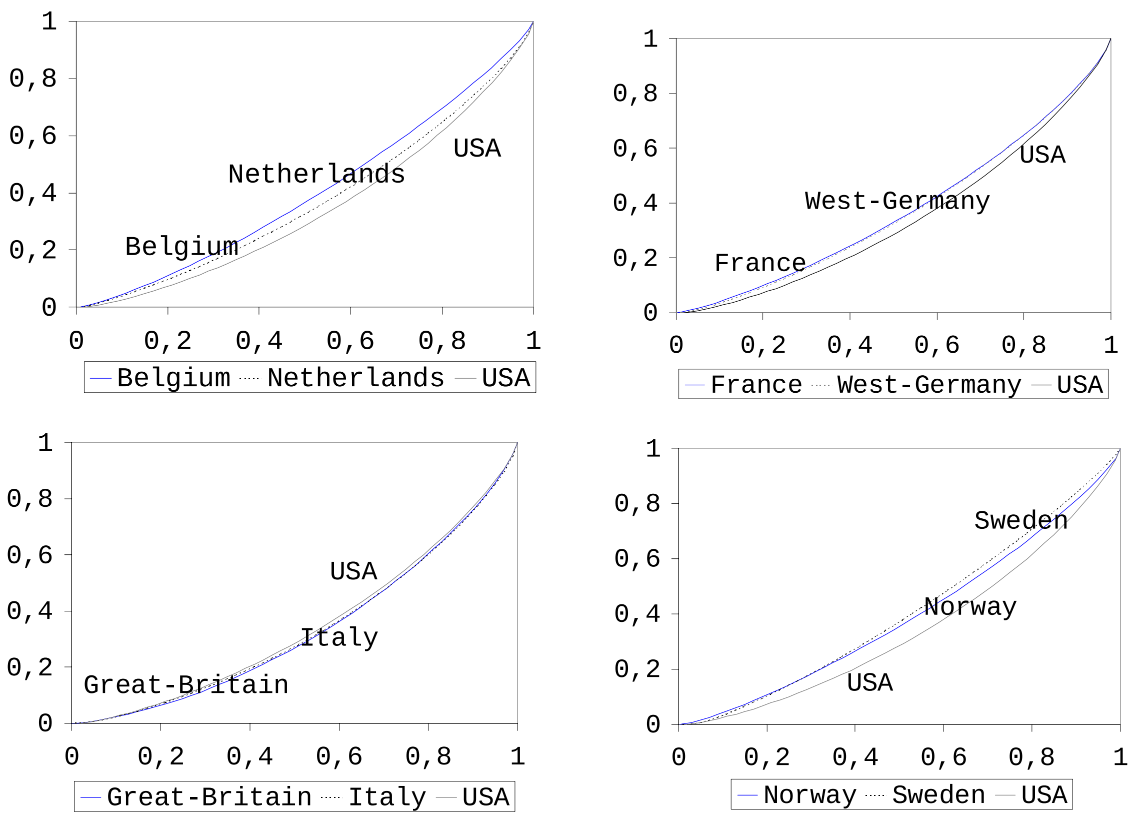

Figure 3.2

This visual inspection is confirmed by the results of the Lorenz dominance tests for each pairwise comparison (table 4.1). These results do not lead to a complete ranking of the countries. However three groups of three countries clearly emerge from those tests. The first group is made of Sweden Norway and Belgium. The second one includes France, Germany and the Netherlands. The third one is composed of GB, Italy and the US. The hierarchy between the three groups is obvious. All countries in the first group Lorenz-dominate the countries of the second and third group, the countries of the second group Lorenz-dominates the countries of the third one. The within-group ranking is less clear. Within the first group, Sweden dominates Norway but not Belgium; Lorenz curves for Belgium and Norway intersect. This apparently low level of inequality in Belgium may partly be ascribed to the fact that our Belgian data do not take asset income into account.

Within the second group and third group, for each pairwise comparison, dominance tests conclude to either equality or crossing of the Lorenz curves.

TAB. 4.1 – Lorenz dominance tests

	Sweden	Norway	Belgium	France	W-Germ	Nether	G-Britain	Italy	USA
Sweden	-	>	?	>	>	>	>	>	>
Norway	-	-	?	>	>	>	>	>	>
Belgium	-	-	-	>	>	>	>	>	>
France	-	-	-	-	?	=	>	>	>
W-Germ	-	-	-	-	-	?	>	>	>
Nether	-	-	-	-	-	-	>	>	>
G-Britain	-	-	-	-	-	-	-	=	=
Italy	-	-	-	-	-	-	-	-	?
USA	-	-	-	-	-	-	-	-	-

The symbols read as follows : > : The row dominates the column. < : the column dominates the row. = : Lorenz curves are identical. ? : Lorenz curves are non comparable.

4.2. Comparison with other studies

In order to assess the reliability of our data, we now compare our results to the ones obtained in other studies, using harmonized income data. In this perspective, we estimate scalar indexes of relative inequality in the nine countries. Estimates are reported in table 7.2 in the appendix, with bootstrapped standard-errors in brackets. For obvious reasons, inequality indexes (Gini, CV) and inter-quantile ratios presented in table 7.2 suggest a ranking of countries that is similar to the one established in the previous section. Within-group differences in inequality indexes are not statistically significant, while between-groups differences are.

One natural benchmark to gauge the reliability of our income data is to compare our results to those obtained in Gottschalk and Smeeding (1997), using data from the Luxembourg Income Study for the early nineties. Three points should be emphasized. First our relative ranking of countries is to a large extent consistent with the results presented in their studies. Second, for most countries, our estimates of inequality indexes are lower than those reported in their studies. This may largely reflect differences in sample selection rules, and in particular the fact that we have restricted our samples to a narrower age interval.[14] Third, two noteworthy differences appear regarding the level of inequality and the ranking of two countries : France and Italy. In our data the former appears less unequal and the latter more unequal than in Gottschalk and Smeeding 2000, both in absolute and relative terms.[15] Regarding France, the difference in measured

[14]For most countries, our samples are restricted to household whose head is aged 25 to 40, while their sample include all non-institutionalized households.

[15]According to the value of Gini coefficients displayed in Gottschalk and Smeeding's study, France

inequality can be explained by the fact that we use data from 1994, against 1989 or 1984 in their study. Moreover, Hourriez and Roux (2001) demonstrates that disposable income inequality decreases slightly between these dates. Regarding Italy, their data refer to 1991, a year for which measured inequality is markedly lower than in adjacent years, in particular 1993, the year used in our study. For Italy as well as more generally, our results seem close to those of other studies, both in terms of levels of inequality and of ranking of the countries : Bertola et al. (2001) find a Gini of 0.348 for disposable income with LIS data in 1994, and rank Italy among the more unequal countries in Europe. The same conclusion emerges from Atkinson (1996), OCDE (1998) and Smeeding and Grodner (2000) who establish a ranking that is similar to ours. Sastre and Trannoy (2001) find very similar results for Gini indexes using LIS data [16].

Overall, our results closely mimic those obtained in various sources our data, which suggests that we should be reasonably confident in the validity of our income data for international comparisons of inequality. We now turn to the analysis of inequality of opportunity.

5. Equality of opportunity for income acquisition

The above conclusions for inequality of outcomes may not prevail for inequality of opportunity. In fact, in a country with limited inequality of opportunity, there can be important differences in individual success (hence important inequality of outcome) if individuals exert very heterogeneous effort levels. At the opposite, a low level of inequality of outcomes is compatible with important differences according to social origin. This would be the case if the level of effort was negatively correlated with social origin. Results of stochastic dominance tests are first presented. Differences in the return and risk of income lotteries conditional on social background are then analyzed. Lastly, using our index of inequality of opportunity, we examine how countries' performance in terms of equality of opportunity compares to their ranking in terms of overall inequality.

5.1. Dominance tests

Figure 3.3 depicts the conditional distributions for primary and disposable income in each country. For each country, income is expressed as a fraction of the country's mean income. It comes as no surprise that having more educated parents is associated with a higher level of income. Indeed in every country but one[17], the CDF for individuals from more privileged origin is always below the CDFs for individuals coming from the two less privileged social backgrounds.

ranks third with a Gini of 0.32 after the United-States (0.36) and Great-Britain (0.34). In our data income inequality is larger in Italy and Netherlands than in France. See table **??**.

[16] They find a Gini of 0.30 for USA, 0.30 for Great-Britain, 0.23 for Norway, 0.22 for Sweden, 0.26 for Germany, and 0.28 for France. See Sastre and Trannoy (2001) table 2 pp.329.

[17] In the case of Germany, the graph of the CDF for Ed_3 is above the one for Ed_2 for incomes greater than 1.5 mean income.

These graphs also reveal important differences between countries in the *magnitude* of the advantage conferred by more privileged backgrounds over less privileged ones. Intuitively, this advantage corresponds to the gap between the CDFs corresponding to the different social backgrounds. As apparent from these graphs, this distance varies strongly from one country to another. For Sweden, the three conditional distributions for Ed_1 to Ed_3 are strikingly close, suggesting that differences in social background translate into very small differences in income. The same holds true, to a lesser extent, in Norway where the gap between the income distributions of the different backgrounds is rather modest.

This stands in marked contrast with the situation in Italy and the US where the gap between the three distributions is important. In Great-Britain, the advantage conferred to the most privileged group is still quite large but the gap between the second most privileged group is less wide than in the US and Italian cases. Moreover, the income distribution of groups Ed_1 and Ed_2 are closer together than in Italy and the US, suggesting more equality of opportunity in this country at the bottom of the social ladder.

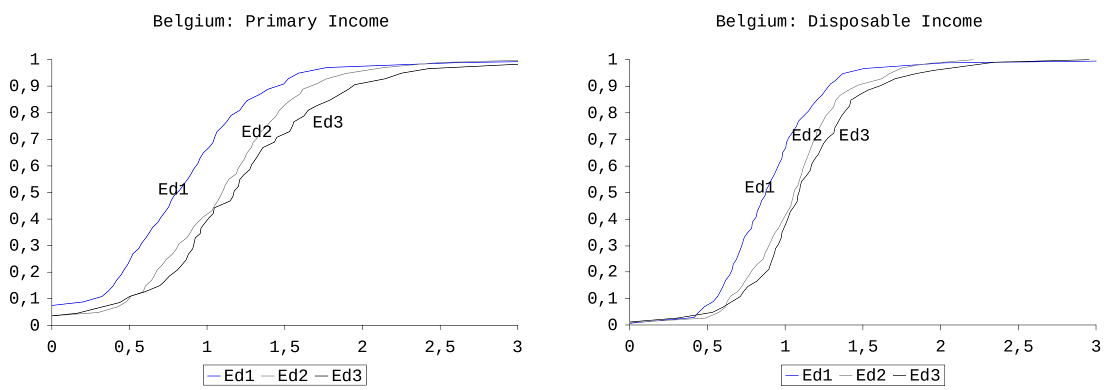

Figure 3.3

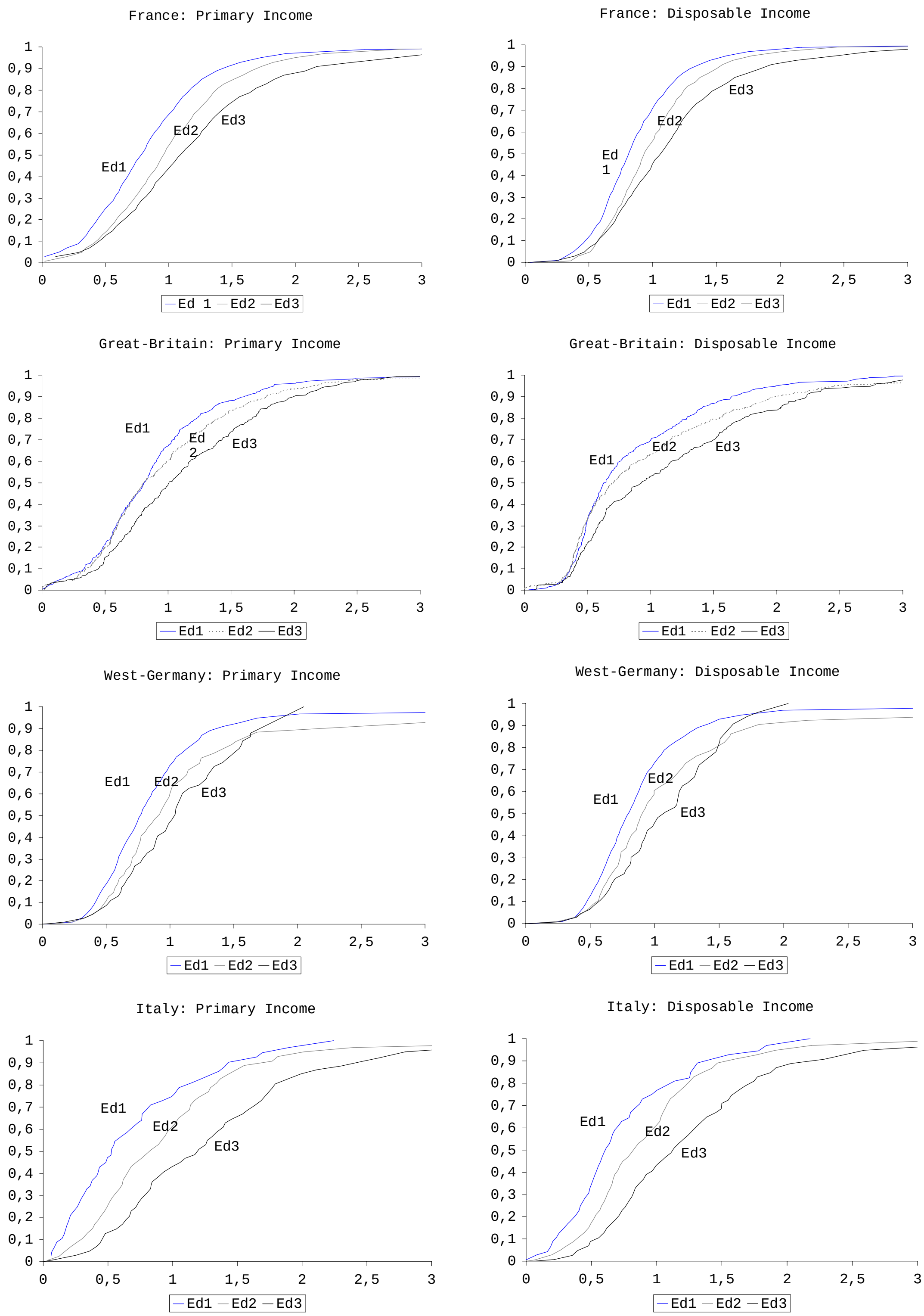

Figure 3.3 (continued)

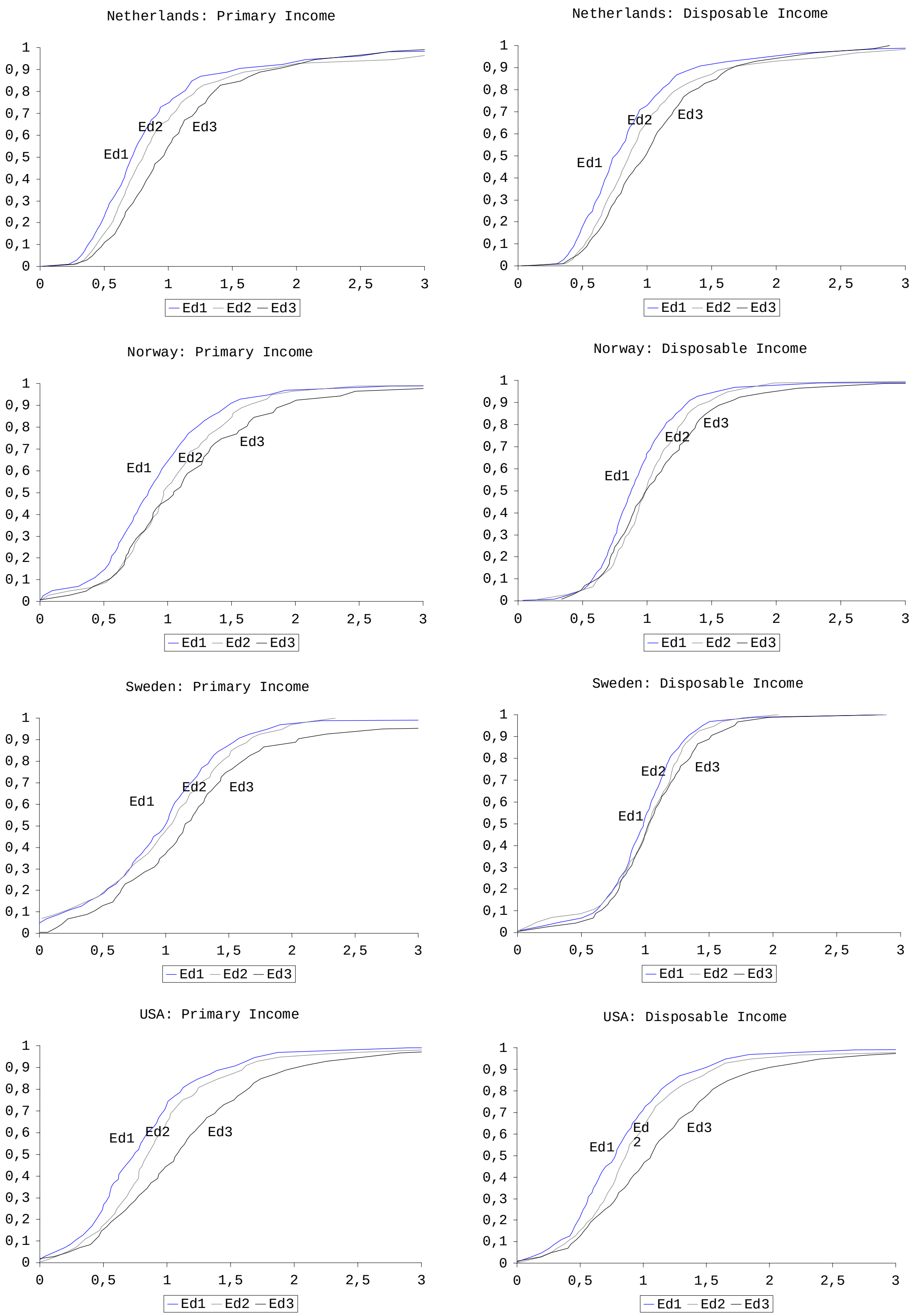

Figure 3.4 (continued)

The rest of the countries in our data (Belgium, France, Germany and Netherlands) exhibit an intermediate degree of inequality of opportunity. There are significant differences in the income distributions offered to individuals according to their social background. However, the distance between these distributions is smaller than in Italy and the US. It should also be noted that in the former group of countries, especially in Belgium and Germany, inequality of opportunity is more pronounced at the bottom of the social hierarchy, to the extent that the gap between the distributions of groups Ed_1 and Ed_2 is larger than the distance between Ed_2 and Ed_3. This contrasts with the situation in Italy, Great-Britain and the US. However, these differences in the locus of inequality of opportunity may partly reflect differences in the classification used to partition our sample according to social background rather than specific national features.

The extent of inequality of opportunity summarized by these graphs can be formally analyzed with stochastic dominance tests. The results of these tests appear in table 5.1. The only country in which our equality of opportunity criterion is satisfied for all groups is Sweden. In fact, this country exhibits a situation described previously as strong equality of opportunity, as the pairwise tests conclude to the equality of the three conditional distributions. It should also be stressed that this strong requirement holds for both primary and disposable income. In all other countries, according to our definition, equality of opportunity does not prevail. There exists at least one social background whose income distribution is dominated by that of another group. It is nevertheless possible to rank these countries according to the number of times the statistical tests conclude to dominance in the three pairwise comparisons. In this respect, when focusing on comparisons of disposable income, Norway is the least unequal (in terms of opportunity) since dominance is detected only in one case and equality prevails in the two other comparisons. Great-Britain and Belgium come next with two cases of dominance and one equality. In the German case, the three tests conclude to dominance, but in two cases, only for second-order stochastic dominance, indicating that the CDFs cross. Lastly, in France, Italy, the Netherlands and the US, the three tests conclude to dominance at the first order, indicating that the hierarchy of social backgrounds apparent on the graphs of the CDF is indeed very robust.

For seven samples out of nine, the results of the dominance tests for primary income are identical to the results for disposable income. This can be interpreted as the weak impact of redistributive policy on equality of opportunity as it is measured here. Hence redistributive policy is not able to fully neutralize the effect of the initial background on the economic success of the next generation. Nevertheless Figure 3.3 reveals that redistributive policies tend to partially offset the impact of social origin on individual income : in all countries, the CDF for primary income, conditional on social background are always further apart than the CDFs for disposable income.

Tab. 5.1 – Stochastic dominance tests

	Belgium						**France**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	$<_1$	$<_1$	-	$<_1$	$<_1$	-	$<_1$	$<_1$	-	$<_1$	$<_1$
Ed_2	-	-	?	-	-	$=$	-	-	$<_1$	-	-	$<_1$
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Great-Britain						**West-Germany**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	$=$	$<_1$	-	$=$	$<_1$	-	$<_1$	$<_1$	-	$<_1$	$<_2$
Ed_2	-	-	$<_1$	-	-	$<_1$	-	-	$=$	-	-	$<_2$
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Italy						**Netherlands**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	$<_1$	$<_1$	-	$<_1$	$<_1$	-	$<_1$	$<_1$	-	$<_1$	$<_1$
Ed_2	-	-	$<_1$	-	-	$<_1$	-	-	$<_1$	-	-	$<_1$
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Norway						**Sweden**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	$=$	$<_1$	-	$=$	$<_1$	-	$=$	$=$	-	$=$	$=$
Ed_2	-	-	$=$	-	-	$=$	-	-	$=$	-	-	$=$
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	U.S.A.					
	Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	$<_1$	$<_1$	-	$<_1$	$<_1$
Ed_2	-	-	$<_1$	-	-	$<_1$
Ed_3	-	-	-	-	-	-

The symbols read as follows : $<_1$: The column dominates the row at the first order. $<_2$: The column dominates the row at the second order.$=$: CdF are identical. ? : CdF curves are non comparable.

5.2. Risk and return of the social lotteries

Using standard tools in risk theory, it is also possible to compare the income lotteries attached to different social background in terms of their return and risk. Since for most countries, the tests conclude to first-order stochastic among social backgrounds, we already know that the expected income (*i.e.* the return) is usually larger for the more favored social background. However, whether the lotteries offered to the more fortunate type are also less risky remains an open question.[18]

An almost equal risk of conditional lotteries

To focus solely on risk, we examine conditional distributions centered around their means, and we draw Lorenz curves of these centered distributions. Comparing two distributions, if the Lorenz curve of the first distribution is above the Lorenz curve of the other then the first distribution will be considered less risky by all risk-averse individuals, whatever the degree of their risk-aversion. Figure 4.6 in the appendix presents the Lorenz curves for the conditional distributions. Table 5.2 contains the results of the Lorenz dominance tests. The testing procedure is similar to the one used for stochastic dominance : we first test for equality of the Lorenz curves, and then test for dominance if equality is rejected.

These results suggest that the degree of risk of the income lotteries associated with social background tend to be rather similar. For most countries, the Lorenz curves of the different types are very close, especially for disposable income. Regarding the tests, there is a surprisingly large proportion of pairwise comparisons for which we conclude to the equality of the Lorenz curves : 19 times out of 27 for primary income and 17 times out of 27 for disposable income. Even if we exclude all cases in which the uncentered distributions are already equal, we conclude to the equality of the Lorenz curves in about half of the cases. In each country there is at least one pairwise comparison for which equality holds. This is true for both primary and disposable income. All three conditional distributions display the same degree of risk in four countries for primary income (France, West-Germany, Sweden and the US) and two countries for disposable income (Sweden[19] and Belgium). For these countries, the equality of risks suggests that the impact of the family background may only be captured by a scale parameter. As a first approximation, in these countries, the distribution of income conditional on social background, takes the following multiplicative form :

$$x_{is} = E(x \mid s)\epsilon_i \qquad (5.1)$$

where x_{is} denotes the income of individual i with social background s, $E(x \mid s)$ is the expectation of income conditional on s and ϵ_i is a random term independent of social

[18] This cannot be deduced from our previous empirical evidence since first-order stochastic dominance is consistent with any behavior of the decision-maker toward risk.

[19] This comes as no surprise regarding Sweden since we had already noted that the conditional distributions are very similar in this country.

background.[20] It should be stressed that equality in the degree of risk of the different distributions is an interesting special case where the ranking of the distributions can be achieved based solely on a comparison of the returns.

When equality of risks does not hold, the tests conclude to the crossing of the Lorenz curves in one third of the cases. When the conditional Lorenz curves can be ranked, the table indicates that less privileged backgrounds face more risky income lotteries than more privileged ones in all cases for primary income, but only in one third of the cases for disposable income. This indicates that redistributive policies tend to lower the risk of the worst social lotteries. For instance France or West-Germany face a situation of perfect equality of risk in primary income, but after income tax and transfers, the lottery corresponding to the more privileged type is riskier than the other two. Suppose that we are ready to assume, following Roemer's suggestion, that the dispersion of incomes within a type is the result of effort only. Then a policy aimed solely at reducing inequality of opportunity should leave the level of risk unchanged. Under the assumption, which is quite strong admittedly, we conclude that the French and German redistributive policies are not solely motivated by equality of opportunity.

Inequality of Return and inequality of Risk

So far our appraisal of risk relies on ordinal comparisons. Resorting to a cardinal measure allows us to exhibit additional empirical evidence, though at the price of lower robustness. We compute two new indexes that measure respectively inequality of opportunity in terms of returns to social lotteries and in terms of risk across social types. Both indexes derive from the Gini-Opportunity (GO) index described in section 2. Our measure of inequality of opportunity in returns to social lotteries (GO-return) is equal to the value of the GO index when within-social-type inequality has been erased (*i.e.* all individuals in a group have an income equal to the average income of that group). Our measure of inequality of opportunity in risk to social lotteries (GO-risk) is equal to the value of the GO index when between-social-type inequality has been erased (*i.e.* all social types have the same mean income, but within-type inequality remains[21]). The values of these indexes are presented in figure 3.4.

[20] For France, this result is robust to a finer partitioning of social background (see Lefranc et al. (2004c) for more details).

[21] More precisely, we equalize between types inequality by a homothetic transformation of the conditional distribution of each type .

Tab. 5.2 – Lorenz dominance tests

	Belgium						**France**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	<	?	-	=	=	-	=	=	-	=	>
Ed_2	-	-	=	-	-	=	-	-	=	-	-	>
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Great-Britain						**West-Germany**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	=	=	-	>	=	-	=	=	-	=	>
Ed_2	-	-	<	-	-	=	-	-	=	-	-	=
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Italy						**Netherlands**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	<	<	-	?	<	-	=	<	-	=	?
Ed_2	-	-	=	-	-	=	-	-	<	-	-	<
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	Norway						**Sweden**					
	Primary Income			Disposable Income			Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	=	?	-	>	=	-	=	=	-	=	=
Ed_2	-	-	=	-	-	=	-	-	=	-	-	=
Ed_3	-	-	-	-	-	-	-	-	-	-	-	-

	U.S.A.					
	Primary Income			Disposable Income		
	Ed_1	Ed_2	Ed_3	Ed_1	Ed_2	Ed_3
Ed_1	-	=	=	-	?	=
Ed_2	-	-	=	-	-	=
Ed_3	-	-	-	-	-	-

The symbols read as follows : < : The column dominates the row. = : Lorenz curves are identical.
? : Lorenz curves are non comparable.

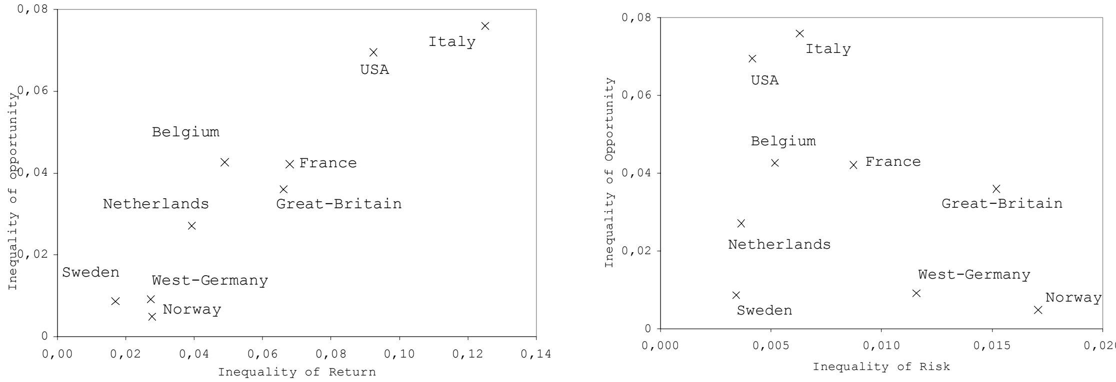

Figure 3.4

It turns out that the dispersion across countries in GO-return is slightly larger than the dispersion in GO-risk. Indeed, the largest value of GO-return is about 7 times greater than the smallest one, while the gap is only 5 times larger for GO-risk. It is instructive to figure out how these two components of inequality of opportunity shape in inequality of opportunity. The two figures illustrate how return inequality and risk inequality are related to inequality of opportunity measured by the Gini-Opportunity index for our sample of countries.

The left panel highlights the positive contribution of inequality of returns to inequality of opportunity. A strong similarity emerges from the comparison of the ranking according to both dimensions. In the making of inequality of opportunity, inequality of returns stands out as the dominant force.

The influence of inequality of risks on the overall result is more complex to figure out. When interpreting the right panel, we have to take into account the fact that a higher risk inequality is not always detrimental to the least favored background. Indeed, in some countries, for instance France, Great-Britain and Norway, the least advantaged social group is less risky than at least one of the two other groups, while in other countries like the Netherlands and Italy the most privileged type is less risky than some other group (see Table 5.2). In the former case, risk inequality mitigates return inequality a little bit, while in the latter risk inequality exacerbates return inequality. Let us take some examples to see how this phenomenon matters to shape the ranking of countries. Norway exhibits a larger return inequality than Sweden and yet in terms of inequality of opportunity the ranking is reversed. Indeed Norway displays a large inequality of risks to the detriment of the most privileged type. The same explanation runs for the comparison of France and Belgium, and of Great-Britain and the Netherlands : for France and Great-Britain, inequality of risk mitigates inequality of returns.

5.3. Inequality of opportunity versus inequality of outcomes

We now address the relationship between inequality of opportunity and inequality of outcomes among countries. To do so, we use the Gini index and the Gini-Opportunity index, since resorting to a cardinal measure of inequality makes comparisons easier. The values of these indexes are presented in table 5.3. Regarding the extent of inequality of opportunity, three groups of country stand out. A first group composed of Sweden, Norway and Germany with the lowest inequality of opportunity. An intermediate group composed of Belgium, France, Great-Britain and the Netherlands. And a group of high inequality of opportunity composed of Italy and the US.

TAB. 5.3 – Index of inequality of Opportunity (GO) and Inequality of outcome (Gini)

	GO	Gini
Sweden	0.009	0.19
Norway	0.005	0.21
Belgium	0.043	0.20
Nether	0.027	0.26
France	0.042	0.25
West-Germ	0.009	0.26
G-Brit	0.036	0.30
Italy	0.076	0.34
USA	0.069	0.31

Figure 3.5 reveals a positive correlation between inequality of opportunity and inequality of outcomes. Sweden and Norway are the least unequal countries according to both concepts while the United-States and Italy are the most unequal ones. The correlation between inequality of opportunity and inequality of outcome is of course far from perfect.[22] If we draw a line that joins the two polar cases, two groups of outliers stand out : Belgium and France lay above the line, Netherlands, Great-Britain and Germany are below.

[22]If we were to exclude the US and Italy from our graph, very little dependence would have been detected between the extent of inequality of outcome and inequality of opportunity. Of course the omission of these two large countries would have hampered the study. This observation tells us that the positive correlation between the two concepts of inequality may depend on which country is included in the sample.

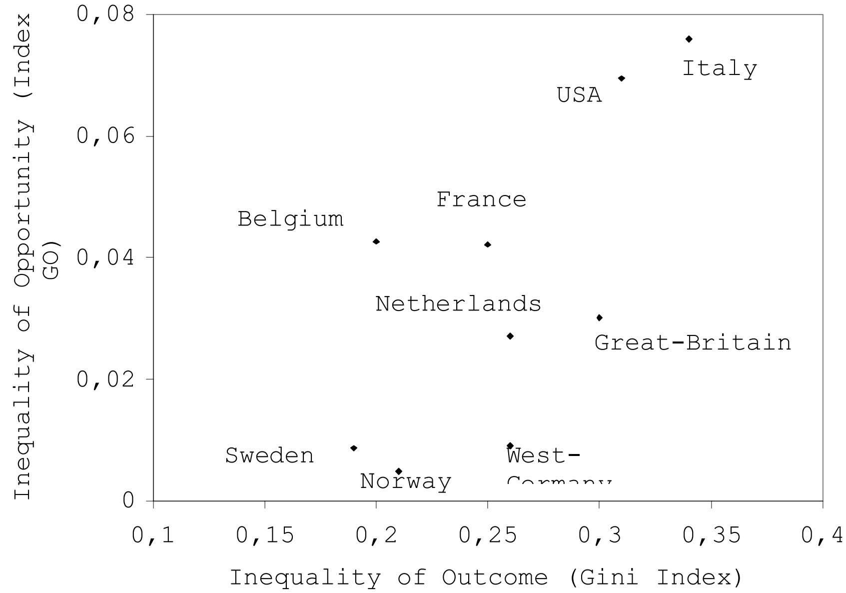

Figure 3.5

Given the size of our sample of countries, these facts should be interpreted with great caution. However, this pattern of outliers might reflect attitude towards individual responsibility rooted in religious and cultural ethics. European countries of catholic tradition, here Belgium, France and Italy, apparently favor equality of outcome over equality of opportunity : in terms of opportunity, they are the most unequal countries among our sample of European countries. The opposite seems true for European countries with a protestant tradition, here the Netherlands, Germany and Great-Britain. This echoes a well known theme in the sociology of religion. Weber (1904-1905) held that the devotion to work that was one of the fundamental elements of capitalism and modernity derived, at least in part, from the Puritan effort to turn work into a spiritual vocation. The respect of effort which lies at the heart of the equality of opportunity doctrine and which leads Dworkin and others philosophers to prescribe the principle of natural reward may take its root in the Protestantism. Consequently the idea of equality of opportunity would be more easily absorbed by countries routinely exposed to the idea of respect of effort that country that are not.

Obviously the poor ranking of the US in terms of inequality of opportunity as well as the preeminent position of Scandinavian countries in terms of equality of outcomes tells us that factors other than religion shape social and economic policy.

6. Conclusion

We started by claiming that confining analysis to inequality of outcomes is unduly restrictive. If inequality of opportunity were perfectly related to inequality of outcome, the interest of focusing on opportunity would have been greatly reduced, given the considerable amount of results already collected regarding differences across countries in income inequality. Fortunately, our results suggest that inequality of outcome is far from perfectly correlated with inequality of opportunity. On the one hand, countries that exhibit very high (low) levels of inequality of outcome also experience high (low) levels of equality of opportunity. On the other hand, the ranking of countries according to the two criteria are not identical, particularly for countries ranked in the middle of the pack. Obviously, more countries should be analyzed to obtain a more complete and definite picture of the potential contrast or congruence between inequality of outcome and inequality of opportunity among the developed world.

This rather complex picture already suggests two lines for further investigation. First, some policy instrument may achieve reduction in inequality of both outcome and opportunity. For instance, by reducing inequality of opportunity for education, by giving more resources to schools located in poor neighborhoods, equality may be enhanced in the long run on both dimensions. It may explain the achievement of equality of opportunity in Sweden as well as the remoteness of this goal in the US. Analyzing the impact of such policies may help to understand the extent of the correlation between inequality of outcome and inequality of opportunity. This calls for further modeling of the mechanisms through which inequalities of different types have been generated. It is clearly out of the scope of this article but it may be pursued in further research. Then, our results also suggest that the relative emphasis put on the two objectives of equality may vary across countries. A better knowledge of the political debate about redistributive issues in each country may shed light on international differences in this respect.

Lastly, in view of the rather crude description of the family background adopted here, our results must be taken with a grain of salt. Using a more detailed description of individual social background may affect the results. However, it is hard to guess whether the rankings obtained here are robust or not to such a refinement of the analysis.

7. Appendix

7.1. Statistical tests

The testing procedure has been developed in Davidson and Duclos (2000). It can be applied to any order of stochastic dominance. In this appendix we illustrate the case of second order stochastic dominance test. First, we estimate the Lorenz or the General Lorenz curves with their non-parametric estimator. From a sample of size N_A, $\mathbf{L}_A$ represents the estimated Lorenz curve of distribution A, and Σ_A its variance-covariance matrix. To compare the Lorenz curves of distributions A and B, we compute the difference of the two estimated vectors, noted $\gamma = \mathbf{L}_A - \mathbf{L}_B$. Insofar as the distributions A and B are independent, the global variance-covariance matrix is given by : $\Sigma = \Sigma_A + \Sigma_B$.

To test the equality of the Lorenz curves : the nul hypothesis is given by $\mathbf{H}_0 : \boldsymbol{\gamma} = 0$. It is then possible to show (see for example Beach and Davidson (1983) and Davidson and Duclos (2000)) that under $\mathbf{H}_0$ the estimated vector $\hat{\gamma}$ is asymptotically normal, then :

$$\hat{\boldsymbol{\gamma}} \sim \mathcal{N}(0, \frac{\mathbf{\Sigma_A}}{\mathbf{N_A}} + \frac{\mathbf{\Sigma_B}}{\mathbf{N_B}})$$

The asymptotic distribution of the statistic T_1, under the nul hypothesis of equality :

$$T_1 = \hat{\boldsymbol{\gamma}}'(\frac{\Sigma_A}{N_A} + \frac{\Sigma_B}{N_B})^{-1}\hat{\boldsymbol{\gamma}} \sim \chi^2_k$$

To test equality of the two Lorenz curves A and B, one only need to compare the value of the statistic T_1 with a χ^2 at five or one percent.

To test relative dominance (ie : L_A dominates L_B), the two hypotheses are $\mathbf{H}_0 : \boldsymbol{\gamma} \in \mathbb{R}^k_+$ against $\mathbf{H}_1 : \boldsymbol{\gamma} \notin \mathbb{R}^k_+$. The Wald test statistic with inequality constraints has been developed by Kodde and Palm (1986) and Wolak (1989). The statistic T_2 defined by :

$$T_2 = \min_{\gamma \in \mathbb{R}^k_+} ||\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}||$$

with $||x|| = x'\Sigma^{-1}x$. Kodde and Palm (1986) have demonstrated that T_2 follows a mixture of χ^2 distributions :

$$T_2 \sim \Sigma^k_{j=0} w(k, k-j, \Sigma) Pr(\chi^2_j \geq c)$$

with $w(k, k-j, \Sigma)$ represents the probability that $k-j$ elements of $\boldsymbol{\gamma}$ be strictly positive. The distribution of this mixture of χ^2 is not tabulated but upper and lower bounds of critical values are given in Kodde and Palm. It is either possible, if lower and upper bounds do not enable to conclude to estimate critical values of the statistic T_2 by a Monte-Carlo procedure[23].

[23] It is necessary to draw 10,000 normally multivariated vectors with expectation 0 and variance-covariance matrix Σ, then to compute the proportion of vectors that have j positive elements (for $j \in (0, k)$), the proportion is an estimator of the weight $w(k, j, \Sigma)$.

7.2. Data

TAB. 7.1 – Income variables by country

	Belgium	France	G-Britain	Germany	Italy	Nether.	Norway	Sweden	USA
Activity									
Wages and Salaries	Y	Y	Y	Y	Y	Y	Y	Y	Y
Mandatory employee contrib.	S	S	S	S	S	S	Y	Y	S
Farm/non farm self. emp. income	N	Y	Y	Y	Y	Y	Y	Y	Y
In-kind earnings	N	N	N	N	Y	Y	Y	Y	N
Mand. contrib. for self-emp.	N	S	S	S	S	S	Y	Y	S
Patrimony									
Cash property income (rents, interests, dividends)	Y	Y	Y	Y	Y	Y	Y	Y	Y
Noncash property income (imputed rent from own house)	N	N	Y	Y	Y	Y	Y	Y	N
Market value of residence	N	N	N	N	Y	Y	Y	Y	Y
Taxes									
Income taxes	S	Y	S	S	S	S	Y	Y	S
Property or wealth taxes	S	Y	S	S	S	S	Y	Y	S
Other direct taxes	N	Y	N	N	N	S	Y	Y	S
Transfers									
Sick, accident, disability pay	Y	Y	Y	Y	Y	Y	Y	Y	Y
Social retirement benefits	Y	Y	Y	Y	Y	Y	Y	Y	Y
Child or family allowances	Y	Y	Y	Y	N	Y	Y	Y	Y
Unemployment compensation	Y	Y	Y	Y	N	Y	Y	Y	Y
Maternity allowances	N	Y	Y	Y	N	Y	Y	Y	Y
Military/vet/war benefits	N	Y	Y	Y	Y	Y	Y	Y	Y
Other social insurance	N	Y	Y	N	Y	Y	Y	Y	Y
Means-Tested cash benefits	Y	Y	Y	Y	Y	Y	Y	Y	Y
Private pensions	Y	N	Y	Y	N	Y	Y	Y	N
Alimony or child support	Y	Y	Y	Y	Y	Y	Y	Y	Y

S : Simulated Y : source of income presented in the basis, N : source of income not available in the basis. For the definition of any variable see LIS webpage :http ://www.lisproject.org/techdoc/variabdef.htm

7.3. Indexes of inequality of outcome

Table 8: Inequality indexes for disposable income

	Gini	P90/P10	P75/P25	P50/P10	P90/P50	Var. Log.	Theil	Coef. Var.
Belgium	0.20	2.44	1.57	1.63	1.50	0.14	0.06	0.35
	(.007)	(.036)	(.026)	(.032)	(.022)	(.022)	(.004)	(.015)
France	0.25	2.97	1.73	1.72	1.73	0.22	0.11	0.53
	(.005)	(.030)	(.010)	(.014)	(.013)	(.010)	(.006)	(.029)
West-Germany	0.26	3.00	1.75	1.76	1.72	0.37	0.12	0.56
	(.009)	(.052)	(.017)	(.023)	(.021)	(.052)	(.012)	(.048)
Great-Britain	0.30	4.32	2.05	2.20	1.94	0.44	0.16	0.61
	(.008)	(.097)	(.062)	(.045)	(.025)	(.048)	(.011)	(.043)
Italy	0.34	4.66	2.19	2.24	2.08	0.46	0.19	0.67
	(.008)	(.107)	(.074)	(.031)	(.093)	(.033)	(.012)	(.031)
Netherlands	0.26	2.96	1.77	1.73	1.72	0.22	0.11	0.51
	(.005)	(.033)	(.015)	(.015)	(.015)	(.013)	(.006)	(.017)
Norway	0.21	2.40	1.56	1.60	1.50	0.19	0.08	0.45
	(.011)	(.045)	(.017)	(.027)	(.017)	(.027)	(.012)	(.057)
Sweden	0.19	2.26	1.47	1.62	1.38	0.27	0.06	0.34
	(.007)	(.058)	(.013)	(.027)	(.011)	(.039)	(.005)	(.013)
USA	0.31	4.48	2.14	2.24	1.98	0.43	0.16	0.62
	(.008)	(.111)	(.033)	(.048)	(.030)	(.067)	(.011)	(.036)

Var.log.: variance of logarithms. Coef. Var.: coefficient of variation. In brackets: standard-errors estimated by bootstrap (200 replications).

7.4. Conditional Lorenz curves

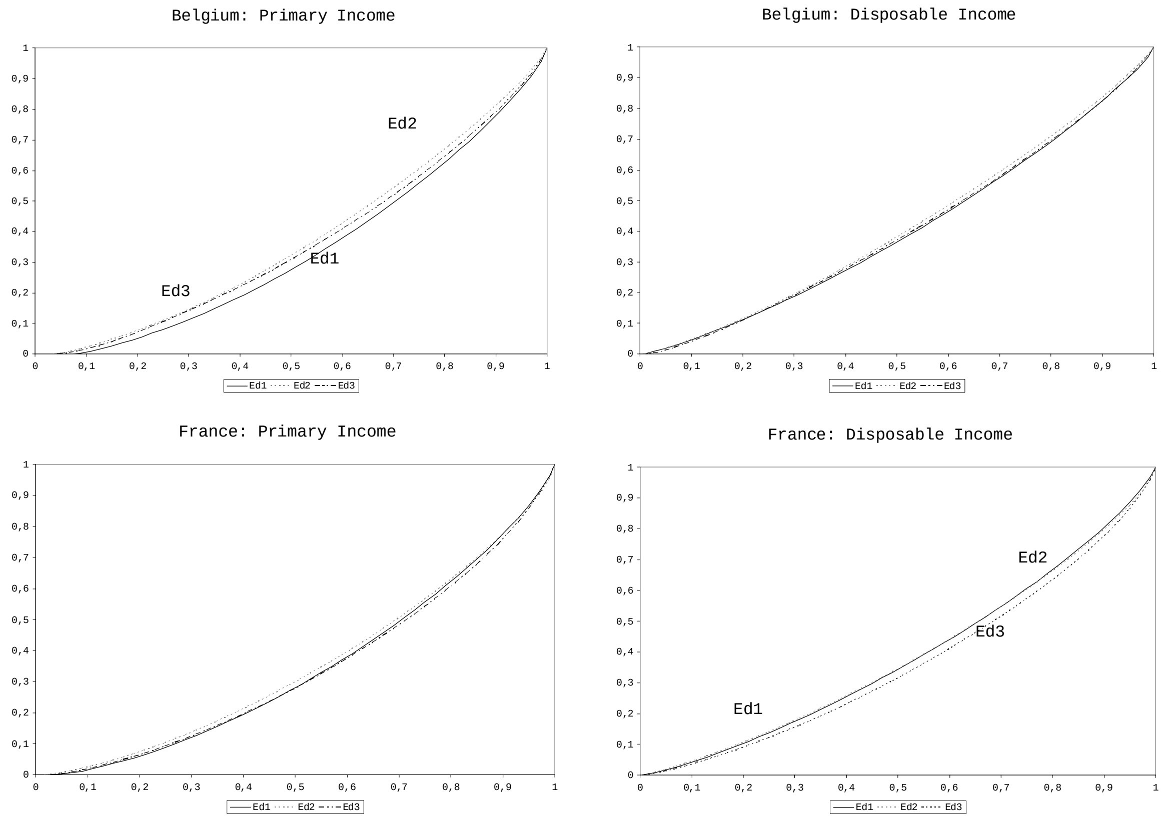

Figure 3.6

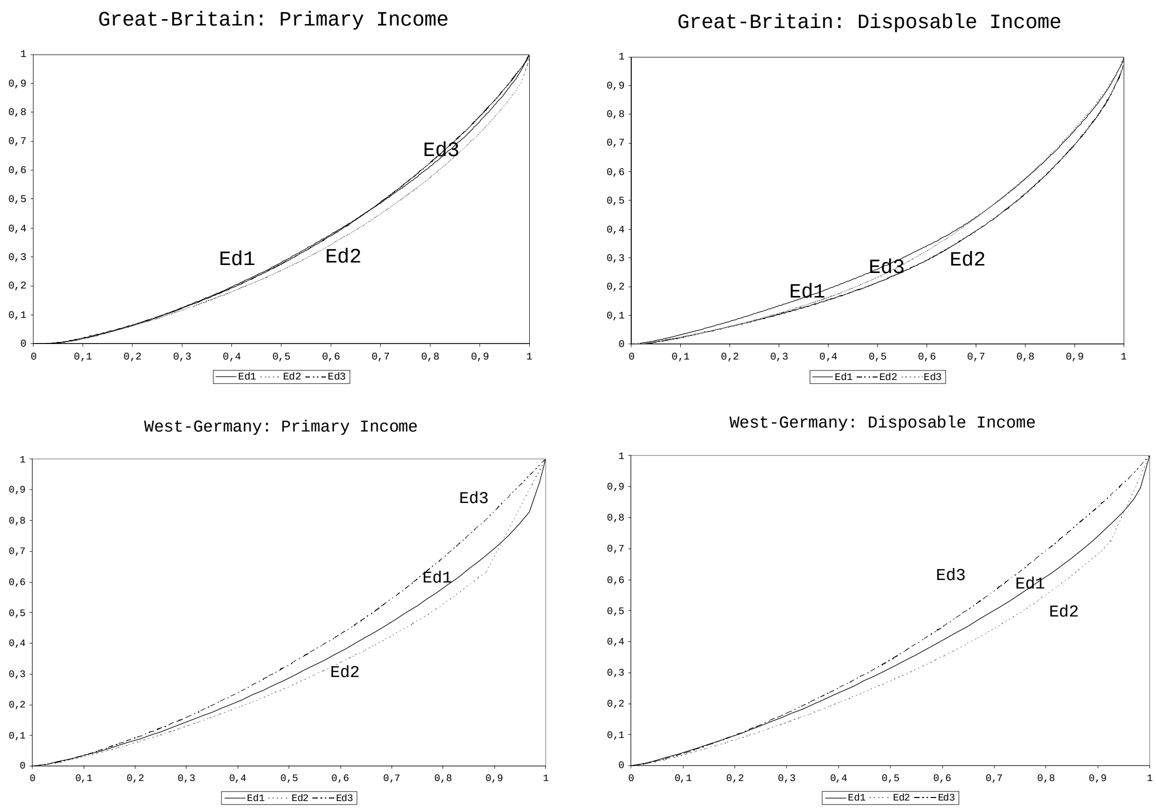

Figure 3.6 (continued)

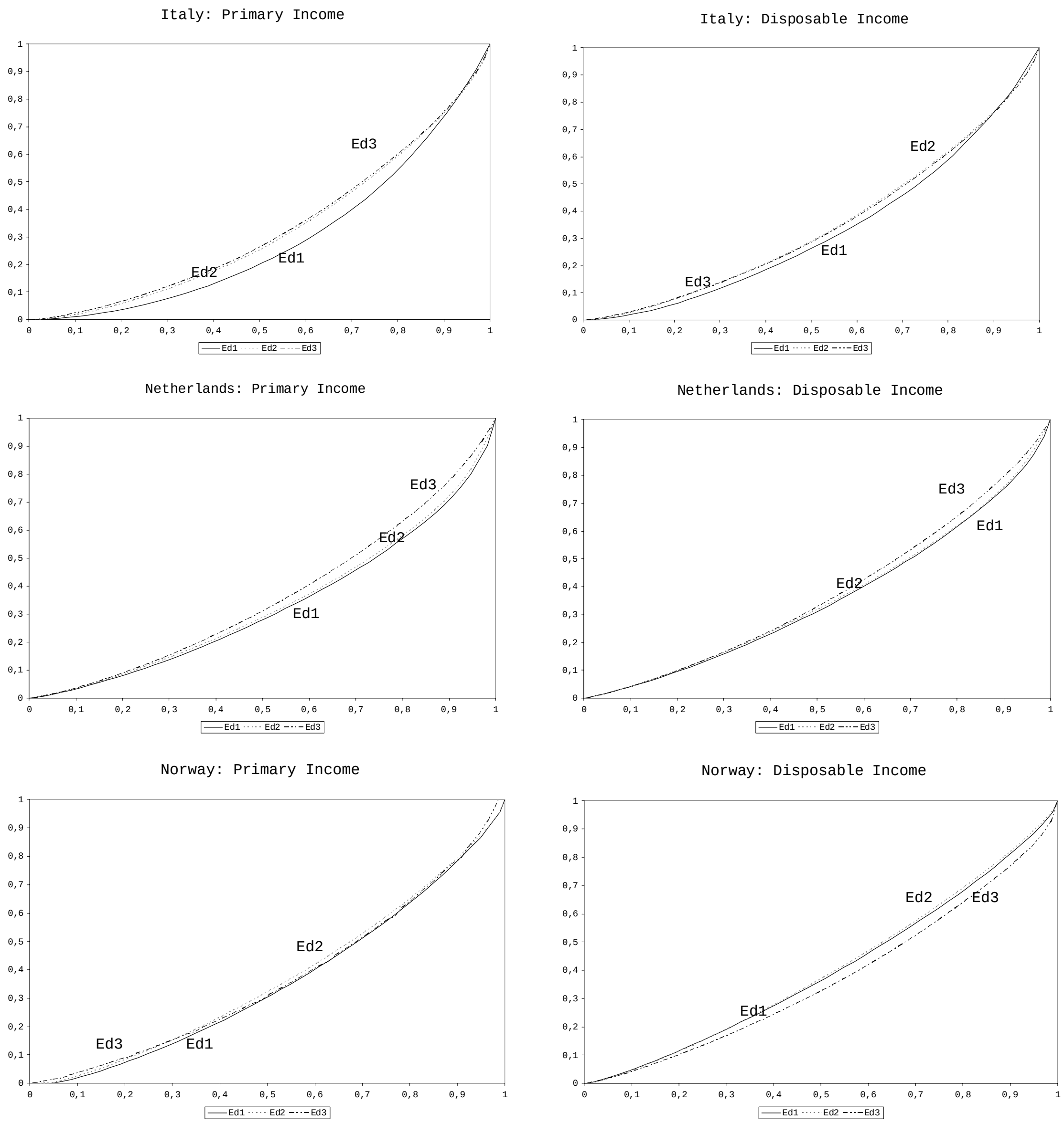

Figure 3.6 (continued)

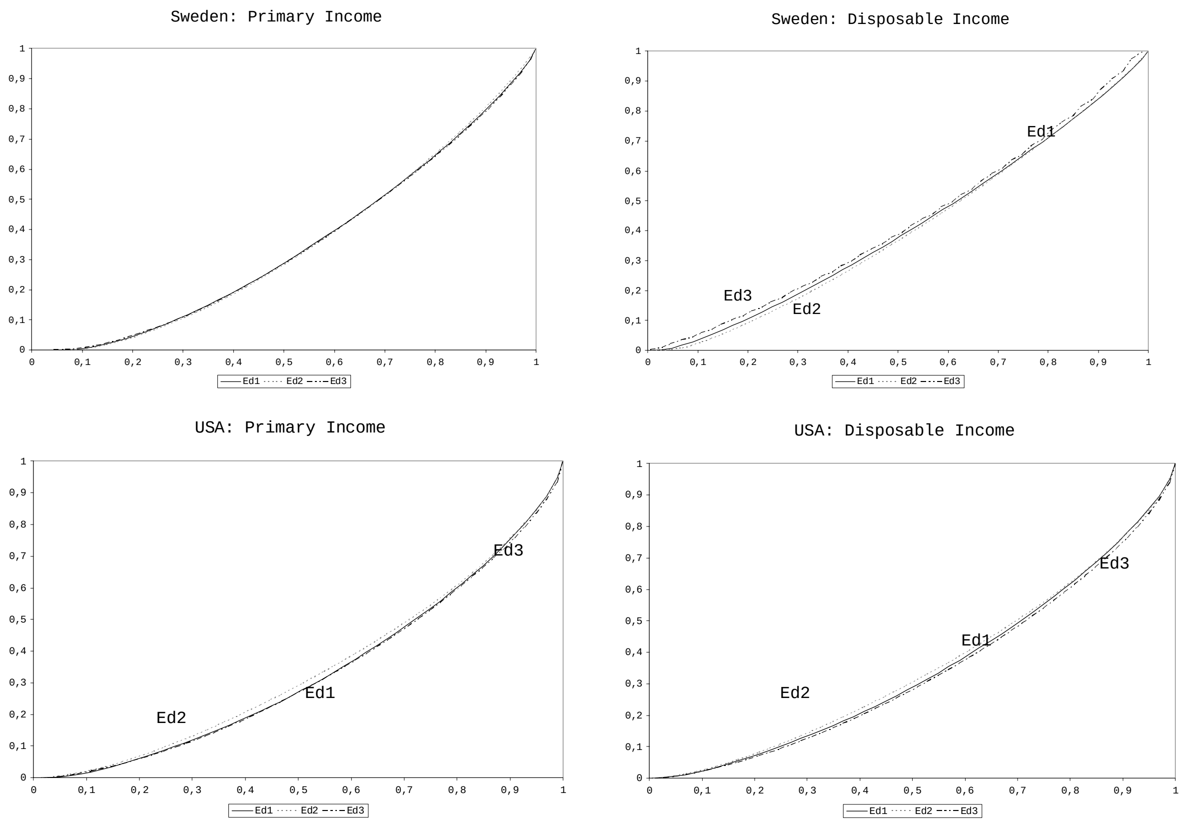

Figure 3.6 (continued)

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