Is India better off today than 15 years ago ? A robust multidimensional answer*

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Abstract

This paper provides a robust multidimensional normative evaluation of the growth episode that India has experienced in the last 15 years. Specifically, the paper compares the evolution, between 1987, 1995 and 2002 of the distribution of several individual attributes on the basis of ethically robust dominance criteria. The individual attributes considered are real consumption (measured at the individual level), literacy rate, under 5 mortality and violent crime rates (all measured at the district levels). District level variables are interpreted as (local) public goods which, along with consumption, contribute to individual well-being. The robust criteria used are generalizations, to more than two attributes, of the first and second order dominance criteria of Atkinson and Bourguignon (1982) and coincide with the unanimity of utilitarian value judgements taken over a specific class of individual utility functions. The main result of the empirical analysis is that all

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utilitarian rankings of distributions of the four attributes who assume that individual utility functions satisfy the assumptions of second order dominance aggree that India is better off in 2002 than in 1987 or 1995 but that these rankings disagree as to how to rank 1987 and 1995. Furthermore, if one removes crime from the list of attributes, the dominance is shown to apply steadily over the whole period.

1. Introduction

In the last fifteen years, the Indian economy has grown at an average of around 7% per year (about 3.5% per capita). This spectacular growth, which seems to be connected to the liberalization reforms introduced in the late eighties, has immensely modified the lives of the billion individuals living in this country. The object of this paper is to provide a *robust* normative appraisal of this modification. Specifically, we seek to provide a robust answer to the basic question: is India a better place to be now than it was fifteen years ago ?

There have been for sure numerous attempts to providing answers to this question in the recent literature. Many of them have examined the impact of Indian growth on pecuniary poverty and/or inequality (see e.g. Datt & Ravallion (2002), Deaton & Drèze (2005) and the various contributions contained in the collective volume of Deaton & Kozel (2005)). Yet, interesting as they are, most of these attempts have suffered from *two* basic insufficiencies.

First they have focused on specific poverty (e.g. headcount ratio, poverty gap, squared poverty gap, etc.) or inequality (typically Gini coefficient) indices. Poverty analysis based on a specific poverty index is fragile because it rides heavily on the choice of a poverty line, a choice that is known to be very difficult (see e.g. Lipton & Ravallion (1998)). Inequality analysis based on a specific index suffers from the same lack of robustness with respect to the choice of the index (i.e. would the conclusions obtained from comparing Gini coefficients remain valid for the coefficient of variation or for the Theil index ?).

The second, and in our view more important, limitation of the existing attempts to normatively appraise the recent growth in India is that they have taken a *unidimensional* perspective of focusing only on *pecuniary* variables. Yet it has long been recognized (see for instance Kolm (1977), Atkinson & Bourguignon (1982), Atkinson & Bourguignon (1987), Rawls (1971), Sen (1987) and Sen (1992)) that monetary income or consumption is not the only individual attribute that is relevant for normative evaluation. Attributes such as health, education, protection against crime and pollution (to mention just a few) are also important contributors to individual well-being and the distributions of these attributes, along with that of pecuniary consumption, is of key importance for the normative evaluation of the development path of a country. While this multidimensionality of economic development is becoming increasingly acknowledged, it has failed so far to give rise to successful empirical implementations. With some recent exceptions (see e.g. Crawford (2005), Duclos *et al.* (2006) and Gravel *et al.* (2008)) much of the current applied work on multidimensional normative appraisal aggregates the various individual attributes into a single index - for instance the emblematic Human Development Index (HDI) - and looks at the distribution of this one-dimensional index. Such an approach obviously suffers from the arbitrariness of the aggregation procedure.

To some extent, the lack of empirical studies that perform robust comparisons of distributions of several attributes results from the insufficient development of the theory of multidimensional normative evaluation. Foundational results such as the Hardy-Littlewood-Polya theorem (see e.g. Kolm (1969) and Sen (1973)) that have given firm justifications to the use of Lorenz curve and to several inequality indices for one-dimensional comparisons, are not yet available for the multidimensional case. Yet the (slow) progress that have been made in the fields in the last twenty years or so, following notably the work of Atkinson & Bourguignon (1982), do not make us completely deprived either. Hence we have the theory and methods for appraising, in an ethically robust matter, the impact of India's growth on the distribution of well-being through the evolution of the distribution of several attributes. The attributes considered in this paper are individual consumption (as obtained from the National Sample Survey NSS) of India in the rounds 1987-1988, 1995-1996 and 2002) and three attributes measured at the level of the district of residence of each household: literacy, under 5 mortality and violent crimes. We interpret these three attributes as local public goods. For instance, the district literacy rate can be interpreted as the probability that an individual living in the district encounters someone who is literate. This is obviously a plausible indicator of the "quality" of the (district) environment in which the individual lives. Similarly the child (under five) mortality rate that prevails in a district can be interpreted as the probability that a decision to have a child results in it's demise before the age of five. This probability is meant to be a *gross output* of the health system of the district, output which depends upon both the information available to prevent child mortality (by having regular medical examination during and after the pregnancy for instance) and the quality of hospitals and doctors. Finally, the fraction of the district population that has been the victim of a violent crime is obviously an indicator of the "public safety" that prevails in the district and is a clear contributor to individual well-being.

The main conclusion of the analysis is that the joint distribution of district literacy, under five mortality and individual consumption in India in 2002 and 1995 dominates that of 1987 for the first order multidimensional criterion and that the distribution of 2002 dominates that of 1995 for the second order dominance criterion. Hence, in a rather robust sense, there has been a steady improvement of social welfare in India on the period 1988-2002. This is a *strong* dominance result since it is based, at least for 1987-1995 and 1987-2002, on a first order and three-dimensional argument. In a nutshell, all anonymous and Paretian welfarist social planners who assume that individuals convert district child mortality, district literacy and individual consumption into well-being by the same utility function satisfying rather mild properties, given below, agree to say that India has been steadily improving over the considered period. The only attribute whose introduction sometimes breaks dominance verdicts is crime, whose average level has been increasing between 1987 and 1996, before starting a descent from 1996 to 2002. Yet, if one abstracts from the comparisons of 1995 and 1987, and focuses on the comparison of 2002 with either 1995 or 1987, one finds that the joint distribution of all four attributes in 2002 dominates that of 1996 or 1988 at the second order, and that the dominance is at the first order when one compares 2002 with 1988. While a bit less strong than the previous ones, this dominance also contributes to make one relatively optimistic about the appraisal of the recent Indian growth on the distribution of well-being.

The plan of the rest of the paper is as follows. In the next section, we present the theoretical criteria used to perform the comparisons. Section 3 discusses the data, the statistical methodology and the results of the comparisons and section 4 provides some conclusion.

2. Presentation of the criteria

While the analysis is conducted herein with four attributes (individual consumption, district infant mortality, district literacy and district crime), many theoretical results on multidimensional dominance have been derived for two attributes only. We introduce in this section the criteria used in this paper to compare distributions of any given number, k say, of attributes between a given number, n say, of households [1] indexed by i. We also provide normative foundations for these criteria by showing that each of them is equivalent to the ranking that would be agreed upon by all utilitarian planers who assume that households convert attributes into well-being by the same utility functions satisfying a given set of properties.

We depict any distribution z of the k attributes between the n individuals as a $k \times n$ matrix of non-negative numbers[2] which we write as:

$$z = \begin{bmatrix} z_{11} & z_{21} & \cdots & z_{n1} \\ z_{12} & z_{22} & \cdots & z_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ z_{1k} & z_{2k} & \cdots & z_{nk} \end{bmatrix}$$

where, for every $i = 1, ..., n$ and $j = 1, ..., k$, z_{ij} represents the amount of attribute j received by individual i in the distribution z. We also denote by z_i the ith column of the matrix z, that we interpret as the bundle of the k attributes that describes the situation of household i in distribution z.

All normative comparisons of distributions of k attributes considered in this section are based on the *symmetric utilitarian criterion*. Let $U : \mathbb{R}_+^k \to \mathbb{R}$ be a utility function that transforms the attribute into individual well-being. For the utility function U, the utilitarian criterion ranks x above y if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$. The symmetric requirement that households use the same function to convert attributes into well-being is somewhat natural in the multidimensional context considered herein. If two individuals were different in their ability to convert attributes into well-being, the reason for this difference should be accounted for and included in the analysis, as an additional variable of the utility function. Obviously, the assumption that the social planner has the information required to measure utility cardinally and perform interpersonal comparisons of utility differences that underlies utilitarianism (see e.g. D'Aspremont & Gevers (1977)) is a strong one. A more acceptable assumption, which lies at the heart of

[1] We focus the discussion on the case where the number of households is the same. As is well-known, cases where the number of houeholds differ between distributions can be transformed into cases with the same number of households after appropriate replications of the distributions.

[2] The assumption that attributes quantities are non-negative numbers is not essential.

the dominance approach adopted herein, is that the social planner is willing to measure utility cardinally and to perform interpersonal comparisons of utility differences, but does not know which exact function to use. It only knows that the function satisfies some basic properties and, being careful, it only accepts to make rank two distributions when the symmetric utilitarian criterion ranks them in the same fashion for all utility functions satisfying the properties.

This leads to a notion of utilitarian dominance for a class $\mathbb{U}$ of utility functions that we define as follow.

Definition 1. ***(Utilitarian dominance)***. *We say that x utilitarian dominates y for the class of functions $\mathbb{U}$, denoted $x \succeq_{\mathbb{U}} y$, if and only if $\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$ for all utility functions U in $\mathbb{U}$.*

To define the properties satisfied by the utility functions considered in this paper, it is convenient, but not necessary, to assume that the utility function is differentiable with respect to its k arguments to the required degree (actually only discrete definitions of derivatives are required for the proof). For every function Ψ of k variables ($k \geq 2$), we denote by $\Psi_j(z)$ its jth partial derivative evaluated at the $k-$dimensional vector z. With this notation, the class of utility functions considered are.

$$\mathbb{U}^{M1} = \{U : \mathbb{R}_+^k \to \mathbb{R}\colon\ (-1)^{\#H} U_{h_1 h_2 \ldots h_{\#H}}(z) \leq 0 \text{ for all } z \in \mathbb{R}_+^k \text{ and all } H \subseteq \{1, 2, ..., k\} \text{ with } H = \{h_1, ..., h_{\#H}\}\}.$$

:and

$$\mathbb{U}^{M2} = \mathbb{U}^{M1} \cup \{U : \mathbb{R}_+^k \to \mathbb{R} : (-1)^{\#H \cup J} U_{h_1 h_2 \ldots k_{\#H} j_1 j_2 \ldots j_{\#J}}(z) \leq 0 \text{ for all } z \in \mathbb{R}_+^k$$

$$\text{and all subsets } H = \{h_1, ..., h_{\#H}\} \text{ and } J = \{j_1, ..., j_{\#J}\} \text{ of } \{1, 2, ..., k\}\}$$

Functions in $\mathbb{U}^{M1}$ have the property of being *non-decreasing* with respect to every attribute. This property emerges from the formal definition of $\mathbb{U}^{M1}$ by taking $H = \{j\}$ for every $j \in \{1, ..., k\}$. Yet, in addition to this one-dimensional property, functions in $\mathbb{U}^{M1}$ satisfy other conditions that specify the way by which the marginal utility of every attribute varies with the level of the other attributes. These conditions reflect assumptions made on the substitutability/complementarity between any two attributes, and the way by which this pairwise substitutability/complementarity varies with the level of the other attributes, and the way by which this cross-attribute variation of the substitutability/complementarity between attributes vary with other attribute, and so on, until one exhausts the list of attributes. Specifically, we are assuming that any two attributes are *substitute* to each other and, therefore, that the marginal utility of one attribute is decreasing with respect to any other attribute (condition $U_{hj}(z) \leq 0$, obtained from the formal definition of $\mathbb{U}^{M1}$ by considering $H = \{h, j\}$ for every $h, j \in \{1, ..., k\}$). Functions in $\mathbb{U}^{M1}$ also satisfy the assumption that the *decrease* in marginal utility of an attribute with respect to another is *itself* decreasing with respect to any other attribute (condition $U_{ghj}(z) \geq 0$) and that this decrease in the decrease of the marginal utility of one attribute is also decreasing with respect to the other remaining attribute, and so on. Unless one assumes additive separability of the individual utility function, it is important that one specifies the *connections* that exist between these attributes. In the class $\mathbb{U}^{M1}$, we connect in the fashion just described ,all *first order* own derivatives.

In addition to imposing properties on cross-dimensional behavior of the first own derivatives, the class $\mathbb{U}^{M2}$ impose analogous properties on the cross-dimensional behavior of the *second order* own derivatives, assumed to be negative just like their standard one-dimensional counterpart. The properties on the cross-dimensional behavior of the second own derivatives are obviously more difficult to understand intuitively. They roughly say that the decrease in the marginal utility of each attribute should be decreasing with respect to another attribute, and that this decrease should be also decreasing with respect to another attribute, and so on. All in all, functions in $\mathbb{U}^{M2}$ satisfy the properties that the impact of anything that happens in one or several dimensions should be decreasing with respect to the other dimensions. As for the class $\mathbb{U}^{M1}$, the sign of the derivative are alternating with the number of terms involved (negative when there is an even number of terms, positive when the number of terms is odd).

Atkinson & Bourguignon (1982) have proposed, in the case of two attributes only, two operational criteria that, as it turns out, are equivalent to the rankings provided by all utilitarian planners who assume that individual utility functions are in $\mathbb{U}^{M1}$ and $\mathbb{U}^{M2}$ respectively. The definitions of these criteria for the k dimensional case are as follows.

Definition 2. ***(Multidimensional Headcount Poverty dominance)*** *Distribution x dominates distribution y for the Multidimensional Head-*

count Poverty criterion, denoted $x \succeq_{MHP} y$ if, for every list $(t_1, t_2, ...t_k)$ of k poverty lines, one has:

$$\#\{i : (x_{i1}, x_{i2}, ..., x_{ik}) \leq (t_1, t_2, ...t_k)\} \leq \#\{i : (y_{i1}, y_{i2}, ..., y_{ik}) \leq (t_1, t_2, ...t_k)\}.$$

In words, x headcount poverty dominates y in a multidimensional sense if, for every list of poverty lines (one such line for every attribute), the number of individuals who are poor with respect to all attributes is lower in x than in y. This criterion is a straightforward generalization of the one-dimensional poverty headcount dominance one where people can be poor with respect to several attributes. It can be noted that if x headcount poverty dominates y in a multidimensional sense, then x headcount poverty dominates y in the one-dimensional sense for every attribute in isolation but that the converse does not hold.

The second criterion, first introduced by Atkinson & Bourguignon (1982) in the case of two-attributes distributions, can be viewed as a generalization of the well-known one-dimensional poverty gap dominance criterion.

Definition 3. *(**Multidimensional Poverty Gap dominance**) Distribution x dominates y for the Multidimensional Poverty Gap criterion, denoted $x \succeq_{MPG} y$, if, for all vectors $(t_1, t_2,..., t_k)$ of poverty lines , and all non-empty subsets K of $\{1, ..., k\}$, one has:*

$$\sum_{i=1}^{n} \prod_{j \in K} \max(t_j - x_{ij}, 0) \leq \sum_{i=1}^{n} \prod_{j \in K} \max(t_j - y_{ij}, 0) \tag{2.1}$$

In words, x poverty gap dominates y in the multidimensional sense if, for all lists of poverty lines (one such list for every attribute), and all (non-empty) combinations of attributes, the *product* of the amounts of the attributes that would be necessary to eliminate the poverty defined by the lines is lower in x than in y. Notice carefully that the implementation of the multidimensional poverty gap criterion requires, because of the need to consider $K = \{j\}$ for every j, the usual one dimensional poverty gap criterion to hold on every dimension.

We now establish, in the following two propositions, the equivalence between each of the two operational criteria and their utilitarian dominance counterpart. To the very best of our knowledge, these are not available in the literature for the general k-dimensional case. Atkinson & Bourguignon (1982) have provided, in the two dimensional case, a proof of one direction for each of the two equivalences and Hadar & Russell (1974) have provided, for the general k-dimensional case, a proof of one direction of the first equivalence. In a recent paper Anderson (2008) has also provided, for the three-dimensional case, a proof of one direction of each of the two equivalences. The proofs of these two propositions are provided in the appendix.

Proposition 1. *For every two distributions x and y of k attributes , $x \succeq_{\mathbb{U}^{M1}} y \Leftrightarrow x \succeq_{MHP} y$.*

Proposition 2. *For every two distributions x and y of k attributes , $x \succeq_{\mathbb{U}^{M2}} y \Leftrightarrow x \succeq_{MPG} y$.*

In view of these two propositions, as well as the definitions of $\mathbb{U}^{M1}$ and $\mathbb{U}^{M2}$, it is clear that $x \succeq_{MHP} y$ implies $x \succeq_{MPG} y$ but that the reverse implication does not hold. Hence multidimensional poverty gap dominance is more discriminant than multidimensional headcount poverty dominance. As usual with dominance analysis, the increase in discriminatory power gained from switching from one criterion to the other must be balanced against the decreasing plausibility of the properties of the individual utility function assumed in the corresponding utilitarian dominance criterion. The class $\mathbb{U}^{M2}$ may seem particularly exhausting in this respect, especially if many attributes (such as literacy, infant mortality or crime) are not cardinally measurable and if, therefore, a second (or larger order) derivative taken with respect to them has no clear meaning.

3. Empirical implementation

3.1. Data

We compare, over time, joint distributions of individual equivalent consumption expenditure, district level literacy, under 5 mortality rates and violent crime rates. We interpret the latter three variables as local (district) public goods that affect all households living in the district, and that contribute to individuals' well-being[3].

[3]We choose public goods instead of individual attributes like individual literacy as consumption is strongly correlated with individual literacy. Instead we focus on the public goods aspect of education by treating the literacy of the district as an externality.

Household consumption figures over all India (excluding troubled north-eastern states and Jammu-Kashmir) are obtained from the 43rd (1987-1988), 52nd (1995-1996) and 58th rounds (2002) of consumption expenditure surveys conducted by the Indian National Sample Survey Organization (NSSO). Individual consumption expenditures have been derived from households' ones using the Oxford equivalence scale[4] and are in 2002 Indian rupees[5]. Our analysis is based on 131,511 individuals in 2002, 203,228 individuals in 1995-96 and 563,931 individuals in 1987-88.

We have assigned to each individual the literacy, under 5 mortality rate and violent crime rate of his or her district of residence as provided by NSSO data. Literary and under 5 mortality rates have been obtained from the census for the years 1981, 1991 and 2001[6].Under 5 mortality rates (number of children who die before the age of five per thousand births) have been calculated, for the same census years, from the Census of India by the International Institute for Population Science. Violent crime rates (number of murders, attempted murders, and rapes per million individuals) have been obtained, for the same years as NSS data, from the National Crime Record Bureau. We have restricted our attention to the most violent and extreme form of crime to reduce the risk of trend biases due to the evolution of the reporting behavior of the victims of crimes (or their families). Our assumption is that this bias, while still present, is less severe in the case of violent crimes as they tend to be reported to the police less selectively than robberies, burglaries, and other form of criminal acts.

In the realm of health, we are constrained by the lack of individual level data. Hence we choose district level under 5 mortality rate as a rough measure of the level of health.We are constrained to define public goods at the district level as the consumption expenditure data does not give any geographical identifiers finer than a district.

[4]The Oxford Equivalence Scale is an adjustment suggested by OECD to account of economies of scale within a household. It puts a value of 1 to the household head, 0.5 to each adult member and 0.3 to each child.

[5]Price deflators are the Urban Non Manual Employees price index for urban data and Agricultural Labourers price index for rural ones. Comparisons or pooling between urban and rural data are performed using Deaton (2005) (table 17;3) ideal Fisher index.

[6]Due to subdivisions in the district areas that have taken place in India over the 1981-2001 period, there are more districts in 2001 and 1991 than in 1981. In order to make the comparisons consistent, we have aggregated data for 1991 and 2001 to adhere to the original, and coarser, 1981 districts partition.

3.2. Statistical methodology

In order to account for the fact that the compared distributions of disposable income are samples drawn from a larger population, we perform statistical inference based on either Intersection-Union (IU) and (UI) methods as advocated, respectively, by Howes (1994) and Bishop *et al.* (1989). As seen in section 2, checking for dominance involves verifying if a finite number, m say, of inequalities hold. Each such inequality can be seen as a statistical hypothesis and the sequence of these inequalities can also be seen as a statistical hypothesis. The UI procedure suggested by Bishop *et al.* (1989) (see also Bishop *et al.* (1992)) tests dominance in two steps. The first step consists in testing m subhypotheses of the nature:

$$\begin{aligned} H_0^i &: \gamma_i^A = \gamma_i^B \\ H_A^i &: \gamma_i^A \neq \gamma_i^B \\ \text{for } i &= 1, ..., m \end{aligned}$$

where γ_i^j can be either headcount poverty or the poverty gap for the distribution j $(j = A, B)$ for the poverty line i, .The overall null hypothesis of equality, $H_0 : \gamma^A = \gamma^B$ is the logical intersection of all the m subhypotheses, while the overall alternative is the logical union of the alternative sub hypotheses. Given this, we reject the overall hypothesis H_0 if and only if, H_0^i is rejected for some i. The second step of the procedure, if H_0 has been rejected, requires us to differentiate between dominance and non comparability. The suggestion provided by Bishop *et al.* (1989) for making this differentiation is that if at least one of the poverty difference $\gamma_i^A - \gamma_i^B$ is significantly negative and none of these differences are significantly positive, then we should conclude that A significantly dominates B. On the other hand, if we observe both significantly positive and significantly negative differences, we conclude A and B are non comparable.

The procedure requires the construction of a test statistic for the poverty measure γ_i^j. To this aim, let T_i be defined by:

$$T_i = \frac{\widehat{\gamma}_i^A - \widehat{\gamma}_i^B}{\left(\frac{\widehat{\omega}_{ii}^A}{N^A} + \frac{\widehat{\omega}_{ii}^B}{N^B}\right)^{\frac{1}{2}}}$$

where $\widehat{\gamma}_i^j$ is the sample estimate of γ_i^j , $\widehat{\omega}_i^A$ is the variance estimates of $\widehat{\gamma}_i^j$ and N^j is sample size of distribution j $(i = 1, ..., m$; $j = A, B$. Variance estimates are derived in Davidson & Duclos (2000) for one-dimensional headcount ratio and the poverty gap dominance criteria and in Duclos *et al.* (2006) for their multi-dimensional generalizations according the following formula:

$$\widehat{\omega}_i = \frac{1}{N} \sum_{\{h:y_h<t\}} \left[(t_1 - y_{h1})^{s-1} (t_2 - y_{h2})^{s-1} ... (t_k - y_{hk})^{s-1}\right]^2 - (\widehat{\gamma}_i)^2$$

for k-dimensional poverty (for any $k \geq 1$) where s denote the order of dominance ($s = 1$ for headcount poverty and $s = 2$ for poverty gap).

Since the subhypotheses must be tested simultaneously, we test each T_i using a studentized maximum modulus distribution (see Stoline & Ury (1979)) with m and infinite degrees of freedom. We perform inference tests by comparing T_i to its critical value for a significance level C_α, taken to be 95% herein.

The Bishop *et al.* (1989) UI inference rule adopted in this paper is to:

1. Reject the equality of distributions if there is any $|T_i|. \geq C_\alpha$
2. Given rejection in Step 1, accept dominance of A over B if there is at least one T_i is significant negative T_i. and no T_i is significantly positive.

Bishop *et al.* (1989) UI inference rule can be contrasted with the intersection-union (IU) inference rule, initiated by Howes (1994) and Kaur *et al.* (1994), for which the null of non- dominance is tested against the alternative of dominance. Given our notation, this implies that *every* T_i has to be significantly negative for rejecting of the null of non-dominance of A over B. If this somewhat conservative test is adopted, each of the T_i statistics follows a standard Student distribution.

This testing methodology has been the object of an extensive examination in Davidson & Duclos (2006) who show that it is impossible to reject the null hypothesis of non-dominance when the population distributions are continuous in their tails. As a solution to this problem, Davidson & Duclos (2006) have advocated testing dominance on restricted domains of the distributions through a procedure which involves censoring distributions at the tails. They have also shown, through Monte Carlo simulations, that bootstrapping tend to lead to a better inference in that case. This approach, which is not adopted in this paper, has been applied to the evaluation of multidimensional poverty in six African countries in Batana & Duclos (2008).

3.3. One-dimensional comparisons

While we are concerned with the evolution over time of the joint distribution of the four attributes, we first provide some descriptive comments on the behavior of the (marginal) distribution of each attribute in isolation. After all, having one-dimensional dominance of every attribute in isolation is a necessary condition for having multidimensional dominance.

3.3.1. Distributions of consumption

Figure 3.1 compares the ordered vectors of 10 000 individual consumptions in India for the three periods.[7]. As suggested by this figure, there has been an almost secular rise of equivalent expenditure over the years. While this is definitely true for individuals with low rankings in the distributions, it is, surprisingly, not true for higher ranked individuals. As can be seen, there is some crossing in the right tail of the distributions. We suspect that this is a result of the thinness of the sample in 2002, and the under representation of high income households that is notorious in NSSO data.[8]. Indeed, except for implausibly high poverty lines, it appears that 2001 weakly dominates 1996 and strictly dominates 1988 for the headcount poverty dominance criterion.

The problem with the right tail of the distribution disappears when one switches to poverty gap dominance (equivalent to generalized Lorenz dominance for one dimensional comparisons) as illustrated on figure 3.2 Hence, if one uses poverty gap as a measure of poverty, there is no debate whatsoever to have on the appropriate poverty line order to appraise the poverty trends in India. Pecuniary poverty has gone down in India no matter what the line used to define it is.

[7] These 10 000 individual consumptions levels have been selected randomly from the underlying sample distributions. Expenditures have been slightly discretized by putting them into intervals of 50 and by assigning to all expenditures in the same interval the median value of that interval.

[8] It is a well-documented fact (see e.g. Banerjee & Piketty (2005)) that the consumption expenditures measured by NSS tend to underestimate the consumption expenditures as defined in National Accounting data and, more importantly for our purpose here, that this downward bias has increased significantly during the nineties. The reasons for this increasing underestimation, by NSS, of average consumption expenditure are not fully understood.

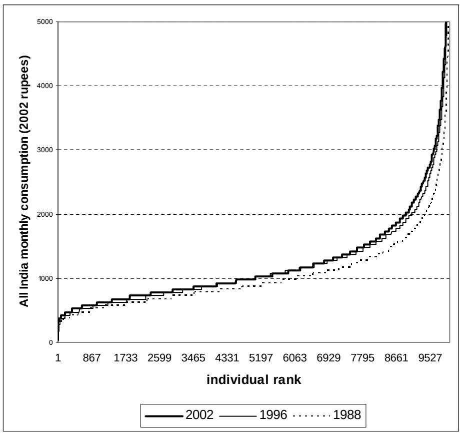


Figure 3.1: Orderered consumption vectors.

3.3.2. Distributions of district public goods

Figure 3.3 shows ordered vectors of literacy rates in the individuals' districts of residency (with individuals increasingly ordered in terms of the literacy rate of their district of residence). Except for individuals who live in the most literate districts (where the room for improvement is small), there has been a sharp increase in district literacy during the whole period for any individual position. There is a small (statistically insignificant) crossing at the very upper tail of the distributions between the ordered vectors of 1991 and 2001.progress has been more important for individuals located in the center of the ordered vectors than for those located at the extreme.

Figure 3.4 depicts the (decreasingly) ordered vectors of district under 5 mortality rate. The trend here is very similar - if not even more spectacular - to that of observed for literacy.

Figure 3.5 shows ordered vectors of district public safety levels (1000 minus the number of violent crime per million). As can be seen, crime in the safest districts has gone up over the period. This increase in crime has been particularly strong between 1987 and 1995-96, where it has concernedmany districts. From 1995-96 on, the risk of crime has gone down in most districts. Yet the safest district in 2002 is still worse off, crime-wise, than the best off district in 1995-1996 and 1987-1988. Because of this, there can not be headcount poverty dominance of the distribution of public safety in 2002

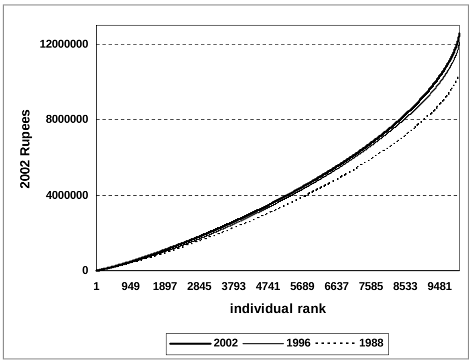


Figure 3.2: Consumption Lorenz curves

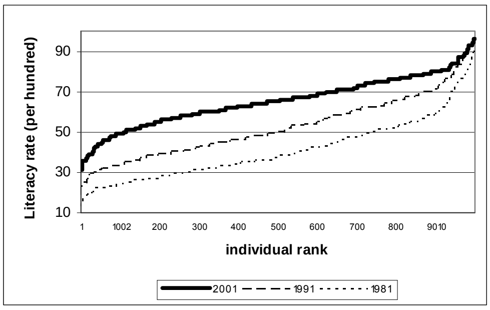


Figure 3.3: Ordered literacy vectors

over those of either 1996 and 1988.

Figure 3.6 shows the generalized Lorenz curves of public safety for the the same years. The figure suggests a dominance of 2002 over both 1996 or 1988 by the Generalized Lorenz dominance, and therefore the poverty gap, criterion.

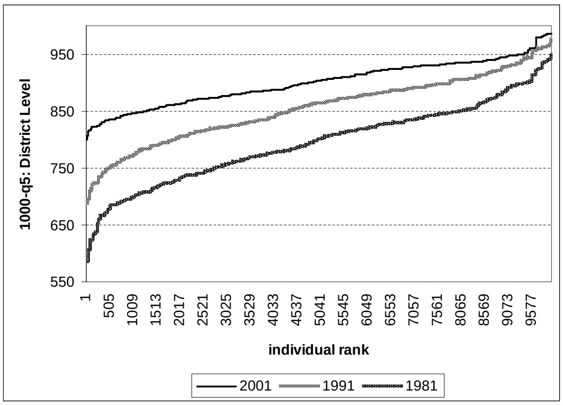


Figure 3.4: Ordered Vector of Under 5 Mortality Rates

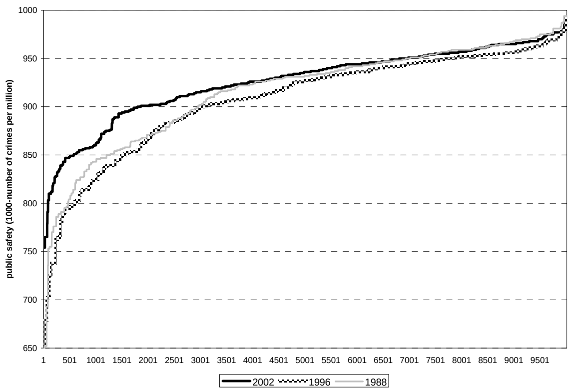


Figure 3.5: Ordered vectors of public safety

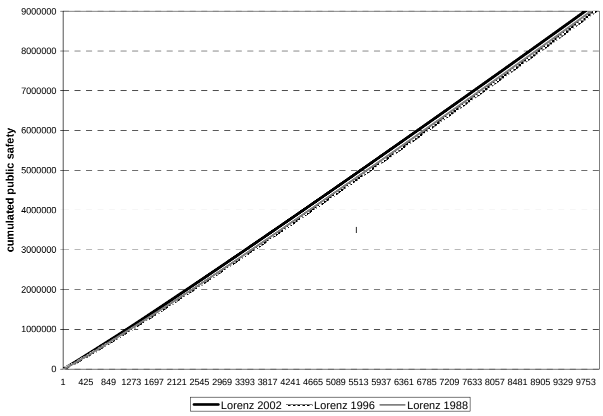


Figure 3.6: Lorenz curves of public safety

3.4. Multidimensional comparisons

We start with the discussion of the very demanding four-dimensional test. As discussed earlier, implementing four-dimensional dominance analysis requires the verification of as many inequalities as there are logically conceivable combinations of observed values of the four attributes in the two distributions. As this number of inequalities can become huge very quickly, we have resorted to the expedient of reducing the number of different combinations of observed values by "rounding off" slightly crime, literacy and under 5 mortality figures. Essentially, we have rounded-off under 5 mortality and crime at the closest hundred (587 becomes 600 for instance), and the literacy figure at the nearest 10 (e.g. 17 becomes 20). Rounding off has one important impact for public safety. If we round off, there seems to be first order dominance of 2002 over 1987-88 and 1995-96. Of course our rounding-off is much more less severe than the aggregation of attributes values into deciles averages commonly done in the literature. Hence, despite round-off, we had to check for some 180,000 inequalities!

The following table reports the best (i.e. the least ethically demanding) dominance results, if any, for all pair of years using Bishop *et al.* (1989) UI inference method. Details of the tests are provided in appendix B.

Year	**1987-1988**	**1995-1996**	**2002**
1988	-		
2002	2002($\mathbb{U}^{M1}$)	2002($\mathbb{U}^{M2}$)	-

This table hence tells us that all utilitarian planners who assume that Indians transform individual expenditure, district public safety, district literacy and district protection against the risk of loosing one's child into well-being by the same utility functions in $\mathbb{U}^{M2}$ agree to say that India is a better place to be in 2002 than it was in either 1995 or 1987. This is not true for 1996 over 1987 however. As suggested by one-dimensional analysis, this non-dominance seems to result from the sharp rise in crime rate between 1987 and 1996 that destroys any hope of getting four-dimensional dominance over that period. Moreover, we notice that if one restricts the comparison to 2002 and 1987, the unanimity for considering the period as an improvement in social welfare is even stronger since it covers the wider class of utility functions $\mathbb{U}^{M1}$. We emphasize that these dominance results are strong ones. Obtaining four-dimensional poverty gap dominance is difficult, as there is a very large family of social welfare judgements that need to agree on that.

While we could not reach this dominance conclusion with the more conservative IU inference methodology, we would be very close to it. As a matter of fact, first order dominance of 2002 over 1987-88 fails at only 1.1 percent of points of our domain. In the case of second order dominance, strict dominance of 2002 over 1987 fails only because of 0.03 percent of points in the domain, even though all T_i statistics have the right sign. Similarly second order strict dominance of 2002 over 1996 fails because of insignificant (but correct signs) of 0.3 percent of points in the domain. For headcount poverty dominance, most dominance failure occur either at very high levels of literacy (96% of all dominance failure occur at the two highest observed values of literacy rate) and very low levels of consumption expenditure. For poverty gap, the failure to achieve dominance for the IU methodology occurs only at very low individual expenditure levels.

Let us check now what happens if we drop crime rate, whose blocking power seems to be responsible for the failure of obtaining dominance of 1996 over 1988. The following table indicates the dominance verdict by the UI tests.

Year	1987-1988	1995-1996	2002
1988	-		
1996	1996 $(\mathbb{U}^{M1})$		
2002	2002$(\mathbb{U}^{M1})$	2002$(\mathbb{U}^{M2})$	-

There is, by this criterion, a clear trend in improvement over the period 1987-2002 that is recognized as by all utilitarian planners who believe that individual utility belong to $\mathbb{U}^{M1}$, with the exception of the period 1996-2002, where the unanimity is limited to those utilitarian planners who suppose individual utility to be in the smaller class $\mathbb{U}^{M2}$. Abstracting for crime therefore, all utilitarian planners who believe that Indians transform identically district level protection against risks of under 5 mortality and illiteracy and individual consumption into well-being by a function in $\mathbb{U}^{M2}$ agree to say that social welfare in India has increased steadily over the period. Again, these dominance verdicts are rejected by IU methodologies, but by a very small margin. For instance, the the failure of 2001 to dominate 1987 for the IU inference method arises only because of insignificant (but correct) signs at only 0.09 percent of our grid points. Moreover, 76% of these failures to get dominance occur at low expenditure levels.

Another interesting observation that can be taken from this analysis is that, over the years, the changes that have taken place in that period in the correlation between the attributes (viz. the fact that richer individual tend to live in better districts in terms of local public goods) do not appear to play any role in the normative evaluation. For example, it is never the case that there is dominance of all marginal distributions (by the UI tests)[9] for some poverty criterion but no dominance when we consider the joint distribution.

Given the failure of dominance above if we follow IU tests, it would interesting to know if there is any domain over which we get verdicts of domination. We find that there is indeed strict first dominance if we restrict our analyses to univariate dimensions. If we consider the marginal distributions of all India expenditure, we find 2002 strictly dominates 1987-88 and 1995-96 unambiguously. Similarly, if we look at All India literacy in isolation, there is strict dominance of 2002 over 1987-88 and of 1995-96 over 1987-88. In terms of risks of under 5 mortality, we find that 2002 dominates 1995-96 and 1987-88. The distribution in1995-96, in turn dominates 1987-88. In the case of public safety, the distribution function in 2002 dominates

[9] Tests results available on request.

both 1995-96 and 1987-88. Therefore if one were to look at these dimensions in isolation, there would seem to be rapid improvements even with the stringent IU test of dominance.

4. Conclusion

Is India better off today than 15 years ago ? The answer that we give to this question in this paper is a qualified, but robust, yes. In view of the importance of India in the world, and the importance of the changes that this country has gone through in the last fifteen years or so, we believe this answer to be of intrinsic general interest. But more importantly, the point of the paper was also to illustrate the fruitfulness of robust multidimensional methodologies for answering questions like this. When one looks at individual consumption, district literacy, district under five mortality and district crime (the later three variables being interpreted as local public goods) either separately, or jointly, there seems to be little doubt that the distribution of well-being in India has improved over the period no matter what are the assumptions made on the function that transform these attributes into well-being, provided that it is in the class $\mathbb{U}^{M2}$. As it turns out, in the case of India, there is not much point in looking at the joint distribution of the attributes as the ranking of the distributions that has been obtained is the one that results from the intersection of all rankings based on every dimension in isolation. This, obviously, could not be guessed at first glance.

We interpret our results as saying that someone who would have normative doubts about the direction taken by India in the last fifteen year would need to question these doubts somehow. Of course, we have clearly not considered all individual attributes that are normatively relevant. Environmental indicators are, in particular, lacking and it would be nice to obtain good data on those. Moreover, the district level at which non-pecuniary attributes are measured is probably not fine enough to capture truly local public good effects.

Yet we would like to emphasize that, if our results push toward some optimism with respect to the normative direction taken by India in the last fifteen years, they do not *in themselves* say much about the normative appraisal of the liberalization reforms launched in the eighties, and which are believed by some to be partly responsible for the increase in growth observed over the period as compared to the pre-eighties situation. For in order to normatively appraise such liberalization reforms, one would need to compare the current distribution of the attributes with the (counterfactual) one that would have prevailed now had the reforms not been implemented and had India continued to grow on the pre-eighties path. The analysis in this paper does not provide any answer as to what the verdict of this counterfactual comparison would be.

Appendix A. Proofs of propositions 1 and 2

Proof of proposition 1

Necessity.

Assume $x \succeq_{\mathbb{U}^{M1}} y$. Then, we have:

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i) \tag{.1}$$

for all U in U^{M1}. Consider the family of functions $\Phi^t : R_+^k \to R$, defined, for every $a \in R_+^k$, by:

$$\begin{aligned} \Phi^t(a) &= 1 \text{ if } a_j > t_j \text{ for some } j \\ &= 0 \text{ otherwise.} \end{aligned}$$

and indexed by t, where $t = (t_1, ..., t_k) \in R_+^k$ is a given vector of poverty lines. While non-differentiable (and non-continuous), it can be checked that Φ^t belongs to U^{M1} for every vector t. In particular, Φ^t is non-decreasing in each of the argument and, that the conditions on the cross partial (discrete) derivative is also satisfied. Since Φ^t belongs to U^{M1}, and $x \succeq_{\mathbb{U}^{M1}} y$, we must have, for all vector t of poverty lines:

$$\sum_{i=1}^{n} \Phi^t(x_i) \geq \sum_{i=1}^{n} \Phi^t(y_i)$$

$$\Leftrightarrow$$

$$\#\{i : x_{ij} > t_j \text{ for some } j\} \geq \#\{i : y_{ij} > t_j \text{ for some } j\}$$

$$\Leftrightarrow$$

$$n - \#\{i : x_{ij} > t_j \text{ for some } j\} \leq n - \#\{i : y_{ij} > t_j \text{ for some } j\}$$

$$\Leftrightarrow$$

$$\#\{i : x_{ij} \leq t_j \text{ for all } j\} \leq \#\{i : y_{ij} \leq t_j \text{ for all } j\}$$

as required by multidimensional headcount poverty dominance.

Sufficiency.

For any vector $a \in R_+^k$, define the (discrete) densities:

$$f^x(a) = \frac{\#\{i : x_i = a\}}{n} \text{ and}$$
$$f^y(a) = \frac{\#\{i : y_i = a\}}{n}$$

With this notation, the condition (.1) can be written as:

$$\int_0^{\overline{z}_1} \dots \int_0^{\overline{z}_k} [f^x(z) - f^y(z)]U(z)d_z \geq 0 \qquad (.2)$$

for some appropriate definition of integration (which could be the Lebesgue one or, if one wants to stick to the discrete setting, the Abel identity formula (see e.g. eq. 2.49 in Fishburn & Vickson (1978)) and where $\overline{z}_j$ for $j = 1, ..., k$ is an upper bound for the attribute j in the two distributions. The proof of the sufficiency of multidimensional headcount poverty dominance for utilitarian dominance over the class U^{M1} can then be obtained by integrating by parts expression (.2). The result of this integration by part are provided in equation (5.5') in Hadar & Russell (1974) and the statement of the sufficiency of the condition is the content of their theorem 5.8.

Proof of proposition 2

Necessity.

Assume that $x \succeq_{\mathbb{U}^{M2}} y$ and, therefore, that the inequality:

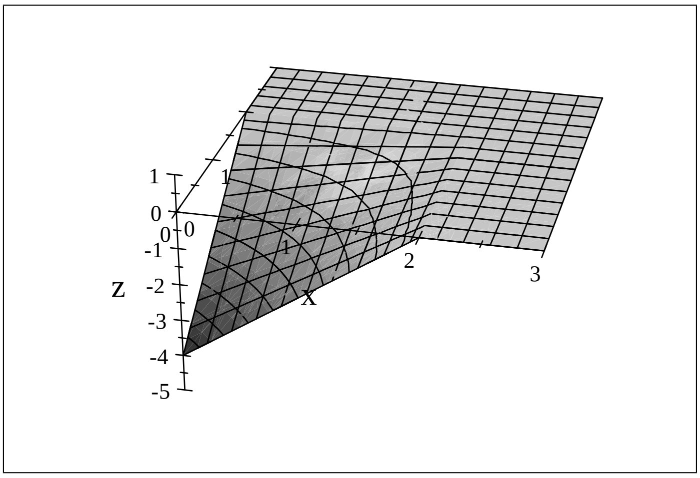
$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$$

holds for all U in U^{M2}. Consider the family of functions $\Phi^{tH} : R^k \to R$, defined, for every non-empty subset H of $\{1, ..., k\}$, every a and $t \in R_+^{\#H}$, by:

$$\Phi^{tH}(a) = -\prod_{h \in H} (-\min(a_h - t_h, 0)).$$

It should be noted that this function is defined over R_+^k even though, for several specifications of H, it does not depend at all upon the dimensions whose index lies outside H. A graphical representation of this function (for the case where $H = \{1, 2\}$ and $t_1 = t_2 = 2$) is provided on figure 4. We first show that, for every $t \in R^{\#H}$ and $H \subset \{1, ..., k\}$, this function belongs to U^{M2}. To see this, consider first the behavior of the function when viewed as a function of the arguments indexed by H in the interior of the set $\times_{h \in H}[0, t_h]$ (where the min operator does not apply). At any point a in this set, the function Φ^{tH} writes:

$$\Phi^{tH}(a) = -(-1)^{\#H} \prod_{h \in H} (a_h - t_h)$$



Consider any $j \in H$. In the interior of $\times_{h \in H}[0, t_h]$, the sign of $\prod_{h \in H, h \neq j} (a_h - t_h)$ is negative if $\#H$ is even and positive if $\#H$ is odd. Hence $\Phi_j^{tH}(a) \geq 0$ for any $j \in H$. Similar arguments can establish that all the derivative properties of the functions in U^{M2} are satisfied. The argument can be adapted, with some care, to the case where the min operator enters into the picture. Since Φ^{tH} belongs to U^{M2} and $x \succeq_{\mathbb{U}^{M2}} y$ holds, we have:

$$\sum_{i=1}^{n} \Phi^{tH}(x_i) \geq \sum_{i=1}^{n} \Phi^{tH}(y_i)$$

$$\Leftrightarrow$$

$$-\sum_{i=1}^{n} \prod_{h \in H} (-\min(x_{ih} - t_h, 0)) \geq -\sum_{i=1}^{n} \prod_{hq \in H} (-\min(y_{ih} - t_h, 0))$$

$$\Leftrightarrow$$

$$\sum_{i=1}^{n}\prod_{h\in H}(\max(t_h - x_{ih},0)) \leq \sum_{i=1}^{n}\prod_{h\in H}(\max(t_h - y_{ih},0))$$

which, applied to every t and every H, is precisely the definition of the multidimensional poverty gap dominance of y by x.

Sufficiency.

We prove that $\sum\limits_{i=1}^{n}\prod\limits_{j\in K}\max(t_j - x_{ij},0) \leq \sum\limits_{i=1}^{n}\prod\limits_{j\in K}\max(t_j - y_{ij},0)$ for every $t \in R_+^k$ and $K \subset \{1,...,k\}$ is sufficient for the inequality:

$$\sum_{i=1}^{n} U(x_i) \geq \sum_{i=1}^{n} U(y_i)$$

to hold for all utility functions in U^{M2}. As in the proof of proposition 2, this inequality can be written as:

$$\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}[f^x(a) - f^y(a)]U(a)d_z \geq 0 \tag{.3}$$

with $f^x(a) = \frac{\#\{i:x_i=a\}}{n}$ and $f^y(a) = \frac{\#\{i:y_i=a\}}{n}$ being the discrete joint density corresponding to x and y, and the integration being the appropriate one (for instance the Abel discrete decomposition of eq. 2.49 in Fishburn & Vickson (1978)), which we write as an integral, to alleviate the notation). As in the proof of proposition 1, $\overline{z}_j$ is an upper bound for the attribute j that is relevant for the comparison of x and y. Let $\Delta f(a) = f^x(a) - f^y(a)$ for every $a \in R_+^k$. Furthermore, for any two vectors v and w in $\mathbb{R}_+^k$ and any index set $K \subset \{1,...,k\}$, we denote by $(v_K; w_{-K})$ the vector in $\mathbb{R}_+^k$ whose coordinate that are indexed by K are as in v and all the other coordinate are as in w. Furthermore, when the number of coordinates is small, we write $(v_{hij}; w_{-hij})$ instead of $(v_{\{hij\}}; w_{-\{hij\}})$. If one integrates by part the left hand side of (.3) once for every integrand, one obtains, after lengthy manipulations :

$$\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta f(a)U(a)d_z = -\sum_{h=1}^{k}\int_0^{\overline{z}_h}\Delta F_h(a_h)U_h(a_h;\overline{z}_{-h})da_h$$

$$+\sum_{h=1}^{k-1}\sum_{i=h+1}^{k}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\Delta F_{hi}(a_h,a_i)U_{hi}(a_h,a_i;\overline{z}_{-hi})da_h da_i$$

$$-\sum_{h=1}^{k-2}\sum_{i=h+1}^{k-1}\sum_{j=i+1}^{k}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\Delta F_{hij}(a_h,a_i,a_j)U_{hij}(a_h,a_i,a_j;\overline{z}_{-hij})da_h da_i da_j$$

....

....

$$+(-1)^k\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta F(a_1,...,a_k)U_{12...k}(a_1,...,a_k)da_1...da_k \qquad (.4)$$

where:
$\Delta F(a) = \int_0^{a_1}...\int_0^{a_k}\Delta f(\alpha)d\alpha_1...d\alpha_k$ denotes the difference in the cumulative distribution, and,
ΔF_{hi} denote the difference in the cumulative joint distribution of the attributes h and i (the value of the other attributes being fixed at their upper bound; a similar interpretation holds for ΔF_h, ΔF_{hij}, etc.)
This expression was obtained in Hadar & Russell (1974) (equation 5.5'). It shows that, if the utility function is in U_1, then the condition $\Delta F(a) \leq 0$ for every $a \in [0,\overline{z}_1]\times ...\times[0,\overline{z}_k]$ (headcount poverty dominance for every combinations of poverty lines) is sufficient for the inequality (.3). If we now integrate by part every term of (.4) with respect to every integrand, we get:

$$\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta f(a)U(a)d_z = -\sum_{h=1}^{k}[\int_0^{\overline{z}_h}\Delta F_h(a_h)da_h U_h(\overline{z}_1...,\overline{z}_k)$$
$$-\int_0^{\overline{z}_h}\int_0^{a_h}\Delta F_h(\alpha_h)d\alpha_h U_{hh}(a_h;\overline{z}_{-h})da_h]$$

$$+\sum_{h=1}^{k-1}\sum_{i=h+1}^{k}[\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\Delta F_{hi}(a_h,a_i)da_h da_i U_{hi}(\overline{z}_1,...,\overline{z}_k)$$

$$-\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{a_i}\Delta F_{hi}(a_h,\alpha_i)da_h d\alpha_i U_{hii}(a_i;\overline{z}_{-i})da_i$$

$$-\int_0^{\overline{z}_i}\int_0^{\overline{z}_h}\int_0^{a_h}\Delta F_{hi}(\alpha_h,a_i)d\alpha_h da_i U_{hih}(a_h;\overline{z}_{-h})da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{a_h}\int_0^{a_i}\Delta F_{hi}(\alpha_h,\alpha_i)d\alpha_h d\alpha_i U_{hihi}(a_h,a_i;\overline{z}_{-hi})da_h da_i]$$

$$-\sum_{h=1}^{k-2}\sum_{i=h+1}^{k-1}\sum_{j=i+1}^{k}[\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\Delta F_{hij}(a_h,a_i,a_j)da_j da_i da_h U_{hij}(\overline{z}_1,...,\overline{z}_k)$$

$$-\sum_{g=h}^{j}\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_g}\Delta F_{hij}(\alpha_g;a_{-g})d\alpha_g da_{-g} U_{hijg}(a_g;\overline{z}_{-g})da_g$$

$$+\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{a_i}\int_0^{\overline{z}_j}\Delta F_{hij}(\alpha_h,\alpha_i,a_j)da_j d\alpha_i d\alpha_h U_{hijih}(a_h,a_i;\overline{z}_{-hi})da_i da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\Delta F_{hij}(\alpha_h,a_i,\alpha_j)d\alpha_j da_i d\alpha_h U_{hijjh}(a_h,a_j;\overline{z}_{-hj})da_j da_h$$

$$+\int_0^{\overline{z}_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\int_0^{a_i}\Delta F_{hij}(a_h,\alpha_i,\alpha_j)da_h d\alpha_j d\alpha_i U_{hijji}(a_i,a_j;\overline{z}_{-ij})da_j da_i$$

$$-\int_0^{\overline{z}_h}\int_0^{a_h}\int_0^{\overline{z}_i}\int_0^{\overline{z}_j}\int_0^{a_j}\int_0^{a_i}\Delta F_{hij}(\alpha_h,\alpha_i,\alpha_j)d\alpha_h d\alpha_j d\alpha_i U_{hijjih}(a_h,a_i,a_j;\overline{z}_{-hij})da_j da_i da_h]$$

....

....

$$(-1)^k[-\sum_{g=1}^{k}\int_0^{\overline{z}_g}\Delta H(a_g;\overline{z}_{-g})U_{12...kg}(a_g;\overline{z}_{-g})da_g$$

$$+\sum_{f=1}^{k-1}\sum_{g=h+1}^{k}\int_0^{\overline{z}_f}\int_0^{\overline{z}_g}\Delta H(a_f,a_g;\overline{z}_{-fg})U_{12...kfg}(a_f,a_g;\overline{z}_{-hi})da_f da_g$$

$$-\sum_{f=1}^{k-2}\sum_{g=f+1}^{k-1}\sum_{h=g+1}^{k}\int_0^{\overline{z}_f}\int_0^{\overline{z}_g}\int_0^{\overline{z}_h}\Delta H(a_f,a_g,a_h;\overline{z}_{-fgh})U_{12...kfgh}(a_f,a_g,a_h;\overline{z}_{-fgh})da_f da_g da_h$$

....

....

$$+(-1)^k\int_0^{\overline{z}_1}...\int_0^{\overline{z}_k}\Delta H(a_1,...,a_k)U_{12...k12....k}(a_1,...,a_k)da_1...da_k]$$

where, for every $a \in [0,\overline{z}_1]\times ...\times[0,\overline{z}_k]$,

$$\begin{aligned}\Delta H(a) &= \int_0^{a_1}....\int_0^{a_k}\Delta F(\alpha_1,...,\alpha_k)d\alpha_1...d\alpha_k\\ &= \int_0^{a_1}....\int_0^{a_k}(a_1-\alpha_1)....(a_k-\alpha_k)\Delta f(\alpha_1,...,\alpha_k)d\alpha_1...d\alpha_k\end{aligned}$$

Hence, for utility functions satisfying:

$$\begin{aligned}U_i &\geq 0 \text{ for all } i\\ U_{ij} &\leq 0 \text{ for all } i,j \text{ not necessarily distinct}\\ U_{hij} &\geq 0 \text{ for all } h,i,j\text{, two of which at least being distinct}\\ U_{ghij} &\leq 0 \text{ for all } g,h,i,j\text{, at most 2 pairs of which being identical}\\ U_{fghij} &\geq 0 \text{ for all } f,g,h,i,j\text{, at most 2 pairs of which being identical}\\ U_{efghij} &\leq 0 \text{ for all } e,f,g,h,i,j \text{ at most 3 pairs of which being identical}\\ &....\\ U_{11...kk} &\leq 0\end{aligned}$$

the condition that $\Delta H(a) \leq 0$ for all a and that:

$$\begin{aligned} &\int_{j\in K}^{a} \cdots \int \Delta F_K(\alpha) d\alpha \\ = &\int_{j\in K}^{a} \cdots \int \prod_{j\in K} (a_j - \alpha_j) \Delta f_K(\alpha) d\alpha \\ \leq &\ 0 \end{aligned}$$

for all non-empty $K \subset \{1, 2, ..., k\}$ is sufficient for the inequality (.3) to hold.

Appendix B. Details of statistical tests

.1. Four dimensional comparisons.

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.70	5.46	REJECT	-288.06
2002 Vs 1988	4.25	5.46	ACCEPT	-800.15
1996 Vs 1988	68.72	5.46	REJECT	-315.644
Second Order Dominance				
2002 Vs 1996	-0.32	5.46	ACCEPT	-325.16
2002 Vs 1988	-1.18	5.46	ACCEPT	-849.15
1996 Vs 1988	65.20	5.46	REJECT	-414.903

SM distribution with degree of freedom (214498,∞).

.2. Three dimensional comparisons without crime

	Maximum *t*	Critical Value	Verdict	Minimum *t*
First Order Dominance				
2002 Vs 1996	8.70	5.45	REJECT	-290.3
2002 Vs 1988	4.21	5.45	ACCEPT	-846.5
1996 Vs 1988	2.26	5.45	ACCEPT	-330.4
Second Order Dominance				
2002 Vs 1996	-0.33	5.45	ACCEPT	-333.1
2002 Vs 1988	-1.26	5.45	ACCEPT	-866.5
1996 Vs 1988	-0.25	5.45	ACCEPT	-412.5

SM distribution with degree of freedom (195622,∞).

References

Anderson, G. 2008. The empirical assessment of multidimensional welfare, inequality and poverty: Sample weighted multivariate generalization of the kolmogorov-smirnov two sample tests for stochastic dominance. *Journal of economic inequality*, **6**, 73–87.

Atkinson, A. B., & Bourguignon, F. 1982. The comparison of multi-dimensioned distribution of economic status. *Review of economic studies*, **49**, 183–201.

Atkinson, A. B., & Bourguignon, F. 1987. Income distributions and differences in needs. *In:* Feiwel, G. R. (ed), *Arrow and the foundation of the theory of economic policy*. London: Macmillan.

Banerjee, A., & Piketty, T. 2005. Are the rich growing richer? evidence from indian tax data. *Pages 520–529 of:* Deaton, A., & Kozel, V. (eds), *The great indian poverty debate*. Macmillan India, Delhi.

Batana, Y. M., & Duclos, J. Y. 2008. *Multidimensional poverty dominance: Statistical inference and an application to west africa*. CIRPE working paper no 08-08.

Bishop, C. M., Formby, J. P., & Thistle, P. D. 1989. Statistical inference, income distributions and social welfare. *In:* Slotje, D. J. (ed), *Research on economic inequality*. JAI Press.

BISHOP, C. M., FORMBY, J. P., & THISTLE, P.D. 1992. Convergence of the south and the non-south income distributions, 1969-1979. *American economic review*, **82**, 262–272.

CRAWFORD, I. 2005. *A nonparametric test of stochastic dominance for multivariate distributions*. University of Surrey, Institute for Fiscal Studies.

D'ASPREMONT, C., & GEVERS, L. 1977. Equity and the informational basis of social choice. *Review of economic studies*, **46**, p.199–210.

DATT, G., & RAVALLION, M. 2002. Is india economic growth leaving the poors behind ? *The journal of economic perspectives*, **16**, p.89–108.

DAVIDSON, R., & DUCLOS, J. Y. 2000. Statistical inference for stochastic dominance and for the measurement of poverty and inequality. *Econometrica*, **58**, p.1435–1465.

DAVIDSON, R., & DUCLOS, J. Y. 2006. *Testing for restricted stochastic dominance*. CIRPE working paper no 06-09.

DEATON, A. 2005. Prices and poverty in india. *Pages 412–427 of:* DEATON, A., & KOZEL, V. (eds), *The great indian poverty debate*. Delhi: Macmillan India.

DEATON, A., & DRÈZE, J. 2005. Poverty and inequality in india: A reexamination. *Pages 428–465 of:* DEATON, A., & KOZEL, V. (eds), *The great indian poverty debate*. Delhi: Macmillan India.

DEATON, A., & KOZEL, V. 2005. *The great indian poverty debate*. Delhi: Macmillan India.

DUCLOS, P. Y., SAHN, D., & YOUNGER, S. D. 2006. Robust multidimensional poverty comparisons. *The economic journal*, **116**, 943–968.

FISHBURN, P. C., & VICKSON, R. G. 1978. Theoretical foundations of stochastic dominance. *In:* WITHMORE, G. A., & FINDLAY, M. C. (eds), *Stochastic dominance*. Lexington Books.

GRAVEL, N., MOYES, P., & TARROUX, B. 2008. Robust international comparisons of distributions of disposable income and access to regional public goods. *Economica*. forthcoming.

HADAR, J., & RUSSELL, W. 1974. Stochastic dominance in choice under uncertainty. *In:* BALCH, M. S., MCFADDEN, D. L., & WU, S.Y. (eds), *Essays on economics behavior under uncertainty*. Amsterdam, UK: North Holland.

HOWES, S. 1994. *Testing for dominance: Inferring population rankings from sample data*. unpublished paper, policy research department, World Bank.

KAUR, A., PRAKASAO, R., & SINGH, H. 1994. Testing for second-order stochastic dominance for two distributions. *Econometric theory*, **10**, 849–866.

KOLM, S. C. 1969. The optimal production of social justice. *In:* GUITTON, H., & MARGOLIS, J. (eds), *Public economics*. Macmillan, London.

KOLM, S. C. 1977. Multidimensional egalitarianisms. *Quarterly journal of economics*, **91**, 1–13.

LIPTON, M., & RAVALLION, M. 1998. Poverty and policy. *Pages 2551–2657 of:* BEHRMAN, J., & SRINIVASAN, T. N. (eds), *Handbook of development economics*. Elsevier, Amsterdam.

RAWLS, JOHN. 1971. *A theory of justice*. Cambridge, Massachusett: Berknap Press of Harvard University Press.

SEN, A. K. 1973. *On economic inequality*. Oxford, Clarendon.

SEN, A. K. 1987. *The standard of living*. Cambridge, UK: Cambridge University Press.

SEN, A. K. 1992. *Inequality reexamined*. Cambridge, MA: Harvard University Press.

STOLINE, M. R., & URY, H. K. 1979. Tables of the studentized maximum modulus distributions and an application to multiple comparisons among means. *Technometrics*, **21**, 87–93.