This paper analyzes tax competition in a simple dynamic framework. We expand the classical conclusions of the tax competition literature examining the effects of imperfect capital mobility. Especially, we are concerned about Wildasin (2003) findings that "The locally-optimal tax on mobile factors is lower, the faster the speed with which factors adjust to fiscal policy", and we show that it may not be the case in the long run, suggesting that government attempts to restrict capital movements may be counterproductive. Additionally, we examine the determination of tax bands for tax harmonization.

Keywords: Tax competition, Tax bands harmonization.

JEL: H00; H87; F2.

In this article we keep the structure and classical conclusions of the tax competition literature, i.e. "independent governments engage in wasteful competition for scarce capital through reductions in tax rates and public expenditure level" (Wilson, 1999). Nevertheless, we expand those conclusions asking about the effects of imperfect capital mobility in a tax competition environment. Especially, we are concerned about Wildasin (2003) findings that "The locally-optimal tax on mobile factors is lower, the faster the speed with which factors adjust to fiscal policy", and we show that it may not be the case in the long run, suggesting that government attempts to restrict capital movements may be counterproductive. As a direct consequence, we argue that an integration process (such as in the EU) may increase the taxation level and public good provision in the long run even though national governments continued to work independently in the tax policy area.

The second point we address in this work is the existence of tax bands. They have been emphasized, without much result, as a mean to corporate tax

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harmonization in the EU. Tax bands are thought to establish a level playing field for free and fair competition between Member States (see European Commission (2001)[8]). For instance, the Commission’s proposal of 1975 sets a band for the corporate tax rate between 45 and 55%, and later on, the Ruding Report (1992) recommended a minimum statutory tax rate of 30% and a maximum of 40%. Although those bands can be justified on political terms, they are less justifiable on economics grounds. Peralta and van Ypersele (2002)[16] analyzed fiscal coordination among countries and showed that a corporate tax range will be approved by unanimity, while a minimum tax will not. Here we show how tax bands may arise as natural long run limits of the tax competition model and movements between them may be interpreted as changes in the way companies perceive the outcome of public investments1.

The theoretical literature on tax competition is large and covers a wide range of situations and modelling approaches. The most complete surveys on the area are Wilson (1999)[26] and Wilson and Wildasin (2004)[27], although new works are constantly published. Several papers have examined different aspects of intertemporal fiscal competition. Notably among them is Wildasin (2003)[25] who addresses the dynamics when it is costly to adjust the stock of a production factor. Another study with the same spirit in a two-period horizon is Lee (1997)[13].

So far, the tax competition literature does not suggest we should see tax rates converging in any sense, although they do converge2. The purpose of this paper is to think about competition and convergence as interrelated phenomena derived from the same economic forces. Tax competition tries to attract capital through reductions in tax rates, but given that capital is neither a perfectly mobile nor an immobile factor, the adjustment process requires some time that companies need to anticipate for taking a correct investment decision and governments need to consider for a gradual move of the tax.

Baldwin and Krugman (2004)[3] begin one of their articles giving some stylised facts about the taxation level in the EU. They describe a group of 6 countries (the core: Germany, Benelux, France and Italy) and a group of 4 countries (the periphery: Spain, Portugal, Ireland and Greece), and observe that "Over a period during which the integration of the European economy was steadily increasing, the average corporate tax rate in the industrial core was fairly steady, fluctuating between 7% and 10%. If anything, the average has been climbing in recent years. The rate in the poor countries, on the other hand, fell from 1965 to 1984, but has climbed dramatically ever since". Let us reconsider that assertion and plot in Figure 1 the corporate tax revenue as percentage of GDP since 1985, because Spain and Portugal are only members of the European Union since 19863.

In this case, the tax rates in nations with good infrastructure (the core) level off while tax rates in countries with less developed services (the periphery)
converge upward. Baldwin and Krugman argue that "These trends certainly suggest that something more complex than the kind of tax competition that would produce a race to the bottom is going on", and then, they turn to a new economic geography model. Instead, we will turn to the old tax competition model to observe those more complex things.

The paper proceeds as following. In Section 1 we present a classical static model. In Section 2 we introduce some simple dynamics through the anticipation of tax changes. This model will adopt a particular form in Section 3. Section 4 discusses tax bands and some policy results, and finally, Section 5 sums up the main conclusions of the article.

1 The classical static model

Let me start presenting a static (or atemporal) model of tax competition based on Wildasin (1988)[23] and Sinn (2003)[19]. It will serve to introduce notation and as a benchmark to state results comparable with those of the dynamic model. We consider a world with $n > 2$ identical countries with benevolent governments, each of them producing an homogeneous good, having as production factor capital $k_i$ which is internationally mobile and, without loss of generality, it can represent capital per worker in jurisdiction $i$. In the absence of a tax, firms will invest up to the point where the marginal product of capital is equal to the world market net return $r$ ($f'_k = r$). As usual, $f(k)$ is assumed to be
increasing and concave \( (f''_k < 0 < f_1') \)\(^4\). If the government sets a tax \( t \) per unit of capital at source and there appear other costs \( C \), the capital will leave the country until the net marginal product after taxes and costs is equal to the world market net return.

\[
f'_k - t_i - C_i(k_i, g_i; X_i) = r \quad i = 1, ..., n \tag{1}
\]

Note that the per unit of capital costs are assumed to be constant at the firm level but not at the aggregate of the economy\(^5\). Additionally, we take the capital stock as fixed for the whole world economy. Therefore, the following condition must hold:

\[
\sum_{i=1}^{n} k_i = k \tag{2}
\]

Sinn assimilates the cost function \( C(k, g; X) \) to a congestion cost. It can be interpreted as the cost of any impure public good\(^6\) such as highways, communication systems, information networks, the electromagnetic spectrum, schools, etc. And we assume that \( C_k > 0 \) (introducing a new machine increases the teaching costs in the employees) and \( C_g < 0 \) (increasing the education level provided by the government reduces the costs of the company). Furthermore, there are other factors, which also affect the private cost, such as the legal system, the corruption level, etc. All those factors are condensed in the vector \( X \).

If the units of public good are chosen appropriately, we can measure public good provision in terms of public expenditure \( w \) (i.e. in terms of the numeraire: units of private good). And we assume, for the moment, that this is equal to the public technology developed by the government \( g \).

\[
w_i = t_ik_i = g_i \tag{3}
\]

Equations (1), (2) and (3) define a system of equations that can be used to solve implicitly the equilibrium amount of capital in each jurisdiction and the net return to capital as a function of taxes and other factors \( X \). Letting \( \tau \) represent the vector of tax rates, we have \( k(\tau, X) \) and \( r(\tau, X) \).

The private good consumption of a representative household of this economy is the following:

\[
y_i = f(k_i) - k_if'_1(k_i) + \theta_iKr \tag{4}
\]

where the remuneration to labour is per-capita output less the remuneration to capital and the household also receives the remuneration of its own share of capital \( \theta_iK \).

Each country chooses its tax rate maximizing the welfare of their residents over private and public consumption

\[
\max_{t_i} U_i(g_i, w_i) \quad \text{s.t. (3) and (4)}
\]

\(^4\)Additionally, we assume the following Inada conditions \( f'(0) = \infty, f'(\infty) = 0, f(0) = 0 \) and \( f''(k) \) is finite for finite and non-zero values of \( k \).

\(^5\)This is a fundamental diﬀerence with Wildasin (2003) model. There he assumes adjustment costs at the ﬁrm level, which depend on the rate of gross investment.

\(^6\)We are considering club goods whose beneﬁts are excludable but they are partially non-rival.
Taking that function in an additive form to have perfect substitution between private and public goods, and after some straightforward manipulations we obtain

\[ U_i = f(k_i) + (\theta_i \overline{K} - k_i) - k_i C(k_i, t_i, k_i; X_i) \]  

(5)

where the last term represents the total cost per capita of capital. Observe that the government’s objective function is the value of the domestic product plus the net contribution from abroad (i.e. per capita GNP) adjusted by the externality generated by capital. Solving from the system of equations (1) and (2), we have

\[ \frac{\partial k_i}{\partial t_i} = \frac{1 + C_i}{n} \frac{n - 1}{n} < 0, \quad \frac{\partial k_i}{\partial t_j} = -\frac{\partial k_i}{\partial \tau_i}, \quad \frac{\partial r}{\partial t_i} = -\frac{1 + C_i}{n} < 0, \]

provided of course we assume \( f''' < 0 \) (decreasing marginal product) and \( 0 > C_i > -1 \) (a change in the tax reduces the congestion cost less than proportionally). These classical results display the engine of the tax competition discussion. An increase in the tax rate promotes a decline in the tax base, due to the effect of attraction that alternative low tax locations produce in the mobile capital. Additionally, this capital movement is higher the more numerous the countries in competition, given that the reaction on net returns declines to zero as \( n \) increases.

Taking taxes in other countries as given, the necessary first order condition yields:

\[ \frac{\partial U_i}{\partial t_i} = t_i \frac{\partial k_i}{\partial t_i} + \frac{\partial}{\partial t_i} \left( \theta_i \overline{K} - k_i \right) - k_i \frac{\partial r}{\partial t_i} - k_i \frac{\partial C}{\partial t_i} = 0 \]

(6)

which follows from using (1). Laussel and Le Breton (1998)[12] give sufficient conditions for the existence of the Nash equilibrium in Wildasin’s (1988) model. In this case, those conditions are strengthened and we need to assume as sufficient conditions that \( f'''(k_i) = 0 \) and \( C_i'' = 0 \). Observe that under those assumptions \( \psi_i(t_i, \tau_{-i}) < 0 \) with \( \psi_i(0, \tau_{-i}) > 0 \) and \( \psi_i(t_i, \tau_{-i}) < 0 \) for \( t_i \) so high that \( k_i = 0 \). Therefore, there exists one value of \( t_i \) such that \( \psi_i(t_i, \tau_{-i}) = 0 \).

The slope of the best-reply function is

\[ \frac{\partial n}{\partial \tau_{-i}} = -\frac{\partial r}{\partial \tau_{-i}} \left( 1 - k_i \frac{\partial C}{\partial t_i} \right) \]

indicating that tax rates are strategic complements \((0 < \frac{\partial n}{\partial \tau_{-i}} < 1)\). Expression (6) is equivalent to:

\[ t_i = \frac{C_i k_i^2 + C_i k_i \frac{\partial k_i}{\partial t_i} - \frac{\partial r}{\partial \tau_i} (\theta_i \overline{K} - k_i)}{\frac{\partial C}{\partial t_i} (1 - k_i C_i)} \]

(7)

An equilibrium of the non-cooperative game is determined by the solution of the system of the \( N \) best-reply functions as in (7). For simplicity and symmetry with the example of Section 3, let us assume that the cost function is homogeneous of degree zero in \( k \), such that we can write \( C(t_i) \), and given the assumption of identical countries, \( t_i = t_j = \ldots = t_n \) must hold in the Nash equilibrium, implying that capital is evenly distributed \( \theta_i \overline{K} = \theta_j \overline{K} = \ldots = \theta_n \overline{K} = k_i = k_j = \ldots = k_n = k^* \). Hence, expression (7) reduces to

\[ t^* = \frac{k_i C t_i}{\frac{\partial C}{\partial t_i}} \]

(8)
which is positive. In other words, a positive tax requires not only that an increase in its value reduces the cost of using the public good but it should also foster the capital to flee the country\(^7\). The tax plays a dual role in this model. It raises revenue to finance the public good, while helping to reduce capital that generates a congestion cost. Additionally, we can directly show that for all \(t > t^*\) welfare can be improved, implying that the Nash equilibrium (8) is sub-optimal. From (5) we have

\[
U(\tau, X) = f(k) - kC(k, tk; X) > U(\tau^*, X) = f(k) - kC(k, t^*k; X) \tag{9}
\]

which highlights the welfare loss of fiscal competition\(^8\). Tax competition implies a capital tax that is too low from a social point of view. If all jurisdictions agreed to raise their tax rates, they would increase the level of the public technology or amenities. Simultaneously, given the assumption of identical jurisdictions and a fixed world capital stock, such a tax increase would not have detrimental effects on the level and allocation of capital among countries. Therefore, countries cooperating in the tax setting would maximize welfare choosing the maximum possible tax rate.

From (7) we can write the best reply function that constitute the base for an empirical estimation (see Brueckner, 2002\(^4\)), as

\[
t_i = \phi(\tau - i, X) \tag{10}
\]

1.1 A graphical presentation

The results developed above for the fiscal externality can be better understood graphically. The government will choose the tax level at the point where the marginal cost is equal to the marginal benefit of a change in the tax. Taking \(r\) constant and the cost function linear for clarity of exposition, we can represent the equilibrium tax rate as in Figure 2, where we assume that the following modified Inada condition is satisfied: \(\lim_0 \frac{\partial(f(k^i) - k^ir)}{\partial t} = 0\). In other words, we assume that the inflow of capital will stop when the marginal return is equal to the marginal opportunity cost of the investment. Moreover, deriving the total cost function with respect to \(t\), we have:

\[
\frac{\partial C_k}{\partial t} = \frac{\partial C_k}{\partial t} + k_t \frac{\partial C_t}{\partial t} + k_g \frac{\partial C_g}{\partial t} + t \frac{\partial C_t}{\partial t},
\]

where from the last term in brackets we see that a necessary condition for this to be negative is that we were on the positive slope of Laffer’s curve.

The area above the Social Benefit function and below the segment NP represents the Social loss generated by tax competition and point P is the Pareto optimum given the current form of the cost function. Therefore, any upward harmonization of the capital tax rate can produce a Pareto improvement.

Let us use Figure 2 to depict a potential situation that all governments seem to be aware of. We are referring to a harmonization in the capital income

\(^7\)A review of the empirical literature analysing whether company taxation has an impact on the allocation of foreign direct investment is given by de Mooij and Ederveen (2003)\(^9\).

\(^8\)The analysis differs from that of Sinn in assuming that the owners of immobile factors are not taxed to finance the services given by the government. If that were the case, we would be back to Sinn’s idea that systems competition is efficient.
tax. All tax rates at the right of $t^*$ are welfare increasing, which follows from (9). Nevertheless, Frey and Eichenberger (1996)[11] argue that governments have incentives to leave a "harmonization cartel" "... as long as such action promises high rents and cannot be sanctioned or remains undetected by the other members of the politicians cartel. As a result, taxes will be de facto less harmonized than de jure and some competition between jurisdictions is thereby reinstalled". The effective tax rate literature (see for ex. Sorensen (2004)[22]) characterizes the real tax standing on a country as the junction of the statutory tax rate, the tax base and some economic variables such as inflation, technology growth and risk. It is clear then that a simple harmonization of the statutory tax does not suffice for coordination across countries and a complete harmonization is nearly impossible, suggesting that what can be claimed at most is a competitive equilibrium.

Suppose countries agree a harmonized tax at level $t_H$ in Figure 3. At that point country $i$ or $j$ has an incentive to cheat given that the marginal social benefit is higher than the marginal social cost for a decrease in the tax rate. If country $i$ decreases the effective tax rate, the capital will flow to the country increasing the social benefit to point B. For country $j$ the reduction of tax by his neighbour implies an outflow of capital, and therefore a lower social benefit level at $t_{jH}$. The dilemma now for $j$ is to accept the game proposed by $i$ or to propose a new agreement. If $j$ opts for the game, this country will also decrease its tax to attract capital and the game will continue until no country has an incentive to adjust its tax rate. That occurs at a point $t^*$ where the marginal social benefit is equal to the marginal social cost. On the other hand, if $j$
chooses a new agreement, the country will be afraid of being fooled again by a false promise and it will demand very hard clauses or a harmonized equilibrium tax $t^*$. This simple description contains the general arguments why tax harmonization has been proposed in the EU and it has failed in reaching an agreement. The harmonization today in the corporate income tax is scarce and different proposals did not have much success, as reported by Cnossen (2003)[7], European Commission (2001)[8], Devereux (2004)[10], Sorensen (2004)[21], European Parliament (2002)[15] and others.

2 A dynamic model

If it is too costly for a government in terms of international credibility or local budget constraints to jump to point $t^*$, we can think of a gradual move, which will constitute the optimal tax path. However, this tax setting will be anticipated by enterprises and taken into account in the design of the optimal capital path. Therefore, the standard equation for the cost of capital (see for example eq. (4.12) in Auerbach (1983)[2] or eq. (6.10) in Alworth (1988)[11]) must be modified accordingly.

For developing the intuition, let us simply consider an enterprise determining the optimal capital size that maximizes the wealth of its owners and which can move capital without cost to continue its production elsewhere. Trying to keep close to the cost of capital formulation, capital is expected to evolve according to the investment ($Inv.$) minus the part of capital consumed by an immediate change in tax. A shift in the tax rate will use up a part of current capital, depending on the degree of factor mobility ($x$). If $x = 0$ there is a perfect capital mobility, and a change in tax does not consume capital; the company may flee the country. If $x = 1$ all current capital will support the tax change. Thus,

$$\dot{K} = Inv. - xK\phi(t) \quad (11)$$
where $\phi(0) = 0$, $\phi'(\cdot) > 0$, $0 \leq x \leq 1$ and $K$ is capital in a company. Maximizing with respect to $K$

$$V = \int_0^\infty [(1-t) F(K) - Inv.] e^{-rt} dt \quad \text{s.t.} \quad (11)$$

the Euler condition defines now a new cost of capital$^9$:

$$F'(K) = r + \frac{rt + x\phi(t)}{1-t} \quad (12)$$

which is higher the more rapidly the increase in tax is expected to take place. Firms will anticipate the action of the government and will decrease the investment in the country accordingly. In the opposite case, if taxes are decreasing, firms will impulse a flow of investment to the country. Note that this intuition is similar to the one obtained by Wildasin (2003), although he assumes that the stock of capital will gradually adjust over time and the fiscal policy perturbation is once-and-for-all permanent. On the contrary, we assume here that the effective tax rate will gradually change, which does not constitute a sub-realistic idea given that tax policy is one of the favourite topics of every government, leading to frequent changes in a variety of parameters such as tax bases and tax rates. And even after new tax legislation is introduced it goes through years of further debate and modification.

The second term on the RHS of (12) can be interpreted as an effective tax ($\Upsilon$) depending on the current taxation and the anticipated tax shift. Hence, we can rewrite equation (1) as:

$$f_{k_i} - \left\{ t_i + \frac{r}{1-t} \kappa \right\} - C_t(k_i, g_i; X_i) = r \quad i = 1, \ldots, n \quad (13)$$

where $\kappa$ represents the percentage of the change in the tax that can affect the current capital level before it can leave the country. If $\kappa$ is equal zero the capital can instantaneously fly out and there is no reason for firms to change their investment decisions. If $\kappa$ is equal one all the tax change will affect current capital and firms should consider a higher effective tax today. We can think of $\kappa$ as a parameter reflecting the level of capital immobility (or irreversibility) or the degree of capital restrictions existing in the country. Given that capital may be perfectly mobile in the long run but it is only imperfectly mobile in the short run, the planned tax path should consider these restrictions in the adjustment process.

For the representative household the present value of private consumption over his infinite planning horizon is

$$\int_0^\infty y_i e^{-\rho t} dt = \int_0^\infty f(k_i) - k_i f'_i(k_i) + \theta_i e^{-\rho t} dt$$

$^9$The inclusion of a constant congestion cost does not substantially change the expression.
Similarly, the government budget constraint must consider the effect of a change in revenue.

\[ w_{it} = k_{it}(t_{it} + t_{i}\iota) \]

And the rate of public technology development is

\[ g_{it} = k_{it}(t_{it} + t_{i}\iota I) \] (14)

where \( t_{i}\iota \) is instantaneous public investment per unit of capital and \( I \) is the rate of public investment instantaneous success. This last variable represents whether a change in tax contributes to the development of the public technology. If \( I = 0 \) the new tax revenue does not have any effect on the construction of highways or schools in the country. On the other hand, if \( I = 1 \), each new euro in tax revenue improves immediately the infrastructure level. In other words, the rate of public instantaneous success represents whether public technology is sticky.

In this way, the maximization problem can be written

\[
\max U_i(t_i) = \int_0^{\infty} f(k_i) + (\theta_i k_i - k_i) r - k_i C(k_i, g_i; X_i) e^{-\rho_i} dt
\]

We leave aside for a moment the determination of the optimal tax path to observe the long run equilibrium. It is straightforward to show that when the tax change immediately affects the cost function with \( I = 1 \), the equilibrium level of \( t \) for this problem is equal to (7). However, economic intuition should indicate that something is wrong; public technology is somehow sticky. Highways are not constructed in one day (hardly in one year), better education does not bring better workers next day, etc. For this reason, we should consider the case when the speed of change in taxes does not affect the cost through the provision of the public technology, i.e. when \( C(k_i, k_i t_i) \). In that case, the long run equilibrium does not remain the same. Instead, it becomes

\[
\tau = \frac{C_{k_i} k_i^2 + C_{k_i} k_i^3}{(1 - k_i C_{k_i})} \left( \frac{\partial k_i}{\partial t_i} + \rho \frac{\partial k_i}{\partial t_0} \right)
\]

where the tax level is lower than in the static case for \( \frac{\partial k_i}{\partial t_0} < 0 \).

We can resume these observations in the following proposition in a linear exposition for the sake of clarity.

**Proposition 1** In a tax competition environment, if a change in tax has an immediate effect on costs in the form \( C(t + t{\iota}) \), the long run equilibrium is equal to the static case \( t^* \); and the long run equilibrium is lower than \( t^* \) when costs are not affected, i.e. when public technology is sticky, \( C(t) \).
Proof. For the first case, the long run equilibrium is equal to \( t_d = \frac{(1+\kappa)k_iC_\Upsilon}{\partial k_i/\partial t_i} \) and given that \( \frac{\partial k_i}{\partial t_i} = \chi \frac{\partial k_i}{\partial t_i}, t_d = t^* \). For the second case, \( \tilde{T} = \frac{k_i C_\Upsilon}{\partial k_i/\partial t_i} \) and because \( \frac{\partial k_i}{\partial t_i} = \frac{\chi(1+C_\Upsilon)}{\tilde{T}} \frac{n-1}{n} > \frac{\partial k_i}{\partial t_i} = \frac{\chi}{\tilde{T}} \frac{n-1}{n}, t^* > \tilde{T} \).

A second result can be derived observing whether public technology is sticky or not.

**Proposition 2** If a change in the capital tax does not impulse an immediate change in costs (sticky public technology), the best government strategy is to eliminate as much capital barriers as possible. With a Ramsey’s planner the capital barriers are irrelevant.

Proof. The first part follows from \( \kappa \rightarrow 0, \frac{\partial k_i}{\partial t_i} \rightarrow 0 \) and therefore, \( \tilde{T} \rightarrow t^* \). The second part follows immediately from \( \rho = 0 \).

The intuition behind is simple and clear. Firstly, because firms will consider any possible change in the tax, the real cost of capital will be higher and they will end up investing less, unless a lower tax is proposed. Therefore, governments trying to improve the well-being of current society will set a low capital tax in order to attract and keep capital. Secondly, when capital can immediately move abroad, there is not any reason to anticipate any change and the tax equilibrium level will be the same as in the static case. Finally, when a government is indifferent between current and future generations the tax equilibrium remains at level \( t^* \) of Section 1, because the planner does not seek an increase in tax trying to favour current society and therefore, there is not any expected change in the tax for the companies. On the other hand, Proposition 2 contradicts Proposition 2 of Wildasin (2003) in the long run, although here \( \kappa \) is not endogenously determined and the tax policy is anticipated, implying that no quasi-rents can be captured.

2.1 The tax path

The determination by the government of the optimal tax path greatly simplifies assuming that \( f''(k) = 0 \) and \( C''(\Upsilon) = 0 \). And using the Euler equation it is...

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\(^{10}\) Ramsey argues that it is ethically indefensible for the current-generation planner to discount the utility of future generations: "we do not discount later enjoyments in comparison with earlier ones, a practice which is ethically indefensible and arises merely from the weakness of the imagination" (Ramsey (1928)[17], p. 543).
easily found that
\[ t''_i + t'_i \left( \frac{1}{3} - x_\rho - f'' \frac{\partial k_i}{\partial t_i} \right) - t_i \left( \frac{\partial k_i}{\partial t_i} + \rho \frac{\partial k_i}{\partial t_i} \right) \left( \frac{2x}{3} - f' \frac{\partial k_i}{\partial t_i} \right) = \left( k_i - k^* \right) \left( 1 + x_\rho - f'' \frac{\partial k_i}{\partial t_i} + \rho \frac{\partial k_i}{\partial t_i} \right) - k^* C_{\tau}^\prime \left( 1 + I x_\rho \right) \]

which is a second-order linear differential equation with a constant term. Its general solution is known to be
\[ t(\tau) = A_1 e^{r_1 \tau} + A_2 e^{r_2 \tau} + t. \]

Given that \( k_i \) is function of \( \tau \) and \( \tau \), we turn now to some specific functions for more concrete results.

### 3 Special case

The model presented above gives the base for the derivation of reaction functions for specific production and cost functions. Starting with the static problem of the government, let us work with a structure simple enough to obtain analytic results. We assume the quadratic production function\(^{11}\)
\[ f(k) = \beta k_i - \gamma k_i^2 \] with \( \beta, \gamma > 0 \), and the cost function equal to
\[ C_i = a - b k_i g_i \] with \( a, b > 0 \) and \( b < 1 \).

Thus, defining \( \overline{k}_n = k^* \), we can derive the amount of capital in country \( i \), which is
\[ k_i = k^* + \frac{(t_j - t_i)(1 - b)(n - 1)}{2n\gamma} \]
and easily we can see that \( \frac{\partial k_i}{\partial t_i} = -\frac{(1-b)(n-1)}{2n\gamma} < 0 \), \( \frac{\partial k_i}{\partial t_j} = -\frac{\partial k_i}{\partial t_i} > 0 \) and \( \frac{\partial r}{\partial t_i} = -\frac{(1-b)}{n} < 0 \). The latter expressions show that the tax base declines when the tax rate in the country increases, due to the fact that capital leaves the country to equalize net returns. When the number of countries increases the effect on capital is bigger given that the change on net return declines to zero as \( n \) tends to infinity.

Replacing and maximizing (5) we obtain country \( i \)'s reaction function
\[ t_i = \frac{-2b_\gamma k^* n^2 + t_j (1 - 2b + b^2 - n + 3b n - 2b^2 n - 2n^2 + b^2 n^2)}{(-1 + b)(-1 + b - 2bn + n^2 + bn^2)} \]

which for a two-country case \( (n = 2) \) is equal to
\[ t_i = \frac{8b_\gamma k^* + t_j (1 - b^2)}{4 - (1 + b)^2} \]

\(^{11}\)This particular production function was already used by Bucovetsky (1991), Grazzini and van Ypersele (1997) and Peralta and van Ypersele (2002). It fits in the assumptions imposed except for \( f'(0) = \infty \) and \( f'(\infty) = 0 \). These conditions have to be replaced by \( \overline{T} < \frac{d}{2\gamma} \) in order to ensure positive marginal productivity.
The second order condition for the maximization problem is satisfied under the assumption \( b < 1 \). The Nash equilibrium is located at the intersection of both reaction functions as illustrated in Figure 4, where taxes are strategic complements (\( \frac{\partial t_i}{\partial t_j} > 0 \)).

In a dynamic setting things change a bit, given that firms will anticipate tax changes affecting the investment decisions in the country. The maximization problem of the government must now take (13) into account in the selection of the tax path that maximize the social utility for the current generation as well as for all generations to come.

3.1 Case 1: Flexible public technology. The instantaneous public investment affects the cost function

Let us consider first the situation when the cost function is immediately affected by a change in the tax \((I = 1)\), i.e. when \( C_i = a - b(t_i + t_j) \). Hence, the Euler
equation is
\[ t_{ii}'' - \rho t_{ii}' - t_i \frac{(1 + \kappa \rho)}{\kappa^2} = \frac{-8b\gamma k^* (1 + \kappa \rho) - t_j (1 - b^2)(1 + \kappa \rho) - \kappa t_j' (1 - b^2)(1 + \kappa \rho)}{\kappa^2 (4 - (1 + b)^2)} \]

with general solution
\[ t_i(i) = A_1 e^{r_1 i} + A_2 e^{r_2 i} + t_d \]

and characteristic roots equal to
\[ r_{11}, r_{21} = \frac{1}{2} \rho \pm \frac{\rho^2 + 4 \frac{(1 + \kappa \rho)}{\kappa^2}}{2} \]

The characteristic roots are both real (because the expression in brackets under the square-root sign is positive). Moreover, we have \( r_1 > 1 + \rho > 1 \) and \( r_2 < -1 \). On the other hand, the particular integral is positive, representing the equilibrium level of \( t \), which in a stationary state of tax in \( j \), is
\[ t_d = \frac{8b\gamma k^* + t_j (1 - b^2)}{4 - (1 + b)^2} \] (17)

\[ \text{For } n \text{ countries the Euler equation is} \]
\[ t_{ii}'' - \rho t_{ii}' - t_i \frac{(1 + \kappa \rho)}{\kappa^2} = \frac{2b\gamma k^* n^2 - t_j + \kappa t_j' 1 - 2b - n + 3b n - b n^2 + b^2 (1 - n)^2}{\kappa^2 (1 - 2b + b^2 + 2b - 2b n - n^2 + b n^2)} \]

\[ \text{The second order condition for an absolute maximum requires that the integrand function } F(t,t,t) \text{ is concave in the variables } (t,t) \text{ and that the } \lim_{\iota \to \infty} F_\iota(t,t^*) \leq 0. \text{ Applying the sign semi definiteness test we thus have} \]
\[ D_{\iota i}^2 > 0 \Rightarrow F_{\iota i} = \frac{(-4 + (1 + b)^2) \kappa^2}{8\gamma \kappa p} < 0 \]
\[ D_{\iota i}^2 = F_{\iota i} = \frac{(4 - (1 + b)^2) \kappa^2}{8\gamma \kappa p} < 0 \]

For the supplementary condition
\[ F_t = \frac{\mu}{8\gamma} \kappa^{-3} \gamma k + \kappa - \left( \frac{4 + (1 + b)^2}{\gamma \kappa} \right) \kappa^3 \end{equation} t_i + \kappa t_i + 1 - b^2 \kappa t_j + \kappa t_j e^{-\rho t} \]

which clearly tends to zero as \( t \) becomes infinite. As for the second component \( (t - t^*) \) the social benefit is concave, which suggests that as \( t \) tends infinite the deviation of any neighbouring path from the optimal path is bounded. Thus, the vanishing of \( F_t \) guarantees that the supplementary condition is satisfied.
To obtain the definite solution we make use of the initial condition \( t(0) = t_0 \). Since we are expecting the tax to converge to a long run equilibrium and neither \(+\infty\) nor \(-\infty\) is acceptable as the terminal value of \( t \) on economic grounds, we set \( A_1 = 0 \), and then the optimal tax path chosen independently by the government is

\[
t(\iota) = (t_0 - t_d)e^{r_2\iota} + t_d
\] (18)

where \( t_d \) emerges as the optimal terminal tax rate, taking the tax rate in the other country as given. The general shape of the tax is given by the following Figure.

Observe that if \( \kappa \) is equal to zero capital adjusts immediately and taxes should immediately adjust too (since when \( \kappa \to 0, r_2 \to -\infty \)), showing that the greater the speed with which factors adjust to fiscal policy, the quicker the government should look for the long run equilibrium. On the other hand, when \( \kappa \) is higher than zero the capital tax should attain the long run equilibrium following a smooth path.

### 3.2 Case 2: Sticky public technology. The instantaneous public investment does not affect the cost function

Here we consider the case when a change in tax does not immediately affect the cost function \( (I = 0) \), although it affects the revenue of the government. The logic for this case is that an immediate change in tax cannot drive an immediate change in infrastructure, it remains sticky. In other words, we conserve the cost function equal to \( C_i = a - bt_i \).
Thus the Euler equation

\[
t''_i - \rho t'_i + t_i \left( \frac{-4 + (1 + b)^2 - \kappa \rho(3 - b)}{3 \kappa^2} \right) = -8 \sigma r k^* - t_j (1 - b^2 + \kappa \rho(1 - b)) - x d'(1 + b + \kappa \rho)
\]

with characteristic roots

\[
\hat{A} \quad \hat{s} \quad \hat{t} \quad \hat{1}
\]

\[
r_1 = \frac{\hat{A}}{2} \quad \rho \pm \frac{\rho^2 + 4 \left(1 - b^2 + \kappa \rho(3 - b) \right)}{3 \kappa^2}
\]

where \( r_1 > \rho > 0 \) and \( r_2 < 0 \). Observe that the government should increase (decrease) its tax rate slower than when the rate of public investment success is one \((r_{21} < r_{22})\) because the effect of a change in tax does not reduce the congestion cost. In the same way, the long run equilibrium under a stationary state of tax in \( j \) is

\[
\tilde{t} = \frac{8 \sigma r k^* + t_j (1 - b^2 + \kappa \rho(1 - b))}{4 - (1 + b)^2 + \kappa \rho(3 - b)}
\]

The similarity with expression (17) is obvious. The optimal tax path is similar to Case 1, but attaining an inferior level \( \tilde{t} \). Also note the higher \( \kappa \) the lower the long run tax equilibrium level, given that companies will not invest as much as they should do in a situation with free capital exit, forcing the government to reduce even more the capital tax. Moreover, we can easily corroborate that with a Ramsey’s planner both expressions are equal, showing the irrelevance of capital barriers when governments are committed to respect current tax levels.

4 Long run equilibriums as bands for the tax

The analysis in the last two sections has shown two long run equilibriums, which can be interpreted as thresholds dividing the tax in three intervals. Over the interval \((0, \tilde{t})\), the capital tax rate is so low that investments will flow to the country attracted by high net returns and welfare could be increased with only augmenting the tax rate. With values over \( t^* \), capital is leaving the country and tax competition will impulse a decrease in the tax rate. This leaves as the relevant interval \((\tilde{t}, t^*)\), where public investment instantaneous success and tax competition will determine the equilibrium level. (See Figure 5).

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\[^{14}\text{Observe that the integrand is locally concave in } t \text{ along the optimal path, satisfying the Legendre second-order necessary condition}

\[ F_{tt} = \frac{-3 \kappa^2}{8 \rho e^m} \leq 0 \]
4.1 Changing perceptions

Like Rome, a project cannot be built in a day or at least there is a perception it cannot be done. In the same way, whether or not public technology is sticky is also a matter of perceptions. Let us assume in this section that the perception of public technology is only sticky in the short run. Therefore, for the instantaneous public investment to be successful there will be a previous level of infrastructure, unknown for the government and determined by business preferences. The accumulated investment per unit of capital over the equilibrium level is depicted by the area $\int_0^\infty (t_0 - \bar{t}) e^{r^2} dt$ in (18). Let $I$ in (14) be an indicator function, taking value 1 when investment attains the threshold for business $S = \int_0^T (t_0 - \bar{t}) e^{r^2} dt$ and 0 when it does not reach that value, $S \neq \int_0^T (t_0 - \bar{t}) e^{r^2} dt$. For a given $t_0 > \bar{t}$, we can think the optimal path as a succession of a decreasing tax when the threshold $S$ is not reached and an increasing tax when businesses perceive an enough development of the public technology created by government or, simply, a positive flexible relation between public technology and taxes paid.

Certainly, individuals and companies observe the government and what the government does with the tax revenue. Their perception is essential for the level of investment in the country and it will finally influence the taxation level. Each firm has its particular threshold of demands before investing, and we can suppose each country will use its resources to satisfy those investment requisites. For instance, a country trying to capture international business needs a correct communication system, one looking for technical industries needs good engineers, for Europe to take the lead in innovative industries or to become "the most competitive and dynamic knowledge-based economy in the world" by 2010 (Lisbon Council, 2000), several instruments have been established such as increasing the EU’s overall spending on R&D to 3% of GDP by the year 2010 (Barcelona European Council, 2002).

One alternative way to overcome the lack of infrastructure or direct incentives is bringing down tax rates, but as we have seen in Section 3, this situation is even worse than what a simple tax competition model may predict. There-
fore, we can consider as a normal situation governments promoting programs in favour of companies during periods of time when an exit of capital towards alternative locations occurs. Nevertheless, those actions may be reverted once companies observe the advantages the program produced in the current location. In the same way, we can think of a future time when the current accumulated investment losses effectiveness to attract capital (or the business threshold moves up) and the process with \( I = 0 \) begins again.

### 4.2 Asymmetric countries

When the rate of public investment success differs across countries, in a way such that \( I_i = 1 \) and \( I_j = 0 \), we can easily see from (17) and (19) that equilibrium is found in a point C in Figure 7 rather than in point A where perceptions are good in both countries (\( I_{i,j} = 1 \)) or in point B where business perceives a lack of correspondence between taxes and public investment (\( I_{i,j} = 0 \)). This analysis suggests that it would be in countries' own interest to develop an equivalent level of infrastructure or accumulated investment to allow them to attain a position A. Certainly, we cannot assume that a competition centre-periphery will allow countries to tax more. We need a centre-centre game in the model to have a Pareto improvement in a tax competition environment. And for that reason, rich governments (those with a well developed infrastructure) should have incentives to aid less developed countries. However, even when equilibrium tends to C, the tax in country \( j \) is higher than \( t \) implying that it may obtain resources to develop infrastructure in a future in order to attract capital and attain a long run equilibrium A. This result also tells us that initially the tax competition between a country with a well developed infrastructure and a poorer one may be temporally harmful for the more advanced country, until the time the perception
of infrastructure in the later changes.

Beyond what has been said, we can add that in an integration event (such as the European Union) capital movement limitations are by definition eliminated or reduced to a minimum, and as a direct consequence of Proposition 2, the distance between tax bands are also reduced, driving the tax rates to converge towards a high tax level $A$.

5 Conclusions

In this work we have examined a simple model of tax competition. Departing from the classical conclusions of the static model, we analysed the role played by imperfect capital mobility in the determination of the capital tax steady-state equilibrium, and how companies through the anticipation of the tax, capital restrictions and a preferred level of public infrastructure condition such equilibrium.

We developed a simple idea of how governments interact in the determination of the capital tax. Rather than taking the alarming OECD omen of "race-to-the-bottom" or a Leviathan argument, we sustain the thesis of a relevant interval. With tax values over the upper limit we should observe a downward race in the classical sense, while with too low values we should have the opposite effect. The central region should remain relevant in the long run. And even though on this area the competition between states still takes place, it will be the perception by enterprises of government investments, which will finally influence the tax
equilibrium. Obviously, this perception would be unnecessary if companies can fold up their activities in one place and immediately open them in another without costs.

References


