Problems set 1 (production)

Nicolas Gravel, University of the Mediterranean

Septembre 2008

Problem 1: Let f be a production function for a technology with two inputs having constant returns to scale. Show that if the average product of one of the factor is increasing with respect to the level of use of this factor, then the marginal product of the other factor must be negative.

Problem 2. Which conditions (if any) must be imposed on the numbers a et b in order for the technology represented by the production set $Y = \{(y_1, y_2) \in \mathbb{R}^2 : y_1 \leq \ln(a - y_2) - b\}$ to satisfy the conditions of irreversibility, impossibility of free production, possibility of no production, free disposal and convexity. ?

Problem 3: True or false ? If a technology is represented by a production function $f : \mathbb{R}_+^l \to \mathbb{R}$ that is quasi-concave et that satisfiest $f(0^l) = 0$, then this technology can not have increasing returns to scale. (We recall that a production fonction is quasi-concave if, for any level of production y, the set $V(y) = \{x \in \mathbb{R}_+^l : f(x) \geq y\}$ is convex).

Problem 4 For every of the following production functions, (where $a, b \in \mathbb{R}_{++}$), indicates if the technology that it represents satisfies the 6 assumptions that we have seen in class (except irreversibility).

(i) $f(x_1, x_2) = e^{\min(ax_1, bx_2)}$

(ii) $f(x_1, x_2) = ax_1 + x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} + bx_2$

(iii) $f(x_1, x_2) = ax_1 - x_1^{\frac{1}{2}} x_2^{\frac{1}{2}} + bx_2$

(iv) $f(x_1, x_2) = \min(1 - a/x_1, 1 - b/x_2)$

Problem 5 Show that if a technology is represented by a production function $f : R_+^l \to R$ is additive in the sense that $f(x + z) = f(x) + f(z)$ for every x, $z \in R_+^l$, the technology will be convex if inputs are perfectly divisible.

Problem 6: For every production function, find the profit function that corresponds to it.

(a) $f(x) = \ln x$ si $x \geq 1$

$= 0$ otherwise

(b) $f(x_1, x_2) = 100 x_1^{\frac{1}{2}} x_2^{\frac{1}{4}}$

(c) $f(x_1, x_2) = (ax_1 + bx_2)^{\frac{1}{2}}$ (pour a, $b \in \mathbb{R}_{++}$)

(d) $f(x_1, x_2) = (\min(x_1, x_2))^a$. Which property must satisfy the number a in order for the profit function to be well-defined ?

Problem 7: What does the producer's surplus, defined as the integral under the supply curve of the firm calculated between two arbitrary prices of output, measure ?

Problem 8: A statistician has collected the following informations on the behavior of a firm producing one output with two inputs.

	1st trimester	2nd trimester	3nd trimestre
output price	2	3	2,5
price of input 1	1	0,5	2
price of input 2	1	3	2
quantity of output	100	100	90
quantity of input 1	75	75	50
quantity of input 2	75	55	50

Can this behavior be coming from a firm that maximises its profit in a perfectly competitive environment ? (Justify).

Problem 9 An econometrician has estimated the following output supply and factor demands (respectively) of a firm evolving in a perfectly competitive environment and producing one output (y) using two inputs (x_1 and x_2)..

$$\begin{aligned} y^*(p, w_1, w_2) &= p[\ln(p - w_1) - \ln w_1 + \frac{p}{2w_2}] \\ x_1^*(p, w_1, w_2) &= \frac{p - w_1}{w_1} \\ x_2^*(p, w_1, w_2) &= \frac{p^2}{4w_2^2} \end{aligned}$$

where p is the output price and w_1 and w_2 are, respectively, the prices of inputs 1 and 2. Can these demand and supply functions be resulting from a profit maximizing ? Justify.