

# Does endogenous formation of jurisdictions lead to wealth stratification ?<sup>1</sup>

Nicolas Gravel<sup>2</sup> and Sylvie Thoron<sup>3</sup>

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<sup>2</sup>CSH & IDEP-GREQAM, 2, Aurangzeb Road, 11 0011 Delhi, India  
nicolas.gravel@csh-delhi.com

<sup>3</sup>University of Toulon & GREQAM, Centre de la Vieille Charité, 2, rue de la Charité, 13 002 Marseille Cedex, France, thoron@ehess.univ-mrs.fr

### **Abstract**

Conditions under which endogenous processes of jurisdiction formation entail wealth-stratification are examined in a model where unequally wealthy households with identical preferences form jurisdictions in order to produce a public good financed by proportional taxation. We define a stable jurisdiction structure to be a partition of the households into jurisdictions that is immune to individual deviation. We define a jurisdiction structure to be wealth-stratified when each jurisdiction is composed of households who form an interval with respect to the ordering of their wealth. We show that a necessary and sufficient condition for the stratification of any stable jurisdiction structure is for the individual preferences for the public and the private good to exhibit a relation of gross substitutability/complementarity between the public good and the private good that is independent from prices and wealth.

# 1 Introduction

A widely discussed feature of Tiebout’s [1] seminal work on endogenous jurisdiction formation is the sorting property of the mechanism. If households choose their jurisdiction of residence and affect the local bundle of public goods and taxes by “voting with their feet”, then, so the intuition goes, the resulting equilibrium jurisdiction structure will be characterized by *homogeneous* jurisdictions inhabited by households with “similar” characteristics. The purpose of this note is to investigate the validity of this intuition in plausible models of jurisdiction formations.

While the literature devoted to models of that sort has been abundant in the last three decades (see e.g. [2], [3], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]), it is fair to say that it has been more concerned with the delicate and important question of existence and/or Pareto-efficiency of various notions of equilibria for these models than with the identification of the positive properties possessed by these equilibria (like their possible stratification).

An exception to this is Ellickson [4] who discusses a model in which unequally wealthy households have the same preference for housing, private consumption and a local public good whose production is financed by proportional *housing* taxation. Ellickson [4] defines a stable jurisdiction structure to be a partition of households into jurisdictions that is immune to household’s deviation. The paper *assumes* that such stable jurisdiction structures involve the wealth-stratification of households in the precise sense that if two households with different wealth belong to one jurisdiction, then so do all households with wealth in between that of the two households. Although Ellickson acknowledges that stratification is not a necessary feature of a stable jurisdiction structure and requires further restrictions on households preferences and on the collective choice mechanism used to choose taxes, he does not provide these restrictions.

Another important paper that deals with stratification is Westhoff [11] who proves, in a model with a continuum of households, an arbitrary finite number of jurisdictions and no housing market, the existence of a partition of households into jurisdictions that is *stable* in the same sense as [4]. In Westhoff [11], each jurisdiction chooses a proportional tax rate on the households’ wealth by *majority voting* and uses the taxes collected to produce a public good. Households differ by both their private wealth and their preference for the public and private good. In order to prove existence of a stable jurisdiction structure, Westhoff *imposes* on preferences the property that the *marginal rates of substitution* between tax rate and public good can be completely ordered according to some *a priori* ranking of the households. While Westhoff [11] uses this assumption to prove existence of a stable jurisdiction structure, it also establishes that, under this assumption, any stable jurisdiction structure is stratified, in the same sense as in [4], with respect to the *a priori* given ordering of households (which, in Westhoff [11], needs

not be that of wealth). Again, Westhoff [11] does not identify the properties of the household's preferences for public good and private consumption that give rise to this *a priori* ordering of households in terms of their marginal rates of substitution.

An analogous property of ordering of households preferences is examined by Greenberg and Weber [6] in a cooperative game theoretic setting. Greenberg and Weber [6] are interested in proving the existence of a  $C$ -stable coalition structure (see [7]) defined as a partition of households into coalitions that is immune to coalition deviation. The paper assumes a finite number of individuals and supposes that a coalition can provide its members with any package of public good and taxes that satisfies the coalition's budget constraint. Within this framework, Greenberg and Weber claim that if households preferences are additively separable with respect to the private and public good, then households can be ordered in such a way that if two households  $i$  and  $j$  have the same ranking of two packages of taxes and public good, then so do all households that are ordered between  $i$  and  $j$ . Given this property, Greenberg and Weber show the existence and the stratification of a  $C$ -stable jurisdiction structure.

The object of this note is to clarify the role played by the similar looking conditions of Greenberg and Weber [6] and Westhoff [11] for obtaining wealth-stratified equilibria. The model examined has one private good and one local public good. There is a continuum of households who have possibly different endowments of the private good but the same preferences for the public good and the private good. We focus on wealth heterogeneity because it is usually this type of stratification that is at stake in most discussions of the sorting properties of Tiebout models.<sup>1</sup> It is assumed that local public good provision is financed by proportional wealth taxation and that each jurisdiction selects a tax rate that is favoured by at least one member of the jurisdiction. A well-known particular case of such an intra-jurisdiction rule for choosing the tax rate is *majority voting* where the tax rate that beats any other by a majority of votes is the most preferred one of the *median* household. But our results do not ride on the particular choice of the median as the jurisdiction's "dictator". In this setting, we focus on *stable jurisdiction structures* which are defined, as in Ellickson [4] and Westhoff [11], as partitions of households into jurisdictions that are immune to *individual* deviations.

We provide a condition on households' preferences for public and private good that is necessary and sufficient to ensure the wealth-stratification of any stable jurisdiction structure. The condition requires households to consider public good to be *always* either a gross complement, or a gross substitute to the private good. In particular therefore this condition precludes

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<sup>1</sup> For example, the recent empirical test of the Tiebout model in [14] is casted in terms of income heterogeneity.

the possibility for public good to be a complement to private good at some price configurations, but a substitute for private good at others. As it turns out, this condition amounts at requiring the household’s most preferred tax rate to be a *monotonic* function of its wealth, given the jurisdiction’s wealth. Although not unreasonable (it is for instance satisfied by CES preferences), this condition represents a significant restriction on preferences which, as illustrated in an example, can be violated even by additively separable preferences such as those assumed in Greenberg and Weber [6].

The rest of the note is organized as follows. The next section describes the formal framework. Section 3 provides an example of a standard economy where households have additively separable preferences in which a stable jurisdiction structure is not wealth stratified. Section 4 contains the proof of our main results concerning the necessity and the sufficiency of our condition for the wealth stratification of any stable jurisdiction structure and Section 5 concludes.

## 2 Formal framework

We consider economies with a continuum of households represented by the  $[0, 1]$  interval. An economy consists of four elements. First, there is a Lebesgue measure  $\lambda$  on  $[0, 1]$ . Given a Lebesgue measurable subset  $I$  of  $[0, 1]$ , we interpret  $\lambda(I)$  as “the fraction of households” in  $I$ . The second ingredient is a *wealth distribution* modeled as a Lebesgue measurable bounded from above function  $\omega : [0, 1] \rightarrow \mathbb{R}_{++}$  which associates to each household  $i \in [0, 1]$  its private strictly positive wealth  $\omega_i$ . The third ingredient in the description of an economy is a specification of the households’ preferences. We assume that households have the same preferences for a public good ( $Z$ ) and a private good ( $x$ ) that are represented by a twice differentiable, strictly increasing and strictly concave<sup>2</sup> utility function  $U : \mathbb{R}_+^2 \rightarrow \mathbb{R}$ . Given any bundle of public and private good  $(\bar{Z}, \bar{x}) \in \mathbb{R}_+^2$ , we define  $MRS(\bar{Z}, \bar{x})$ , the *marginal rate of substitution of public good to private good* evaluated at  $(\bar{Z}, \bar{x})$  by:

$$MRS(\bar{Z}, \bar{x}) = \frac{\partial U(\bar{Z}, \bar{x}) / \partial Z}{\partial U(\bar{Z}, \bar{x}) / \partial x} \quad (1)$$

We also denote by  $Z^M(p_Z, p_x)$  and  $x^M(p_Z, p_x)$  the *normalized Marshallian demands* for the public and the private good (respectively) when the prices for these two goods (expressed as fractions of wealth) are  $p_Z$  and  $p_x$ . Normal-

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<sup>2</sup>Our terminology for concavity and quasi-concavity of a function  $f : A \rightarrow \mathbb{R}$  ( $A \subseteq \mathbb{R}^k$ ) is as follows:  $f$  is strictly concave if, for every  $\alpha \in ]0, 1[$  and for every distinct  $a, b \in A$ ,  $f(\alpha a + (1 - \alpha)b) > \alpha f(a) + (1 - \alpha)f(b)$  and  $f$  is quasi-concave if, for every  $a, b, x \in A$ , and every  $\alpha \in [0, 1]$ ,  $f(a) \geq f(x)$  and  $f(b) \geq f(x)$  imply  $f(\alpha a + (1 - \alpha)b) \geq f(x)$ .

ized Marshallian demand functions are the (unique) solution of the program

$$\max_{Z,x} U(Z,x) \text{ subject to } p_Z Z + p_x x \leq 1$$

Given the assumptions on  $U$ , normalized Marshallian demands are differentiable functions of their arguments (except, possibly, at the boundary of  $\mathbb{R}_+^2$ ).

We further assume that the normalized Marshallian demand for public good satisfies the following additional *regularity* condition.

**Condition 1** ( $Z^M(\bar{p}_Z, p_x) = Z^M(\bar{p}_Z, p'_x)$  for some  $\bar{p}_Z \in \mathbb{R}_{++}$  and all  $p'_x, p_x \in I$  for some non-degenerate interval  $I$  of  $\mathbb{R}_{++}$ )  $\Leftrightarrow Z^M(p_Z, p_x) = h(p_Z)$  for all  $(p_Z, p_x) \in \mathbb{R}_{++}^2$  for some function  $h : \mathbb{R}_{++} \rightarrow \mathbb{R}_+$ .

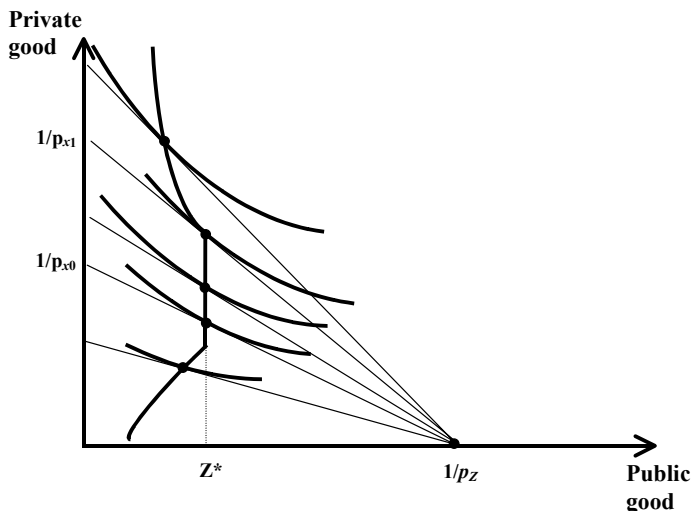


figure 1

Condition 1 rules out cases, such as that depicted on figure 1, where Marshallian demand for public good is, at some price  $p_Z$ , independent from the price of private good on some non-degenerate interval  $[p_{x_0}, p_{x_1}]$  but yet depends on the price of private good outside that interval or at other price than  $p_Z$ . Condition 1 is, however, perfectly compatible with preferences such as Cobb-Douglas that do not exhibit cross-price effects. We let  $\mathbb{U}$  denote the set of all utility functions satisfying the above properties.

Finally, the fourth element of our description of an economy is a *common finite* set  $\mathbb{L}$  of locations (whose typical elements are denoted by  $l, l', l''$ , etc.) available to households.

A *jurisdiction structure* for the economy  $(\lambda, \omega, U, \mathbb{L})$  is a Lebesgue measurable function  $S_J : [0, 1] \rightarrow \mathbb{L}$ . A jurisdiction structure  $S_J$  induces a finite partition of the set  $[0, 1]$  into Lebesgue measurable sets  $J_l$  defined by  $J_l = S_J^{-1} = \{i \in [0, 1] : S_J(i) = l\}$ . Set  $J_l$  contains all households who live at location  $l$  in the jurisdiction structure  $S_J$ . In the same vein, we let, for every  $l \in \mathbb{L}$ ,  $n_l^J = \lambda(J_l)$  denote the “fraction of households living at  $l$ ” in the jurisdiction structure  $S_J$ . The possibility that  $n_l^J = 0$  for some  $l$  is, of course, not ruled out. Since, for every jurisdiction structure  $S_J$ , the sets  $J_l$  partition  $[0, 1]$ , we clearly have that  $\sum_{l \in \mathbb{L}} n_l^J = \lambda([0, 1])$ . Given a jurisdiction structure  $S_J$  and a location  $l$ , we denote by  $\omega_l^J$  the *jurisdiction  $l$ 's wealth* defined by  $\omega_l^J = \int_{J_l} \omega_i d\lambda$ .<sup>3</sup> Clearly  $\omega_l^J = \int_{J_l} \omega_i d\lambda = 0$  if  $n_l^J = 0$ . Hence a jurisdiction containing a set of measure 0 of households would have a jurisdiction's wealth of 0 and would therefore not be able to provide any positive amount of public good to its inhabitants. We adopt the convention that if such a jurisdiction is to choose a tax by a collective choice mechanism to finance public good production, the only choice for such a tax will be 0 (why tax if no public good is to be provided?).

Each jurisdiction selects a tax rate  $t \in [0, 1]$  that is applied to the jurisdiction's wealth and which finances local public good production. Hence, if a jurisdiction structure  $S_J$  assigns household  $i$  to location  $l$  where the tax rate is  $t$ , household  $i$  has access to  $t\omega_l^J$  units of public good and has  $(1 - t)\omega_i$  units of wealth available for private consumption. Denote by  $\Phi(t, \omega_l^J, \omega_i) = U(t\omega_l^J, (1 - t)\omega_i)$  the utility received by a household of wealth  $\omega_i$  assigned to location  $l$  in the jurisdiction structure  $S_J$  if the tax rate is  $t$ .

We record a few properties of the function  $\Phi : [0, 1] \times \mathbb{R}_+^2 \rightarrow \mathbb{R}$  so defined in the following lemma whose (straightforward) proof is omitted.

**Lemma 1** *If  $U$  belongs to  $\mathbb{U}$ ,  $\Phi$  is a twice differentiable function of its three arguments, is strictly increasing and concave with respect to  $\omega_i$  and  $\omega_l^J$  (taking  $t$  as fixed) and is strictly concave and single peaked<sup>4</sup> with respect to  $t$  (taking  $\omega_i$  and  $\omega_l^J \in \mathbb{R}_+$  as fixed).*

An important (and well-known) property of  $\Phi$  is its strict single peakedness. It implies that a household with wealth  $\omega_i$  has a unique favorite tax rate  $t^*(\bar{\omega}, \omega_i)$  in any jurisdiction with wealth  $\bar{\omega}$  to which it may belong. This unique favorite tax rate is the solution of the program:

$$\max_{t \in [0, 1]} \Phi(t, \bar{\omega}, \omega_i) \tag{2}$$

and is, for this reason, a continuous function of  $\bar{\omega}$  and  $\omega_i$ .

<sup>3</sup>This Lebesgue integral is well-defined if  $\omega$  is a bounded and measurable function.

<sup>4</sup>A function  $f : A \rightarrow \mathbb{R}$  ( $A \subset \mathbb{R}$ ) is strictly single peaked if, for all  $a, b$  and  $c \in A$  such that  $a < b < c$ ,  $f(c) > f(b) \Rightarrow f(b) > f(a)$  and  $f(a) > f(b) \Rightarrow f(b) > f(c)$ .

We assume that each jurisdiction’s choice of tax rate is minimally democratic in the sense that it selects the favorite tax rate of *some* member of the jurisdiction. The rule for selecting this member is inconsequential for the results. In many models of endogenous jurisdiction formation with public good provision where voting is assumed, the selected household is that whose favorite tax rate occupies the *median position* in the jurisdiction distribution of favorite taxes. While our results apply to this particular rule of selection of the “dictator” within the jurisdiction, they are valid for other selection rules as well. Denote by  $t_l^J$  the tax rate prevailing in jurisdiction  $l$  in jurisdiction structure  $S_J$ . For every such jurisdiction structure, we therefore assume that, at every location  $l$ ,  $t_l^J = t^*(\omega_l^J, \omega_h)$  for some household  $h \in J_l$ .

We are now equipped to define what is meant by a *stable jurisdiction structure*.

**Definition 1** *A jurisdiction structure  $S_J : [0, 1] \rightarrow \mathbb{L}$  is stable in the economy  $(\lambda, \omega, U, \mathbb{L})$  if for all  $l \in \mathbb{L}$  and all  $i \in J_l$ ,  $\Phi(t_l^J, \omega_l^J, \omega_i) \geq \Phi(t_{l'}^J, \omega_{l'}^J, \omega_i)$  for every  $l' \in \mathbb{L}$ .*

A jurisdiction structure is stable if it gives no single household an incentive to move from its location. This notion of stability corresponds to what is usually referred in the literature on coalitions formation (see e.g. [3]) as a *free mobility equilibrium*. It can be contrasted with the strongest notion that is also encountered in the literature (for example in [6]) and which concerns stability with respect to *group* deviations.

Notice that, since a single household has measure 0, one household’s move to some location affects neither the jurisdiction’s wealth nor tax rate. This allowed “infinitesimality” of the household with respect to jurisdiction’s characteristics is, actually, our main justification for using a continuous framework rather than a discrete one.

As we seek to provide a necessary and sufficient condition on households preferences which ensures that any stable jurisdiction structure is *wealth-stratified*, we now define what is meant by that.

**Definition 2** *A jurisdiction structure  $S_J$  in the economy  $(\lambda, \omega, U, \mathbb{L})$  is wealth-stratified if, for every  $l, l' \in \mathbb{L}$ , and  $i, j, k \in [0, 1]$ , ( $i, k \in J_l, \omega_i < \omega_j < \omega_k$  and  $j \in l'$ )  $\Rightarrow (t_l^J, \omega_l^J) = (t_{l'}^J, \omega_{l'}^J)$*

In words, a jurisdiction structure is wealth-stratified if, whenever a jurisdiction contains two households  $i$  and  $k$  with different levels of wealth, it also contains all households whose wealth are strictly in between that of  $i$  and  $k$  or, if it does not contain those households, it is because they are at location  $l'$  that is indistinguishable from  $l$  in terms of tax rate and jurisdiction wealth. Except for this later proviso (unavoidable in the continuous setting where stable jurisdiction structures may involve the formation

of many jurisdictions with the same tax rate and jurisdiction wealth), this property of *stratification* is called “*consecutiveness*” in Greenberg and Weber [6] where it is given the more general meaning of designating a structure where each jurisdiction (coalition) form an *interval* with respect to an *a priori* ordering of the players which needs not coincide with that of wealths. Yet when Greenberg and Weber interpret this notion of consecutiveness in a setting analogous to ours (with coalitions asking their members endowed with identical *additively separable* preferences to finance a public good by a proportional wealth taxation), they claim that the *a priori* ordering of the players could be that wealths. However, as we shall see in Remark 1, this claim is false.

### 3 An example of a stable jurisdiction structure without stratification

Although the intuition that stable jurisdiction structures involve wealth-stratification seems plausible, its validity requires that households’ preferences satisfy a significant condition. To illustrate this, we first provide a simple example of an economy in which a stable jurisdiction structure is *not* wealth stratified.

**Example 1** *There is a continuum of mass 14 households divided into three groups defined by some strictly positive real number  $\varepsilon$ . A first group consists of a continuum of mass 3 households with wealth uniformly distributed on the interval  $[2 + \sqrt{2} - \varepsilon, 2 + \sqrt{2} + \varepsilon]$ . The second group contains a continuum of mass 3 households with wealth levels uniformly distributed on the interval  $[2 - \sqrt{2} - \varepsilon, 2 - \sqrt{2} + \varepsilon]$  while the third group is made of a continuum of mass 8 households with wealth uniformly distributed on the interval  $[\frac{3}{2} - \varepsilon, \frac{3}{2} + \varepsilon]$ . Households’ preferences are represented by the utility function  $U(Z, x) = \ln Z + 4x - x^2$  which satisfies all of the above properties (it is in particular strictly increasing with respect to the private good if  $x \leq 2$ , which will be the case here).<sup>5</sup> The favorite tax rate  $t^*$  of a household with wealth  $\omega_i$  living at location where the jurisdiction wealth is  $\bar{\omega}$  solves:*

$$\max_t \ln t\bar{\omega} + 4(1-t)\omega_i - (1-t)^2\omega_i^2$$

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<sup>5</sup>Although this preference is not increasingly monotonic with respect to private good if  $x > 2$ , the example would work just as well for a preference represented by the increasing (but not twice differentiable) utility function:

$$\begin{aligned} U(Z, x) &= \ln Z + 4x - x^2 \text{ if } x \in [0, 2] \\ &= \ln Z \text{ otherwise} \end{aligned}$$

and is therefore given by:

$$t^* = \frac{\omega_i - 2 + \sqrt{(\omega_i - 2)^2 + 2}}{2\omega_i}$$

Hence,  $t^* = \frac{1}{2}$  if  $\omega_i = 2 - \sqrt{2}$  or  $\omega_i = 2 + \sqrt{2}$  and  $t^* = \frac{1}{3}$  if  $\omega_i = \frac{3}{2}$ . We note that, in this example,  $t^*$  is independent from  $\bar{\omega}$ . The graph of  $t^*$  with respect to  $\omega_i$  is depicted on figure 2. Consider now the jurisdiction structure  $S_J$  defined, for some locations  $l$  and  $l' \in \mathbb{L}$  ( $l \neq l'$ ), by:

$$\begin{aligned} S_J(i) &= l \text{ if } \omega_i \in [2 - \sqrt{2} - \varepsilon, 2 - \sqrt{2} + \varepsilon] \cup [2 + \sqrt{2} - \varepsilon, 2 + \sqrt{2} + \varepsilon] \text{ and} \\ S_J(i) &= l' \text{ if } \omega_i \in [\frac{3}{2} - \varepsilon, \frac{3}{2} + \varepsilon] \end{aligned}$$

This jurisdiction structure is clearly not wealth-stratified since

$$2 - \sqrt{2} + \varepsilon < \varepsilon - \frac{3}{2} < \frac{3}{2} + \varepsilon < 2 + \sqrt{2} - \varepsilon$$

for a suitable choice of  $\varepsilon$ . Yet it is easy to see that it is stable if  $t_l^J = \frac{1}{3}$  and  $t_{l'}^J = \frac{1}{2}$ . A household located at  $l'$  and endowed with a wealth of  $\frac{3}{2}$  enjoys a utility level of:

$$3 + \ln(4) \simeq 4.3863$$

while a move to  $l$  (where the tax is  $\frac{1}{2}$ ) would provide this household with a utility of :

$$3 - 9/16 + \ln(6) \simeq 4.2293$$

As the inequality  $3 + \ln(4) > 3 - 9/16 + \ln(6)$  is strict, it is robust to the replacement of the household of wealth  $\frac{3}{2}$  by any household in the interval  $[\frac{3}{2} - \varepsilon, \frac{3}{2} + \varepsilon]$  for a suitably small  $\varepsilon$ . Hence no household located at  $l'$  has incentive to move to  $l$ . Analogous calculations show that households with wealth  $2 + \sqrt{2}$  and  $2 - \sqrt{2}$  located at  $l$  have a strict preference for staying at  $l$  rather than moving to  $l'$  and that this preference is robust to an  $\varepsilon$  change if  $\varepsilon$  is small enough.

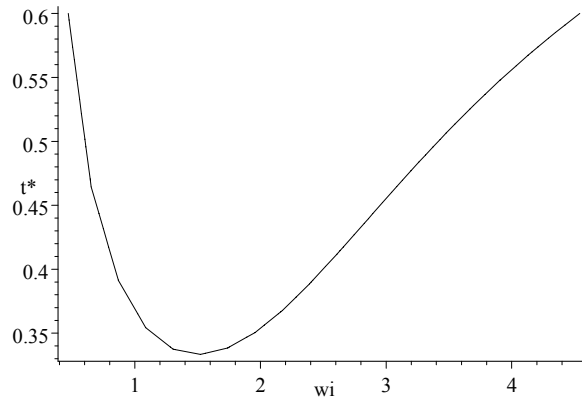


figure 2

The preferences used in the example are quite standard. They are additively separable, strictly concave, differentiable and so on. What is therefore the property of these preferences that enables the construction of this example? This property, exhibited on figure 2, is the *non-monotonicity* of the relationship between the household's favorite tax rate and its private wealth. If households have preferences of the kind described in the example, their most preferred tax rates are first decreasing with respect to their wealth and, for wealth level above  $3/2$ , increasing with wealth. If households most preferred tax rates are *not* monotonic with respect to their wealth in a jurisdiction, the ordering of households according to their most preferred tax rates does *not* coincide with the ranking of individuals according to their wealth. As it turns out, it is precisely this property that must *necessarily* hold for wealth stratification to be a characteristic of any stable jurisdiction structure.

We define as follows this property of strict monotonicity of  $t^*$  with respect to  $\omega_i$ .

**Definition 3** *The function  $t^* : \mathbb{R}_{++}^2 \rightarrow [0, 1]$  that solves (2) is monotonic with respect to  $\omega_i$  if, for any jurisdiction wealth level  $\bar{\omega}$ , and every two household wealth levels  $\hat{\omega}_i$  and  $\tilde{\omega}_i$ , either  $\hat{\omega}_i \geq \tilde{\omega}_i \Rightarrow t^*(\hat{\omega}_i, \bar{\omega}) \geq t^*(\tilde{\omega}_i, \bar{\omega})$  or  $\hat{\omega}_i \geq \tilde{\omega}_i \Rightarrow t^*(\hat{\omega}_i, \bar{\omega}) \leq t^*(\tilde{\omega}_i, \bar{\omega})$ .*

As example 1 makes clear, the non-monotonicity of the household's most preferred tax rates with respect to wealth is perfectly compatible with additively separable preferences. This gives us the opportunity to make the following remark.

**Remark 1** *Greenberg and Weber [6] claim (see part b in the proof of lemma 1) that if households' preferences can be represented by an additively separable utility function, then, for all quantities  $\bar{Z}$  and  $\hat{Z}$  of public good, all pairs of jurisdiction's wealth  $\bar{\omega}$  and  $\hat{\omega}$  and all triplets of individuals  $i, j$  and  $k$  such that  $\omega_i < \omega_j < \omega_k < \min[\bar{\omega}, \hat{\omega}]$ , if*

$$f(\bar{Z}) + g(\omega_i(1 - \frac{\bar{Z}}{\bar{\omega} + \omega_i})) < f(\hat{Z}) + g(\omega_i(1 - \frac{\hat{Z}}{\hat{\omega}})) \quad (3)$$

and

$$f(\bar{Z}) + g(\omega_k(1 - \frac{\bar{Z}}{\bar{\omega} + \omega_k})) < f(\hat{Z}) + g(\omega_k(1 - \frac{\hat{Z}}{\hat{\omega}})) \quad (4)$$

then

$$f(\bar{Z}) + g(\omega_j(1 - \frac{\bar{Z}}{\bar{\omega}})) < f(\hat{Z}) + g(\omega_j(1 - \frac{\hat{Z}}{\hat{\omega} + \omega_j})) \quad (5)$$

To show the falsity of such a claim, take households  $i, j$  and  $k$  to be just as in example 1 above in every respect (preferences, private wealths  $\omega_i = 2 - \sqrt{2}$

$< \omega_j = \frac{3}{2} < \omega_k = 2 + \sqrt{2}$  and  $\bar{\omega} = \hat{\omega} = 120000$ ) and let  $\hat{Z} = \frac{1}{2} \times 120\,000 = 60\,000$  and  $\bar{Z} = \frac{1}{3} \times 120\,000 = 40\,000$ . Assume also that the utility functions are just as in example 1 with . It can be checked easily that, for these values of  $\omega_i, \omega_j, \omega_k, \bar{\omega}, \hat{\omega}, \hat{Z}$  and  $\bar{Z}$  (and taking  $f(Z) = \ln Z$  and  $g(x) = 4x - x^2$ ), both (3) and (4) are satisfied while (5) is violated.

## 4 Main results

As mentioned, the monotonicity of  $t^*$  with respect to household wealth is the key element for guaranteeing the wealth stratification of stable jurisdiction structures. This monotonicity of the household's most preferred tax rate as per definition 3 can be expressed conveniently in terms of standard consumer theory. In order to do this, we first establish the following lemma.

**Lemma 2** *Let  $(\bar{\omega}, \omega_i) \in \mathbb{R}_{++}^2$ . Then for all  $U \in \mathbb{U}$ ,  $\frac{Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\bar{\omega}}$  is the solution of (2).*

**Proof.** Since  $\frac{Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\bar{\omega}} \geq 0$  and  $\frac{x^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\omega_i} \geq 0$ , the fact that  $Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})$  satisfies the budget constraint  $\frac{Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\bar{\omega}} + \frac{x^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\omega_i} = 1$  implies obviously that  $\frac{Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\bar{\omega}} \in [0, 1]$ . Suppose by contradiction that  $\frac{Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})}{\bar{\omega}}$  does not solve (2). That is, suppose that there exists  $\hat{t} \in [0, 1]$  such that  $U(\hat{t}\bar{\omega}, (1-\hat{t})\omega_i) > U(Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i}), x^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i}))$ . Since the bundle  $(\hat{t}\bar{\omega}, (1-\hat{t})\omega_i)$  satisfies the budget constraint  $\frac{\hat{t}\bar{\omega}}{\bar{\omega}} + \frac{(1-\hat{t})\omega_i}{\omega_i} = 1$ , this inequality is incompatible with the very definition of  $Z^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})$  and  $x^M(\frac{1}{\bar{\omega}}, \frac{1}{\omega_i})$ . ■

Lemma 2 states that, in a jurisdiction of wealth  $\bar{\omega}$ , the favorite tax rate of a household with wealth  $\omega_i$  can be viewed as the *expenditure* (using the fraction of the household's purchasing power as the *numéraire*) that the household would like to devote to public good if its price was  $\frac{1}{\bar{\omega}}$  and the price of private good was  $\frac{1}{\omega_i}$ . Interpreted in this light, the property of monotonicity of  $t^*$  with respect to  $\omega_i$  is *equivalent* to the monotonicity of normalized Marshallian demand for public good with respect to the price of the private good. In the language of standard consumer theory, this is equivalent to requiring the public good to be, at any price of public good, either always a gross *complement* to (if  $Z^M$  is monotonically *decreasing* with respect to  $p_x$ ) or always a gross *substitute* for (if  $Z^M$  is monotonically *increasing* with respect to  $p_x$ ) the private good.

Without further assumption on the household's preference, it is possible for the public good to be always a gross substitute of the private good at some price of the public good while being always a gross complement to the private good at some other price of the public good. As it turns out

however, this possibility is ruled out if condition 1 is imposed on the utility function. Specifically, the following lemma establishes that if Marshallian demand for public good is a monotonic function of the price of the private good at every price of public good, then the public good is either *always* a gross substitute for or *always* a gross complement to the private good, as it may be. The straightforward proof of this lemma is omitted.

**Lemma 3** *Assume that Marshallian demand  $Z^M$  is a monotonic function of  $p_x$  at every public good price  $p_Z \in \mathbb{R}_{++}$  and that households' utility function belongs to  $\mathbb{U}$ . Then, for every  $p_Z, p'_Z, p_x, p'_x \in \mathbb{R}_{++}$ ,  $[Z^M(p_Z, p_x) > Z^M(p_Z, p'_x)] \Leftrightarrow [Z^M(p'_Z, p_x) > Z^M(p'_Z, p'_x)]$*

Combining lemmas 2 and 3, one obtains immediately.

**Lemma 4** *For every  $U \in \mathbb{U}$ , the function  $t^*$  that solves (2) is monotonic with respect to  $\omega_i$  for any given jurisdiction level  $\bar{\omega}$  as per definition 3 if and only if the public good is always either a gross complement to, or a gross substitute for, the private good.*

Let us refer to this property according to which the substitutability/complementarity relationship between the public and private good is independent from all possible prices as to the **Gross Substitutability/ Complementarity (GSC) condition**. Although not unreasonable (it is for instance satisfied by commonly used preferences such as CES<sup>6</sup>), the GSC condition is nonetheless a significant restriction that, as illustrated in remark 1, can be violated even by additively separable preferences.

We now proceed to establish that the GSC condition is necessary and sufficient for guaranteeing the wealth-stratification of any stable jurisdiction structure.

Establishing the sufficiency part of this result requires knowledge of the structure of the households' indifference curves in the tax-jurisdiction's wealth space as obtained from the function  $\Phi$  defined previously. As usual, the indifference curve of a household of wealth  $\omega_i$  passing through some point  $(\bar{t}, \bar{\omega})$  such that  $\Phi(\bar{t}, \bar{\omega}, \omega_i) = \bar{\Phi}$  is the graph of the implicit function  $f^{\bar{\Phi}} : [0, 1] \times \mathbb{R}_{++} \rightarrow \mathbb{R}_+$  defined by  $\Phi(t, f^{\bar{\Phi}}(t, \omega_i), \omega_i) \equiv \bar{\Phi}$ . The assumption imposed on  $U$  guarantees that the function  $f^{\bar{\Phi}}$  exists, is derivable everywhere and has partial derivative  $f_t^{\bar{\Phi}}(\bar{t}, \bar{\omega}_i)$  with respect to  $t$  evaluated at

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<sup>6</sup>As CES preferences are represented by the utility function  $U(Z, x) = [\beta Z^\rho + (1-\beta)x^\rho]$  for some  $\beta \in ]0, 1[$  and  $\rho \in -\infty, 1[\setminus\{0\}$  (with  $\lim_{\rho \rightarrow 0} [\beta Z^\rho + (1-\beta)x^\rho] = \beta \ln Z + (1-\beta) \ln x$  for every  $(Z, x) \in \mathbb{R}_+^2$ ), public good is a gross substitute to private good if  $\rho \in ]0, 1[$ , a gross complement to private good if  $\rho \in -\infty, 0[$  and is independent from the price of private good if  $\rho = 0$ .

$(\bar{t}, \bar{\omega}_i)$  given by:

$$f_t^{\bar{\Phi}}(\bar{t}, \bar{\omega}_i) = \frac{1}{\bar{t}} \left[ \frac{\bar{\omega}_i}{MRS(\bar{t}\bar{\omega}, (1-\bar{t})\bar{\omega}_i)} - \bar{\omega} \right] \quad (6)$$

where  $\bar{\omega} = f^{\bar{\Phi}}(\bar{t}, \bar{\omega}_i)$ . Using (6), one can represent the indifference maps of households with different levels of wealth as in figure 4 in the proof of theorem 1. Specifically, indifference curves of a household with private wealth  $\omega_i$  are  $U$ -shaped and reach a minimum at this household's most preferred tax rate for the corresponding jurisdiction wealth level. It can be seen that, at the minimum of an indifference curve, the term within the bracket of (6) is zero thanks to the first order conditions of (2)). Despite what figure 4 suggests, indifference curves need *not* be globally convex. The only property that indifference curves possess is that of being "single caved" (monotonically decreasing at the left of the minimum and monotonically increasing at the right).

We first establish in the following lemma that the ordering of the *slopes* of these indifference curves at every point in the tax- jurisdictions wealth space coincides with the ordering of the households' wealth if preferences for the public and the private good satisfy the GSC condition.

**Lemma 5** *Assume that households preferences are represented by a utility function in  $\mathbb{U}$ . Then, if  $Z^M$  is everywhere a gross substitute (resp. everywhere a gross complement) to the private good, we have, at any  $(\bar{t}, \bar{\omega}) \in ]0, 1[ \times \mathbb{R}_{++}$ ,  $f_t^{\bar{\Phi}^j}(\bar{t}, \omega_j) \leq$  (resp.  $\geq$ )  $f_t^{\bar{\Phi}^k}(\bar{t}, \omega_k)$  for every  $j, k$  such that  $\omega_j < \omega_k$  where, for every  $i$ ,  $\bar{\Phi}_i = \Phi(\bar{t}, \bar{\omega}, \omega_i)$ .*

**Proof.** *We provide the argument for the case of gross substitutability (the complementarity case being symmetric). Assume therefore that  $Z^M$  is everywhere increasing with respect to  $p_x$  and let  $(\bar{t}, \bar{\omega}) \in ]0, 1[ \times \mathbb{R}_{++}$  be a pair of tax rate and jurisdiction wealth and  $j$  and  $k$  be two households such that  $\omega_j < \omega_k$ . Define  $\bar{\omega}(j)$  and  $\omega_i(j)$  to be the numbers (depicted on figure 3) that generate public good and private good normalized prices  $1/\bar{\omega}(j)$  and  $1/\omega_i(j)$  which would lead a consumer with a wealth of 1 to choose the bundle of public and private good  $(\bar{t}\bar{\omega}, (1-\bar{t})\omega_j)$ . Hence  $\bar{\omega}(j)$  and  $\omega_i(j)$  satisfy the standard tangency and budget equality conditions:*

$$MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_j) = \frac{\omega_i(j)}{\bar{\omega}(j)}$$

and

$$\frac{\bar{t}\bar{\omega}}{\bar{\omega}(j)} + \frac{(1-\bar{t})\omega_j}{\omega_i(j)} = 1$$

Combining these two equations yields:

$$\frac{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_j)}{\omega_j} = \frac{(1-\bar{t})}{\bar{\omega}(j) - \bar{t}\bar{\omega}} \quad (7)$$

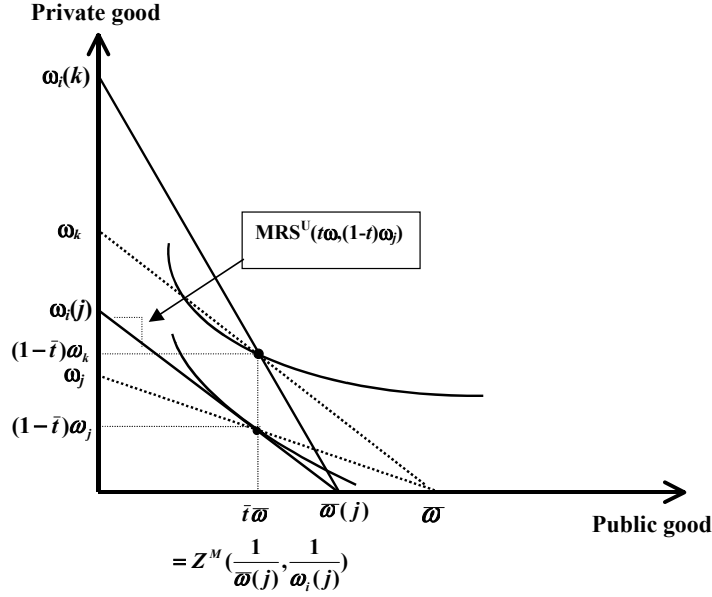


figure 3

Define now  $\omega_i(k)$  to be a level of household wealth which would generate the private good price  $1/\omega_i(k)$  that is just sufficient to enable a consumer with wealth 1 and facing public good price  $1/\bar{\omega}(j)$  to afford the bundle  $(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k)$ . This  $\omega_i(k)$  (which is clearly larger than  $\omega_i(j)$  if  $\omega_j < \omega_k$ ) is defined by the budget constraint equality:

$$\begin{aligned} \frac{\bar{t}\bar{\omega}}{\bar{\omega}(j)} + \frac{(1-\bar{t})\omega_k}{\omega_i(k)} &= 1 \\ &\iff \\ \omega_i(k) &= \frac{\bar{\omega}(j)(1-\bar{t})\omega_k}{\bar{\omega}(j) - \bar{t}\bar{\omega}} \end{aligned} \quad (8)$$

Now, since  $Z^M$  is increasing with respect to  $p_x$ , we must have  $Z^M(\frac{1}{\bar{\omega}(j)}, \frac{1}{\omega_i(j)}) \geq Z^M(\frac{1}{\bar{\omega}(j)}, \frac{1}{\omega_i(k)})$  and, therefore, the slope of the indifference curve passing through the bundle  $(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k)$  must be, in absolute value, less than the price ratio  $\omega_i(k)/\bar{\omega}(j)$ . Formally, this amounts to say that:

$$MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k) \leq \frac{\omega_i(k)}{\bar{\omega}(j)}$$

or, using (8):

$$\frac{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k)}{\omega_k} \leq \frac{(1-\bar{t})}{\bar{\omega}(j) - \bar{t}\bar{\omega}} \quad (9)$$

Combining inequality (9) and equality (7), we get:

$$\begin{aligned} \frac{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k)}{\omega_k} &\leq \frac{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_j)}{\omega_j} \\ &\iff \\ \frac{\omega_k}{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_k)} &\geq \frac{\omega_j}{MRS^U(\bar{t}\bar{\omega}, (1-\bar{t})\omega_j)} \end{aligned}$$

which, using the definition of  $f_t^{\bar{\Phi}_j}$  provided by (6), establishes the result. ■

This lemma, which implies that indifference curves of households with different wealth are *single crossing* in the tax rates-jurisdiction wealth space, completes the list of auxiliary results that are needed for the following theorem.

**Theorem 1** *A stable jurisdiction structure of an economy  $(\lambda, \omega, U, \mathbb{L})$  where  $U$  belongs to  $\mathbb{U}$ , is well stratified for every measure of households  $\lambda$ , distributions of wealth  $\omega$  and finite set of location  $\mathbb{L}$  if and only if  $U$  represents a preference that satisfies the GSC condition.*

**Proof. Necessity.** Using lemma 4, assume by contradiction that  $t^*$  is not monotonic with respect to  $\omega_i$ . This implies that, for some jurisdiction wealth level  $\bar{\omega}$ , there exists individual wealth levels  $a, b, c \in \mathbb{R}_{++}$  such that  $a < b < c$  and such that either

$$(i) \ t^*(\bar{\omega}, a) = t^*(\bar{\omega}, c) > t^*(\bar{\omega}, b)$$

or

$$(ii) \ t^*(\bar{\omega}, a) = t^*(\bar{\omega}, c) < t^*(\bar{\omega}, b)$$

Assume case (i). Consider an economy where  $\lambda(\{i \in [0, 1] : \omega_i = a\}) = A$ ,  $\lambda(\{i \in [0, 1] : \omega_i = c\}) = C$  and  $\lambda(\{i \in [0, 1] : \omega_i = b\}) = B$  and where  $\lambda([0, 1]) = A + C + B$  where  $A, B$  and  $C$  are such that  $aA + cC = bB = \bar{\omega}$ . Consider then the jurisdiction structure  $S_J$  defined by  $S_J(i) = l$  if  $\omega_i = a$  or  $\omega_i = c$  and  $S_J(i) = l'$  if  $\omega_i = b$ . As the two jurisdictions with a non-zero measure of households have the same wealth  $\bar{\omega}$ , jurisdiction  $l$  obviously chooses  $t^*(\bar{\omega}, a) = t^*(\bar{\omega}, c)$  as its tax rate while the choice of jurisdiction  $l'$  is  $t^*(\bar{\omega}, b)$ . By the very definition of  $t^*$  and the uniqueness of the solution of the program (2), this jurisdiction structure is stable. The proof for the case (ii) is similar.

**Sufficiency.** We sketch the argument for the case where the public good is everywhere a gross complement to the private good. Assume  $Z^M$  to be decreasing with respect to  $p_x$  and, by contradiction, let  $S^J : [0, 1] \rightarrow \mathbb{L}$  be a jurisdiction structure for the economy  $(\lambda, \omega, U, \mathbb{L})$  that is not wealth-stratified and is, therefore, such that, for some locations  $l$  and  $l' \in \mathbb{L}$  (with  $l \neq l'$ ), and households  $i, j$  and  $k \in [0, 1]$  with  $\omega_i < \omega_j < \omega_k$ ,  $l = S_J(i) = S_J(k)$ ,

$S_J(j) = l'$  and  $(t_l^J, \omega_l^J) \neq (t_{l'}^J, \omega_{l'}^J)$ . Denote by  $(t, \omega)$  and  $(t', \omega')$  the tax-jurisdiction wealth packages in jurisdiction  $l$  and  $l'$  respectively. We need to show that such a jurisdiction structure is not stable. Instability is clear if  $t = t'$  as, in that case, there is unanimous strict preference for going to the jurisdiction with the largest wealth. If  $t \neq t'$ , stability of  $S^J$  requires:

$$\bar{\Phi}_i = \Phi(\omega_i, t, \omega) \geq \Phi(\omega_i, t', \omega')$$

$$\bar{\Phi}_j = \Phi(\omega_j, t, \omega) \leq \Phi(\omega_j, t', \omega')$$

$$\bar{\Phi}_k = \Phi(\omega_k, t, \omega) \geq \Phi(\omega_k, t', \omega')$$

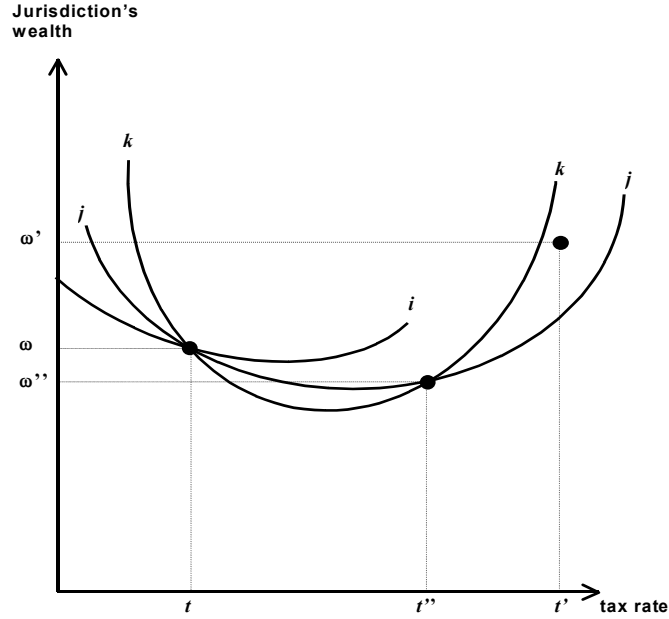


figure 4

Since  $t^*$  is increasing with respect to private wealth, we have that  $t^*(\omega_i, \omega) < t^*(\omega_j, \omega) < t^*(\omega_k, \omega)$ . Assume that  $t \in [t^*(\omega_i, \omega), t^*(\omega_j, \omega)]$  (we leave the case where  $t \in [t^*(\omega_j, \omega), t^*(\omega_k, \omega)[$  to the reader). Assume first that  $t' > t$ . By lemma 5, the slopes of households indifference curves are ordered in such a way that  $f_t^{\bar{\Phi}_k}(t, \omega_k) < f_t^{\bar{\Phi}_j}(t, \omega_j) < f_t^{\bar{\Phi}_i}(t, \omega_i)$  and are as on figure 4. Notice that, for any tax rate and jurisdiction wealth  $(\bar{t}, \bar{\omega})$  such that  $\bar{t} > t$  in a neighborhood of  $(t, \omega)$ , one has:

$$\begin{aligned} \Phi(\omega_j, t, \omega) &< \Phi(\omega_j, \bar{t}, \bar{\omega}) \Leftrightarrow \bar{\omega} > f_t^{\bar{\Phi}_j}(t, \omega_j) > f_t^{\bar{\Phi}_k}(t, \omega_k) \\ &\Rightarrow \Phi(\omega_k, t, \omega) < \Phi(\omega_k, \bar{t}, \bar{\omega}) \end{aligned}$$

Hence, for household  $k$  to weakly prefer staying at  $l$  rather than moving to  $l'$ , there must be a package of tax rate and aggregate wealth  $(t'', \omega'')$  (with  $t'' \in [t, t']$ ) such that  $\Phi(\omega_k, t, \omega) = \Phi(\omega_k, t'', \omega'')$  and  $\Phi(\omega_j, t, \omega) = \Phi(\omega_j, t'', \omega'')$ .

Without loss of generality, let  $(t'', \omega'')$  be such that  $t'' \leq \tilde{t}$  for every tax rate  $\tilde{t} \in [t, t']$  such that  $\Phi(\omega_j, \tilde{t}, \tilde{\omega}) = \Phi(\omega_j, t, \omega)$  and  $\Phi(\omega_k, \tilde{t}, \tilde{\omega}) = \Phi(\omega_k, t, \omega)$  for some  $\tilde{\omega} \in \mathbb{R}_{++}$ . Since  $f_t^{\bar{\Phi}^k}(t, \omega_k) < f_t^{\bar{\Phi}^j}(t, \omega_j)$  we must have, by definition of  $t''$ , that

$$f_t^{\bar{\Phi}^k}(t'', \omega_k) > f_t^{\bar{\Phi}^j}(t'', \omega_j) \quad (10)$$

where  $\bar{\Phi}_h'' = \Phi(\omega_h, t'', \omega'')$  for  $h = j, k$ . Yet inequality (10) is incompatible with lemma 5. The proof for the other cases is similar. ■

## 5 Conclusion

The main object of this note was the identification of a condition on households' preferences for public and private good that is necessary and sufficient to guarantee the wealth stratification of the result of endogenous processes of jurisdiction formation. Using a simple model of jurisdiction formation in which the only trade off made by households is that between the tax they pay and the public good they get, we find a clear and easy-to-check condition: Households' preferences must consider the public good to be either always a gross complement to, or always a gross substitute for, the private good.

Yet the model we use provides a somewhat too stylized representation of the process by which jurisdictions actually form in the “real world”. A clearly missing ingredient is the housing market. While endogenous models of jurisdiction formation with housing markets and intra-jurisdiction collective decision making with respect to taxes and public good have been constructed and studied, notably by [15], [16], [8] and [10], the efforts in the literature have focused here again on the question of existence and Pareto optimality of a (general) equilibrium for these models and have not specifically examined the positive properties of this equilibrium with respect to stratification. Such an examination is a priority for future research as it could also make more precise the intuitive analysis of this case provided in Ellickson [4].

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