The optimal choice of a jurisdiction structure

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Abstract

This paper discusses the problem of designing an optimal jurisdiction structure from the view point of a welfarist social planner when households with identical utility functions for non-rival public spending and private spending have private information on their contributive capacities. In the specific case where households have logarithmic utilities, it provides a complete characterization of this optimal jurisdiction structure in the two-types case and discusses the issues that arise in the three type case.

Preliminary and incomplete sketch. Not to be quoted nor circulated

"*C'est pour unir les avantages divers qui résultent de la grandeur et de la petitesse des nations que le fédératif a été créé.*" (Alexis de Toqueville)

1 Introduction

In many countries, one finds significant cross-regions variation in the bundles of public goods and taxes available to households. This is clearly the case in federal countries such as Canada and United States where the provinces and the states have the power to decide the provision of specific public goods (for instance education) and to collect taxes. But this phenomenon is also observed in "unitary" countries such as France or UK where cities have specific powers in terms of public good production (for instance the financing of primary school infrastructure) and taxations (local taxes). This heterogeneity in the package of public goods and taxes offered to the citizens of a same country is sometimes perceived as the source of unacceptable inequalities. For this reason, attempts are sometimes made by central governments to correct these by means of various cross-jurisdictions equalizing payments schemes. But one may wonder why the central government does not push further this equalizing logic by carrying itself the task of providing its citizens with the same package of public goods and taxes instead of maintaining these distinct jurisdictions. As the recent north

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American episodes of city mergers in large agglomerations (Boston, Montreal, Toronto) illustrate, this centralizing solution is sometimes adopted. Yet, the decision to merge many cities into one large agglomeration that is responsible for providing the same public goods and taxes package to all its inhabitants has been received with great skepticism by many. Is this skepticism justified ? Is there some argument that can justify cross-citizens heterogeneity in public goods and taxes packages from a normative standpoint or, to put it bluntly, federalism ?

This question is not new. It was underlying the above quote from Alexis de Toqueville, and was framed by Wallace Oates (1972) as follows:

> "In the absence of cost-savings from the centralized provision of a [local public] good and of interjurisdictional externalities, the level of welfare will always be at least as high (and typically higher) if Pareto-efficient levels of consumption are provided in each jurisdiction than if any single, uniform level of consumption is maintained".[1]

In this paper, we formulate in a precise model Oates's and Toqueville's intuition that a federal provision of public goods in separated jurisdictions can be normatively better than a centralized provision "in the absence of cost-saving from the centralized provision". Specifically, we provide a model in which even when the cost-saving case in favour of a centralized provision of a public good is maximal - namely when the public good is non-rival - it can be efficient and optimal to organize its provision in a federal system when the social planner does not have all the information to tax optimally its citizens. The formal architecture of the model is as follows. There is a collection of households who have the same preferences for one public good and one private good. Each household has an exogenous pecuniary wealth that is unobserved by the central government. The public good can be provided locally in distinct jurisdictions or centrally in one grand jurisdiction and is financed by taxes in such a way that the budget is balanced within the federation (but not necessarily within a given jurisdiction). The central government chooses bundles of local public good provision and tax payment (one such a bundle for every jurisdiction) in such a way as to provide incentive to households to reveal their willingness to pay for the public good (unknown to the government) by their locational choice. As in Tiebout (1956) therefore, households "vote with their feet" (see also Wildasin (1987)). More specifically, the government chooses bundles of public good and taxes that maximizes a social welfare function under two constraints: 1) a preference revelation constraint (each household must prefer the combination assigned to it by the government to any other) and 2) a budget constraint (the taxes raised in all jurisdictions must be sufficient to finance public goods that are locally provided in all of them). This problem is somewhat reminiscent of the classical Mirlees optimal income taxation problem (see for instance Mirlees (1971), Mirlees (1976), Mirlees (1986)) with leisure replaced by public good. Yet this "replacing" significantly modifies the nature of the problem. Since leisure is a

[1] A nice survey of the literature on fiscal federalism is provided by Oates (1999).

purely private good (the fact that Bob has 24 hours of leisure *per* day can not be used to improve Mary's utility) there is no intrinsic benefit for the social planner to pool together households with different characteristics. Hence, in the Mirlees setting, optimal income tax schedules will involve significant *separation* of workers with different characteristics in order to provide workers with the proper incentives to reveal their type. With public goods, the central planner do benefit from *pooling* together individuals with different characteristics since a given quantity of a pure public good may benefit to many individuals at no extra cost. Because of this the central planner must make a trade-off between the benefit of pooling individuals in order to achieve larger quantity of public good at lower cost and the informational cost associated to the fact of providing the same package of public goods and taxation to a large number of heterogeneous households. The nature of this trade off determines the optimal level of heterogeneity in public good and taxation package for a community. It also determines the choice between a centralized provision of the public good within a single "grand" jurisdiction or its provision within separated jurisdictions between which the federal government organizes optimal equalization transfers (see e.g; Boadway and Flatters (1982), Buchanan (1950), Flatters, Henderson, and Mieszcowski (1974) or Gravel and Poitevin (2006)).

While a few models have examined fiscal federalism issues under asymmetric information in the recent literature, including those of Bordignon, Manasse, and Tabellini (2001), Breuillé and Gary-Bobo (2007), Cornes and Silva (2000) and Cornes and Silva (2002), none that we are aware have considered the problem of choosing the appropriate jurisdiction structure - federal or centralized as it may be.

The rest of the paper is organized as follows. In the next section, we set up the notation and examine in its full generality the problem of the optimal choice of a jurisdiction structure under asymmetric information. In section 3, we examined the problem in the somewhat more specific setting in which there are only two types of households, and we provide a complete characterization of the solution of the problem in the important case where the social planner is utilitarian and would like, in the federal solution, to transfer wealth from the rich jurisdiction to the poor one. Section 3 discusses the extension of the analysis to the three-type case and section 4 concludes.

2 The General Structure of the problem

2.1 Notation

There are n households taken from a finite set N. Household i has a monetary wealth $w_i \in \mathbb{R}_+$ and consumes a public good (z) and a private good (x). Households are ordered by their wealth in such a way that $\omega_i \geq \omega_{i+1}$ for $i = 1, ..., n-1$. The public good is non-rival in consumption but excludable. Exclusion can be performed by partitioning the set of households N into pairwise disjoints sets

N_j for $j = 1, ..., l$ for some $l \in \{1, ..., n\}$ such that $\bigcup_{j=1}^{l} N_j = N$. Every such partition of N into a collection $\{N_j\}_{j=1}^{l}$ of sets for some $l \in \{1, ...n\}$ is interpreted as a *jurisdiction structure* and every set within such a collection is interpreted as a *jurisdiction* and For any jurisdiction structure $\{N_j\}_{j=1}^{l}$ J, we define an allocation of private and public goods $(x_1, z_1, ..., x_n, z_n) \in \mathbb{R}_+^{2n}$ to be *feasible* for that jurisdiction structure if it verifies, for all $j \in \{1, ..., l\}$, $z_h = z_i$ for all h and $i \in N_j$ (all members of a given jurisdiction consume the same quantity of the public good) and if $\sum_{i \in N} x_i + \sum_{j \in \{1,...,l\}} z_j \leq \sum_{i \in N} w_i$ (Federation budget constraint) where, for every j, z_j is defined by $z_j = z_i$ for all $i \in N_j$. If $(x_1, z_1, ..., x_n, z_n)$ is a feasible allocation of private and public goods, we denote by $T_i = \omega_i - x_i$ the *tax paid by household* i. An equivalent reformulation of the budget constraint is of course that $\sum_{j \in \{1,...,l\}} z_j \leq \sum_{i \in N} T_i$ (tax collected must be sufficient to finance the quantities of the public good provided to the citizens).

Households convert alternative combinations of private and public good into welfare by the same continuously differentiable, strictly increasing and concave utility function $U : \mathbb{R}_+^2 \to \mathbb{R}$ (with image u). A large part of the paper also assumes, in addition to the above properties, that U is *additively separable* so that it can be written, for every bundle $(\overline{z}, \overline{x}) \in \mathbb{R}_+^2$, as:

$$U(\overline{z}, \overline{c}) = f(\overline{z}) + h(\overline{x})$$

for some twice continuously differentiable increasing and concave functions f and h from $\mathbb{R}_+$ to $\mathbb{R}$. For further use, we denote by V ($V : \mathbb{R}_{++}^3 \to \mathbb{R}$).the indirect utility function defined by

$$V(p_z, p_c, R) = \max_{z,c} U(z, x) \text{ subject to } p_z z + p_c x \leq R \tag{1}$$

We also denote by $z^M(p_z, p_c, R)$ and $x^M(p_z, p_c, R)$ the (Marshallian) demands for public good and private consumption (respectively) when the prices for these two goods are p_z and p_c and when the wealth of the household is R. These Marshallian demands are defined, as usual,as the solution of program (1). Given the assumptions imposed on U, it can be seen easily that Marshallian demands and indirect utility are differentiable functions of prices and wealth. We denote by $\mathcal{U}$ the class of all direct utility functions that satisfy all these properties and by $\mathcal{U}_A$ the subset of $\mathcal{U}$ consisting of those functions that are additively separable.

The criterion used by the central government to compare alternative allocations of private and public goods from a social viewpoint is represented by a social evaluation function $S : \mathbb{R}_+^{2n} \to \mathbb{R}$ (with the interpretation that $S(x_1, z_1, ..., x_n, z_n) \geq S(x_1', z_1', ..., x_n', z_n')$ if and only if the allocation $(x_1, z_1, ..., x_n, z_n)$ is socially better than the allocation $(x_1', z_1', ..., x_n', z_n')$). We specifically assume that the social criterion results from the aggregation of individuals' utility function under a *symmetric* and *monotonically increasing* social welfare function $W : \mathbb{R}^n \to \mathbb{R}$ so that, for every allocation $(x_1, z_1, ..., x_n, z_n) \in \mathbb{R}_+^{2n}$,

$S(x_1, z_1, ..., x_n, z_n) = W(U(x_1, z_1), ..., U(x_n, z_n))$. We further assume that individuals' utility functions are cardinally meaningful and interpersonally comparable and that the social welfare function W is independent with respect to unconcerned individuals (using Deschamps and Gevers (1978) terminology). Hence, thanks to the main result of Deschamps and Gevers (1978), W is bound to be either utilitarian, or Leximin. While we shall perform much of the analysis with a utilitarian objective, we shall briefly discuss the Leximin case in the last section.

2.2 The first best problem

If households utilities and wealth are public information, the problem solved by the social planner is easy. Since the public good is non-rival in consumption, it is a waste to have different individuals consuming different quantities of the public good. For if $(x_1, z_1, ..., x_n, z_n) \in \mathbb{R}_+^{2n}$ is a feasible allocation of private and public goods for a jurisdiction structure $\{N_j\}_{j=1}^{l}$ with $l > 1$, one can improve everyone's utility by providing everyone with $\overline{z} = \max_{h \in N} z_h$ units of the public good and $x_i + \frac{\sum_{j \in \{1,...,l\}} z_j - -\overline{z}}{n}$ units of the private good. For this reason, in a first best environment, there is no loss of generality in writing a feasible allocations as $(x_1, x_2..., x_n, z) \in \mathbb{R}_+^{n+1}$ and in assuming these feasible allocations to satisfy $\sum_{i \in N} x_i + z \leq \sum_{i \in N} \omega_i$ (or $z \leq \sum_{i \in N} T_i$). In a first best world, there is no dispute as to the superiority of a central provision of a non-rival public good over a federal one and there is, accordingly, no need to provide different households with different quantity of the public good. There is, of course, a (distributive) need to have different households paying different taxes for the public good. More specifically, the social planner would choose a distribution of taxes that solves the following program:

$$\max_{(T_1,...,T_n)} W(U(\sum_{i \in N} T_i, \omega_1 - T_1), ..., U(\sum_{i \in N} T_i, \omega_n - T_n,)) \tag{2}$$

Such a distribution individual taxes -call it $(T_1^*, ..., T_n^*)$- would satisfy the well-known Samuelson's condition that the sum of the households marginal rates of substitution between the private and the public good equal 1. It is clear here that if W is Schur concave, the central government would solve problem (2) by equalizing utility levels, an equalization which can only be achieved, given the same level of public good provided to all, by equalizing private consumptions.

2.3 The general second-best problem

If information on households characteristics is private, the central planner is no longer able to levy *different* taxes on households who consume the same quantity of the public good because these households would become indistinguishable from the social planner point of view. The social planner could, however, decide to provide different households with different packages of taxes and public good levels based on, say, their place of residence, in order to make them reveal their characteristics. Households would then "vote with their feet" and choose to live at the place of residence offering them their favorite package of public spending and tax, thus revealing their "type" to the social planner. Of course separating households in this way in different jurisdictions is costly because it requires the use of much more resource to provide the public good than what would be needed under households pooling.

We consider herein the case where the private information concerns the household's wealth, and where the social planner is utilitarian. The wealth of a particular household is unknown to the social planner but the cumulative density of the wealth within the population (e.g. the number of households who have a wealth no greater than any real number) is assumed to be known. With this knowledge, the planner chooses a jurisdiction structure $\{N_j\}_{j=1}^l$ for some $l \in \{1, ..., n\}$ and, for every jurisdiction N_j in this structure, a tax payment and a public good provision pair (T_j, z_j) that generates a feasible allocation of private and public goods that maximizes its social objective, subject to the constraint that every household prefers the package of public good and tax provided in the jurisdiction to which it is assigned to any other. This problem is, in its full generality, rather complex, and proceeds in two steps.

In the *first* step, the social planner, solves, for any jurisdiction structure $\{N_j\}_{j=1}^l$ for some $l \in \{1, ..., n\}$, the program:

$$\max_{z_1, T_1, ..., z_l, T_l} \sum_{j=1}^{l} \sum_{i \in N_j} U(z_j, \omega_i - T_j,) \tag{3}$$

subject to the budget constraint:

$$\sum_{j=1}^{l} z_j \leq \sum_{j=1}^{l} \# N_j T_j \tag{4}$$

and, for every $j \in \{1, ..., l\}$, and every $i \in N_j$, the incentive compatibility constraint:

$$U(\omega_i - T_j, z_j) \geq U(\omega_i - T_{j'}, z_{j'}) \text{ for every } j' \in \{1, ..., l\} \tag{5}$$

Let $\Phi(\{N_j\}_{j=1}^l)$ denote the value of the objective function of the social planner at the solution of program (3) under the constraints (4) and (5). The *second* step of the central planner's problem consists in choosing the jurisdiction structure $\{N_j\}_{j=1}^l$ for some $l \in \{1, ..., n\}$ that maximizes the value of $\Phi(\{N_j\}_{j=1}^l)$. This second step is clearly a discrete problem since there is only a finite number of different possible partitions of N into jurisdiction structures.

An important jurisdiction structure to consider in the second step is the "centralized" one in which $l = 1$ and $N^1 = N$ (everybody is put into one jurisdiction). This jurisdiction structure is important because it is a very plausible candidate for an optimal choice in the second step, as the non-rivalry in consumption makes a strong case in favour of a centralized solution. We therefore below refer to the jurisdiction structure where $l = 1$ and $N^1 = N$ as "centralized" one and to any jurisdiction structure $\{N_j\}_{j=1}^{l}$ for some $l > 1$ as "federal".

Studying program (3) under the centralized jurisdiction structure is easy because there are no constraints (5) to worry about. In that case program (3) writes (after substituting the budget constraint (4) into the objective function)::

$$\Phi(N) = \max_{T \in [0, \omega_n]} \sum_{i \in N} U(nT, \omega_i - T) \tag{6}$$

The necessary (and sufficient by concavity of U) first order condition for an interior solution T^* of this program can be written as:

$$n \frac{\overline{U}_Z^*}{\overline{U}_x^*} = 1 \tag{7}$$

where $\overline{U}_j^* = [\sum_{i \in N} U_j(nT^*, \omega_i - T^*)]/n$ for $j = Z, x$ is the average marginal utility of good j at the optimal choice. This condition looks very much like a Samuelson condition. It states that the optimal allocation of public good in the case where everybody is constrained to pay the same tax (because the government can not distinguish between individuals) equalizes a sum of households marginal rate of substitution between private and public good to the marginal rate of transformation of 1. Yet the marginal rates of substitution whose sum is equalized to one are not the actual ones but are those of an abstract "average individual" whose marginal rate of substitution is the ratio of the average marginal utility of the public good over the average marginal utility of the private good. Because of this, the centralized second best solution is *a priori* far from first best optimality.

Yet there is an obvious case where condition (7) coincides with the standard Samuelson condition and where, as a result, $\Phi(N)$ is the maximal sum of individual utilities that would obtain in the first best world if the government had all the relevant information. This case is when the households utility function is *quasi-linear* so that it writes, for every z and x, $U(z, x) = f(z) + x$ for some increasing and concave function f. Indeed with quasi-linear utility, condition (7) writes:

$$n \frac{\partial f(nT^*)}{\partial z} = 1 \tag{8}$$

which is nothing else than the Samuelson's condition associated with quasi-linear utility. Hence, with quasi-linear utility, it is possible to achieve a first best allocation of private and public good by pooling everybody in the same jurisdiction. Because of this, it is clear that $\Phi(N) \geq \Phi(\{N_j\}_{j=1}^{l})$ for every other jurisdiction structure $\{N_j\}_{j=1}^{l}$ that the central planner could consider for some $l \in \{1, ..., n\}$ in the quasi-linear case. In a quasi linear world, it would not be optimal to create more than one jurisdiction. With quasi-linear utility functions, there is no case in favour of federalism.

In the following example, developed further in the second part of the paper, we show however that this conclusion is highly dependant upon the quasi-linear assumption, and that a federal jurisdiction structure can be better than the centralized one from a social welfare point of view.

Example 1 *There are 100 households, 90 of which are rich - and have a wealth of 10 - and 10 of them are poor (and have a wealth of 1). Their common utility function is* $U(z,x) = \ln z + \ln x$*. The social planner uses a utilitarian social welfare function. If the central planner decides to create only one jurisdiction, it solves*

$$\max_{T\in[0,10]} \ln(100T) + 0.9\ln(10-T) + 0.1(\ln(1-T)$$

The necessary FOC (7) in this case is

$$\frac{1}{T^{*}} - \frac{0.9}{10-T^{*}} - \frac{0.1}{1-T^{*}} = 0$$

a condition that is satisfied by $T^{*} = 0.90108$*. This tax rate yields a production of* 90.108 *units of public good. But the social planner can do better than this by considering the jurisdiction structure where the 90 rich households are gathered together in one jurisdiction with tax and public good provision* T_R *and* z_R *and the poor in another with tax* T_P *and public good* z_P*.For instance, the social planner could select* $z_R = 409.5$*,* $T_R = 5.45$*,* $Z_P = 45.5$ *and* $T_P = -3.55$ *(poor receive a subsidy!!). Such an allocation is feasible for the federal jurisdiction structure because* $z_R + z_P = 409.5 + 45.5 = 455 = n_R T_R + n_P T_p = 90 \times 5.45 - 10 \times 3.55 = 455$*. Moreover, any rich prefers the bundle of public and private good* $(409.5, 4.55)$ *obtained in the rich province of the federation to the bundle* $(90.108, 9.09892)$ *received with the central solution (indeed* $\ln 409.5 + \ln 4.55 \simeq 7.5301 > 6.7092 \simeq \ln 90.108 + \ln 9.09892$*). Similarly, a poor household prefers what it gets in the federal solution than in the centralized one because* $\ln 45.5 + \ln 4.55 \simeq 5.3328 > 2.1876 \simeq \ln 90.108 + \ln 0.09892$*. Hence the federal jurisdiction structure is Pareto superior to the Centralized one. Finally, it can be checked that the federal structure is "viable" in the sense that each of the two categories of households prefers its jurisdiction to that of the other. This is clear for the poor who could not even pay the tax of* 5.45 *that is charged in the rich jurisdiction. But that is also true for the rich who would get, were it to move to the poor jurisdiction, a utility of* $U(z^{P}, 10 - T_P) = \ln 45.5 + \ln 14.55 \simeq 6.4953$*, which is smaller than the utility of* 7.5301 *obtained in its jurisdiction.*

The question is thus: under which conditions is a centralized jurisdiction structure optimal ? How can we characterize optimal jurisdictions structures in general contexts ? These questions are clearly difficult to answer in general. For one thing, the two-step program described above is difficult to solve because if the number of households with different wealth levels is large, then so is the number of jurisdiction structures that are to be compared. Despite its complexity, a couple of simple general remarks can be made on the program 3 for the case where there the jurisdiction structure is federal.

This case requires that constraints (5) be handled properly. It is easy to see that the preference of a household with wealth ω_i in the space (z, T) of all public good and tax packages that it can face are represented by the utility function $\Phi^{\omega_i} : \mathbb{R}^2 \to \mathbb{R}$ defined by $\Phi^{\omega_i}(z,T) = U(z, \omega_i - T)$. These preferences are convex, increasing in z and decreasing in T and, if the public good is normal, satisfies the single crossing property that $MRS^{\Phi^{\omega_1}}(\overline{z}, \overline{T}) > MRS^{\Phi^{\omega_2}}(\overline{z}, \overline{T})$ at every $(\overline{z}, \overline{T})$ where, for $i = 1, 2$, $MRS^{\Phi^{\omega_1}}(\overline{z}, \overline{T})$ is the slope of the indifference curve of a type-ω_i household at $(\overline{z}, \overline{T})$ defined by

$$MRS^{\Phi^{\omega_1}}(\overline{z}, \overline{T}) = \frac{\partial T^{a_i}(\overline{z})}{\partial z}$$

where the implicit function T^{a_i} (for some real number a_i satisfying $\Phi^{\omega_i}(\overline{z}, \overline{T}) = a_i$) is defined in the usual way by

$$\Phi^{\omega_i}(z, T^{a_i}(z)) = a_i \tag{9}$$

Using (9), one sees that

$$MRS^{\Phi^{\omega_i}}(\overline{z}, \overline{T}) = \frac{\partial U(\overline{z}, \omega_i - \overline{T})/\partial z}{\partial U(\overline{z}, \omega_i - \overline{T})/\partial x}$$

and, therefore, since the public good is normal, that $MRS^{\Phi^{\omega_i}}(\overline{z}, \overline{T})$ is increasing with respect to ω_i everywhere.

The indifference curves of two households with wealth a and b (with $b > a$) are represented in figure 1.

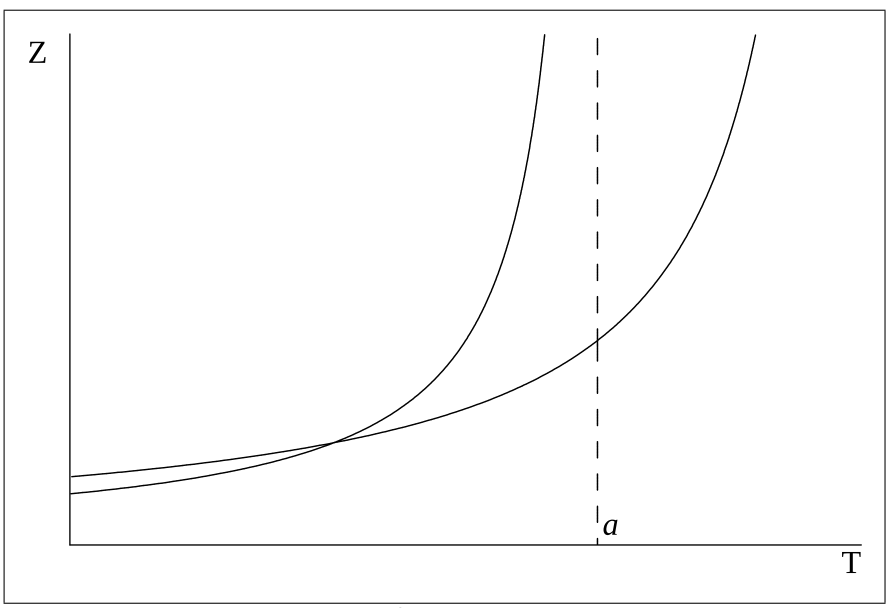

figure 1

This standard single-crossing property simplifies somehow the solution of the program 3 in at least two ways. First it guarantees that two incentive constraints can never be simultaneously violated. Second, if there are more than two types of households, it also guarantees that the sets N_j in the jurisdiction structures will all be *consecutive* (see e.g. Greenberg (1983)) in the sense that if two households with different wealth levels are put in a jurisdiction N_j, then so will all households with wealth in between of those two. This means than in any optimal federation chosen by the social planner, there will be perfect segregation in the sense that in any two provinces with different *per capita* wealth, the poorest household living in the richest province will be richer than the richest individual living in the poorer province.

The difficulty of studying problem (3) in the general case leads us to consider the (much) simpler problem of characterizing optimal jurisdiction structure when there are only a small number of types of households. In the next section, we restrict attention to the two-type cases, before considering the much more complex three-type problem.

3 A "simple" second best problem with two types of households

We assume here that the n households can be split in two types: n_1 households of type 1 and n_2 households of type 2. A household of type $i = 1, 2$ has a private wealth w_i such that $w_1 \geq w_2$. In this setting the choice of the optimal jurisdiction structure made by the social planner amounts to comparing the value of its objective function if only one jurisdiction form and everybody pay the same tax with the situation where the households are split into two jurisdictions: one inhabited by the poor, and the other by the rich.[2] Let us denote by (N_1, N_2) the (federal) jurisdiction structure in which the rich and the poor live in two distinct jurisdictions and by N the centralized jurisdiction structure in which all households live in the same jurisdiction.

Consider first the later jurisdiction structure. Let us examine the structure of the *utility possibility set* in this situation. The Pareto frontier of this set is easy to characterize. It is a curve lying between two points: One where each poor gets his ideal utility associated to his most preferred tax and the rich gets the utility associated with the poor's most preferred tax rate and the other extreme situation where the rich gets her ideal utility and the poor gets the utility level associated to the fact of paying the rich most preferred tax rate. The most preferred tax rate T_2^* of the poor is the solution of the program

$$\max_{T \in [0, \omega_2]} U(nT, \omega_2 - T)$$

It is immediate to see that this ideal tax rate is given by $T_2^* = \frac{z^M(\frac{1}{n}, 1, \omega_2)}{n}$ (the most preferred tax rate of the poor is the expenditure (using private good

[2] It is clear that, beside this segregated federation in which all type-1 households live in a jurisdiction and all type-2 households live in another, there are no partition of the set of households into disjoint subsets that can maximize a Pareto-inclusive objective while satisfying the incentive compatibility constraints.

as the *numeraire*) that the poor would like to devote to the public good if he was facing a Lindahl price of the public good of $\frac{1}{n}$). Hence the poor's ideal utility level is $V(\frac{1}{n}, 1, \omega_2)$. For the rich, things are slightly different because of the fact that, when deciding her most preferred tax rate, she must take into account the ability to pay of the poor. The most preferred tax rate of the rich T_1^* is in effect the solution of the program:

$$\max_{T\in[0,\omega_2]} U(nT, \omega_1 - T)$$

so that there are two cases that can occur:

1: $T_1^* = \omega_2$ (the rich chooses to tax all the poor's income) or

2:$T_1^* = \frac{z^M(\frac{1}{n},1,\omega_1)}{n}$ (as for the poor, the most preferred tax rate of the rich is the expenditure that she would like to devote to the public good if she had the opportunity to purchase it at the Lindahl price of $1/n$.)

In short $T_1^* = \min(\frac{z^M(\frac{1}{n},1,\omega_1)}{n}, \omega_2)$ and the rich ideal utility level U_{SB1}^{1*} is defined by(

$$\begin{aligned} U_{SB1}^{1*} &= V(\frac{1}{n}, 1, \omega_1) \text{ if } \frac{z^M(\frac{1}{n}, 1, \omega_1)}{n} \leq \omega_2 \text{ and} \\ &= U(n\omega_2, \omega_1 - \omega_2) \text{ otherwise} \end{aligned}$$

It is immediate to see that, if the public good is normal , $T_1^* > T_2^*$ and that both

$$U_{SB1}^{1*} > U(z^M(\frac{1}{n}, 1, \omega_2), \omega_1 - T_2^*) > V(\frac{1}{n}, 1, \omega_2) > U(nT_1^*, \omega_2 - T_1^*)$$

Hence the two extreme points $a = (U(z^M(\frac{1}{n}, 1, \omega_2), \omega_1 - T_2^*), V(\frac{1}{n}, 1, \omega_2))$ and $b = (U_{SB1}^{1*}, U(nT_1^*, \omega_2 - T_1^*)$ of the Pareto frontier will lie in the area where the utility of the rich is larger than that of the poor. It will therefore be difficult to be egalitarian in this second best world in one jurisdiction. Between its extreme points $a = (U(z^M(\frac{1}{n}, 1, \omega_2), \omega_1 - T_2^*), V(\frac{1}{n}, 1, \omega_2))$ and $b = (U_{SB1}^{1*}, U(nT_1^*, \omega_2 - T_1^*)$ the Pareto frontier is defined by the function $\Psi : \mathbb{R} \to \mathbb{R}$ defined by $\Psi(u_1) = U(nT^{SB}(u_1), \omega_2 - T^{SB}(u_1)$ where the function $T^{SB} : [U(z^M(\frac{1}{n}, 1, \omega_2), \omega_1 - T_2^*), U_{SB1}^{1*}] \to [0, T_1^*]$ is defined implicitly, for every $u_1 \in [U(z^M(\frac{1}{n}, 1, \omega_2), \omega_1 - T_2^*), U_{SB1}^{1*}]$ by

$$U(nT^{SB}(u_1), \omega_1 - T^{SB}(u_1)) = u_1$$

It is easy to verify by usual implicit function arguments that the function Ψ is decreasing and concave on its domain.

This frontier is illustrated on figure 1 in a situation where $U_{SB1}^{1*} = V(\frac{1}{n}, 1, \omega_1)$ (the rich ideal tax is less than the poor's wealth).

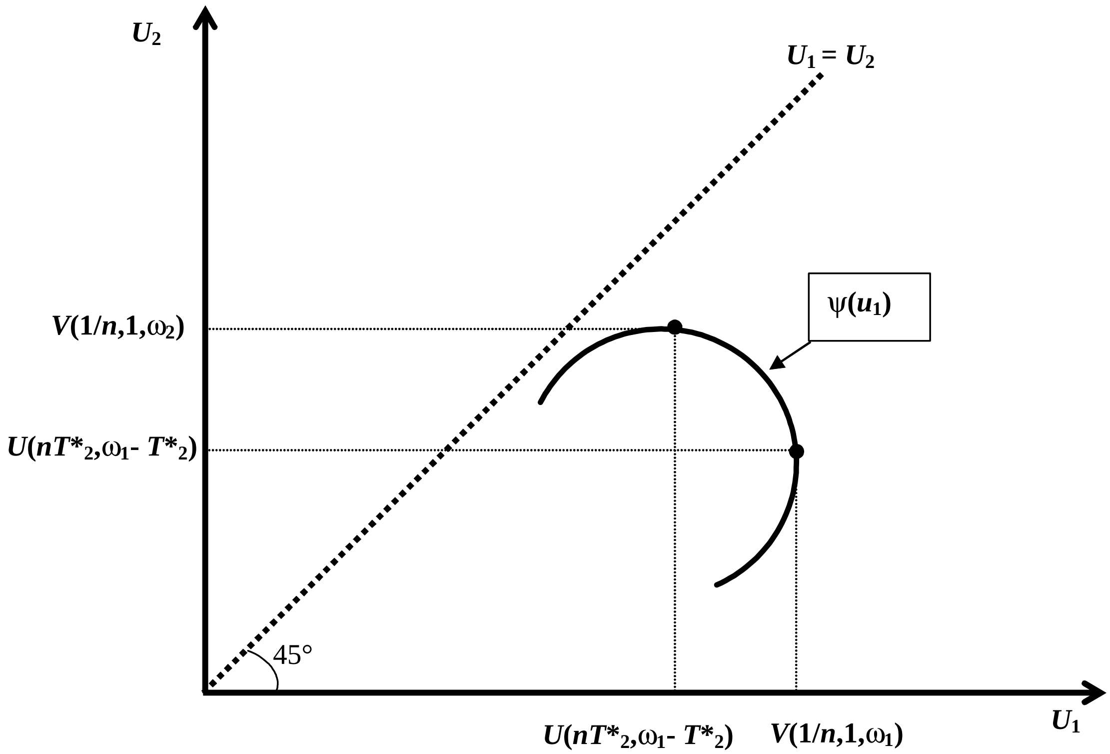

Figure 2

Notice that T_2^* is the solution that will be selected by a Leximin or a Maxmin social planner.

Let us now consider the (federal) jurisdiction structure (N_1, N_2) in which the central government "separates" the two types into two different jurisdictions. While the single-crossing property rules out the possibility that the incentive constraints for the two types of household be simultaneously violated, it does not rule out the possibility that they be simultaneously satisfied. Suppose for the moment that this the case. Then one can solve program (3) by ignoring the two incentive constraints. This program, studied in the general case in Gravel and Poitevin (2006), describes how a welfarist constitution maker would like to organize equalization payments in a federal system with an immobile population. Let $B(n_1, n_2, \omega_1, \omega_2) = \{(z_1, T_1, z_2, T_2) \in \mathbb{R}^4 : n_1T_1 + n_2T_2 \geq z_1 + z_2$ and $T_i \leq \omega_i$ for $i = 1, 2\}$ denote the set of two-jurisdiction packages of public goods and taxes that are feasible for a two jurisdiction segregated federation notwithstanding the IC constraints. It is easy to prove the following lemma.

Lemma 2 *Let U be a utility function in $\mathcal{U}$. Then $(z_1^*, T_1^*, z_2^*, T_2^*)$ is Pareto-efficient in the set $B(n_1, n_2, w_1, w_2)$ if and only if there exists $s_i^* \in [-w_i, +\infty$*

(for $i = 1, 2$) satisfying $n_1 s_1^* + n_2 s_2^* = 0$ such that $z_i^* = z^M(\frac{1}{n_i}, 1, w_i + s_i^*)$ and $T_i^* = w_i - x^M(\frac{1}{n_i}, 1, w_i + s_i^*)$.

This lemma hence tells us that any Pareto-efficient allocation of public goods and tax burden in a two-jurisdiction federation - ignoring the incentive compatibility constraints - can be thought of as resulting from a two-step procedure analogous to that discussed in Gravel and Poitevin (2006):A *first* step in which a federal government selects a pair of *per capita* equalization subsidies s_1 and s_2 (one such subsidy for every jurisdiction, the subsidies summing to 0) and a *second* step in which each household allocates its wealth - increased by the subsidy received - between public and private good expenditure assuming that it faces a price of public good given by the inverse of the population size of its jurisdiction of residence. An efficient federal provision of public goods and tax burdens that satisfies the incentive constraint (5) is depicted on figure 3.

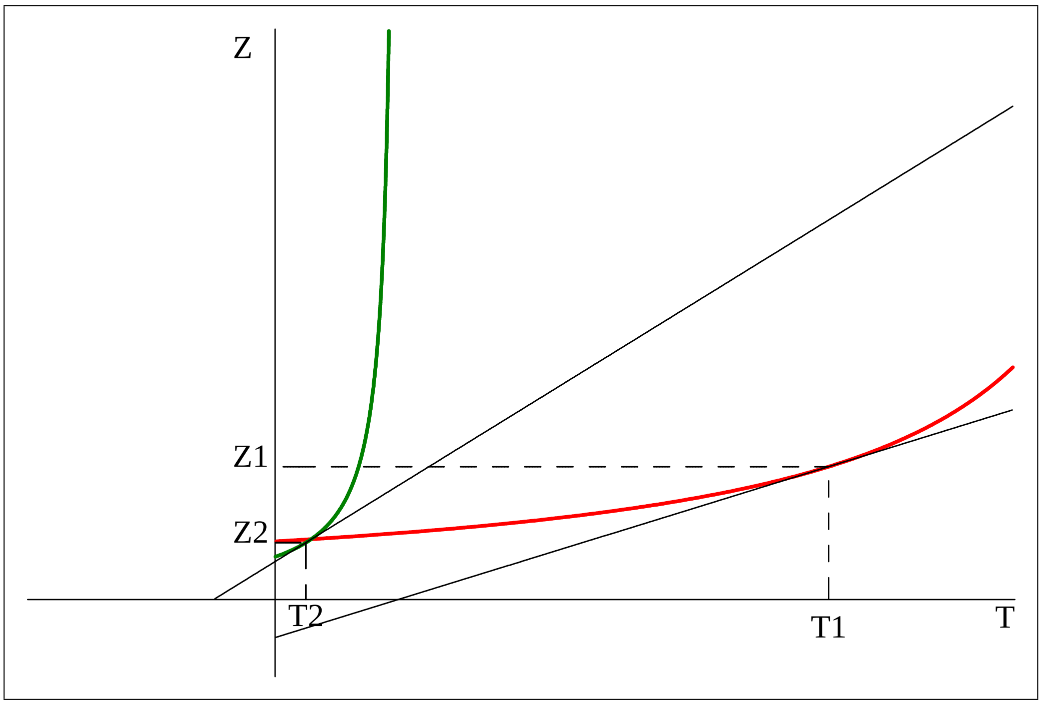

figure 3

As discussed in Gravel and Poitevin (2006), for general utility functions (even additively separable ones), it is possible for the subsidies to be *regressive* in the sense of being increasing with respect to households wealth (in a two jurisdiction case, regressivity would amount to have the poor giving a subsidy to the rich). This may arise if the indirect utility function induced by the additively separable utility function is not additively separable between income and the price of the public good. In Gravel and Poitevin (2006), it was shown that, for additively separable utility functions, the requirement that the optimal - from the view point of the utilitarian ethics - subsidies be progressive for every distributions of wealth and population sizes implies that the utility function is either quasi-linear or logarithmic with respect to the public good. As mentioned above, quasi-linear utility makes the second best problem trivial. Because progressive equalization payments across jurisdictions are rather common in federations that we are aware of, we find useful to pursue the analysis in the case where households utility is logarithmic with respect to the public good. This case also simplifies significantly the handling of incentive compatibility constraints in the study of the optimal jurisdiction structure. For, it can be easily seen that, in the two-type case, if a pair of packages of local public goods and taxes that is Pareto efficient in the federation for a progressive subsidies scheme violates the incentive constraint of the poor, then the centralized jurisdiction structure will Pareto dominates the federal one. We state this useful fact formally as follows.

Claim 3 *Let $B(n_1, n_2, \omega_1, \omega_2) = \{(z_1, T_1, z_2, T_2) \in \mathbb{R}^4 : n_1T_1 + n_2T_2 \geq z_1 + z_2$ and $T_i \leq \omega_i$ for $i = 1, \dot{2}\}$ and let $(z_1^*, T_1^*, z_2^*, T_2^*)$ be efficient in $B(n_1, n_2, \omega_1, \omega_2)$ with respect to a scheme of subsidies s_1 and s_2 as per lemma 2 satisfying $s_1 < 0 < s_2$. Then, if $U(z_2^*, w_2 - T_2^*) < U(z_1^*, w_2 - T_1^*)$ (incentive compatibility constraint is violated), $\Phi(N) > \Phi(N_P, N_R)$.*

Proof. *Assume that $(z_1^*, T_1^*, z_2^*, T_2^*)$ is efficient in $B(n_1, n_2, \omega_1, \omega_2)$ with respect to a scheme of subsidies s_1 and s_2 as per lemma 2 satisfying $s_1 < 0 < s_2$. and is such that:*

$$U(z_2^*, w_2 - T_2^*) < U(z_1^*, w_2 - T_1^*) \tag{10}$$

Because the per capita subsidies s_1 and s_2 that support this Pareto-efficient federal provision of public good as per lemma 2 are progressive, $s_1 = \frac{z_1^*}{n_1} - T_1^* < 0$ so that $T_1^* > 0$ (since $z_1^* > 0$). Consider then merging jurisdictions 1 and 2 and providing every one with z_1^* unit of public good while asking a tax payment of T_1^*. If it was feasible, such a centralized provision of the public good would Pareto dominate the federal provision associated with $(z_1^*, T_1^*, z_2^*, T_2^*)$ because type 1 individual would be indifferent and, because of inequality (10), type 2 households would be strictly better off. Let us show that the centralized provision (z_1^*, T_1^*) is feasible. That is, let us show that:

$$(n_1 + n_2)T_1^* \geq z_1^*$$

We know already that, since $(z_1^*, T_1^*, z_2^*, T_2^*)$ is feasible and efficient in $B(n_1, n_2, \omega_1, \omega_2)$, one has:

$$\begin{aligned} n_1T_1^* + n_2T_2^* &= z_1^* + z_2^* \\ &\Longleftrightarrow \\ n_1T_1^* + n_2T_2^* - z_2^* &= z_1^* \end{aligned}$$

which implies, since $n_2T_2^* - z_2^* = n_2s_2 > 0$, that $n_1T_1^* > z_1^*$ and, therefore, that $(n_1 + n_2)T_1^* > z_1^*$ (since $T_1^* > 0$). Because this inequality is strict, it is actually *possible to increase public good provision for all without increasing taxes. That is, it is possible to increase the utility level of all.* ■

Simple as it is, this claim facilitates the analysis of the two-type case by restricting attention, in the study of the optimal federal provision of public good, to the incentive compatibility constraint of type-1 households (as any violation of the incentive constraint of type-2 household in the optimal federal provision would indicate that a central provision would then be preferable). Of course, the progressivity of the equalization subsidies that support an efficient federal provision of public goods is crucial for this claim.

3.1 The two-type cases with logarithmic utility functions

In what follows, we provide a complete characterization of the two-type case for a utilitarian social planner when the household's utility function writes:

$$U(Z, x) = \ln Z + \ln x \tag{11}$$

Of course this case is more specific than only assuming the logarithmic form for the public good. But it is (relatively) easily tractable.

As discussed above, a useful preliminary step to the full blown study of the optimal choice of a jurisdiction structure under private information is the examination of "pseudo" first best case where information is public but where households live in two distinct jurisdictions. In this case, each jurisdiction has its own level of public good and it is excludable to households in the other jurisdiction. More specifically, the government solves the following program

$$\max_{z_1,T_1,z_2,T_2} n_1U(\omega_1-T_1, z_1)+n_2U(\omega_2-T_2, z_2) \quad \text{s.t.} \quad z_1+z_2 \leq n_1T_1+n_2T_2. \tag{12}$$

In each jurisdiction, the optimal level for the public good satisfies the Samuelson condition that the sum of households' marginal rate of substitution between the private and the public goods be equal to 1, the marginal cost for the public good. Since the government can transfer income across jurisdictions and utility is additively separable between the two goods, private consumption is equalized across jurisdictions.

$$n_i\frac{U_z(\omega_i - T_i^{fb2}, z_i^{fb2})}{U_c(\omega_i - T_i^{fb2}, z_i^{fb2})} = 1 \;\text{ and }\; U_c(\omega_1 - T_1^{fb2}, z_1^{fb2}) = U_c(\omega_2 - T_2^{fb2}, z_2^{fb2})$$

The solution to the first-best problem with two jurisdictions is

$$\begin{aligned}
z_1^{fb2} &= \frac{n_1(n_1w_1 + n_2w_2)}{2(n_1 + n_2)},\\
z_2^{fb2} &= \frac{n_2(n_1w_1 + n_2w_2)}{2(n_1 + n_2)},\\
T_1^{fb2} &= \frac{n_1w_1 - n_2w_2 + 2n_2w_1}{2(n_1 + n_2)},\\
T_2^{fb2} &= \frac{n_2w_2 - n_1w_1 + 2n_1w_2}{2(n_1 + n_2)}.
\end{aligned}$$

Notice that private good consumption - equal to the Marshallian demand for the private good thanks to lemma 2- is equalized across jurisdictions because the household's wealth (net of the subsidy received) is equalized across jurisdictions and, thanks to the (Cobb-Douglas) structure of the households preferences, Marhallian demands for the private good does not depend upon the price of the public good. Notice that the equalization of wealth achieved here result from the fact that indirect utility of the household is additively separable between the wealth and the price of the public good. However, the quantity of the public good consumed in the two jurisdictions differ because the population size (equal to the inverse of the price of the public good) differ. Note that the level of public good in one jurisdiction is increasing in that jurisdiction's population and decreasing in the other jurisdiction's population.

We can compute the (aggregate) subsidy as per lemma 1 which, given the progressivity associated with this preferences, comes from the rich (type 1) to the poor (type 2). The subsidy is nothing else than the difference between tax revenues and public good expenditure in the rich jurisdiction:

$$s^{fb2} \equiv n_1 T_1^{fb2} - z_1^{fb2} = \frac{n_1 n_2 (w_1 - w_2)}{n_1 + n_2}.$$

The subsidy is increasing in the difference in income between the two types of households, which is to be expected for a progressive subsidy. The subsidy ensures that the marginal utility of income is equalized across jurisdictions. As mentioned above, additively separable utility function that is logarithmic with respect to public good is necessary and sufficient for a utilitarian social planner who use an additively separable utility function to choose a subsidy that does go from the rich to the poor for any value of the parameters of the problem..

The subsidy is also increasing with respect to the population size of each jurisdiction. When population increases in one jurisdiction, the demand for public good provision increases. For the assumed utility function, the increase in public good is exactly financed by the increase in population so that taxes do not have to increase. When n_2 increases, the per capita subsidy to the poor decreases so that the marginal utility of income for the poor has increased. To restore the equality of marginal utility of income across jurisdictions, the subsidy has to increase. When n_1 increases, the per capita subsidy from the rich decreases so that their marginal utility of income has increased. The subsidy then has to increase for optimal redistribution.

We now address the problem of the optimal choice of an optimal jurisdiction structure in the case where the social planner can not observe the household's income. In the two-type case, the government has two jurisdiction structures to compare. The centralized one in which all households are pooled in the same jurisdiction and pay the same tax and the "decentralized" one where types are "separated" into two jurisdictions and are offered differential tax package.

We first characterize the optimal allocation under each jurisdiction structure. Second, we compare social welfare under each structure.

If all households are put into one single grand jurisdiction, the central government solves the following problem.

$$\max_T n_1 U(\omega_1 - T, nT) + n_2 U(\omega_2 - T, nT)$$

The first-order condition 7 writes here:

$$n_1 U_z(\omega_1 - T^{sb1}, z^{sb1}) + n_2 U_z(\omega_2 - T^{sb1}, z^{sb1}) = \frac{n_1 U_c(\omega_1 - T^{sb1}, z^{sb1}) + n_2 U_c(\omega_2 - T^{sb1}, z^{sb1})}{n}.$$

which, as discussed above, is akin to a Samuelson condition where the aggregate marginal benefit for the public good is equal to the *average* marginal cost in utility terms.

The solution the second-best problem with one jurisdiction is

$$\begin{aligned} z^{sb1} &= \frac{n_1 w_1 + n_2 w_2 + 2n_1 w_2 + 2n_2 w_1 - g(n_1, n_2, w_1, w_2)}{4}, \\ T^{sb1} &= \frac{z^{sb1}}{(n_1 + n_2)}, \end{aligned}$$

where

$$g(n_1, n_2, w_1, w_2) = (-8(n_1 + n_2)^2 w_1 w_2 + (n_2(2w_1 + w_2) + n_1(w_1 + 2w_2))^2)^{1/2}.$$

In this solution, all households consume the same amount of the public good which is provided to all. No redistribution is possible and all households pay the same tax. If the one-jurisdiction structure allows to economize on the cost of providing the public good, it does poorly on redistribution grounds.

We now characterize the solution with two jurisdictions, which amounts to the solution of the following program:

$$\begin{aligned} \max_{z_1, T_1, z_2, T_2} \quad & n_1 U(\omega_1 - T_1, z_1) + n_2 U(\omega_2 - T_2, z_2) \\ \text{s.t.} \quad & \sum_{j=1}^{J} z_j \leq \sum_{j=1}^{2} n_j T_j \\ & U(\omega_1 - T_1, z_1) \geq U(\omega_1 - T_2, z_2) \\ & U(\omega_2 - T_2, z_2) \geq U(\omega_2 - T_1, z_1) \end{aligned}$$

Some differences with a standard screening problem are worth keeping in mind. In a standard problem, each type faces an IR constraint. Combined with the single-crossing property, the IR constraint usually determines which type is the "good" one and, hence, which of the two IC constraints is binding. This allows to characterize the solution relatively easily. In the problem here, the IR constraints are replaced by an aggregate budget constraint. Even though the single-crossing property holds, it is not easy to determine which IC is binding. It turns out that either one could be binding depending on the value of the parameters (income and population). This features also makes the program somewhat different than an optimal taxation one *à la* Mirlees (1986).

They key for identifying which IC constraint binds is to see which one is violated at the pseudo first-best allocation. Recall that the pseudo first best represents the situation where there are two jurisdictions and incomes are public information. It therefore constitutes the appropriate "first-best" benchmark to consider when analyzing the two-jurisdiction case with private information.

We evaluate the incentive constraint of the rich at the $fb2$ solution. We can show that

$$U(\omega_1 - T_1^{fb2}, z_1^{fb2}) \geq U(\omega_1 - T_2^{fb2}, z_2^{fb2})$$

is equivalent to

$$\frac{n_1^2 w_1 + n_2^2(w_2 - 2w_1) + 3n_1 n_2(w_2 - w_1)}{n_2(3n_1 w_1 + 2n_2 w_1 - 2n_1 w_2 - n_2 w_2)} \geq 0. \tag{13}$$

Since the denominator is positive, only the numerator matters. Fortunately, it is possible to restrict the number of parameters by replacing n_1 by $a * n_2$ and w_1 by $b * w_2$ where $a = n_1/n_2$ and $b = \omega_1/\omega_2$. By assumption, $a > 0$ and $b > 1$. Condition (13) can then be rewritten as

$$n_2^2 w_2(1 - 3a(b-1) - 2b + a^2 b) \geq 0,$$

or, equivalently,

$$\text{IC}_1(a, b) \equiv a^2 b - 3(b-1)a + 1 - 2b \geq 0. \tag{14}$$

When inequality (14) holds, the incentive constraint for the rich household is satisfied at the pseudo first-best solution.

We can do similar computations for the incentive constraint for the poor at the $fb2$ solution.

$$U(\omega_2 - T_2^{fb2}, z_2^{fb2}) \geq U(\omega_2 - T_1^{fb2}, z_1^{fb2})$$

is equivalent to

$$\text{IC}_2(a, b) \equiv a^2(b-2) + 3(b-1)a + 1 \geq 0. \tag{15}$$

When this inequality holds, the incentive constraint for the poor household is satisfied at the pseudo first-best solution.

Figure 4 provides a graphical representation of the two incentive constraints in (a, b)-space. As mentioned above, the single-crossing property rules out the possibility for the two incentive constraints (14) and (15) to be simultaneously violated.

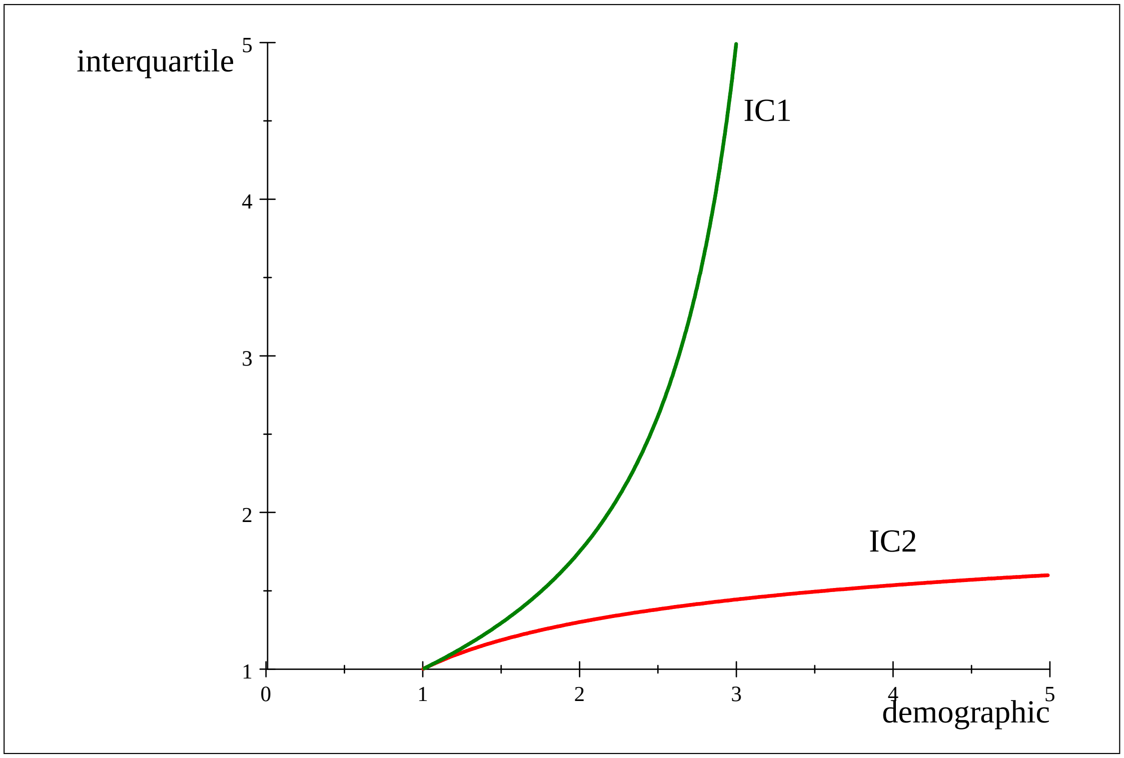

Figure 4

We can show that the two curves IC_1 and IC_2 intersect at $a = b = 1$, which is to be expected. When $b = 1$, all households have the same income. At the pseudo first best, they only differ in their consumption of the public good, which depends on the population of their jurisdiction. At $a = 1$, both jurisdictions have the same population, hence the same consumption of the public good. Both incentive constraints are therefore satisfied at this point. Furthermore, these two curves are tangent at this point.

For all values of $a < 1$ the incentive constraint of the rich household is violated at the pseudo first best. That is to say that the rich household always has an incentive to mimic the poor household when there are more poor households than there are rich ones. In this case, a poor household has a larger consumption of the public good than a rich household has and the poor household pays less taxes than the rich household does. The rich household therefore has the incentive to mimic the poor household (or move to the poor household's jurisdiction).

We can also show that the incentive constraint of the poor type is always satisfied for $b \geq 2$. Although this precise value depends on the functional form of preferences, in general, the poor household does not mimic the rich household when there is a large income disparity since then taxes paid by the rich household are relatively high. In general, the poor household has an incentive to mimic the rich when b is relatively small and a is relatively large so that income differences are small and there are a lot of rich. But of course, as shown by the claim above, the incentive constraint of the poor shall not be a source of worry for the choice of the optimal jurisdiction structure.

We now solve for the second-best allocation when constraint (14) is not satisfied at the $fb2$ allocation. In that case, the welfare optimizing problem becomes

$$\begin{aligned} \max_{z_1,T_1,z_2,T_2} \quad & n_1 U(\omega_1 - T_1, z_1) + n_2 U(\omega_2 - T_2, z_2) \\ \text{s.t.} \quad & \sum_{j=1}^{J} z_j \leq \sum_{j=1}^{2} n_j T_j \\ & U(\omega_1 - T_1, z_1) \geq U(\omega_1 - T_2, z_2) \end{aligned} \tag{16}$$

The solution to this problem entails that the allocation of the rich household is not distorted (no distortion at the top) and that of the poor household is distorted with a suboptimal level of the public good. To obtain a more precise characterization, it is instructive to compute how the subsidy from the rich to the poor varies with the introduction of private information.

To compute the optimal subsidy in the pseudo-first best without incentive constraints, we solve the following maximization problem.

$$\begin{aligned} \max_{z_1,T_1,z_2,T_2} \quad & n_1 U(\omega_1 - T_1, z_1) + n_2 U(\omega_2 - T_2, z_2) \\ \text{s.t.} \quad & n_1 T_1 - s \geq z_1 \\ & n_2 T_2 + s \geq z_2 \end{aligned}$$

The variable s represents the aggregate subsidy paid by the rich to the poor. This problem gives the optimal level of taxes $T_i^*(s)$ and public good $z_i^*(s)$ for a given subsidy s. We can then solve for the optimal subsidy with

$$\max_s n_1 U(\omega_1 - T_1^*(s), z_1^*(s)) + n_2 U(\omega_2 - T_2^*(s), z_2^*(s)). \tag{17}$$

The optimal subsidy is

$$s^{fb2} = n_1 T_1^{fb2} - z_1^{fb2} = \frac{n_1 n_2 (w_1 - w_2)}{n_1 + n_2}.$$

This sequence of maximization problems is equivalent to problem (12) in that $T_i^*(s^{fb2}) = T_i^{fb2}$ and $z_i^*(s^{fb2}) = z_i^{fb2}$. For subsequent use, we define $U_i^*(s) \equiv U(\omega_i - T_i^*(s), z_i^*(s))$.

We want to compare the optimal subsidy with public information s^{fb2} to that with private information when constraint (14) is binding, denoted s_1^{sb2}. With private information (and when the binding incentive constraint is (14)), the following sequence of problems is equivalent to problem (16).

$$\begin{aligned}\max_{z_1,T_1,z_2,T_2} \quad & n_1U(\omega_1 - T_1, z_1) + n_2U(\omega_2 - T_2, z_2) \\ \text{s.t.} \quad & n_1T_1 - s \geq z_1 \\ & n_2T_2 + s \geq z_2 \\ & U(\omega_1 - T_1, z_1) \geq U(\omega_1 - T_2, z_2)\end{aligned}$$

This problem gives the optimal level of taxes $T_i^{\text{IC1}}(s)$ and public good $z_i^{\text{IC1}}(s)$ for a given subsidy s. We can then solve for the optimal subsidy with

$$\max_s n_1U(\omega_1 - T_1^{\text{IC1}}(s), z_1^{\text{IC1}}(s)) + n_2U(\omega_2 - T_2^{\text{IC1}}(s), z_2^{\text{IC1}}(s)). \tag{18}$$

Define $U_i^{\text{IC1}}(s) \equiv U(\omega_i - T_i^{\text{IC1}}(s), z_i^{\text{IC1}}(s))$.

The allocation of the rich household under asymmetric information is not distorted. This is the familiar “no distortion at the top" result. This implies that, for a given s, $T_1^{\text{IC1}}(s) = T_1^*(s)$ and $z_1^{\text{IC1}}(s) = z_1^*(s)$. We then have $U_1^{\text{IC1}}(s) = U_1^*(s)$. Problem (18) can be rewritten as

$$\max_s n_1U(\omega_1 - T_1^*(s), z_1^*(s)) + n_2U(\omega_2 - T_2^{\text{IC1}}(s), z_2^{\text{IC1}}(s)). \tag{19}$$

We seek to compare the subsidies s_1^{sb2} and s^{fb2} to understand how the $sb2$ allocation differs from the $fb2$ one. The first-order condition for problem (17) is

$$n_1\frac{\partial U_1^*(s^{fb2})}{\partial s} + n_2\frac{\partial U_2^*(s^{fb2})}{\partial s} = 0. \tag{20}$$

The first-order condition for problem (18) is

$$n_1\frac{\partial U_1^{\text{IC1}}(s_1^{sb2})}{\partial s} + n_2\frac{\partial U_2^{\text{IC1}}(s_1^{sb2})}{\partial s} = 0. \tag{21}$$

To compare the subsidies s_1^{sb2} and s^{fb2}, it suffices to evaluate the first-order condition (21) at $s = s^{fb2}$. Using (20) and the fact that $U_1^*(s) = U_1^{\text{IC1}}(s)$, we obtain that

$$n_1\frac{\partial U_1^{\text{IC1}}(s^{fb2})}{\partial s} + n_2\frac{\partial U_2^{\text{IC1}}(s^{fb2})}{\partial s} = n_2\frac{\partial U_2^{\text{IC1}}(s^{fb2})}{\partial s} - n_2\frac{\partial U_2^*(s^{fb2})}{\partial s}.$$

Because of concavity and because s_1^{sb2} solves condition (21), it is possible to write

$$\begin{aligned}s_1^{sb2} < s^{fb2} \quad \text{if} \quad & \frac{\partial U_2^{\text{IC1}}(s^{fb2})}{\partial s} - \frac{\partial U_2^*(s^{fb2})}{\partial s} < 0; \\ s_1^{sb2} = s^{fb2} \quad \text{if} \quad & \frac{\partial U_2^{\text{IC1}}(s^{fb2})}{\partial s} - \frac{\partial U_2^*(s^{fb2})}{\partial s} = 0; \\ s_1^{sb2} > s^{fb2} \quad \text{if} \quad & \frac{\partial U_2^{\text{IC1}}(s^{fb2})}{\partial s} - \frac{\partial U_2^*(s^{fb2})}{\partial s} > 0.\end{aligned}$$

We can show that
$$\frac{\partial U_2^{\mathrm{IC1}}(s^{fb2})}{\partial s} - \frac{\partial U_2^*(s^{fb2})}{\partial s} < 0$$
for all parameter values for which constraint (14) is binding. This implies that the subsidy from the rich to the poor under asymmetric information is smaller than that under symmetric information, that is, $s_1^{sb2} < s^{fb2}$.

We can show that
$$T_1^*(s) = \frac{n_1\omega_1 + s}{2n_1} \quad \text{and} \quad z_1^*(s) = \frac{n_1\omega_1 - s}{2}.$$
We then have that $T_1^*(s_1^{sb2}) < T_1^*(s^{fb2})$ and $z_1^*(s_1^{sb2}) > z_1^*(s^{fb2})$. Under asymmetric information, the rich household pays less taxes and consumes more of the public good than under symmetric information. By increasing the utility of the rich household, it is possible to reduce the distortion on the poor household's allocation to satisfy the incentive constraint for the rich one.

The distortion on the poor household's allocation is such that $z_2^{\mathrm{IC1}}(s_1^{sb2}) < z_2^*(s^{fb2})$, that is, the consumption of public good is reduced. This makes the poor household's allocation less attractive to the rich household and hence, relaxes the latter incentive constraint. In general, it is not possible to determine whether the taxes on the poor household are larger or smaller under asymmetric information than under symmetric information. On the one hand, less needs to be raised since less public good is provided. On the other hand, the subsidy from the rich households is smaller so that more taxes need to be raised. The aggregate effect is ambiguous.

To recapitulate, when the incentive constraint of the rich household is binding, that is, when there are relatively more poor households, it is optimal to increase the rich household's utility by lowering its taxes and increasing its level of public good. It is also optimal to distort the poor household's allocation by lowering its consumption of the public good.

3.2 Optimal jurisdiction structure

We now look at the choice of the optimal jurisdiction structure in the presence of asymmetric information.

The first comparison we make is between the one-jurisdiction structure and the two-jurisdiction one when both incentive constraints (14) and (15) are satisfied. In that case, we compare the allocations $sb1$ and $fb2$.

The social welfare at the $sb1$ allocation is
$$SW^{sb1} = n_1 U(\omega_1 - T^{sb1}, z^{sb1}) + n_2 U(\omega_2 - T^{sb1}, z^{sb1}) \tag{22}$$
The social welfare at the $fb2$ allocation is
$$SW^{fb2} = n_1 U(\omega_1 - T^{fb2}, z^{fb2}) + n_2 U(\omega_2 - T^{fb2}, z^{fb2}) \tag{23}$$
After some manipulations (again performed by *Mathematica*), we can show that
$$\begin{aligned} SW^{fb2} - SW^{sb1} \propto \quad & 4^{1+a} a^a (1+ab)^{2+2a} - (1+a)^{1+a} \left(1 + 2b + a(2+b) - h(a,b)\right)^{1+a} \times \\ & \left(3 + 2a - 2b - ab + h(a,b)\right) \left(-1 - 2a + 2b + 3ab + h(a,b)\right)^a \end{aligned}$$

where a = n_1/n_2 , b = ω_1/ω_2 , and $h(a,b) = \sqrt{-8(1+a)^2 b + (1 + 2b + a(2+b))^2}$. The sign of $SW^{fb2} - SW^{sb1}$ is the same as the sign of the expression on the right-hand side. This is quite a messy expression but because it depends on only two parameters, it is possible to plot it. We do this on figure 5.

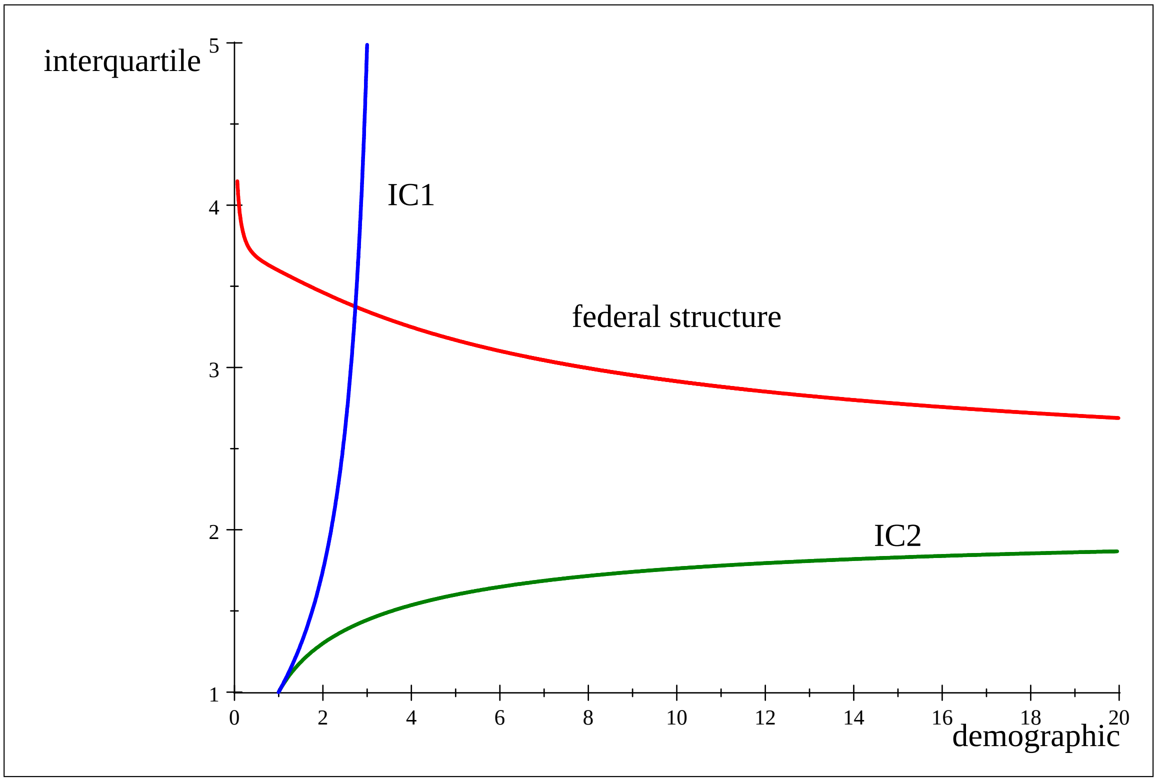

Figure 5

Over the region where both incentive constraints are satisfied, that is, when the allocation $fb2$ is the solution to the second-best problem, we can show that the allocation $sb1$ is preferred to the allocation $fb2$ when income disparities are not too large or when the are relatively more rich households than poor ones. When income disparities are large, there is a strong motive for income redistribution. This cannot be achieved under the one-jurisdiction structure. The two-jurisdiction structure is therefore optimal. When there are relatively more rich households, their weight in social welfare increases. Since they consume more of the public good, social welfare increases. Furthermore, an increase in the number of rich households leads to an increase in the subsidy paid to the poor households, making them better off. Figure 5 also illustrates claim 1 above since the boundary curve on the North East of which the federal solution dominates the centralized one never intercept the boundary of the zone where the incentive constraint of the poor is violated.

We can also use the Pareto criterion to compare the two structures. In Figure 6, we add two curves. The first curve represents the locus for which

$$U(\omega_1 - T^{sb1}, z^{sb1}) \leq U(\omega_1 - T^{fb2}, z^{fb2}).$$

so that rich households prefer the $fb2$ allocation for all parameter values to the right of this curve. The second curve represents

$$U(\omega_2 - T^{sb1}, z^{sb1}) = U(\omega_2 - T^{fb2}, z^{fb2}).$$

so that poor households prefer the $fb2$ allocation for all parameter values to the right of this curve. Of course these two curves intersect on the indifference curve for the symmetric utilitarian social welfare. There is a region where all households prefer the two-jurisdiction structure to the one-jurisdiction structure. Alternatively, there is also a region where the opposite is true.

<TO BE CONTINUED>

4 The optimal jurisdiction structure with three type of individuals

<TO BE PROVIDED>

5 Conclusion

<TO BE PROVIDED>

References

Boadway, R., and F. Flatters (1982): "Efficiency and equalization payments in a federal system of government: A synthesis and extension of recent results," *Canadian Journal of Economics*, 15, 613–633.

Bordignon, M., P. Manasse, and G. Tabellini (2001): "Optimal Redistribution under Asymetric Information," *American Economic Review*, 91, 709–723.

Breuillé, M.-L., and R. J. Gary-Bobo (2007): "Sharing Budgetary Austerity under Free Mobility and Asymmetric Information : an Optimal Regulation Approach to Fiscal Federalism," *Journal of Public Economics*, 91, 1177–1196.

Buchanan, J. M. (1950): "Federalism and Fiscal Equity," *American Economic Review*, 40, 583–599.

Cornes, R. C., and E. C. Silva (2000): "Local Public Goods, Risk Sharing and Private Information in Federal Systems," *Journal of Urban Economics*, 47, 39–60.

——— (2002): "Local Public Goods, Interegional Transfers and Private Information," *European Economic Review*, 46, 329–356.

Deschamps, R., and L. Gevers (1978): "Leximin and Utilitarian Rules: A Joint Characterization," *Journal of Economic Theory*, 17, 143–163.

Flatters, F., V. Henderson, and P. Mieszcowski (1974): "Public goods, efficiency, and regional fiscal equalization," *Journal of Public Economics*, 3, 99–112.

Gravel, N., and M. Poitevin (2006): "The Progressivity of Equalization Payments in Federations," *Journal of Public Economics*, 90, 1725–1743.

Greenberg, J. (1983): "Local Public Goods with Mobility: Existence and Optimality of a General Equilibrium," *Journal of Economic Theory*, 16, 137–150.

Mirlees, J. A. (1971): "An Exploration in the Theory of Optimal Income Taxation," *Review of Economic Studies*, 38, 175–208.

——— (1976): "Optimal Tax Theory: A Synthesis," *Journal of Public Economics*, 6, 327–358.

——— (1986): "The Theory of Optimal Income Taxation," in *Handbook of Mathematical Economics*, ed. by K. J. Arrow, and M. D. Intriligator. North Holland, Amsterdam.

Oates, W. E. (1972): *Fiscal Federalism*. Harcourt Brace Jovanovich, New York.

——— (1999): "An Essay on Fiscal Federalism," *Journal of Economic Literature*, 37, 1120–1149.

Tiebout, C. M. (1956): "A Pure Theory of Local Expenditures," *Journal of Political Economy*, 64, 416–424.

Wildasin, A. (1987): "Theoretical Analysis of Local Public Economics," in *Handbook of Regional and Urban Econonomics*, ed. by E. Mills. North Holland.