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ADMISSION, TUITION, AND FINANCIAL AID POLICIES IN THE MARKET FOR HIGHER EDUCATION

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We present an equilibrium model of the market for higher education. Our model simultaneously predicts student selection into institutions of higher education, financial aid, educational expenditures, and educational outcomes. We show that the model gives rise to a strict hierarchy of colleges that differ by the educational quality provided to the students. We also develop a new estimation procedure that exploits the observed variation in prices within colleges. Identification is based on variation in endowments and technology. It does not rely on observed variation in potentially endogenous characteristics of colleges such as peer quality measures and expenditures. We estimate the structural parameters using data collected by the National Center for Education Statistics and aggregate data from Peterson's and the National Science Foundation.

KEYWORDS: Higher education, peer effects, college competition, nonlinear pricing, equilibrium analysis, estimation.

1. INTRODUCTION

OVER THE PAST SEVERAL YEARS, research has investigated normative and positive consequences of competition in primary, secondary, and higher education, and the likely effects of policy changes, including vouchers, public school choice, and changes in education financing.[2] Some of this research has relied on general equilibrium models. Given the absence of large scale policy experiments, these models have been a primary tool for evaluating the impact of a variety of education reform measures. To date, the predictions of these models

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[2]Recent theoretical studies include Benabou (1996), Caucutt (2002), de Bartolome (1990), Epple and Romano (1998, 2003), Fernandez and Rogerson (1998), Manski (1991), Nechyba (2000), and Rothschild and White (1995). Some recent empirical studies include Bergstrom, Rubinfeld, and Shapiro (1982), Carlton, Bamberger, and Epstein (1995), Downes and Greenstein (1996), Ferreyra (2003), Fuller, Manski, and Wise (1982), Hoxby (2000), and Rouse (1998).

have been subjected to little formal empirical testing. This paper provides an integrated approach to estimation and inference based on this class of models. For this purpose, we focus on the market for higher education. Colleges and universities provide a promising environment for developing this approach because a variety of data sets collected by the National Center for Education Statistics and commercial companies such as Peterson's are available to researchers.

We present a general equilibrium model of the market for higher education that extends earlier work on competition in the market for primary and secondary education. In our model, colleges seek to maximize the quality of the educational experience provided to their students.[3] The quality of the educational experience depends on peer ability and income of the student body, and on instructional expenditures per student. If peer "quality" is an important component of college quality, students and their parents will seek out colleges where the student body offers high quality peers.[4] Likewise, colleges will attempt to attract students who contribute to improving peer quality. In higher education, colleges have the latitude to choose tuition and admission policies to attempt to attract a high quality student body. Our model thus yields strong predictions about the hierarchy of colleges that emerges in equilibrium, about the allocation of students by income and ability among colleges, and about the pricing policies that colleges adopt.[5] We complete the theoretical development with an analysis of existence of equilibrium.

We also provide a new approach for identifying and estimating this class of differentiated product models. Our approach differs significantly from the previous empirical literature on differentiated products.[6] First, our data allow us to impute costs. We can, therefore, estimate cost functions without relying on strong functional form assumptions or assumptions about the demand side of the model. Second, our model endogenously generates a distribution of student types among colleges, as well as a distribution of college characteristics, given colleges' posted maximum tuitions and financial endowments. Our estimation procedure differentiates between variation in fundamentals (such as

[3]We refer to institutions of higher education as "colleges," having in mind inclusion of the undergraduate division of universities.

[4]There is a large, growing, and controversial literature by social scientists on peer effects. Methodological issues are discussed in Manski (1993), Moffitt (2001), and Brock and Durlauf (2001). Limiting discussion to recent research on peer effects in higher education, Sacerdote (2001) and Zimmerman (2003) find peer effects between roommates on grade point averages. Betts and Morell (1999) find that high-school peer groups affect college grade point average. Arcidiacono and Nicholson (2005) find no peer effects among medical students. Dale and Krueger (1998) have mixed findings.

[5]An insightful overview of the college quality hierarchy and its determinants is provided by Winston (1999).

[6]Recent studies include Berry, Levinsohn, and Pakes (1995), Epple and Sieg (1999), Epple, Romer, and Sieg (2001), Nevo (2001), Petrin (2002), Blundell, Browning, and Crawford (2003), Ekeland, Heckman, and Nesheim (2004), and Bajari and Benkard (2005).

college endowments) and variation in product characteristics. Identification is primarily based on variation in fundamentals. Observed (and potentially endogenous) characteristics such as expenditures and peer quality measures are treated as endogenous latent variables in estimation. Third, identification of demand-side parameters of the model largely depends on the observed variation in prices within colleges. In contrast, almost all previous empirical papers have relied solely on variation in prices among products, ignoring for all practical purposes issues related to price discrimination. One central part of the estimation strategy is to match financial aid policies observed in the data to those predicted by our equilibrium model. This approach requires us to characterize optimal pricing functions for each college and hence compute the equilibrium of the model at each step of the estimation algorithm. We develop a new maximum likelihood estimator that controls for the fact that some of the most important student characteristics, such as income and ability, are likely to be measured with errors in the data. The result is, we believe, the first instance in which an equilibrium model with endogenous product characteristics, sorting, and price discrimination along multiple dimensions has been estimated.

To implement the empirical analysis, we obtained and merged three data bases to unite student-level and college-level data. These include a data base that provides individual-level information for a sample of students and two data bases, one proprietary and one public, that provide complementary college-level information about the universe of colleges and universities in the United States.

Our findings suggest that the market for higher education is competitive. Colleges at low and medium quality levels have close substitutes in equilibrium and thus a limited amount of market power. Admission policies are largely driven by the "effective marginal costs" of educating students of differing abilities and incomes. Colleges with high quality have significantly more market power because they do not face competition from higher quality colleges. Hence, they can set tuitions above effective marginal costs and generate additional revenues that are used to enhance quality. A main component of college quality is, of course, instructional expenditures, but colleges at all quality levels spend a significant amount of resources on merit aid. This finding is consistent with our modeling approach that assumes that peer quality is increasing in the ability of the mean student body. We also find that colleges at all levels link tuition to student (household) income. Some of this pricing derives from the market power of each college. This allows colleges to extract additional revenues from students who are inframarginal consumers of a college. However, as noted above, our empirical findings suggest that the market power of most colleges is limited, which further suggests that pricing by income may be driven by other causes. We, therefore, permit colleges to place value on having a less elite student body, because colleges want average income to approach that of the population. Our empirical findings here indicate that colleges are, in fact, concerned about the relative affluence of their students and thus tend to subsidize students from families with lower incomes.

Finally, we use our model to evaluate some policy experiments. We consider a ban on price discrimination by colleges and compute the equilibrium that arises in our model when colleges are forced to adopt prices that equal effective marginal costs. We find that such ban is likely to have a significant impact on the sorting of students and thus the composition of colleges. Since many students are near the margin of switching colleges, modest changes in tuition result in large changes in attendance. Higher ranked colleges are more attractive under effective marginal cost pricing and thus they have a larger market share. We also consider a fairly radical change in the federal financial aid formula that is targeted toward lower income students. This policy redistributes resources to lower income households. We find that the income effect associated with this policy causes some of the poorer students to attend higher quality colleges and some of the richer students to do the reverse. As a consequence, the policy increases the access to high quality education of lower income households.

The paper is organized as follows. Section 2 lays out the general equilibrium model and derives a set of college-level conditions that characterize the equilibrium allocation. Section 3 defines equilibrium, analyzes existence, and characterizes properties of market equilibrium. Identification and estimation are discussed in Section 4. Section 5 provides information about our data set, which is created by drawing together information from the National Center for Education Statistics, the National Science Foundation, and Peterson's. An online supplement (Epple, Romano, and Sieg (2006)) provides further details on the programs and data. The empirical results are discussed in Section 6 and then used in some policy analysis. Section 7 presents the conclusions of the analysis and discusses future research.

2. A THEORETICAL MODEL OF HIGHER EDUCATION

2.1. *Preferences and Technologies*

In this section, we develop our theoretical model of provision of undergraduate higher education. There is a continuum of potential students who differ with respect to their household income y and their ability level b.

ASSUMPTION 1: *The joint distribution of income and ability $F(b, y)$ is continuous with convex support $S \subset \mathbb{R}_+^2$ and joint density $f(b, y)$.*

There are J colleges and each student chooses among the subset of the colleges to which the student is admitted (as described below) and an outside option.

ASSUMPTION 2: *The outside option denoted by 0 provides an exogenous quality q_0 with zero cost to the student and is available to any student.*

The quality of college $j = 1, 2, \ldots, J$ is given by

$$q_j = q(I_j, \theta_j, d_j), \tag{2.1}$$

where θ_j is a peer-student measure, which is equal to mean ability level in the student body, I_j is the expenditure per student in excess of minimal or "custodial costs," and d_j is a measure of the (relative) mean income of the student body.[7]

ASSUMPTION 3: *The quality function $q(\cdot)$ is a twice differentiable, increasing, quasi-concave function of its arguments.*

As noted in the Introduction, the data on financial aid suggest that a college's objective includes a term that measures the income of the college's students. Denote mean income of students in college j by μ_j^y and define

$$d_j = \frac{1}{\mu_j^y}. \tag{2.2}$$

Anecdotal evidence suggests that colleges in the United States make an effort to attract greater numbers of lower income students.[8] Our main motivation for using mean income as a measure in the quality index is empirical. Reduced form regressions reported in Epple, Romano, and Sieg (2003) suggest that colleges engage in a significant amount of price discrimination by income, i.e., they provide need-based aid. Some price discrimination by income arises in our model even if we do not introduce a measures of the relative income position. Since there are only finitely many differentiated colleges in equilibrium, they have a limited amount of market power over inframarginal students and can extract additional revenues from them. However, this type of price discrimination is not sufficient to explain the amount of pricing by income observed in the data. As shown below, the income measure used above implies the type of monotonic pricing by income that is prevalent in our data.[9]

[7]We henceforth omit ranges of subscripts and the like when they are obvious from the context.

[8]Concern for socio-economic diversity of student bodies is cited by Harvard president Lawrence Summers as motivating Harvard's recent announcement that no tuition will be charged to students from households with less than \$40,000 in annual income: "When only 10 percent of the students at elite higher education come from the lower half of the income distribution, we are not doing enough" (*New York Times*, Feb. 29, 2004, p. 14). See the discussion in the text for how our simple specification of d_j relates to socio-economic diversity.

[9]The stylized fact that colleges can extract so much revenue from higher income households is clearly an empirical puzzle given that many colleges compete for students. Reduced form analysis also confirms that this fact is robust to a number of changes in the specification of the reduced form model. For example, adding race or ethnicity to the model does not fundamentally alter this finding. More future research is needed to find other compelling explanations for this puzzle.

A less obvious consequence of including this income measure in the quality index is that it may lead to more income diversity within a college in equilibrium. This result may be surprising because d_j does not measure income diversity per se. The intuition for this finding is that colleges—especially the most selective ones—will penalize higher income students and subsidize lower income students. As consequence, the most selective schools will have lower mean income levels in equilibrium than in a model without the income penalty term.[10] For now, we assume that students have the same quality index as colleges. In Section 6.3, we examine the alternative of having students indifferent to the incomes of their peers, where then student and college perceptions of "quality" differ.

The college cost function is

$$C(k_j, I_j) = F + V(k_j) + k_j I_j, \tag{2.3}$$

where k_j is size of the college j's student body and I_j is per student expenditure on quality enhancing inputs. Schooling costs include components that are independent of educational quality, the "custodial costs" mentioned above.

ASSUMPTION 4: *The term $V(k)$ is an increasing and convex function that is twice differentiable in k, i.e., $V', V'' > 0$.*

Note that the average cost function, C/k, is U-shaped in k and denote the efficient scale by k^*.

Substantial financial aid to many undergraduates in the form of grants, loans, and work–study funding is provided by the federal government and, to a lesser extent, by other entities that are also independent of the student's college. We refer to such aid as noninstitutional aid. Let p_j^m denote the maximum tuition at college j. We presume that the value of noninstitutional aid to the student at college j, denoted a_j, can be written as

$$a_j = a(b, y, p_j^m). \tag{2.4}$$

Much of this aid is based on the federal government's calculation of the family's ability to pay. Such aid is need based, implying that it depends on student income and tuition of the college attended. Some noninstitutional aid seems to be meritorious, so we will allow it to depend on ability as well. More aid is also given to students who attend more expensive colleges. We, therefore, make the following assumption.

ASSUMPTION 5: *The term $a(\cdot)$ is increasing in b and p_j^m, and decreasing in y.*

[10]Including mean income in the quality index may lead to increased diversity if income and willingness to pay are not perfectly correlated.

We assume that the decision to attend college is made by the student's household. Household utility depends on numeraire consumption, educational quality, and the student's ability. Household utility from attendance at college j is given by the (conditional) utility function

$$U(b, y - p_j + a_j, q_j), \tag{2.5}$$

where p_j is tuition paid by the student and numeraire consumption is $x = y - p_j + a_j$. We assume that U satisfies standard regularity conditions. In addition, we assume that the utility function satisfies the following single-crossing conditions:

ASSUMPTION 6: *The utility function satisfies*

$$\frac{\partial\left(\frac{\partial U/\partial q}{\partial U/\partial x}\right)}{\partial x} > 0, \tag{2.6}$$

$$\frac{\partial\left(\frac{\partial U/\partial q}{\partial U/\partial x}\right)}{\partial b} \geq 0. \tag{2.7}$$

Thus the demand for college quality is increasing in income and nondecreasing in ability.[11] One interpretation of the utility function has utility increasing in the numeraire good and educational attainment of household's student, $at(q, b)$, with at increasing in its arguments. This interpretation is consistent with the utility function that we estimate below.

Households choose among colleges to which they are admitted or no college, taking as given college qualities and their tuition and admission policies.

2.2. *The Decision Problem of a College*

Colleges are assumed to maximize quality.[12] Their choices must satisfy a profit constraint, with revenue equal to the sum of all tuition from students plus other college earnings, the latter denoted R_j. We refer to R_j as college j's endowment earnings, but it includes also nontuition revenues like state subsidies. The number of colleges and their endowments are taken as exogenous.[13] We do

[11]Assumption (2.7) can reasonably be challenged. If it fails to hold, then the sorting pattern our model predicts might be disrupted. As detailed below, our empirical specification of the model treats the boundary case of (2.7), which has zero ability elasticity of demand for college quality. It is of interest to examine alternatives in future research.

[12]See Epple and Romano (1998, 2003) for an analysis of profit maximization by (secondary) schools in a related model. Another preliminary analysis of achievement maximization, using the college mean of the $at(\cdot)$ function discussed above, demonstrates that it leads to predictions that are similar to the ones implied by quality maximization.

[13]An interesting extension that we do not pursue in this paper would be to endogenize endowment income.

not allow for entry and restrict attention to cases where all J colleges can cover their costs in equilibrium. Although colleges will condition tuition on student characteristics, we presume that college j charges a maximum tuition p_j^m. We do not have an explicit theory to explain or determine the magnitude of p_j^m, so we treat it as exogenous. Our motivation for introducing these price caps is empirical. We interpret the price cap as the college's posted tuition, with lower tuition framed as financial aid, a scholarship, or, perhaps, a fellowship. To derive certain properties of the equilibrium allocation, we use the following requirement:

ASSUMPTION 7: *We assume* $0 \leq R_1 < \cdots < R_J < \infty$ *and* $0 < p_1^m < \cdots < p_J^m < \infty$.

To avoid the problem that a college may operate with infinite per student expenditure levels, we also need the next assumption:

ASSUMPTION 8: *We have* $F > \max_j R_j = R_J$.

We observe in the data that a fraction of students in each college pay the posted tuition (price cap). Price caps are thus an important feature of currently used financial aid and pricing policies. Introducing price caps thus seems to be necessary if the objective is empirical analysis. As discussed below, price caps somewhat alter the predictions of the model. One important implication of the price cap is that the more selective colleges will define minimum ability thresholds that students have to satisfy to be admitted (see Figure 1 in Section 2.2). In this model, we do not differentiate between administrators (agents) who operate the college and trustees (principals) who set objectives for the college. We thus assume that there are no informational asymmetries and that administrators implement policies preferred by the trustees. In a more general model, there may be some agency problems. Trustees may, for example, be concerned that a significant fraction of high-income–low-ability students may harm the reputation of the college. Administrators may have some private incentives to admit these students (e.g., to associate with very wealthy households). A binding price cap that limits the ability of administrators to admit these type of students may then result in equilibrium. An interesting extension of our model, which we do not explore in this paper, would be to formalize these ideas.[14] For the rest of the analysis, we treat price caps as predetermined while acknowledging that this is a limitation of the analysis.

Colleges take type (b, y)'s alternative utility as given when maximizing quality. This assumption conforms to a competitive model and is further discussed

[14]In a dynamic model, price caps may also arise because colleges may charge lower tuition rates because they expect to receive donations from wealthy alumni in the future.

below. Let $r_j = r_j(b, y + a_j, q_j)$ denote college j's beliefs about a student's reservation price for attending a college of quality q_j. That is, r_j, satisfies

$$U(b, y - r_j + a_j, q_j) = U_j^A(b, y), \tag{2.8}$$

where $U_j^A(b, y)$ denotes the beliefs that college j holds about the maximum alternative utility that type (b, y) can attain if the household does not attend college j. Equilibrium requires that these beliefs are consistent with optimization by other colleges as detailed below.

College j chooses an admission function

$$\alpha_j(b, y) \in [0, 1] \tag{2.9}$$

that indicates the proportion of type (b, y) in the student population admitted and chooses a tuition function $p_j(b, y) \leq p_j^m$. Let $P_j = \{\succ_j, =_j, \prec_j\}$ describe college j's preferences as to whether an admitted student matriculates, where $\succ_j$ signifies strict preference of college j, etc. Let

$$\alpha_j^m(b, y) \in [0, 1] \tag{2.10}$$

denote college j's beliefs about the proportion of type (b, y) that will matriculate conditional on admittance, implying the college believes $\alpha_j(b, y)\alpha_j^m(b, y)$ of the type (b, y) will actually attend. In equilibrium, college j's beliefs are

$$\alpha_j^m(b, y) = \begin{cases} 1, & \text{if } U(y - p_j + a_j, q_j, b) > U_j^A(b, y), \\ 1, & \text{if } U(y - p_j + a_j, q_j, b) = U_j^A(b, y) \text{ and } \succ_j, \\ 0, & \text{if } U(y - p_j + a_j, q_j, b) = U_j^A(b, y) \text{ and } =_j, \\ 0, & \text{if } U(y - p_j + a_j, q_j, b) < U_j^A(b, y). \end{cases} \tag{2.11}$$

Note that beliefs about matriculation make sense only if $\alpha(b, y) > 0$; hence beliefs are not specified if $\prec_j$. Beliefs (2.11) are consistent with utility maximization by students and must be satisfied in equilibrium almost everywhere as shown below. Optimal tuition satisfies

$$\left[p_j(b, y + a_j, q_j) - \min\{r_j(b, y + a_j, q_j), p_j^m\}\right]\alpha_j(b, y)\alpha_j^m(b, y) = 0. \tag{2.12}$$

Tuition to admitted students lower than the minimum in (2.12) would imply the student has a strict preference for matriculating, and college j could increase tuition and thus I_j and q_j while still attracting that student. Tuition to nonadmitted students is arbitrary, but it is convenient to specify that p_j equals the minimum in (2.12) for all students when solving college j's optimization problem. Note that the fourth line of (2.11) is then irrelevant.

Given beliefs (2.11) about matriculation, college j's optimization problem is

$$\max_{\alpha_j(b,y), p_j(b,y+a_j,q(\cdot)), k_j, \theta_j, \mu_j^y, d_j} q(\theta_j, I_j, d_j) \tag{2.13}$$

subject to the following constraints.

Optimal Pricing:

$$p_j(b, y+a_j, q(\cdot)) = \min\{r_j(b, y+a_j, q(\cdot)), p_j^m\}. \tag{2.14}$$

Identity Constraints:

$$k_j = \int\int_S \alpha_j^m(b, y)\alpha_j(b, y) f(b, y)\, db\, dy, \tag{2.15}$$

$$\theta_j = \frac{1}{k_j}\int\int_S b\alpha_j^m(b, y)\alpha_j(b, y) f(b, y)\, db\, dy, \tag{2.16}$$

$$\mu_j^y = \frac{1}{k_j}\int\int_S y\alpha_j^m(b, y)\alpha_j(b, y) f(b, y)\, db\, dy, \tag{2.17}$$

$$d_j = \frac{1}{\mu_j^y}. \tag{2.18}$$

Budget Constraints:

$$F + V(k_j) + k_j I_j = T_j + R_j, \tag{2.19}$$

$$T_j = \int\int_S p_j(b, y+a_j, q(\cdot))\alpha_j^m(b, y)\alpha_j(b, y) f(b, y)\, db\, dy. \tag{2.20}$$

Feasibility Constraint:

$$\alpha_j(b, y) \in [0, 1]. \tag{2.21}$$

The first-order condition for the optimal admission function is

$$\alpha_j(b, y)\begin{pmatrix} = 1 \\ \in [0, 1] \\ = 0 \end{pmatrix} \quad \text{if} \quad p_j \begin{pmatrix} > \\ = \\ < \end{pmatrix} EMC_j(b, y), \tag{2.22}$$

where

$$EMC_j(b, y) = V'(k_j) + I_j + \frac{\partial q_j/\partial\theta}{\partial q_j/\partial I}(\theta_j - b) + \frac{\partial q_j/\partial\mu_j^y}{\partial q_j/\partial I}(\mu_j^y - y). \tag{2.23}$$

The first-order condition for I_j is

$$\frac{\partial T_j}{\partial I_j} - k_j = -\frac{\partial q_j/\partial I_j}{\lambda_j}, \tag{2.24}$$

where $\lambda_j > 0$ is the multiplier on the revenue constraint (2.19). An admission function that is consistent with (2.22) is

$$\alpha_j(b, y)\begin{pmatrix}=1\\=1\\=0\end{pmatrix} \quad \text{if} \quad p_j\begin{pmatrix}>\\=\\<\end{pmatrix}EMC_j(b, y), \tag{2.25}$$

where the second line is shown below to be necessary to find market equilibrium.

To clarify the first-order conditions and the requirements when optimization has multiple solutions, begin with (2.24). It implies that college j spends more on educational inputs than the Samuelsonian level to the extent that it has some market power (i.e., $\lambda_j < \infty$). This is because the college values quality itself, not just because quality increases permit higher tuition. Turning to the admission/matriculation aspects of the solution, consider EMC_j, which we refer to as the effective marginal cost of attendance. From (2.23), effective marginal cost is the sum of the resource cost of a student's attendance, the cost of the ability externality on college quality (the third term on the right-hand side), and the cost of the student's income on college quality (the fourth term on the right-hand side). The ability externality equals the change in the peer measure from attendance of a student of ability b multiplied by the resource cost of maintaining quality. Note that students with higher ability than the mean have negative ability–externality costs. The cost term that reflects the effect on mean incomes of a student's attendance is analogous, but note that it is negative for students with income below the college's mean.[15] This is, of course, because college quality is decreasing in the mean income of the student body.

A college has strict preference for attendance of a student from whom it can obtain tuition greater than the effective marginal cost. All these students are admitted and they all attend in equilibrium ($\alpha_j = \alpha_j^m = 1$). A college has strict preference for nonattendance of a student for whom it cannot obtain tuition that is at least the effective marginal cost. None of these students is admitted and, obviously, none attends ($\alpha_j = 0$).[16] A college is indifferent at the margin to the attendance of a student from whom it might just obtain tuition equal to the effective marginal cost. College optimization does not determine the admission policy for such students, but equilibrium requires that they are all admitted ($\alpha_j = 1$) but (almost) none attends ($\alpha_j^m = 0$) as shown below.

[15]The character of this term in effective marginal cost changes when students are indifferent to the income of their peers is examined below.

[16]The college could, of course, admit these students and charge them a prohibitive price. Any policy for nonadmitted students that makes attendance at college j an inferior or infeasible alternative to their equilibrium policy implies the same equilibrium. We already observed that the price to nonadmitted students is arbitrary.

3. EQUILIBRIUM

3.1. *Definition of Market Equilibrium*

To define a market equilibrium, it is necessary to determine the equilibrium reservation utility function and beliefs about matriculation of each college, and to consider competitive interaction between colleges. We define equilibrium as a competitive equilibrium, assuming that colleges are utility takers and students take college policies and qualities as given. The assumption of utility taking by colleges is a generalization of price taking that has been utilized in the competitive club-goods literature.[17] In equilibrium, the alternative utility and thus reservation price functions of each college and their beliefs about student matriculation must be consistent with utility maximization and the actions of the other colleges. We refer to the set of colleges that admit a student in equilibrium along with the outside option as the effective choice set of a student and we denote it $\tilde{J}(b, y)$. More formally,

$$\tilde{J}(b, y) = \{j | \alpha_j(b, y) = 1\} \cup \{0\}. \tag{3.1}$$

Knowing the colleges in the effective choice set allows us to characterize alternative utilities and impose the equilibrium restriction that alternative utility functions must be consistent with optimal choices. A competitive market equilibrium for this economy can then be defined as follows:

DEFINITION 1: The *economy E* of our model consists of an outside option with quality q_0; a college quality function $q(\cdot)$; a set of colleges $\{1, \dots, J\}$; a vector of endowment incomes $(R_1, \dots, R_J)$ and price caps $(p_1^m, \dots, p_J^m)$; a set of noninstitutional financial aid functions $(a_1(\cdot), \dots, a_J(\cdot))$; a cost function $C(k, I)$; a continuous distribution F of household types (b, y) with convex support S; and a utility function $U(y - p + a, q, b)$.

A *competitive utility-taking equilibrium* for this economy E consists of a set of admission and pricing functions, $\alpha_j(b, y)$ and $p_j(b, y + a_j, q_j)$; an alternative utility function $U_j^A(b, y)$ for each college; a vector of college characteristics $(k_j, \theta_j, \mu_j^y, I_j)$ for each college; a set of matriculation functions $\alpha_j^m(b, y)$; and an allocation of students into the J colleges and no college such that:

(i) Every student (b, y) is allocated to a preferred option in his or her effective choice set.

(ii) Each college chooses its size, peer quality, relative income position, and expenditures, as well as admission and tuition policies, to maximize quality, taking as given its endowment, price cap, alternative utility function, and beliefs about matriculation.

[17]This assumption has substantial precedence in the club-goods literature with nonanonymous crowding. See the discussion in Scotchmer (1994).

(iii) Beliefs about alternative utilities and matriculation that are consistent with optimal choices. For each household type (b, y) and each college j,

$$U_j^A(b, y) = \max\Big[U(b, y, q_0), \max_{i \neq j, i \in \widetilde{J}(b,y)\setminus\{0\}} U(b, y - p_i + a_i, q_i)\Big]. \tag{3.2}$$

Thus the maximum alternative utility is the maximum over the outside option and the next best college alternative, taking qualities, tuitions, and admissions as given.

(iv) Each student attends at most one college, i.e., markets for the J colleges clear:

$$\sum_{j=1}^{J} \alpha_j(b, y)\alpha_j^m(b, y) \le 1, \tag{3.3}$$

where types for whom the inequality is strict are attending no college.

Several issues regarding this definition of equilibrium warrant elaboration. First, we have a lemma:

LEMMA 1: *Equilibrium requires both of the following components*:
(i) *Matriculation as specified in* (2.11).
(ii) *Admission* $\alpha_j(b, y) = 1$ *when* $p_j = EMC_j$.

The proof is given in the Appendix. What deserves some intuitive explanation in Lemma 1 is that students are admitted to a college when the college is indifferent to their matriculation (when $p_j = EMC_j$), while none of them attends (see (2.11)). Colleges must be of different qualities in equilibrium because their endowments differ (as shown below). Admission to another college is needed in equilibrium to determine tuition at the college a student attends, where the attended college strictly prefers their attendance. If students attending one college did not have access to another college, then their college could raise tuition and some other college would then strictly prefer their admission, etc. If students choose to matriculate at the college that offers admission and is indifferent to their attendance, then the college that strictly prefers their attendance would reduce tuition infinitesimally, but no such optimal tuition exists. Equilibrium is then characterized by an attendance (or matriculation) set for each college, $A_j = \{(b, y)|\alpha_j^m(b, y)\alpha_j(b, y) = 1\}$, and an admission set, $B_j = \{(b, y)|\alpha_j(b, y) = 1\}$, with $A_j \subseteq B_j$.[18]

Generally students attending college j will fall into two groups. One group will obtain utility equal to $U_j^A(b, y)$ and pay r_j. The second group attending college j obtains higher utility than $U_j^A(b, y)$, paying tuition p_j^m with $p_j^m < r_j$.

[18]In the cases we examine, for all j, college j will be the best alternative for some college students who do not attend j, so A_j is a proper subset of B_j. There are exceptions. If, for example, there is only one college, then the outside option is a best alternative for every student and $A_1 = B_1$.

We have assumed utility taking by colleges, which is a common assumption in the related club-goods literature. Utility taking corresponds to taking the equilibrium reservation price function as a given, with r_j a function of student type and college j's own quality. Although the reservation price function of a college also depends implicitly on the equilibrium qualities and admission–pricing policies of other colleges (because alternative utility depends, in general, on other colleges' policies), a college treats these as given when making their own choices. An alternative approach would be to examine a Nash equilibrium in pricing and admission policies. In such a model, each college would make its own choices, taking as given the admission and pricing policies of other colleges. The difference is that in the Nash approach a college would take into account the changes in other colleges' qualities that varying its own policies would cause as students change colleges. These changes result from the peer effects in the model. Utility taking implies that colleges take *both* admission–pricing policies *and* qualities of other colleges as given when they make choices. In the Nash approach, colleges would explicitly take into account how the reservation price function varies with other colleges' qualities as their own policies are varied. Utility taking then abstracts from a strategic consideration that might be relevant empirically while being much simpler. Although the strategic element from which we abstract is extremely interesting, we have chosen the simpler model because we believe the empirical relevance of the strategic considerations is not likely to warrant the added complexity.

3.2. *Properties of Equilibrium*

Before we discuss the existence of equilibrium, we discuss necessary properties of equilibria to illustrate some of the features of our model.

PROPOSITION 1: *A market equilibrium satisfies the following four conditions*:

(i) *There is a strict quality hierarchy of colleges in equilibrium. The hierarchy follows the endowment ranking.*

(ii) *Almost every given type* (b, y) *makes the same choice of* $j \in \{0, 1, \dots, J\}$.

(iii) *Given that a positive measure of ability types with income* y *attend college* j, *there exists a locus* $b_j(y)$ *that defines the minimum ability that a student with income* y *must exceed to attend college* j. *For given* y, b_j *satisfies* $EMC_j(b, y) = \min\{p_j(b, y + a_j, q_j), p_j^m\}$ *if there exists such a pair* $(b_j, y) \in S$. *Otherwise* $b_j(y)$ *equals the minimum* b *in* S *for given* y.

(iv) *Choosing among the set of colleges and no college*, *almost every student type* (b, y) *attends the college or no college that would maximize utility if* $p_j = EMC_j(b, y)$ *for all* $j \in \tilde{J}(b, y)$. *College pricing in excess of EMC is to take away consumer surplus* (*constrained by the price cap for some students*).

A proof of Proposition 1 is given in the Appendix.

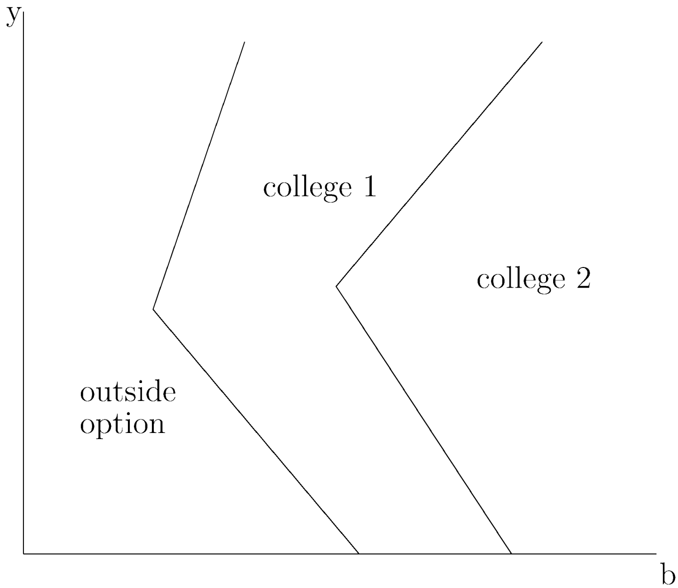

FIGURE 1.—An example of college attendance spaces that illustrates the boundaries of college attendance sets.

Figure 1 shows an example of how the type space is partitioned into colleges, assuming just two colleges, where college 2 is of higher quality. The example assumes that noninstitutional aid is independent of ability as we find empirically. The tipped-L solid lines are the $b_j(y)$ loci of Proposition 1(iii) that separate types into colleges and no college. Those students to the right of the rightmost "boundary locus" attend the higher quality college 2; those between the boundary loci attend college 1; while the rest do not attend college. The upward-sloping part of each boundary locus satisfies $p_i^m = EMC_i(b, y) \leq r_i(b, y + a_i, q_i)$ and the downward-sloping part satisfies $r_i(b, y + a_i, q_i) = EMC_i(b, y) \leq p_i^m$, where i is the number of the college to the right of the locus. The value of r_i depends on the student's type, including the options available to the student.

3.3. *Existence and Computation of Equilibrium*

It should be clear from the discussion so far that there is no hope to solve for equilibrium analytically. A college's optimum will necessarily be on a boundary: prices will be set to equal reservation prices or the price cap, and attendance of each type will satisfy $\alpha_j \alpha_j^m = 1$ for some j. Hence we need to rely on numerical techniques to compute equilibria, which is also quite challenging. To compute equilibrium allocations, it is useful to focus on a slightly broader class of allocations that satisfy the following four conditions: (a) market clearing; (b) utility maximization; (c) consistency of alternative utilities and beliefs; and

(d) the first-order conditions and local second-order conditions of the colleges' optimization problems. Any equilibrium must satisfy these conditions,[19] but an allocation that satisfies these conditions may not be an equilibrium because it may fail to satisfy global optimality of college choices. For lack of a better term, we will refer to these allocations as "local equilibria."

It is not hard to see that one can characterize a local equilibrium for our model based on a vector of college sizes, peer ability and income measures, and instructional expenditures for each college, thus yielding $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. The value of k_0 is given by $1 - \sum_{j=1}^J k_j$. Finding a local equilibrium for this model can be viewed as a classical fixed-point problem of an equilibrium correspondence that maps $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$ into $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. Such a mapping exists.[20] Moreover, we can show that the mapping used in this analysis has a fixed point under suitable regularity conditions. Finally, this mapping can be used to design an algorithm to compute equilibria numerically. To implement this algorithm, we need to evaluate integrals numerically using simulation techniques. We compute fixed points using an algorithm that finds a root to a nonlinear system of equations. In our computations, we use Broyden's method. Once we have found local equilibria, we need only to verify ex post that college decisions satisfy global optimality. Verifying global optimality is a straightforward exercise, but computationally costly, because we need to consider deviations in four strategic variables for each of the J colleges.[21] Our computational analysis suggests that allocations computed as outlined above satisfy global optimality.

4. IDENTIFICATION AND ESTIMATION

4.1. *The Supply Side*

First, we consider the problem of identifying and estimating the parameters of the custodial cost function. We observe $\{T_j, R_j, I_j, k_j\}_{j=1}^J$ for a large sample of colleges of size J. The budget constraint then implies that we can impute custodial costs $C_j^c = T_j + R_j - k_j I_j$. Assuming that custodial costs are quadratic in k_j, we obtain

$$C_j^c = F + c_1 k_j + c_2 k_j^2 + \varepsilon_j, \tag{4.1}$$

where $C_j^c \equiv F + V(k_j)$ and ε_j denotes an error term. To achieve identification, we can assume that the left-hand side cost variable in (4.1) is measured with error (for example, due to measurement error in expenditures). In that case, it is reasonable to assume that the measurement error ε_j is uncorrelated with college size, i.e., $E[\varepsilon_j | k_j] = 0$. Under these identifying assumptions,

[19]We show in Appendix D that local second-order conditions are satisfied.

[20]An outline of the mapping is reported in Appendix C. A detailed discussion of this mapping and its properties is available upon request from the authors.

[21]See Appendix E for an outline of the algorithm that we use to check for global optimality.

we can then estimate the parameters of the cost function using ordinary least squares (OLS). (Of course, we do not need to use a quadratic functional form, we can be fully nonparametric at this stage of the analysis.)

Alternatively, we can follow more recent practice in the industrial organization literature and interpret the error term in the cost function (4.1) as an unobserved shock to the cost function. In that case, the assumption that $E[\varepsilon_j|k_j]=0$ is likely to be violated because differences in costs across colleges will affect their behavior. We may, therefore, expect that ε_j and k_j are correlated, as discussed, for example, in Berry (1994). In that case, we need to find instruments for k_j to estimate the parameters of the cost function. Based on the structure of the model, functions of the rank of endowment income may serve as valid instruments.

The main problem associated with introducing unobserved elements to the cost function is that we need to alter the underlying equilibrium model, i.e., we need to account for idiosyncratic differences in costs among colleges. In the model outlined in the previous section, we have assumed that all colleges have identical cost functions. Although allowing for heterogeneity in cost functions is promising, we do not explore it in detail in this paper. However, we explore instrumental variable (IV) estimation in Section 5 as part of a sensitivity analysis.

4.2. *The Demand Side*

We assume that the joint distribution of log income and ability among students who attend college is bivariate normal:

$$(4.2)\qquad \begin{bmatrix} \ln(y) \\ b \end{bmatrix} \sim N\left[\begin{pmatrix} \mu_{\ln(y)} \\ \mu_b \end{pmatrix}, \begin{pmatrix} \sigma^2_{\ln(y)} & \rho\sigma_{\ln(y)}\sigma_b \\ \rho\sigma_{\ln(y)}\sigma_b & \sigma^2_b \end{pmatrix}\right].$$

We use a Cobb–Douglas specification for q_j:

$$(4.3)\qquad q_j = I_j^{\omega}\theta_j^{\gamma}d_j^{\psi}, \quad \omega, \gamma, \psi > 0.$$

Household preferences are also Cobb–Douglas:

$$(4.4)\qquad U(y-p_j+a_j, q_j, b) = (y-p_j+a_j)q_j b^{\beta}.$$

Note that this specification implies that the own-ability elasticity of demand for quality is zero, so ability sorting that arises in equilibrium is driven by pricing of the ability externality.[22]

Our approach to estimation utilizes both college-level and student-level data. For each college, we observe the market size, the endowment, and the

[22]The elasticity of own ability in the utility function, β, is not identified because it does not affect equilibrium outcomes.

price caps. Moreover, for a subset of colleges in our data set, we also observe a rich cross-sectional sample of students of size N. For each student, we observe an income and ability measure. Because we will assume that income and ability are measured with error, let us denote these observed measures with $\tilde{y}$ and $\tilde{b}$. We also observe the price paid by the student, denoted by $\tilde{p}$, and the college that is attended.

We impose all restrictions that arise from equilibrium in our estimation procedure. We can compute equilibria for (reasonably aggregated versions of) our model for each parameter vector. As above, let $p_j(b, y)$ denote the price predicted by our model, which is a deterministic function of (b, y) given q_j and a_j.[23] Given the structure of our model, tuition is given by

$$p_j(b, y) = \min\{p_j^m, r_j(b, y)\}. \tag{4.5}$$

Hence institutional financial aid is given by

$$a_j^I(b, y) = \max\{0, p_j^m - r_j(b, y)\}. \tag{4.6}$$

We assume that aid is measured with error.[24] We observe

$$\tilde{a}_j^I(b, y) = \max\{0, a_j^I(b, y) + \varepsilon_j^a\}, \tag{4.7}$$

where ε_j^a is independent and identically distributed normally distributed with mean zero, constant variance σ_a^2, and density function $g(\cdot)$. The likelihood of observing $(\tilde{a}_j)$ conditional on (b, y) is given by

$$l(\tilde{a}_j^I|b, y) = \mathbb{1}\{\tilde{a}_j^I = 0\} \Pr\{\varepsilon_j^a \leq -a_j^I(b, y)\} + \mathbb{1}\{\tilde{a}_j^I > 0\} g(\tilde{a}_j^I - a_j^I(b, y)), \tag{4.8}$$

where $\mathbb{1}\{\cdot\}$ denotes an indicator function.

The conditional density of observing an individual with characteristics (b, y) in college j is given by

$$f_j(b, y) = \begin{cases} f(b, y)/k_j, & \text{if } (b, y) \in A_j, \\ 0, & \text{otherwise,} \end{cases} \tag{4.9}$$

where $f(\cdot)$ is the joint density function of (b, y). This likelihood function in (4.9) assigns zero probability to observations $(b, y) \notin A_j$. Hence the likelihood function for any particular sample will not be well defined. To obtain a nondegenerate likelihood function, we assume that income and ability are also measured with error.[25] Let b (y) denote unobserved ability (income) and let $\tilde{b}$ ($\tilde{y}$) denote the observed ability (income), which includes measurement

[23] We suppress q_j and a_j as arguments of $p_j(\cdot)$ and $r_j(\cdot)$ henceforth.
[24] We assume that the price caps of each college are measured without error.
[25] Our approach is thus similar in spirit to work on kinked budget constraints by Hausman (1985).

error. Let $h_b(\tilde{b}|b)$ and $h_y(\tilde{y}|y)$ be the corresponding density functions. The likelihood of observing $(\tilde{a}^I_j, \tilde{y}, \tilde{b})$ in school j is then given by

$$(4.10) \qquad l_j(\tilde{a}^I_j, \tilde{y}, \tilde{b}) = \int\int_{A_j} \frac{l(\tilde{a}^I_j|b, y) h_b(\tilde{b}|b) h_y(\tilde{y}|y) f(y, b)}{k_j}\, db\, dy.$$

The integral on the right-hand side of (4.10) is evaluated numerically.[26] The log-likelihood function for a sample of N students is then simply given by

$$(4.11) \qquad L = \sum_{n=1}^{N}\sum_{j=1}^{J} w_n d_{jn} \ln(l_j(\tilde{a}^I_{jn}, \tilde{b}_n, \tilde{y}_n)),$$

where w_n is the weight associated with observation n, and d_{jn} is equal to 1 if individual n attends school j and 0 otherwise. Weights are necessary to reflect the sampling design of the National Center for Education Statistics (NCES) data set used in the analysis.[27]

The parameters of the likelihood function can be decomposed into the structural parameters of the equilibrium model and the parameters of the distributions of the measurement errors. Maximization of the likelihood function is computationally intensive because we need to solve for the equilibrium of the model for each evaluation of the likelihood function. The estimation procedure consists of an outer loop that searches over the parameter space and an inner loop that computes the equilibrium, boundary loci, and the choice probabilities, and evaluates the likelihood function for each parameter vector.

4.3. *Discussion*

It is fairly intuitive that the parameters of the quality and utility functions are identified from the observed variation in attendance decisions of students and financial aid received by students in our sample. Since colleges engage in price discrimination, we observe variation in institutional aid within and across colleges. One technical difficulty arises because income and ability are assumed to be measured with error. This induces an error-in-variables problem in the analysis. However, the properties of the econometric model allow us to decompose the distribution of observed income and ability into the latent distribution of income and ability, and the distribution that reflects measurement errors. The variances of income and ability determine the distribution of households by income and ability in equilibrium of the model. The variances of the error

[26]For a discussion of simulation in estimation, see, for example, Pakes and Pollard (1989), McFadden (1989), and Gourieroux and Monfort (1993).

[27]We use a trimmed estimator to avoid results driven by outliers.

components do not affect equilibrium. The variances of the measurement errors are thus ultimately identified from variation of choices, holding observed income and ability fixed. If there is much variation in choices at observed income and ability levels, we will need large variances in measurement error to reconcile our model with the observed distribution of students across colleges. Hence we can estimate the variances of the measurement errors and correct for the bias in the pricing and admission equations to obtain a consistent estimator for the parameters of the utility function.[28]

It is also useful to compare our approach for demand side estimation to other commonly used approaches in the literature on differentiated products. There are many similarities, but there are at least four important differences. First, our specification of the model does not contain additively separable idiosyncratic errors in the utility function (as typically assumed in random utility models). Although we could add these error terms to our specification of utility in (4.4), it would significantly alter the structure of the model. The resulting model would more closely correspond to a model developed by Anderson and de Palma (1988) that has different equilibrium properties. We thus cannot add additively separable errors to our utility function in estimation without fundamentally altering the equilibrium properties of the model. To generate a well-behaved likelihood function, we therefore do not rely on additively separable error terms in the utility function, but rather introduce measurement error in income and ability.

Second, most popular aggregate discrete choice estimators, such as Berry, Levinsohn, and Pakes (1995) or Epple, Romer, and Sieg (2001) do not rely on full solution algorithms, i.e., these studies do not compute equilibria of the underlying model to estimate the parameters of the model. Partial solution estimation algorithms have some obvious advantages, but it is hard to see how they can be applied in our context. Much of the identification in our model arises from the observed variation of prices within colleges. To exploit this variation in estimation, we need to characterize optimal pricing functions and hence compute equilibria. Computing equilibria in estimation is of course costly. Hence in this study, we estimate only models in which the choice set is a lot smaller than in applications that do not rely on full-solution algorithms. We discuss these issues in detail in the next section.

Third, and perhaps most importantly, our approach to identification differs from state-of-the-art differentiated product models in the following fundamental way. Previous studies typically treat product specific characteristics as exogenous. Endogenous product characteristics are inherent in our framework in which key "product" characteristics, such as peer quality and expenditures, are determined by choices of students and colleges. Thus, in our approach, we do not exploit the variation in these types of characteristics. Instead we exploit variation in endowments and price caps. In estimation, we compute

[28]A formal analysis of identification is available upon request from the authors.

the equilibrium and treat all other characteristics, such as instructional expenditures and peer quality measures, as latent variables. Much of the recent literature on estimating differentiated products revolves around dealing with unobserved product characteristics. In our approach, it is irrelevant whether a product characteristic is observed or not. Almost all product characteristics are treated as latent variables in estimation.[29]

Finally, most recent papers in industrial organization treat costs as latent variables and identify the parameters of the marginal costs function from the first-order condition that prices, markups, and marginal costs need to satisfy in equilibrium. This approach ultimately rests on functional form assumptions. Moreover, fixed costs are typically not identified. In our data set, total costs can be imputed with error from college-level data. Identification thus exploits variation in observed costs and college sizes within the population of colleges. We assume that there are no unobserved differences in cost functions among colleges.[30] To exploit fully this feature of our approach, it would be desirable to estimate a model with a large choice set, allowing for unobserved differences in initial endowments or technology. We view this as important future research.

5. DATA

5.1. *College Data*

We have assembled a comprehensive data base for public and private colleges and universities in the United States for the academic year 1995–1996. Peterson's conducts a survey of all colleges and universities, obtaining information on faculty resources, financial aid, the distribution of standardized test scores, and a host of other variables. We have supplemented this data set with information on educational expenditures and endowments from the National Science Foundation (NSF) web-accessible computer-aided science policy analysis and research (WebCASPAR) data base. The Peterson's data base contains a total of 1,868 four-year colleges and universities within the United States. We view our model as being better suited to characterize private rather than public institutions. Public universities and colleges are, therefore, not included in our sample.[31] We also do not consider private colleges

[29]Most of the recently proposed estimators that deal with unobserved product characteristics typically rely on large J asymptotics. See, for example, Berry, Linton, and Pakes (2004). In contrast, our estimator of the demand side parameters requires only that N, the number of students in our sample, be large.

[30]One of the advantages of the approach in Berry, Levinsohn, and Pakes (1995) is that it allows for unobserved differences in marginal costs in a simpler supply model with exogenous characteristics and no price discrimination.

[31]Given the presence of a substantial number of selective public institutions, this is an important simplification. Modifying our theoretical framework to reflect objectives and constraints of public institutions is an important task for future research. First, however, it seems prudent to investigate how well the framework we have developed is able to capture the admission and pricing decisions of private institutions.

TABLE I

DESCRIPTIVE STATISTICS: PRIVATE COLLEGES[a]

Variable	Mean	Std. Dev.	Minimum	Maximum
Enrollment	1,951.69	1,935.26	66.00	1,9737.00
Market share	0.001302	0.0012911	0.000044	0.013167
Price cap	12,226.89	3,736.33	6,000.00	22,000.00
Endowment income per student	357.13	865.31	0.00	11,125.88
Instructional expenditures	4,924.22	2,309.43	847.24	17,749.94
Imputed custodial costs	2,894.64	1,673.00	15.76	9,069.95

[a]Sample size is 768.

that are highly specialized, do not have a regular accreditation, have price caps of less than \$6,000, or have missing values for key variables that are the focus of our analysis. This leaves us with a sample of 768 private universities and colleges.

Table I provides some descriptive statistics for the 768 colleges in our sample. In our model, we differentiate between exogenous differences in fundamentals, such as price caps and endowments, and endogenous differences in observed characteristics such as expenditures and peer quality measures. We find that there is large variation in endowment and price caps, as well as in observed characteristics.

One of the advantages of our data is that we can impute (custodial) costs ($V(k) + F$) by using the budget identity of our model as explained in Section 4.1. Table I shows that average custodial costs are approximately \$2,894 with a standard deviation of \$1,673 (in 1995 dollars). Based on the imputed costs, we can estimate cost functions for the colleges in our sample. First we estimate a simple quadratic cost function using OLS. We then consider alternative restricted least squares estimators that impose cost-minimizing college sizes of 0.0025 and 0.003, respectively, where we measure size as a proportion of the student population. We find that these restricted estimators typically perform better than the unrestricted estimators. As part of our sensitivity analysis, we also consider simple IV estimators. Price caps and endowments are used as instruments for college size. However, we find that the IV estimates are less precise than the least squares estimates. Table II summarizes the main results.[32] Given a quadratic cost function, we can define the optimal school size

[32]The cost estimates in Table I and the results reported in Table II are based on the subsample for which imputed costs are strictly positive. Using the full sample lowers the cost estimates and increases the estimated standard errors. We also rescaled the Peterson's financial aid variable to make it comparable to our financial aid estimates based on the National Post-Secondary Student Aid Study sample. The main qualitative results about the shape of the average cost function are not sensitive to this adjustment.

TABLE II

AVERAGE CUSTODIAL COST FUNCTION

	Parameter c_1		Parameter c_2	
Model	Estimate	Std. Error	Estimate	Std. Error
1. OLS: $F = 0.940$ (0.194)	1,919	178	22,783	21,716
2. OLS: $F = [k^*]^2 c_2$ and $k^* = 0.003$	1,905	181	64,001	20,069
3. OLS: $F = [k^*]^2 c_2$ and $k^* = 0.0025$	2,242	172	28,602	22,192
4. OLS: $F = [k^*]^2 c_2$ and $k^* = 0.003$	1,358	635	148,968	77,015
5. GLS: $F = [k^*]^2 c_2$ and $k^* = 0.003$	2,392	258	45,991	22,343
6. IV: $F = [k^*]^2 c_2$ and $k^* = 0.003$	4,440	1,229	−174,754	132,703
First stage of the IV regression: $R^2 = 0.089$				

k^* as well as average and marginal costs as

$$k^* = (F/c_2)^{0.5}, \qquad (5.1)$$
$$AC = F/k + c_1 + c_2 k,$$
$$MC = c_1 + 2c_2 k.$$

The implied average cost functions are initially quite steep, but once college size is larger than 0.002, the curve flattens out considerably.

5.2. *Market Structure and Aggregation*

Identification and estimation of the remaining parameters of the model requires us to solve for the equilibrium of the model. There are 768 colleges and universities in our data set. Solving our model for such a large value of J poses major computational challenges that we have not yet solved. Consequently, we aggregate colleges with similar observed characteristics into six groups, treating each group as one college in the estimation. By aggregating colleges, we also abstract from a number of idiosyncratic factors such as regional preferences that may be important at a disaggregate level, but are likely to be less important at a more aggregate level. We first rank the 768 private colleges by their price caps. We then define six groups of colleges using the following thresholds:

$$\text{group 1:} \quad 6{,}000 \leq p_j^m \leq 11{,}500, \qquad (5.2)$$
$$\text{group 2:} \quad 11{,}500 < p_j^m \leq 13{,}500,$$
$$\text{group 3:} \quad 13{,}500 < p_j^m \leq 15{,}000,$$
$$\text{group 4:} \quad 15{,}000 < p_j^m \leq 18{,}000,$$
$$\text{group 5:} \quad 18{,}000 < p_j^m \leq 20{,}000,$$
$$\text{group 6:} \quad 20{,}000 < p_j^m.$$

This aggregation makes sure that schools within each group have similar price caps, which is important for determining financial aid.

Because we aggregate colleges into college types, we need to construct an aggregate cost function. To illustrate the basic idea, suppose there are n colleges in a group. Treating colleges within a group as identical permits us to derive the cost function for the group from the estimated parameters. Let $K = nk$ be the population of students served by the group of colleges. Then the cost function for the group can be written as a function of the population served,

$$
\begin{aligned}
C^c(K) &= nc^c(k) \\
&= nF + c_1 nk + c_2 nk^2 \\
&= \bar{F} + C_1 K + C_2 K^2,
\end{aligned}
\tag{5.3}
$$

where $\bar{F} = nF$, $C_1 = c_1$, and $C_2 = c_2/n$. Following the preceding logic, we treat the cost function of a college group as quadratic, where the fixed cost component is proportional to the fixed costs of an individual college. To illustrate this approach, consider the parameter estimates of model 4 reported in Table II. For these estimates, we obtain, for $n = 100$: $\bar{F} = 134.10$, $C_1 = 1{,}358$, $C_2 = 1{,}490$, and $k^* = 0.30$. In the next section, we also explore how sensitive the estimates of the remaining parameters of the model are to the choice of the cost function.

5.3. *Attendance and Financial Aid*

Our primary data source is the National Post-Secondary Student Aid Study (NPSAS) obtained from the National Center for Education Statistics (NCES).[33] The NPSAS contains extensive information for a sample of students. Of particular relevance for our work, the NPSAS contains the student's performance on standardized tests (either SAT or ACT), income of the student's family, and information about the financial aid received by the student. We have secured from the NCES a restricted-use version of the NPSAS that contains student-level data for 1995–1996 and links each student in the sample to the college the student attended in academic year 1995–1996. We study four-year private colleges and universities. For a given wave of the NPSAS survey, the NCES chooses a set of colleges and universities. It then selects a sample of students from within each of those institutions. Our sample consists of 1,755 incoming freshman students.[34] We use SAT score as our measure of

[33] A drawback of this data set is that it does not include students who choose not to attend college. As a consequence, we do not model the participation decision in this analysis. Including students who do not attend college in the analysis would be desirable, because many public policies are designed to encourage college attendance of "marginal" students. Due to the data limitations discussed above, we cannot evaluate these types of policy effects in this study.

[34] In selecting our sample of students, we deleted observations for students with athletic scholarships, because their criteria for admission may not conform to the spirit of our analysis.

TABLE III

DESCRIPTIVE STATISTICS BY COLLEGE TYPE[a]

College Number	Number of Students	Price Cap	Endow. Income	Market Share	Mean Income	Mean SAT Score	Expend. per Student	Inst. Aid	Other Aid
1	377	9,101	167	0.284	51,267	957	3,801	2,564	4,018
2	327	12,692	306	0.181	57,692	1,011	5,078	4,803	3,962
3	345	14,423	221	0.183	62,764	1,060	6,484	4,964	4,248
4	263	16,175	438	0.140	64,005	1,076	6,836	6,972	4,083
5	277	19,074	1,132	0.128	67,816	1,132	9,423	8,651	4,576
6	166	21,820	1,906	0.065	73,816	1,237	10,404	7,898	5,465

[a]Expenditures and endowment data are from the NSF WebCASPAR data base. All remaining variables are from the NCES.

student ability, making the standard conversion for those who took the ACT. The mean SAT score is 1,054 with a standard deviation of 196. Mean income is 60,080 with a standard deviation of 39,455. The correlation between income and SAT score is 0.24. We use these estimates as parameters of the distribution of measured income and ability. We then estimate the model, allowing for measurement error in both income and ability.

We match each student in the NPSAS to one of the six groups defined above. Table III reports the weighted means for the six college types in our analysis.[35] The average price caps range from $9,101 in the lowest ranked college group to $21,820 in the highest ranked group. We find that there is a hierarchy in mean income that follows the ranking among colleges. Mean income ranges from $51,267 in the lowest ranked college to $73,816 in the highest college. The same hierarchy holds for tuition, institutional aid per student, expenditure per student, and endowment per student.[36] Students receive substantial amounts of financial aid from the institution they attend. Mean institutional financial aid ranges from $2,564 to $8,651.

Finally, we need a simple representation of the noninstitutional aid formula. Approximating this formula is challenging, however, because noninstitutional aid comes as federal aid, state aid, or private aid, and aid from each source comes in a variety of forms. Noninstitutional financial aid takes the form of grants, loans, work–study aid, and other forms. The NPSAS data include a number of measures of noninstitutional aid received by the sampled students. One measure of noninstitutional aid, a_n, can be defined as a weighted sum of grants, gr_n, loans, lo_n, and work–study aid, ws_n: $a_n = gr_n + 0.25lo_n + 0.5ws_n$. The weights used in this formula are somewhat arbitrary, but are similar to

[35]All empirical results reported in this paper use weights provided by the NPSAS to account for the sampling design used by the NCES.

[36]Mean institutional aid is lower in the highest ranked group than in the next highest, however.

ones used in the literature (Clotfelter, Ehrenberg, Getz, and Siegfried (1991)). We find that the fraction of students in each rank of colleges receiving positive amounts ranges from 0.9 to 0.95. Mean financial aid amounts are similar across college types as reported in Table III. The mean aid ranges from \$3,962 to \$5,465. We pool the data for all students in our sample and regress the total noninstitutional aid amount on a set of regressors to obtain the noninstitutional aid function $a_{jn} = 3978 - 0.024 y_n + 0.121 p_j^m$. This function then yields federal financial aid payments that approximate those observed in the data. We found that merit aid was not significant.[37]

6. EMPIRICAL RESULTS

6.1. *Parameter Estimates*

We estimate the remaining parameters of the model using a full information maximum likelihood estimators. Table IV reports the parameters estimates and estimated standard errors for the model. We explored a number of different approaches for estimating the standard errors of the parameters of the model. We find that standard first-order Taylor expansions yield estimated standard errors that are very small—probably too small to be credible. Higher order asymptotics are problematic in our application because the likelihood function does not have analytical derivatives. We, therefore, report bootstrapped standard errors in Table IV.[38]

We find that the estimate for γ, which measures the effect of peer ability in quality, is positive and takes a value of 0.43. This finding supports our model's prediction that individuals sort based on the perceived quality of their peer group. We find that the estimate for ψ, the coefficient that measures the effect of relative income, is 0.12. To attract students from lower income backgrounds,

TABLE IV
DEMAND SIDE PARAMETER ESTIMATES

	Measurement Errors			Quality Function		
	$\sigma^e_{\ln(y)}$	σ^e_b	σ^e_p	γ	ω	ψ
Parameter estimate	0.531	170.33	3,431.07	0.432	0.084	0.120
Estimated standard error	0.007	2.30	222.02	0.028	0.003	0.012
Cost function: $c_1 = 1{,}358$, $c_2 = 2{,}420$, $F = 134.10$, $k^* = 0.2354$						

[37]In principle, we could have estimated the parameters of the federal aid formula simultaneously with the other parameters of the model. Given the various sources of noninstitutional aid and the inherent complexities of allocating such aid, any formula used in empirical analysis will be a rough approximation of the procedures used in practice.

[38]We use 30 bootstrap iterations. Each iteration takes, on average, 24 hours on a PC.

colleges give financial aid that is inversely related to income as detailed below. The estimate of ω, the coefficient that determines the demand for instructional expenditures, is 0.084. Instructional expenditures are a substantial component of college quality, but have markedly lower elasticity than the peer measure.

The estimates in Table IV also suggest that both income and ability are measured with error. The estimated standard deviation of observed ability (log income) is 195 (0.605). The point estimates for the standard deviation of measurement error in the logarithm of income cluster is 0.53, which implies an estimate of the standard deviation of the latent income distribution of 0.29. Similarly, the point estimates for the standard deviation of the measurement error in ability is 170. Hence the estimated standard deviation of ability is approximately 96. These estimates imply that there is a significant amount of variation in observed ability and income that is not explained by our model. Undoubtedly, income and ability are measured with error. Colleges often use measures of ability to determine admission and financial aid policies that are more comprehensive than simple SAT scores. Nevertheless, our estimates of the measurement components are large, suggesting that our simple equilibrium model may not capture some important aspects of admission and pricing.

Finally, we have also estimated a number of other specifications of our model. In particular, we have explored how sensitive our estimates are to the choice of the cost function and price caps. Table V summarizes the main findings. Using cost functions with less curvature typically increases the point estimates of the parameters in the quality function. Lowering the price caps typically yields slightly lower estimates for the parameters of the quality func-

TABLE V

ESTIMATES FOR ALTERNATIVE SPECIFICATIONS

	I	II	III	IV
Measurement error parameters				
$\sigma^e_{\ln(y)}$	0.5310	0.5344	0.5366	0.5317
σ^e_b	170.33	177.20	180.63	169.74
σ^e_p	3,431.07	3,141.16	3,169.37	3,422.86
Quality function parameters				
γ	0.4317	0.5924	0.6232	0.4760
ω	0.0847	0.0840	0.0950	0.0718
ψ	0.1198	0.1243	0.1261	0.1186
Likelihood function	27,433	27,436	27,427	27,438

Cost function parameters

I: $c_1 = 1{,}358$, $c_2 = 2{,}420$, $F = 134.10$, $k^* = 0.2354$

II: $c_1 = 1{,}386$, $c_2 = 1{,}920$, $F = 124.10$, $k^* = 0.2542$

III: $c_1 = 1{,}358$, $c_2 = 1{,}420$, $F = 124.10$, $k^* = 0.2956$

IV: $c_1 = 1{,}386$, $c_2 = 1{,}920$, $F = 124.10$, $k^* = 0.2542$; price caps minus 1,000 tion. The parameters of the measurement error distributions are similar for all model specifications.

6.2. *Admission and Aid Policies in Equilibrium*

The estimates reported in Table IV correspond to equilibria in the market for higher education. Table VI summarizes some of the key features of the equilibrium predicted by the estimates reported in Table IV. Recall that the variance of latent income (ability) is significantly less than the variance of observed income (ability). As a consequence, the ability and income levels reported in Table VI are not directly comparable to those in Table III. The mean of the (latent) income distribution is approximately $60,000, with a standard deviation of approximately $17,800. We thus find that the lowest ranked college has a mean that is significantly below the mean of the population, while the highest ranked colleges are clearly drawing a fair number of students from the higher percentiles of the income distribution. Similarly, the latent distribution of ability has a mean of 1,053 with a standard deviation of approximately 98. We thus find that the highest ranked college primarily draws students from the tail of the ability distribution. The lowest ranked college largely comprises students with ability levels that are significantly below the mean.

Our model also yields predictions for inputs per student, although we treat inputs as latent variables in estimation. In equilibrium, we find that educational inputs per student range between 5,054 and 10,303. Our model thus explains the inputs for most schools well, except for the lowest ranked school. The estimates imply that college qualities range from 10.72 to 12.08.

We now turn our attention to financial aid policies. As explained in the previous section, the equilibrium of our model reflects noninstitutional financial aid received by students. Table III suggests that individuals in our sample receive on average $4,172 in noninstitutional aid. Table VI shows that our model matches these levels of noninstitutional aid closely. The predicted mean of the distribution of noninstitutional aid is close to the observed mean in each college type.

TABLE VI
EQUILIBRIUM

College	Price Cap	Endowment Income	School Size	Fraction at Cap	Mean Income	Mean SAT	Inputs	Inst. Aid	Other Aid
1	9,101	75	0.331	0.408	56,185	950	5,054	1,560	3,731
2	12,692	222	0.183	0.004	50,446	1,041	6,131	4,252	4,303
3	14,423	322	0.171	0.008	57,907	1,080	6,962	5,228	4,333
4	16,175	521	0.145	0.013	64,471	1,118	7,759	6,304	4,388
5	19,074	786	0.115	0.007	71,937	1,164	8,741	8,323	4,559
6	21,820	2,030	0.054	0.034	86,178	1,227	10,303	9,599	4,550

A challenging part of our analysis is to explain institutional financial aid policies. We observe in the data that a large fraction of students receives quite substantial amounts of institutional aid. We would like to know whether our model can replicate these often generous financial packages. A comparison between Tables VI and III shows that our model explains observed financial aid well, on average.

To get additional insights into the nature of price discrimination predicted by our model, we compute the shadow prices for ability and income. Recall from (2.23) that the shadow price for ability is given by $\frac{\partial q_j/\partial\theta}{\partial q_j/\partial I}$. Similarly, the shadow price for income is $\frac{\partial q_j/\partial\mu_j^y}{\partial q_j/\partial I}$. We find that the shadow price for income ranges between -0.127 and -0.171. The estimated model predicts that a \$10,000 increase in household income raises tuition to students at the margin of switching colleges by \$1,270 in the lowest ranked college and \$1,710 in the highest ranked college. Our model thus predicts that colleges will offer less financial aid to higher income students. Our model also predicts quite significant financial aid for higher ability students. The shadow price for ability ranges between 27.11 and 42.78. The estimated model predicts that 100 SAT points lowers tuition to students at the margin of switching colleges by \$2,711 and \$4,278 at the bottom and top ends of the college hierarchy, respectively.

The ascension along the college quality hierarchy of the shadow values of ability obviously implies that ability is valued at the margin more highly in better colleges, but also is sufficient for "ability stratification." As ability rises for given income, students who switch colleges will always attend higher quality colleges. This is straightforward to show using Assumption 6 on the utility function and the fact that the allocation is consistent with effective marginal cost (EMC) pricing (i.e., Proposition 1(iv)). The ascension in absolute value of the shadow prices on income implies that lower income is valued more highly at the margin as one moves up the college quality hierarchy. The latter effects on the EMCs are more than offset by the unitary income elasticity of demand for quality in (4.4), implying that the boundaries of attendance sets are downward sloping where the price caps are not binding as in Figure 1.

These predictions are qualitatively in line with reduced form estimates. The magnitudes of the marginal effects of income and ability on financial aid are, however, larger than those found in reduced form studies. There are at least two explanations for our estimates. First, reduced form estimates often control for multiple measures of ability such as grade point average in college and high college. Second, and perhaps more importantly, ability is most likely measured with error, which biases reduced form estimates toward zero.

6.3. *An Equilibrium with Different Perceptions of Peer Quality*

Here we consider an alternative specification of the model in which the income peer measure enters the quality measure of the colleges, but does not affect the quality measure of the students. Although both colleges and students are likely to value peer quality abilities, it may be that students are indifferent to the income of their peers. In this model, we can express the college's objective function as $W = W(q_j, \mu_j^y)$, where μ_j^y is mean income and $q_j = q(\theta_j, I_j)$ enters the student utility function. The first-order conditions are the same except that the condition characterizing EMC is modified to be

$$EMC_j(b, y) = V'(k_j) + I_j + \frac{\partial q_j/\partial \theta_j}{\partial q_j/\partial I_j}(\theta_j - b) + \frac{\partial W_j/\partial \mu_j^y}{\partial W_j/\partial I_j} \frac{k_j - \frac{\partial T_j}{\partial q_j}\frac{\partial q_j}{\partial I_j}}{k_j}(\mu_j^y - y),$$

where T_j is college j's revenue function. The difference in the shadow price on the peer income effect is that it is weighted by $(k_j - \frac{\partial T_j}{\partial q_j}\frac{\partial q_j}{\partial I_j})/k_j$. The latter is positive, recall, because colleges provide more inputs than the Samuelsonian amount. However, the magnitude of this term decreases as market power increases. In turn, the power of colleges to provide need-based aid is diminished as competition among colleges increases when students do not care about the income of their peers.

Solving the model numerically is slightly more complicated than before. We need to add a set of equations that iterates on the shadow price of income, thus increasing the numerical complexity of the problem. We recompute the equilibrium using the same parameter values as those reported in Table IV. Table VII summarizes the main results. We find that the qualitative features of the equilibrium are similar to those in the baseline model. However, there are some pronounced differences as well. For example, there is more stratification by income. The mean income in the highest ranked school increases from 86,178 in the baseline model to 95,535 in the alternative specification, which corresponds to slightly more than half a standard deviation of the latent income distribution. Most of this change can be attributed to the fact that the shadow price of income decreases from −0.170 in the baseline model to

TABLE VII

EQUILIBRIUM WITH DIFFERENT PERCEPTIONS OF PEER QUALITY

College	Price Cap	Endowment Income	School Size	Fraction at Cap	Mean Income	Mean SAT	Inputs	Inst. Aid	Other Aid
1	9,101	65	0.383	0.546	51,833	969	5,384	1,149	3,835
2	12,692	234	0.174	0.015	53,073	1,051	7,029	3,349	4,240
3	14,423	333	0.165	0.017	60,542	1,086	7,851	4,338	4,270
4	16,175	547	0.138	0.006	68,447	1,121	8,826	5,233	4,292
5	19,074	875	0.104	0.028	78,142	1,165	10,040	7,010	4,411
6	21,820	3,070	0.036	0.018	95,535	1,224	12,134	7,592	4,325

−0.074 in the alternative specification. With students indifferent to the income of the peers, the college hierarchy is estimated to be more socio-economically stratified.

6.4. *Market Power and Price Discrimination*

In our model, colleges have some degree of market power over inframarginal students, setting financial aid policies such that each student is indifferent between attending his or her first or second best choice (except when constrained by the price cap). Thus the market power of a college is reflected in the extent to which it can extract extra revenues from students by charging prices that exceed effective marginal costs. To assess this, we have computed the ratio of tuition revenue of each college to the revenue that each would obtain if price were set equal to effect marginal cost. This is a version of the Lerner index. We find that colleges in our model have little market power. The highest market power is held by the top-ranked school, because it has no competition from above. Yet even for this college, tuition revenue exceeds revenue with EMC pricing by only 2.7%.

The preceding calculation holds constant the allocation of students that emerges in our equilibrium with price discrimination, exhibited in Table VI. An interesting related experiment is to analyze the equilibrium that emerges if all colleges charge students a price equal to the effective marginal cost. This experiment thus evaluates the extent to which the equilibrium is changed by price discrimination that is unrelated to cost considerations. Table VIII shows that the equilibrium without price discrimination differs significantly from that displayed in Table VI. For example, the mean ability in highest ranked school decreases from 1,227 to 1,207, which equals 0.25 standard deviation. The highest ranked college cannot use surplus tuition to differentiate itself, leading it to have a much larger market share. From this comparison, we see that many students are at the margin of switching colleges, and modest changes in tuition and quality result in large changes in attendance; the relatively flat "demand curves" that colleges face

TABLE VIII

EQUILIBRIUM WITHOUT PRICE DISCRIMINATION

College	Price Cap	Endowment Income	School Size	Fraction at Cap	Mean Income	Mean SAT	Inputs	Inst. Aid	Other Aid
1	9,101	106	0.235	0.094	56,201	928	3,882	2,830	3,730
2	12,692	207	0.196	0.000	48,581	1,020	5,672	4,713	4,348
3	14,423	304	0.181	0.000	56,602	1,064	6,599	5,593	4,365
4	16,175	484	0.156	0.000	62,791	1,102	7,388	6,677	4,428
5	19,074	679	0.134	0.000	68,596	1,143	8,245	8,826	4,640
6	21,820	1,124	0.098	0.000	82,822	1,202	9,693	10,295	4,631

are a further manifestation of the limited market power possessed by colleges.

6.5. *Redistributing Noninstitutional Aid*

We can also use our model to evaluate education reform measures. Under the current noninstitutional aid system, a large fraction of aid is given to families who are reasonably well off. Our estimates of the noninstitutional aid formula suggest that families with income above the sample median receive a significant amount of noninstitutional aid. An interesting policy simulation considers the effects of setting strict limits for the eligibility for noninstitutional financial aid programs.[39]

The effects of increasing federal aid to low-income students are a central concern of policy. The objective of such policies is to increase college attendance by low-income students. It is sometimes alleged, however, that such aid largely has the effect of bidding up college tuitions, offsetting the intended improvement in college access for low-income students. Alternatively, might such aid get passed through to students in the form of increased grants from colleges as they compete for students, again with minimal effect on improving college attendance among the poor, but for a quite different reason? Sorting out these three possible effects of government aid policies cannot be accomplished without recourse to an equilibrium framework.

To gain additional insight into the properties of the model and to measure the potential impact of changes in noninstitutional aid policies, we consider the following policy experiment. We change the noninstitutional aid policies such that households that are above a given threshold (which we set at 1.3 standard deviations above mean income, \$83,836) are ineligible for aid. We then adjust the level of aid to households below the threshold to keep average aid approximately constant. The modified aid formula $a_{jn} = \mathbb{1}\{y_n \leq 83{,}836\}(4{,}288 - 0.024 y_n + 0.121 p_j^m)$ implements this new policy. Table IX summarizes our main findings from this policy experiment.

Our simulations suggest that this policy change would lead to changes in the allocation of households among colleges and related behavioral adjustments of colleges. Comparing Tables VI and IX, we see that with noninstitutional aid redistributed from richer to poorer students, the income effect causes some of the poorer students to attend higher quality colleges and the reverse is true for some of the richer students.[40] Hence, mean income increases in the lower

[39]Keane and Wolpin (1997) analyze the impact of a college tuition subsidy on school attainment and inequality. They find that such a policy had small effects because of unobserved heterogeneity in skill endowments. Carneiro, Hansen, and Heckman (2003) also consider a full subsidy to college tuition. They report that such a policy would primarily affect people at the top end of the high-school earnings distribution. Neither paper considers the type of equilibrium effects considered in this study.

[40]We hold constant total endowment earnings of each college, so the change in per student endowment incomes between Tables VI and IX reflects size changes.

TABLE IX

EQUILIBRIUM WITH DIFFERENT NONINSTITUTIONAL AID FORMULA

College	Price Cap	Endowment Income	School Size	Fraction at Cap	Mean Income	Mean SAT	Inputs	Inst. Aid	Other Aid
1	9,101	80	0.314	0.396	56,782	948	5,007	1,630	3,820
2	12,692	222	0.184	0.003	50,080	1,036	6,012	4,371	4,622
3	14,423	324	0.170	0.008	57,221	1,075	6,815	5,376	4,619
4	16,175	529	0.142	0.010	68,056	1,124	7,873	6,189	3,479
5	19,074	707	0.128	0.000	67,244	1,146	8,093	8,977	4,948
6	21,820	1,793	0.062	0.011	81,194	1,220	9,380	10,555	3,932

quality colleges and decreases in the higher quality colleges. Educational inputs per student move in the same direction as mean income and per student institutional aid moves in the opposite direction. Mean SAT is essentially unaffected.

7. CONCLUSIONS

We have developed an equilibrium model of the market for higher education. We have shown that this model has strong predictions with regard to the sorting of students by income and ability among private colleges and the resulting financial aid policies. Our approach for identification and estimation accounts for the fact that important product characteristics are likely to be endogenous and that student characteristics are measured with error. The techniques developed in this paper allow for price discrimination and thus exploit an important source of variation in the data. We have estimated the structural parameters of the model using a combination of student-level and college-level data. The findings suggest that our equilibrium model replicates many of the empirical regularities observed in our data reasonably well.

One of the limitations of this framework is that we needed to aggregate the choice set and estimate a model of college types. Given the current computational constraints, we focused on a model with only six types. To exploit fully the inherent advantages of the techniques developed in this paper, we would like to consider models with a richer choice set. There are two possible approaches. First, we can simplify the model, ignore price caps, and restrict pricing to be equal to effective marginal costs. Based on our current knowledge, it should be possible to estimate such a model with a much larger choice set given the current set of personal computers. Alternatively, we could explore the use of parallel computing techniques as suggested, for example, by Ferreyra (2003).

We view the methods developed in this paper and our main empirical results as promising for future research. An interesting extension would be to control for additional sources of observed heterogeneity. An extension of our model that controls for minority status is feasible. Epple, Romano, and Sieg

(2002) discuss different strategies of incorporating minority status into a similar equilibrium model.[41] Equilibrium of the model then depends on whether and how racial diversity measures enter the objective functions of colleges and the different types of households. Another focus of future research should be to analyze jointly the markets for public and private education. The benefits from modeling private and public colleges in equilibrium would be substantial from the perspective of policy analysis. It is not particularly difficult to include a public college sector as an outside option in our model. However, devising an equilibrium model that can explain the observed sorting of students among a set of public and private colleges is more complicated and will require significant modifications of the theoretical model used in this study.

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APPENDIX A: PROOF OF PROPOSITION 1

(i) Suppose that $R_i > R_j$. We show a contradiction to either $q_i = q_j$ or $q_i < q_j$. In either of the latter conjectured equilibria, let college i spend on inputs so as to have a balanced budget (which is feasible by Assumption 7). College i could attract the same student body as college j because i would be spending more on inputs than j and thus would be of higher quality. Hence, we have a contradiction to quality maximization by college i in the conjectured equilibrium.

(ii) This condition follows because student choices satisfy (2.22) almost everywhere, college admission choices satisfy (2.25), and we use the market-clearance condition (3.3).

(iii) Fix y. Given that $(b_j, y) \in S$ exists such that $EMC_j(b, y) = \min\{r_j(b, y + a_j, q_j), p_j^m\}$, suppose that $EMC_j(b_j, y) > \min\{r_j(b_j, y + a_j, q_j), p_j^m\}$. By (2.22) and continuity of EMC_j and r_j in b, no types b in the vicinity of b_j will attend college j, a contradiction. Suppose alternatively that $EMC_j(b_j, y) < \min\{r_j(b_j, y + a_j, q_j), p_j^m\}$. If there exists $b < b_j$ in S, then by continuity of EMC_j and r_j, there exists $b' < b_j$ in S for which $EMC_j(b', y) <$

[41] Arcidiacono (2005) considers the effects of affirmative action on financial aid and future earnings.

$\min\{r_j(b', y + a_j, q_j), p_j^m\}$, implying $\alpha_j(b', y) = 1$ with $\succ_j$; hence $\alpha_j^m(b', y) = 1$ by (2.11). Because type (b', y) does not attend college j, $\alpha_j(b', y)\alpha_j^m(b', y) + \alpha_i(b', y)\alpha_i^m(b', y) > 1$ for the alternative $i \neq j$ that type (b', y) chooses in equilibrium, contradicting market clearance (3.3). (If i is the outside option, then $\alpha_i\alpha_i^m$ is interpreted as the proportion of type (b, y) choosing it.) If no $b < b_j$ exists for the present case with $EMC_j(b_j, y) < \min\{r_j(b_j, y + a_j, q_j), p_j^m\}$, then $\alpha_j(b_j, y)\alpha_j^m(b_j, y) = 1$ and, obviously, b_j is the minimum ability type in college j with this income. Now suppose that no $(b_j, y) \in S$ exists for which $EMC_j(b_j, y) = \min\{r_j(b_j, y + a_j, q_j), p_j^m\}$. Arguments similar to those just made preclude that (b_j, y) is in the interior of S. If b_j is on the upper bound of S, then the measure of types with this y attending college j is 0. Hence, b_j must be on the lower bound of S in this case.

(iv) Suppose (iv) does not hold. Then there exists a set with positive measure of student types Z attending college i (or choosing the outside option as discussed below) such that

$$U(y + a_i - EMC_i, q_i, b) < U(y + a_j - EMC_j, q_j, b)$$

for all students in Z and some college $j \neq i$ in $\tilde{J}(b, y)$ (or, in case of the outside option on the right-hand side, having $j = 0 = EMC_j = a_j$). Fix $(b, y) \in Z$. Since $p_i(b, y) > EMC_i$ for student attending i,

$$\begin{aligned} U(y + a_i - p_i, q_i, b) &< U(y + a_i - EMC_i, q_i, b) \\ &< U(y + a_j - EMC_j, q_j, b). \end{aligned}$$

This implies that $r_j(b, y) > EMC_j(b, y)$ and, because $p_j^m \geq EMC_j(b, y)$, that $p_j(b, y) \geq EMC_j(b, y)$. If $p_j(b, y) > EMC_j(b, y)$, then $\alpha_j^m(b, y) = \alpha_j(b, y) = 1$, contradicting market clearance (since $\alpha_i(b, y)\alpha_i^m(b, y) > 0$). If $p_j(b, y) = EMC_j(b, y)$, then utility maximization is violated because all students in Z strictly prefer j. Set $EMC_i \equiv a_i \equiv 0$ and $q_i = q_0$ if i is the outside option; then the proof goes through. (If j is the outside option, then the utility maximization contradiction necessarily arises.)

APPENDIX B: PROOF OF LEMMA 1

(i) The first and fourth lines of (2.11) obviously are necessary for college j's beliefs to be consistent with student choices. If not the second line of (2.11), then $\alpha_j^m < \alpha_j = 1$ for such students by (2.10) and (2.22), where the upper line of (2.22) conforms to $\succ_j$. Then there exists lower tuition that college j would prefer than that in (2.11), a contradiction.

Consider beliefs in the third line of (2.11) and suppose that $\alpha_j^m > 0$ for a positive measure of these students. By (2.12), a positive measure of the latter students must be indifferent to the same alternative $i \neq j$, because there are only a finite number of alternatives.[42] Alternative i cannot be the outside option, because $=_j$ implies that $p_j = EMC_j$ and the measure of students with $U(y - EMC_j + a_j, q_j, b) = U(y, q_0, b)$ equals 0.[43] Hence, i must be another college and it must be that $\alpha_i > 0$ for these students to have i as an alternative. If $\succ_i$, then $\alpha_i = \alpha_i^m = 1$ and thus $\alpha_j \alpha_j^m + \alpha_i \alpha_i^m > 1$, contradicting market clearance (3.3). If $=_i$, then $p_i = EMC_i$, but the measure of students with $U(y - EMC_j + a_j, q_j, b) = U(y - EMC_i + a_i, q_i, b)$ is also 0, a contradiction (again see the footnote 43).

(ii) Suppose that there is a positive measure set of student types (b, y) for whom $p_j(b, y) = EMC_j(b, y)$ with $\alpha_j < 1$ for some college j. By (2.14) and the linearity of EMC_j in b and y (see (2.23)), $r_j(b, y) = EMC_j(b, y)$ for almost all of these students. These students must obtain the same utility as if they are admitted to j at $p_j = EMC_j$, or $r_j > EMC_j$. Given a finite number of differentiated colleges and thus potential alternatives, this positive measure set of student types can be partitioned into positive measure subsets, where each subset has the same best alternative. (This "partition" may be a singleton.) The outside option could never be the best alternative for a subset, because the outside option is inconsistent with $r_j(b, y) = EMC_j(b, y)$ over a positive measure. For college $i \neq j$ to be a best alternative, it must be that $p_i(b, y)$ is such that $r_j(b, y) = EMC_j(b, y)$. However, because the students in this subset do not actually have access to college j, college i could increase tuition while attracting these students. Hence, optimization by college i is contradicted.

REMARK ON LEMMA 1(ii): In equilibrium, there *is* a positive measure set of students for whom $p_j = EMC_j$ and $\alpha_j = 1$ (with then also $\alpha_j^m = 0$ by (2.11)). Because college j admits these students, the alternative college they attend (that strictly prefers their attendance) chooses price to make them indifferent to attending college j.

APPENDIX C: A SKETCH OF THE EQUILIBRIUM MAPPING

In this section we sketch the construction of an equilibrium correspondence that maps $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$ into $(k_j, \theta_j, \mu_j^y, I_j)_{j=1}^J$. Let $x' = T(x)$ denote the equilibrium mapping where $x \in X \subset \mathbb{R}^{4J}$. Define $X = [0, 1]^J \times [0, I_{\max}]^J \times [0, \theta_{\max}]^J \times [0, \mu_{\max}^y]^J$.[44] We can show that the underlying set X, on which we de-

[42]The measure of students for which $\min\{r_j, p_j^m\} = p_j^m$ and $=_j$ holds is 0, so almost everywhere one of these students must have $p_j = r_j$ and be indifferent between j and a best alternative.

[43]We are assuming a_j, which is exogenous, is not contrived to undermine our arguments. More formally, these results hold generically in the space of noninstitutional aid functions.

[44]Variables $\theta_{\max}$ and $\mu_{\max}^y$ are the upper bounds of the ability and income distributions, respectively. The variable $I_{\max}$ is defined as the maximum expenditure per student that can be sustained by the college with the highest endowment. Given that the maximum endowment is too small to cover the fixed cost of operating a college, $I_{\max}$ must be finite.

fine the local equilibrium mapping, is a nonempty, compact, and convex subset of $\mathbb{R}^{4J}$.

We assume that the quality of the outside option q_0, and hence the utility $U_0(y, q_0, b)$, is fixed. The number of households that choose the outside option is given by $k_0 = 1 - \sum_{j=1}^{J} k_j$.

To define the equilibrium mapping, we partition the underlying space X into two sets. The first contains the points at which the measure of households (b, y) for which the set of colleges that are the best choice has more than one element is zero. This set is denoted by X^{ni}. The second set of points is the complement of the first set, denoted by X^i. We can prove that for $x \in X^i$, there exist at least two colleges i and j that are identical in k, I, θ, and μ^y.

For any point $x \in X^{ni}$, we define a mapping $x' = T(x)$ that maps X^{ni} into X. Denote with $k' = T_k(x)$ the projection of $T(x)$ on k. Similarly define the projections $I' = T_I(x)$, $\theta' = T_\theta(x)$, and $\mu^{y'} = T_{\mu^y}(x)$. The local equilibrium mapping is constructed as follows:

(i) Given x, we have θ_j, I_j, k_j, and μ_j^y for each college, including k_0 for the outside option.

(ii) The choice set of a household (b, y) is given by all colleges such that

$$EMC_j(b, y) \leq p_j^m.$$

(iii) For each college in the choice set compute the utility:

$$U_j = U(y - EMC_j + a_j, q_j, b).$$

(iv) Rank colleges by utility, determine first best (fb) and second best (sb) choices for (b, y), and compute attendance sets for each college.

(v) Given the attendance sets of each college, compute θ', k', $\mu^{y'}$, and k_0'.

(vi) Compute the utility of the best alternative (second best choice). If the second best is another college, the reservation utility is given by

$$U^A(b, y) = U(y - EMC_{\text{sb}} + a_{\text{sb}}, q_{\text{sb}}, b).$$

If the second best is the outside option, $U^A(b, y) = U(b, y, q_0)$.

(vii) Compute the reservations price r_{fb}:

$$U(y - r_{\text{fb}} + a_{\text{fb}}, q_{\text{fb}}, b) = U^A(b, y).$$

(viii) Compute the price by comparing the reservation price with the price cap:

$$p_{\text{fb}}(b, y) = \min\{r_{\text{fb}}, p_{\text{fb}}^m\}.$$

(ix) Compute tuition revenues by integrating the reservation prices over attendance sets. Compute I_j' for each college using the budget identity. If $I_j' < 0$, set $I_j' = 0$

This mapping is well defined because the set of households that are indifferent between their first and second best choice has measure zero. In that case, we can arbitrarily assign the indifferent households to one of the two colleges. This will not affect the mapping because it is invariant to changes on a set with measure zero. Thus we have that for each $x \in X^{ni}$, $T(x)$ is a function.

For points $x \in X^i$, there exists a set of households with positive measure that will be indifferent between the first best and the second best. To keep the notation simple, consider the case in which there are two college i and j for which there exists a set of households

$$I_{i,j} = \Big\{(b, y)\big|V_i(b, y|x) = V_j(b, y|x) > \max_{k\neq i,j} V_k(b, y|x)\Big\} \tag{C.1}$$

with $P(I_{i,j}) > 0$, where $V_i(b, y|x)$ is utility for type (b, y) in college i given x. Households that are indifferent can be arbitrarily assigned to any of the colleges in the household's preferred set. Thus any set of policies $\alpha_i^m(b, y|x)$ and $\alpha_j^m(b, y|x)$ that satisfies

$$\alpha_i^m(b, y|x) + \alpha_j^m(b, y|x) = 1 \quad \forall (b, y) \in I_{i,j} \tag{C.2}$$

is feasible and satisfies local equilibrium requirements. Each feasible assignment rule will thus yield a value for $T(x)$. As a consequence, we have for each $x \in X^i$ that $T(x)$ is a correspondence.

A fixed point of this mapping will be an allocation that satisfies (a) market clearing, (b) utility maximization, and (c) the first-order conditions of the college optimization problems. To prove existence, we need to show that, under suitable regularity conditions, the underlying set X is a nonempty compact and convex subset of $\mathbb{R}^{4J}$, and T is a compact and convex-valued upper hemicontinuous correspondence from X into X. Existence of a fixed point then follows from Kakutani's fixed point theorem.

APPENDIX D: SECOND-ORDER CONDITIONS OF THE COLLEGE'S PROBLEM

Here we show that the solution to the college's problem presented in (2.14)–(2.25) is a local maximum. It is simpler and correct here to assume that $\alpha_j^m = 1$ whenever $\alpha_j > 0$, because the college could always reduce tuition marginally below $\min[r_j, p_j^m]$ if $\alpha_j^m < 1$ and the college prefers greater attendance. Put differently, the college can obtain profits infinitesimally close to those where $p_j = \min[r_j, p_j^m]$ for all students with *attendance* $\alpha_j(b, y) \in [0, 1]$ as the college chooses. With slight abuse of notation, letting α_j denote the proportion of students (b, y) who *attend* college j and substituting from (2.18) for d_j, from (2.15) everywhere for k_j, from (2.20) into (2.19), and substituting everywhere (2.14), form a Lagrangian function

$$(D.1)\qquad q(\theta, I, 1/\mu^y) + \lambda_1\Big[\theta \int\int_S \alpha f\, db\, dy - \int\int_S b\alpha f\, db\, dy\Big] + \lambda_2\Big[\mu^y \int\int_S \alpha f\, db\, dy - \int\int_S y\alpha f\, db\, dy\Big] + \lambda_3\Big[\int\int_S \min[r, p^m]\alpha f\, db\, dy + R - F - V\Big(\int\int_S \alpha f\, db\, dy\Big) - I\int\int_S \alpha f\, db\, dy\Big],$$

where we have dropped the subscript j, suppressed the arguments (b, y), and the constraint $\alpha(b, y) \in [0, 1]$ must be taken into account in finding the saddle point. Let $T \equiv \int\int_S \min[r, p^m]\alpha f\, db\, dy$ denote tuition revenues. The first-order conditions are

$$(D.2)\qquad L_\theta = q_\theta + \lambda_1 k + \lambda_3 q_\theta T_q = 0,$$

$$(D.3)\qquad L_I = q_I + \lambda_3(q_I T_q - k) = 0,$$

$$(D.4)\qquad L_{\mu^y} = q_{\mu^y} + \lambda_2 k + \lambda_3 q_{\mu^y} T_q = 0,$$

$$(D.5)\qquad \alpha\begin{pmatrix} = 1 \\ \in [0, 1] \\ = 0 \end{pmatrix} \quad \text{as} \quad L_\alpha \begin{pmatrix} > \\ = \\ < \end{pmatrix} 0 \quad \forall (b, y),$$

where

$$(D.6)\qquad L_\alpha = f\big[\lambda_1(\theta - b) + \lambda_2(\mu^y - y) + \lambda_3(\min[r, p^m] - V' - I)\big]$$

denotes the first variation of L with respect to α.

Substitute from (D.2)–(D.4) into (D.6) and rewrite it as

$$(D.7)\qquad L_\alpha = \lambda_3 f\Big[\min[r, p^m] - \Big(V' + I + \frac{q_\theta}{q_I}(\theta - b) + \frac{q_{\mu^y}}{q_I}(\mu^y - y)\Big)\Big].$$

Using $\lambda_3 > 0$ (by the usual envelope theorem argument applied to dq^*/dR), (D.5) and (D.7) imply the admission criterion reported in (2.14), (2.22), and (2.25). As well, $\lambda_3 > 0$ and (D.3) imply

$$(D.8)\qquad k - T_q q_I > 0,$$

i.e., the result about instructional expenditures reported in (2.24).

Now we show that the solution that satisfies the first-order conditions in the equilibria we study is a local maximum for V'' sufficiently high. First, simplify notation by letting $t \equiv (b, y)$ denote student types. We rewrite the college's problem as a maximization over just $\alpha(t)$ by substituting all the constraints. (Again, we suppress the j subscript.) More specifically, find $\theta(\alpha(t))$ from (2.15) and (2.16). From (2.15), (2.17), and (2.18), find $d(\alpha(t))$. Substitute these into the budget constraint with $p(t) = \min[r(q(\cdot), t), p^m]$, as well as (2.15), and find the maximum $I^*(\alpha(t))$ that satisfies the budget constraint, which is clearly the maximizing I given $\alpha(t)$. Then the college's problem can be written

$$\text{(D.9)} \qquad \max Q(\alpha(t)) \equiv q\big[\theta(\alpha(t)), I^*(\alpha(t)), d(\alpha(t))\big] \quad \text{s.t. } \alpha(t) \in [0, 1] \quad \forall\, t \in S.$$

After some manipulation, one obtains the first variation:

$$\text{(D.10)} \qquad Q_{\alpha(t)} = \frac{f(t)q_I}{k - T_q q_I}\big(\min[r, p^m] - EMC(t)\big).$$

Using (D.8) and imposing the constraint, we obtain, of course, the same admission criterion:

$$\text{(D.11)} \qquad \alpha(t) = 1 \quad \forall\, t \in T_1, \quad T_1 \equiv \big\{t \mid \min[r, p^m] > EMC(t)\big\},$$

$$\text{(D.12)} \qquad \alpha(t) = 0 \quad \forall\, t \in T_2, \quad T_2 \equiv \big\{t \mid \min[r, p^m] < EMC(t)\big\},$$

$$\text{(D.13)} \qquad \alpha(t) = 0 \quad \forall\, t \in T_3, \quad T_3 \equiv \big\{t \mid \min[r, p^m] = EMC(t)\big\},$$

except that we have required $\alpha(t) = 0$ for $t \in T_3$ because this is the equilibrium solution, which we confirm is a local maximum.

Relevant to showing (D.11)–(D.13) implies a local maximum, compute the second variation in $Q(\alpha(t))$. Using (D.10), it can be written

$$\text{(D.14)} \qquad Q_{\alpha(t_1)\alpha(t_2)} = -\frac{f(t_1)q_I}{k - T_q q_I}V'' + \frac{f(t_1)q_I}{k - T_q q_I} \times \frac{\partial[\min[r(q, t_1), p^m] - I - (q_\theta/q_I)(\theta - b) - (q_{\mu^y}/q_I)(\mu^y - y)]}{\partial \alpha(t_2)} + \frac{\partial[f(t_1)q_I/(k - T_q q_I)]}{\partial \alpha(t_2)}\big(\min[r(q, t_1), p^m] - EMC(t_1)\big).$$

Now consider any feasible variation $h(t)$ from $\alpha(t)$ that satisfies (D.11)–(D.13). Given the constraint $\alpha(t) \in [0, 1]$, feasible deviations must satisfy

(D.15) $$h(t) \leq 0 \quad \text{for } t \in T_1; \quad h(t) \geq 0 \quad \text{for } t \in T_2, T_3.$$

Applying Taylor's theorem,

(D.16) $$\begin{aligned}\Delta q &= Q(\alpha(t) + h(t)) - Q(\alpha(t)) \\ &= \int_S Q_{\alpha(t)} h(t) f(t)\, dt \\ &\quad + \frac{1}{2} \int \int_S Q_{\alpha(t_1)\alpha(t_2)} h(t_1) h(t_2) f(t_1) f(t_2)\, dt_1\, dt_2 + h.o.t.\end{aligned}$$

For any variations $h(t)$with $h(t) \neq 0$ over a positive measure of $T_1 \cup T_2$, the first-order term dominates and, by (D.8), (D.10)–(D.12), and (D.15), $\Delta q < 0$. For any other variations, by (D.13) the first-order effect vanishes and we must examine the sign of the second-order effect. For the latter variations, $h(t_1)h(t_2) > 0$ by (D.15) and, using (D.8) and (D.14), we see $\Delta q < 0$ for V'' positive and of sufficient magnitude. Hence, V'' positive and of sufficient magnitude is a sufficient condition for the solution to be a local maximum.

APPENDIX E: GLOBAL OPTIMALITY

To check whether our local equilibrium is in fact an equilibrium, it does not seem to be possible to check all possible deviations. Instead we use the following algorithm.

(i) Consider a possible deviation by school j.

(ii) Hold the choices $(k_i, \theta_i, I_i, \mu_i^y)$ for $i \neq j$ at the levels of the old local equilibrium.

(iii) The old equilibrium defines the reservation utility function of all students who may want to attend j.

(iv) Consider a quality improvement for school j, i.e., values $\tilde{k}_j$, $\tilde{\theta}_j$, $\tilde{I}_j$, and $\tilde{\mu}_j^y$ such that $\tilde{q}_j > q_j$.

(v) Using $\tilde{k}_j$, $\tilde{\theta}_j$, $\tilde{I}_j$, and $\tilde{\mu}_j^y$ as starting values, solve the system of four equation for school j, i.e., try to find a new local equilibrium for school j.

(vi) There are three possible outcomes:

- (a) The algorithm will converge to our old local equilibrium.
- (b) The algorithm will fail to converge from the new starting values.
- (c) The algorithm will converge to a new local equilibrium and thus indicate failure of global optimality.

Finally, by randomly perturbing the quality improvements in step (iv), we trace out alternative local equilibria. Performing a substantial number of such experiments, we found that large deviations tended to provide the second outcome, whereas more modest deviations produced the first outcome. In no case was there a convergence to a new local equilibrium. This suggests that colleges are at the global optimum and hence the allocation computed satisfies all conditions of equilibrium.

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