The econometrics of inequality and poverty

*Lecture 8: Equivalence scales*

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1 Introduction

Surveys are devoted to collecting data on households. Welfare is concerned with individuals. Up to now, we have not made the link between the two. It is evident that in order to reach the same level of welfare a family with two children need more income than a family with no children. And that a couple need more income than a single person, always to reach the same level of welfare. The usual statistical practice consists in dividing the household income by a function of the household size, say $g(n)$, so as to be able to measure the welfare of an adult equivalent, supposing that welfare is equally distributed in the household. For instance, if an household with two adults has an equivalence scale of 1 and an household with two adults an one child has an equivalence scale of 1.2, this means that the second household need 1.2 times more income than the first household in order to get the same level of welfare. Or that the cost of the first child is 20%.

2 Usual scales

Various scales were proposed in the literature and used by statistical agencies. The first equivalence scale that appeared in the literature was the Oxford scale, later named the OECD scale of 1982. This scale, and all subsequent scales give a weight of 1 to the first adult, usual the head of the household, a smaller weight to the second adult (mainly the spouse) and a smaller weight to children. There is a discussion about the age of children. The weights used in the Oxford scale are presented in Table 1.

<table>
<thead>
<tr>
<th>Member</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head of household</td>
<td>1</td>
</tr>
<tr>
<td>Other adults</td>
<td>0.7</td>
</tr>
<tr>
<td>Child</td>
<td>0.5</td>
</tr>
</tbody>
</table>

As a side remark, we can note that the French fiscal system uses a particular equivalence scale for defining the tax burden of an household. The two adults have the same weight of 1, while children have a weight of 0.5, except that the third child has a weight of 1. We shall comment on this later on.

2.1 The need for changing scales

An equivalence scale reflects the fact that there are scale economies in an household. There are collective goods which are consumed by everybody and private goods which are consumed specifically by one individual. The equivalence scale depends on the proportion of collective versus private goods in the household. This proportion can vary over time and across countries.
So we cannot keep the same equivalence scale over time and across countries. This explain the diversity of solutions which were adopted in the past. The OECD modified its equivalence scale in 1994 to modify the weight of the second adult and of the children. The weight of the second adult is decreased while a child is no longer a child after 14 years old and is counted as an adult. Canada Statistics has adopted a similar rule, but fixed the outing of childhood at 16 and gives a weight of 0.4 instead of 0.5 to this class.

The French system of taxation includes something which is similar to an equivalence scale because it introduces shares for computing taxes. Both heads of the household are counted for one share which gives a high incentive to get married. Each child receive a share of 0.5, which is again larger than the common share in OECD equivalence scales. Finally the third and subsequent children have a full share of 1, which is a strong fiscal incentive to have children. We are here in the domain of fiscal incentives and not of finding a clear statistical and economic rule for finding an adult equivalent when measuring income.

Finally, in many studies a simple parametric form is adopted for simplification instead of the linear system described up to now. If \( N \) is the size of the household, an approximation to the linear scale is simply \( N^\alpha \) where \( \alpha \) is a coefficient between 0 and 1. This restriction gives a concave form to the equivalence scale. It implies that the cost of the second child is lower than that of the first and so on. The Luxemburg scale takes \( \alpha = 0.5 \) when more common values are around 0.60.

We can summarise the effect of these different scales in Table 2. The last row of Table 2 gives the implied elasticity of consumption demand with respect to the size of the household. The per capita solution with \( \alpha = 1 \) means that each new member consumes the same amount as the previous members individually. There is no economy of scale. The extreme case \( \alpha = 0 \) means that there is a full economy of scale, which is unrealistic.

### Table 2: Modified OECD (1994)

<table>
<thead>
<tr>
<th>Member</th>
<th>Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head of household</td>
<td>1</td>
</tr>
<tr>
<td>Everybody else aged more than 14</td>
<td>0.5</td>
</tr>
<tr>
<td>Every child below 14</td>
<td>0.3</td>
</tr>
</tbody>
</table>

2.2 The influence of equivalence scales

The FES data set we have and which comes from Flachaire and Nunez (2007) is not useful for us here as it already contains an equivalence scale. In this data set, we have indications concerning the number of children and if the there is a single adult at the head of the household. In the 1979 sample, this case is relatively rare with 177 observations out of 6230.

We want to illustrate the influence of the equivalence scale on the shape of the scaled income distribution. For that, we take the data set Ilocos which is provided in R with the library ineq.
Table 3: Effect of equivalence scale

<table>
<thead>
<tr>
<th>Household size</th>
<th>per capita income</th>
<th>Oxford Old OECD</th>
<th>New OECD scale</th>
<th>Square root scale</th>
<th>Household income</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 adult</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2 adults</td>
<td>2</td>
<td>1.7</td>
<td>1.5</td>
<td>1.4</td>
<td>1</td>
</tr>
<tr>
<td>2 adults, 1 child</td>
<td>3</td>
<td>2.2</td>
<td>1.8</td>
<td>1.7</td>
<td>1</td>
</tr>
<tr>
<td>2 adults, 2 children</td>
<td>4</td>
<td>2.7</td>
<td>2.1</td>
<td>2.0</td>
<td>1</td>
</tr>
<tr>
<td>2 adults, 3 children</td>
<td>5</td>
<td>3.2</td>
<td>2.4</td>
<td>2.2</td>
<td>1</td>
</tr>
<tr>
<td>Elasticity</td>
<td>1</td>
<td>0.73</td>
<td>0.53</td>
<td>0.50</td>
<td>0</td>
</tr>
</tbody>
</table>

It contains 632 observations coming from the *Family and Income and Expenditure Survey* made in one region of the Philippines called Ilocos in 1997. The data contains household income and other information such as sex of the household head, family size, urbanity and province.

We take the most simple scale $N^\alpha$ as we have only information about $N$. We then let $\alpha$ vary between 0 and 1. For each case we estimate the income distribution.

```r
library(ineq)
data(Ilocos)
y = Ilocos$income
n = Ilocos$family.size
alpha = c(0.0,0.3,0.6,1.0)
tit = c("alpha=0.0","alpha=0.3","alpha=0.6","alpha=1.0")
split.screen(c(2,2))
for (i in 1:4){
  screen(i)
yz = y/(n^alpha[i])
yz = yz/max(yz)
plot(density(yz),main=tit[i],xlab="",ylab="",xlim=c(0,0.6))
}
```

The income distribution was normalised so as to have comparable graphs despite the different equivalence scales. When $\alpha = 0$, we have a bimodal density, where a group of richer persons appear. The importance of this group decreases when $\alpha$ is increased.

### 2.3 Family composition in France

Before applying an equivalence scale and looking at the income distribution while forgetting family composition, it is wise to have an idea of how income is distributed among the families.
Figure 1: Influence of the equivalence scale

I have taken the example of France as it is given in the 2011 report of the CONSEIL DES PRÉLÈVEMENTS OBLIGATOIRES, page 187, Table 12. We reproduce this table now and it becomes Table 4 of our chapter.

We must first of all try to figure out the different columns are computed. Net income represents the total disposable income of the household, after payment of social contributions, but before taxes and redistribution. It corresponds to the fiscal income, which means the income that is taken into account by the fiscal authorities. The standard of living corresponds to the same notion, but taking into account family composition. We can try to deduce from the comparison of these two columns which kind of scale has been used. For a couple it corresponds exactly to the new OECD scale, but the correspondance is slightly less precise for the children.

The great advantage of this table is to shed some light on the relation between family composition and income. The number of children is not uniformly distributed with respect to income,
Table 4: Standard of living and net income as a function of family composition

<table>
<thead>
<tr>
<th></th>
<th>Standard of living</th>
<th>Net income</th>
<th>Poverty rate (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Couple without children</td>
<td>27 540</td>
<td>41 310</td>
<td>9</td>
</tr>
<tr>
<td>one child</td>
<td>23 740</td>
<td>44 900</td>
<td>13</td>
</tr>
<tr>
<td>two children</td>
<td>21 550</td>
<td>47 990</td>
<td>17</td>
</tr>
<tr>
<td>three children and more</td>
<td>17 230</td>
<td>45 840</td>
<td>41</td>
</tr>
<tr>
<td>Head of household single</td>
<td>20 580</td>
<td>20 580</td>
<td>22</td>
</tr>
<tr>
<td>one child</td>
<td>15 160</td>
<td>21 420</td>
<td>39</td>
</tr>
<tr>
<td>two children</td>
<td>12 040</td>
<td>21 580</td>
<td>54</td>
</tr>
<tr>
<td>three children and more</td>
<td>6 320</td>
<td>14 470</td>
<td>82</td>
</tr>
<tr>
<td>Ensemble</td>
<td>22 260</td>
<td>34 220</td>
<td>20</td>
</tr>
</tbody>
</table>

Poverty rate is defined as the percentage of households with a standard of living lower than 60% of the median standard of living. INSEE computes the poverty rate as a function of disposable standard of living. After a continuous drop between 1996 and 2002, dropping from 14.5% to 12.9%, it is now stable at 13% in 2008.

so that an equivalence scale cannot remove the influence of the number of children on the standard of living. The rate of poverty is computed with respect to the standard of living and thus takes into account family composition. But even with this adjustment, the proportion of poor households increases with the number of children and becomes dramatic for households with three children and more.

Let us now consider the case of households with a single head. The picture is here even more dramatic. Even if net income is increasing with the number of children as in the previous case (if we drop the case with three children and more), the standard of living is strongly decreasing with the number of children and we reach an incredibly high rate of poverty of 82% for this sub-population.

We can finally try to characterise the dispersion of standard of living. It is much higher among large families and single headed households. The Gini index is 50% higher in this sub-population than in the whole population. This greater dispersion is due to the presence of very low standard of living in this category.

2.4 Taxation and redistribution in France

What is the impact of taxation and redistribution on the standard of living of different types of households? In France, redistribution is very favourable to large families and single headed families. There is nothing abnormal in this as we saw that poverty was concentrated on these family compositions.
Households having two kids or more receive 54% of the total allowances. Single persons without any children receive 17% of the total allowances. Redistribution operates from households without kids and households with one kid toward single headed families and couples with three children or more. From Table 5 (Table 13 page 189 of the report), we see that the effect of taxation and redistribution on standard of living is maximum for single headed families with three kids or more.

<table>
<thead>
<tr>
<th>Family Composition</th>
<th>Net standard of living</th>
<th>Disposable standard of living</th>
<th>Variation</th>
<th>Variation (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Couple without children</td>
<td>27 540</td>
<td>25 580</td>
<td>-1 960</td>
<td>-7</td>
</tr>
<tr>
<td>with 1 child</td>
<td>23 740</td>
<td>22 870</td>
<td>-870</td>
<td>-4</td>
</tr>
<tr>
<td>with 2 children</td>
<td>21 550</td>
<td>21 640</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>with 3 children or more</td>
<td>17 230</td>
<td>19 420</td>
<td>2 190</td>
<td>13</td>
</tr>
<tr>
<td>Isolated without children</td>
<td>20 580</td>
<td>20 030</td>
<td>-550</td>
<td>-3</td>
</tr>
<tr>
<td>with 1 child</td>
<td>15 160</td>
<td>16 840</td>
<td>1 680</td>
<td>11</td>
</tr>
<tr>
<td>with 2 children</td>
<td>12 040</td>
<td>15 020</td>
<td>2 980</td>
<td>25</td>
</tr>
<tr>
<td>with 3 children or more</td>
<td>6 320</td>
<td>12 180</td>
<td>5 860</td>
<td>93</td>
</tr>
</tbody>
</table>

These families are also the poorest ones. Despite this effect of redistribution, the poverty rate of couples with three children or more remains greater than the average (19.7% against 13.0% in 2008). The report notes that this poverty rate has decreased by 8 points since 1996.

On the contrary, the poverty of single headed families is greater than that of every other type of family, whatever the number of children. It has increased since 1996, especially since 2004, passing from 26% to 30% in 2008.

3 Theoretical principles

The various equivalence scales which were given seem to be totally arbitrary. In fact they are not. They are the result of a precise economic theory dealing with utility and household consumption. The key question and difficulty is that we want to compare households which have not the same composition and consequently not the same utility functions. We want to find a number which says by how much the income of a household has to be multiplied if an extra member is added and if this household wants to keep the same level of utility. The individual utility theory does not know how to perform welfare comparisons between households. We must introduce specific assumptions. The theory is explained for instance in Chapter 4, section 4.3 of Deaton (1997) and
also in Hourriez and Olier (1997) at a more elementary level. These are the two sources that we shall use.

3.1 The model of Prais and Houthakker

One fundamental assumption underlying equivalence scales is that there are economies of scale because consumption goods can be divided into collective goods, those which are consumed collectively by all the members of the household and individual consumption goods which are consumed only by one individual. As a collective good, we have mainly housing, as an individual good, we can quote adult clothing or tobacco. The model of Prais and Houthakker (1955) explains the household consumption of various items as a function of income and of the size of the household. Are thus explained the structure of consumption and the influence of the structure of the household from which an equivalence scale can be derived.

Consumption is divided into $K$ different items such as lodging, food, clothing, leisure, ... The size of the household is called $N$ which means the total number of persons which are members of the household. The size effect is introduced both as a deflator of income and as an explanation of a particular consumption item. After various computations which are not reproduced here, the model is written as

$$\log(C_k) = A_k + \alpha_k \log(N) + \beta_k \log(R/N^\alpha),$$

where $C_k$ is the consumption of good $k$, $R$ household income, and $N$ the size of the household. This model can also be expressed in term of budget shares $\omega_k = C_k/R$:

$$\log(\omega_k) = A_k + (\alpha_k - \alpha) \log(N) + (\beta_k - 1) \log(R/N^\alpha).$$

We have as many equations as there are consumption items, but only $K - 1$ equations are independent. This model analyses how the structure of consumption is modified as a function of $N$, when we compare two households which have the same income $R$. We have two effects:

- A size effect. The budget share of individual goods for which $\alpha_k > \alpha$ increases. The budget share of collective goods for which $\alpha_k < \alpha$ decreases.

- An income effect. When $N$ is increased, $R/N^\alpha$ decreases. The structure of consumption is modified. The budget share of luxury goods which have a $\beta_k$ greater than 1 is decreased while the budget share of primary goods which have a $\beta_k$ lower than 1 increases.

This model is not identified, as that can be easily seen by developing it.

$$\log(C_k) = A_k + (\alpha_k - \beta_k \alpha) \log(N) + \beta_k \log(R)$$

We have two regressors and three parameters. As we have only $K - 1$ independent equations, the identification problem cannot be solved by considering the system as a whole. Either we fix $\alpha$ and then, we cannot estimate the equivalence scale we were looking for, or we impose an identification constraint on the $\alpha_k$. 

8
3.2 The identification assumptions of Engel and Rothbarth

The oldest method dates back to Engel (1857). It is based on the assumption that the budget share devoted to food gives a good indication of the level of welfare, independently of the composition of the household. A large and a small family are equally well-off if they devote the same budget share to food. A poor family devotes a very large part of its budget to food. While a rich family devotes a rather small part of its budget to food. This is the first law of Engel. Let us introduce this law in the consumption model of Prais and Houthakker. We suppose that food is the good labeled 1, so that budget share of food noted $C_1/R$ depends only on the household welfare measured as $R/N^\alpha$ and not on $\log(N)$. Consequently, Engel assumption implies that

$$\alpha_1 = \alpha.$$ 

This assumption means that food is a mid-range good, between a strictly individual and a strictly collective good in term of scale economies. This assumption is verified for a poor society, like the one of Engle at his time, but not for the rich western societies of nowadays.

An alternative identifying assumption was proposed by Rothbarth (1943). The budget share of the first adult for his clothing is a good measure of the household welfare. For a given welfare, the clothing consumption of the first adult does not depend on the size of the household. If $C_2$ represent the clothing consumption of that first adult, then, Rothbarth assumption implies that

$$\alpha_2 = 0.$$ 

There are some adjustments depending on the sex of the adult. For instance, we know that a women spend 1.3 times more than a man for clothing. So for a couple, we can take $C_2 = (C_{2M} + C_{2W}/1.3)/2$. An alternative form of the Rothbarth assumption considers the total consumption for clothing of the whole household and says that

$$\alpha_2 = 1.$$ 

There is a second problem concerning clothing consumption. From the Enquête sur le Budget des Familles, it is apparent that individual living alone spend much more money on clothing than individuals living in a couple. Consequently, we can restrict Rothbarth’s identifying assumption to couples. This would mean that clothing consumption for adults living in couples does not depend on the number of children.

3.3 An estimation for France

Hourriez and Olier (1997) report an estimation of the model of Prais and Houthakker (1955) using the Enquête sur le Budget des Familles of 1985, 1989, 1995 using the identifying assumption $\alpha_2 = 0$. Various extra dummy variables were introduced. Some of these variables were socio-economic variables such as localisation and employment status. Other dummy variables concerned the status of the head of the household: single parent households and bachelors; because they have a very different type of consumption. So the scale of equivalence concerns in fact only married couples with or without children.
Table 6: Estimation of the Prais-Houthakker model

<table>
<thead>
<tr>
<th>Consumption</th>
<th>Size elasticity $\alpha_k$</th>
<th>Income elasticity $\beta_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food</td>
<td>0.74</td>
<td>0.67</td>
</tr>
<tr>
<td>Clothing</td>
<td>1.03</td>
<td>0.95</td>
</tr>
<tr>
<td>Lodging</td>
<td>0.46</td>
<td>0.38</td>
</tr>
<tr>
<td>House equipment</td>
<td>0.60</td>
<td>0.55</td>
</tr>
<tr>
<td>Health</td>
<td>0.56</td>
<td>0.40</td>
</tr>
<tr>
<td>Transport-Tel.</td>
<td>0.73</td>
<td>0.49</td>
</tr>
<tr>
<td>Leisure</td>
<td>0.92</td>
<td>0.90</td>
</tr>
<tr>
<td>Misc.</td>
<td>0.90</td>
<td>0.72</td>
</tr>
<tr>
<td>Global scale $\alpha$</td>
<td>0.69</td>
<td>0.58</td>
</tr>
</tbody>
</table>

As the Rothbarth assumption was imposed only for adult clothing, a scale for clothing was estimated. The size elasticity is consistently equal to 1, so clothing a pure individual good. The same can be said for leisure. The good for which the scale economy is the greatest is lodging with a size elasticity of 0.40 on average. This means that a family of four spend $1.7 = 4^{0.4}$ more than a single person for lodging. Leisure on the contrary is an individual consumption because the corresponding $\alpha_i$ is near 1.

The main result is given for $\alpha$ and corresponds to an equivalence scale of $N^\alpha$. The value of $\alpha$ is not constant over time. It is a function of the budget share and of the individual elasticities $\alpha_k$. In the estimations, $\alpha$ is around 0.60. If we go back to Table 3, the Oxford scale corresponds to $\alpha = 0.73$ while the new OECD scale corresponds to $\alpha = 0.53$. The Oxford scale corresponds to a now outdated structure of consumption. At that time, food was the first budget share and lodging had a much smaller share than nowadays. Scale economies were thus much smaller at that time.

When the size of a household is increased, the share of collective goods in total consumption decreases while the share of individual goods increases, just because of the different values of the $\alpha_i$. In France, the size of households has slightly decreases over the years. Consequently, the budget share of housing has increased.

This model might be too simple to describe household consumption as it does not include for instance relative price effects. A large family might decide to increase collective consumption, just because the price of individual goods has increased too much: for instance video cassettes that can be watched collectively versus individual cinema seats. For a single person, the cost can be the same between the two alternatives.
4 Subjective equivalence scales

The estimation of the previous equivalence scales relies on a particular consumption model which is rather restrictive. First of all, the literature has produced more elaborated models, such as the model of Barten, of Deaton and Muellbauer to quote only the main ones. Secondly, there is a fundamental identification problem that the econometrician has to solve by saying what is exactly welfare and how it is measured.

A second approach is possible by a direct questioning of the household, asking them what is their welfare level, most of the time using an indirect question. The task of the econometrician is then to explain the collected answers by the composition and size of the household and the level of income.

4.1 Qualitative questions

This is an important domain in the field of poverty and inequality analysis. Up to now, we have studied what can be called objective criteria which were based on quantitative data. Welfare was identified to a certain level of consumption or income. A vast field of the literature on inequality and poverty has interest on what people feel: for instance how would they define their health status. Here, a survey can ask questions on the intensity of the budget constraint of the household. How do you manage to make the both ends meet is a sensible question for the housekeeper. In the French Enquête sur le Budget des Familles, the following question was asked:

Concerning your budget, which of the following proposition fits the best your budget situation

1. It is difficult
2. you are just, you have to pay attention to your spending
3. your budget situation is fine
4. You experience a good financial ease
5. Do not know

The respondent has to situate on a subjective scale. Several factors can influence his position on that scale, only some of which being of interest for our purpose. We have to take account of the other factors in order to obtain unbiased results. Financial ease is a function of income, but also of wealth. Being the owner of one's apartment makes a great difference. Being in a large town like Paris induces an increase of spending of 28% for Paris for instance in order to get the same level of welfare. And other variables have their importance.

4.2 Ordered logit models

The probability of picking one of the answers is explained by an ordered logit model. There is a great liberty for selecting the explanatory variables at the difference of the Prais-Houthakker
model. We observe an ordered variable $Y$ which can have values 1, 2, 3,... It corresponds to the ordered answers. This observed polychotomous variable is a function of the unobserved level of welfare $Y^*$. The model says that the responder answers 1 if his unobserved level of welfare is below the threshold $\kappa_1$, answers 2 if his unobserved level of welfare is between the thresholds $\kappa_1$ and $\kappa_2$ and so on:

\[
Y_i = 1 \quad \text{if } Y_i^* \leq \kappa_1 \\
Y_i = 2 \quad \text{if } \kappa_1 \leq Y_i^* \leq \kappa_2 \\
Y_i = 3 \quad \text{if } \kappa_2 \leq Y_i^* \leq \kappa_3 \\
Y_i = 4 \quad \text{if } Y_i^* \geq \kappa_3
\]

The latent variable $Y_i^*$ is supposed to be determined by the following linear regression

\[Y_i^* = x_i^\prime \beta + u_i\]

where $u_i$ follows a $(0, \sigma^2)$ normal or logistic distribution. The variance is assumed to be 1 for identification reasons. The logistic distribution is more convenient because it has an analytical cumulative distribution. Choosing the logistic assumption, we have the following ordered logit probabilities:

\[
Pr(Y_i = 1) = Pr(Y_i^* \leq \kappa_1) = \frac{1}{1 + \exp(x_i^\prime \beta - \kappa_1)} \\
Pr(Y_i = 2) = Pr(\kappa_1 \leq Y_i^* \leq \kappa_2) = \frac{1}{1 + \exp(x_i^\prime \beta - \kappa_2)} - Pr(Y_i = 1) \\
Pr(Y_i = 3) = Pr(\kappa_2 \leq Y_i^* \leq \kappa_3) = \frac{1}{1 + \exp(x_i^\prime \beta - \kappa_3)} - Pr(Y_i = 2) \\
Pr(Y_i = 4) = Pr(Y_i^* \geq \kappa_3) = 1 - \frac{1}{1 + \exp(x_i^\prime \beta - \kappa_3)}
\]

There are thus three threshold parameters to estimate, $\kappa_1$, $\kappa_2$ and $\kappa_3$, together with the structural parameters $\beta$.

This model has identification problems due to the fact that unobserved utility has an unknown scale and unknown range. The range is identified by constraining the variance $\sigma^2$ of $u_i$ to 1 and the scale identification is usually solved by constraining one of the $\kappa$ to be zero. Usually we impose $\kappa_1 = 0$. Under this constrain, we can introduce a constant term in the regressors. Otherwise, the constant term has to be omitted.

### 4.3 A subjective equivalence scale model for France

The basic model give the level of unobserved utility or welfare as a function of income and household size

\[Y_i^* = f(R_i, N_i).\]
The equivalence scale \( m(N) \) is found by solving the equation

\[
f(R_i, N_i) = f(R_i/m(N), 1).
\]

Depending on the shape of the function \( f \), the scale \( m(N) \) can be concave or convex. If we decide for

\[
Y_i^* = a + b \log(R_i) + c \log(N_i)
\]

the implied scale is of the form \( N^\alpha \) with \( \alpha = -c/b \) and is concave. There is a decreasing cost for every extra child. Using the answers to the subjective question about financial ease, Hourriez and Olier (1997) obtain an estimated \( \alpha = 0.62 \) in 1995 which is very similar to the answer given by the estimation of the Prais-Houthakker model. So the subjective method is valid and it is simpler to implement. This value is in between the elasticity implied by the old OECD (Oxford) scale and the new OECD scale. In this estimation, the size of the household was taken equal to \( N = Na + 0.55Nc \) where \( Na \) is the number of adults and \( Nc \) the number of children below 14 years.

The second possibility is to use the regression

\[
Y_i^* = a + b \log(R_i) + cN_i.
\]

In this case, the scale is \( A^{N-1} \) with \( A = \exp(-c/b) \). There is an increasing cost of one extra child.

It is difficult to discriminate between the two models, because they are in fact unsatisfactory approximations of a larger model. A larger model would be

\[
Y_i^* = a + b \log(R_i) + c \log(N_i) + dN_i.
\]

In this case, the equivalence scale would be \( N^\alpha A^{N-1} \).

### 4.4 The cost of a child

The cost of a child is not uniform with his age. In order to investigate the cost of a child, Hourriez and Olier (1997) opted for the following regression

\[
Y^* = a + b \log(R) + c_1N_{0-4} + c_2N_{5-9} + c_3N_{10-14} + c_4N_{15-19} + c_5N_{20-24} + c_6N_{\text{adults}}
\]

where \( N_{0-4} \) is the number of children between 0 and 4 years present in the household. The cost of an extra child of age \( i \) is given by

\[
100[\exp(-c_i/b) - 1].
\]

This equation was estimated by Hourriez and Olier (1997), with results given in Table [7]. According to Table [7] the cost of an extra child is between 10% and 20% of the income of his household if the age of the child is below 15 years. Above 15 years, there is a jump in the cost of a child.
Table 7: Cost of a child as a function of his age

<table>
<thead>
<tr>
<th>Age group</th>
<th>1979</th>
<th>1985</th>
<th>1989</th>
<th>1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-4 years</td>
<td>21</td>
<td>20</td>
<td>18</td>
<td>12</td>
</tr>
<tr>
<td>5-9 years</td>
<td>16</td>
<td>15</td>
<td>16</td>
<td>11</td>
</tr>
<tr>
<td>10-14 years</td>
<td>22</td>
<td>18</td>
<td>20</td>
<td>18</td>
</tr>
<tr>
<td>15-19 years</td>
<td>29</td>
<td>34</td>
<td>28</td>
<td>28</td>
</tr>
<tr>
<td>20-24 years</td>
<td>45</td>
<td>38</td>
<td>49</td>
<td>41</td>
</tr>
<tr>
<td>Adult</td>
<td>43</td>
<td>47</td>
<td>45</td>
<td>44</td>
</tr>
</tbody>
</table>

Figures are in percentage.

Hourriez and Olier (1997) said that it is roughly the same as the cost of an extra adult. Consequently, we can simplify the final model and make only the difference between children under 15 years and adults. Thus the size of the household can be simplified so as to be

$$N = N_a + k N_c$$

The value of $k$ can be estimated directly with the following regression

$$Y^* = a + b \log(R) + c_1 N_a + c_2 N_c$$

Then $k$ is given by $c_2/c_1$. The final equivalence scale, whenever we have simply $N$ in a regression will be in fact

$$N^\alpha = (N_a + 0.55 N_c)^\alpha,$$

for instance in the Prais-Houthakker model of Table 6.

4.5 The minimum income question

Another type of question can be asked. It concern the minimum income necessary to make the two ends meet. This question was introduced in the Enquête sur le budget des ménages in 1989 and 1995. And is also present in many different types of survey. This question is phrased as follows in the French survey:

**Quel est, selon vous, le revenu mensuel minimal dont un ménage comme le votre doit absolument disposer pour pouvoir simplement subvenir à ses besoins?** (réponse en clair).

In the German Socio Economic Panel, we find a similar question as reported page 301 of van Praag and Ferrer-i-Carbonell (2008):
what is in your opinion the minimum amount of income that your family in your circum-
stances would need to be able to make ends meet? That would be DM... per month.

Individuals are asked this question and their answer might depend on their income level and
not just on family composition. This is know as preference drift. The RMI was introduced
in France in 1988 and is different for a single person, a couple, a couple with one child, two
children, etc. It was enlarged as the RSA in 2009. In January 2009, the RMI was determined on
the following basis given in Table 8. Individuals do not have necessarily these figures in mind.

<table>
<thead>
<tr>
<th>Number of Children</th>
<th>Single</th>
<th>Couple</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>454.63</td>
<td>681.95</td>
</tr>
<tr>
<td>1</td>
<td>681.95</td>
<td>818.34</td>
</tr>
<tr>
<td>2</td>
<td>818.34</td>
<td>954.73</td>
</tr>
<tr>
<td>per extra child</td>
<td>181.85</td>
<td>181.85</td>
</tr>
</tbody>
</table>

In euros per month.

So their answers are determined by a series of factors on top of family composition and mainly
depend on their level of income. Consequently, the model we have to estimate is

\[
\log(R_{MINI}) = a + b \log(R) + c \log(N)
\]

Parameter \( b \) measures the preference drift. One can think that only the household that have an
income \( R \) near from the reported \( R_{MINI} \) have a correct perception of what is a minimum income
as a function of \( N \). We note this function \( S(N) \). Consequently, we have to solve a fixed point
equation, imposing \( S(N) = R = R_{MINI} \):

\[
\log(S(N)) = \frac{a}{1 - b} + \frac{c}{1 - b} \log(N).
\]

Taking the exponential, the equivalence scale \( m(N) \) is given by

\[
S(N) = S(1)m(N) = e^{\frac{a}{1 - b} N^{\frac{c}{1 - b}}}.
\]

The estimation produced in Hourriez and Olier (1997) gives \( \alpha = 0.37 \), which is a much lower
value than that obtained by the question on financial ease.

The minimum income question is also used to determine a subjective poverty line. It is
present in the EU-SILC (EU Statistics on Income and Living Conditions) survey on minimum
income, and specifically the lowest income to make ends meet variable (Eurostat 2003).

### 4.6 Linear or non-linear equivalence scales?

We come back here on the shape of the equivalence scale. It is an increasing function of \( N \), but
is it concave or convex? With a concave function like \( N^\alpha \), an extra member in the household
increases smaller needs in a large family than in a small family. The cost of the third child is smaller than the cost of the first child. We have increasing economies of scale. When the function is convex as with $A^{N-1}$, this is just the reverse. And we saw that the data could not select one of these models against the other. So a more general form is

$$N^\alpha A^{N-1},$$

which was estimated by Hourriez and Olier (1997) and reported in Table 9. These parameters are

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.16</td>
<td>0.22</td>
<td>0.18</td>
<td>0.25</td>
</tr>
<tr>
<td>$A$</td>
<td>1.27</td>
<td>1.25</td>
<td>1.24</td>
<td>1.20</td>
</tr>
</tbody>
</table>

not directly interpretable, we need to translate them for typical household compositions. This is done in Table 10. Scale economies resulting from forming a couple remained fairly constant over the years. To reach the level of welfare, a couple needs 40% more income than a single person. The cost of the first child seems to remain the same too with a value of 27% (1.70-1.43 = 0.27). On the contrary, the cost of the third child seems to be decreasing over time. Figure 2 show that there are large economies of scale for a married couple compared bachelors. The economies of scale for the children under 14 years are of course lower, they tend to be linear.

### 5 Exercices

1. Using the information contained in Table 8 determine the implicit equivalence scale that was used to determine the RMI in France.
1979 is the black line with circles, 1985 the red upper line, 1989 the green lower line and 1995 the blue lower line.

Figure 2: Convex or concave scales?
References


