Economic Growth and Development

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1 Some evidence

Why do countries differ in income levels?
Why do contries grow at different rates?

2 Neoclassical growth theory

2.1 The Solow Model

$$Y = F(K, L) \quad \rightarrow \quad y = f(k)$$

where $y \equiv Y/L$ and $k \equiv K/L$. F exhibits CRS and f diminishing returns to k. For example

$$Y = K^{\alpha}(AL)^{1-\alpha}$$

Let

$$\left.\begin{array}{l} y = A^{1-\alpha}k^{\alpha} \\ \dot{k} = sy - \delta k \end{array}\right\} g = \alpha\frac{\dot{k}}{k} = \alpha\left(\frac{sA^{1-\alpha}k^{\alpha}}{k} - \delta\right)$$

If $0 < \alpha < 1$ the economy converges to

$$k^* = A\left(\frac{s}{\delta}\right)^{1/(1-\alpha)}$$

and

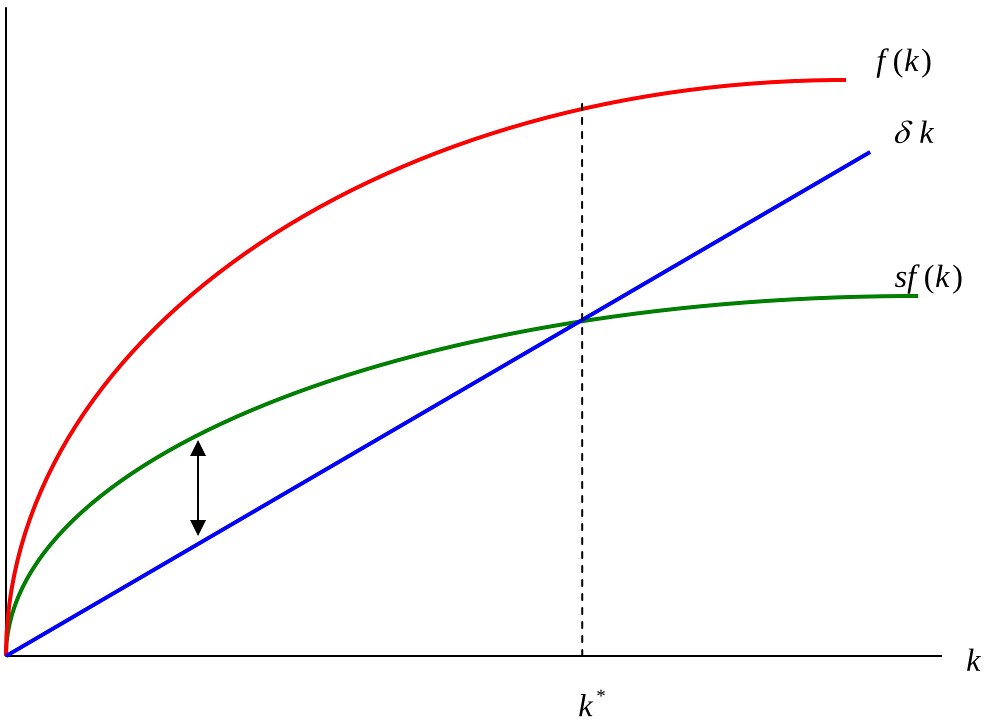
$$g_{\infty} = 0$$

with

$$y^* = A\left(\frac{s}{\delta}\right)^{\alpha/(1-\alpha)}$$

Then we can express the rate of growth as

$$g = \alpha\delta\left(\left(\frac{y^*}{y}\right)^{\frac{1-\alpha}{\alpha}} - 1\right)$$



Output per worker
Capital per worker
Break-even investment per worker

What can prevent the marginal product of capital from falling? Population growth or exogenous technical change.

$$Y(t) = K(t)^{\alpha}(A(t)L)^{1-\alpha} \quad \rightarrow \quad A(t) = A_0 e^{xt}$$

Define

$$\widehat{k}(t) \equiv \frac{K(t)}{A(t)L} = \frac{k(t)}{A(t)}, \widehat{y}(t) \equiv \frac{Y(t)}{A(t)L} = \frac{y(t)}{A(t)}$$

$$Y(t) = K(t)^{\alpha}(A(t)L)^{1-\alpha} \quad \rightarrow \quad \widehat{y}(t) = \widehat{k}(t)^{\alpha}$$

So

$$\frac{d\widehat{y}(t)/dt}{\widehat{y}(t)} = \alpha \frac{s\widehat{y} - (\delta + x)\widehat{k}}{\widehat{k}} = \alpha \left(s\widehat{k}(t)^{\alpha-1} - \delta - x \right)$$

which convergest to zero and defines

$$\widehat{k}^{*} = \left(\frac{s}{\delta + x} \right)^{1/(1-\alpha)}$$

but

$$g = \frac{dy(t)/dt}{y(t)} = \frac{d\widehat{y}(t)/dt}{\widehat{y}(t)} + (1-\alpha)\frac{\dot{A}}{A} = \alpha \left(s\widehat{k}(t)^{\alpha-1} - \delta - x \right) + x$$

and

$$g_{\infty} = x$$

Convergence:

- If the exogenous rate of technical change is the same for all countries, then in the long run they will all grow at the same rate,
- Countries that are further from their steady state grow faster.

2.2 The Ramsey-Cass-Coopmans Model

Endogenizes the savings rate.

2.2.1 Competitive equilibrium

Consumers

$$\max \int_0^\infty \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

$$\begin{aligned} \text{s.t.} \quad \dot{K}(t) &= w(t) + r(t)K(t) - C(t) \\ K(0) &= K_0 \end{aligned}$$

The hamiltonian

$$H(C, K, \lambda, t) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \lambda(w(t) + r(t)K(t) - C(t))$$

The control variable is C and the state variable K.

First-order conditions

$$\begin{aligned} \frac{\partial H}{\partial C} &= 0 \\ \frac{\partial H}{\partial K} &= -\dot{\lambda} \\ \lim_{t \to \infty} \lambda(t)K(t) &= 0 \end{aligned}$$

Then

$$\begin{aligned} e^{-\rho t} C^{-\sigma} - \lambda &= 0 \\ \lambda r &= -\dot{\lambda} \end{aligned}$$

Differentiate the first expression to get

$$-\rho - \sigma \dot{C}/C = \dot{\lambda}/\lambda$$

Then the two equations together imply

$$\frac{\dot{C}}{C} = \frac{r - \rho}{\sigma} \quad .$$

Firms

$$\max \pi = F(K(t), L(t)) - w(t)L(t) - r(t)K(t)$$

$$\Rightarrow \frac{\partial F}{\partial L} = w(t) \text{ and } \frac{\partial F}{\partial K} = r(t)$$

Then

$$\frac{\dot{C}}{C} = \frac{\partial F/\partial K - \rho}{\sigma} \quad .$$

Let $Y(t) = K(t)^{\alpha}(AL(t))^{1-\alpha}$ with $0 < \alpha < 1$. Then

$$\frac{\dot{C}}{C} = \frac{\alpha(AL)^{1-\alpha}K^{\alpha-1} - \rho}{\sigma} \quad ,$$

which goes to zero as the economy accumulates capital. In steady state

$$\frac{\dot{C}}{C} = 0 \text{ and } K^* = AL\left(\frac{\alpha}{\rho}\right)^{1/(1-\alpha)} \quad .$$

No long-run growth.

2.2.2 Welfare analysis

$$\max \int_0^{\infty} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

$$\begin{aligned} \text{s.t.} \quad \dot{K}(t) &= K(t)^{\alpha}(AL(t))^{1-\alpha} - C(t) \\ K(0) &= K_0 \end{aligned}$$

The hamiltonian

$$H(C, K, \lambda, t) = \frac{C(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} + \lambda(K(t)^{\alpha}(AL(t))^{1-\alpha} - C(t))$$

First-order conditions

$$\begin{aligned} e^{-\rho t}C^{-\sigma} - \lambda &= 0 \\ \lambda\alpha(AL)^{1-\alpha}k^{\alpha-1} &= -\dot{\lambda} \end{aligned}$$

and

$$\frac{\dot{C}}{C} = \frac{\alpha A k^{\alpha-1} - \rho}{\sigma} \quad ,$$

Same as in competitive equilibrium

3 Towards Endogenous growth

3.1 Early attempts to endogenise technology

Why should the rate of technical change not depend on economic decisions? Problem: in order to endogenise A, the decisions to increase it must be rewarded. But,

- with CRS, Euler's theorem implies $Y = F_K K + F_L L$,
- if Y has CRS in K and L, it must have IRS in A, L, and K $\rightarrow$ cannot use competitive equilibrium.

Early attempts

1. Arrow 1962 $\rightarrow$ learning-by-doing in production
2. Shell 1973 $\rightarrow$ R&D done by the government
3. Uzawa 1965 $\rightarrow$ A represents human capital that can be produced with labour services

3.2 The AK model

The AK model based on Harrod-Domar

$$\left.\begin{array}{l} Y = AK \\ \dot{K} = sY - \delta K \end{array}\right\} \dot{K} = sAK - \delta K \quad \rightarrow \quad g = sA - \delta$$

Why should we assume constant returns to capital?
Frankel and Romer introduce externalities in a Solow model.

$$\underbrace{Y_j = \overline{A} K_j^{\alpha} L_j^{1-\alpha}}_{\text{firm p.f.}} \quad \Rightarrow \quad \underbrace{Y = \overline{A} K^{\alpha} L^{1-\alpha}}_{\text{aggregate p.f.}}$$

where the N firms are symmetric and $L = \sum_j L_j$ and $K = \sum_j K_j$.
Using the Ramsey set up

$$\max \int_0^{\infty} \frac{C^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{K} = \overline{A} K^{\alpha} L^{1-\alpha} - C$$

$$\Rightarrow \frac{\dot{C}}{C} = \frac{\alpha \overline{A} (L/K)^{1-\alpha} - \rho}{\sigma} \quad .$$

Let the level of technology/knowledge be endogenous: Frankel assumes $\overline{A} = A(K/L)^{\beta}$, while Romer postulates $\overline{A} = AK^{\beta}$ and lets $L = 1$. Following Romer,

$$g = \frac{\alpha AK^{\alpha+\beta-1} - \rho}{\sigma} \quad .$$

1. $\alpha + \beta < 1$: Solow model
2. $\alpha+\beta = 1$: CONSTANT SOCIAL RETURNS TO CAPITAL. The aggregate production function is of the form $Y = AK$, and

$$g = \frac{\alpha A - \rho}{\sigma} \quad .$$

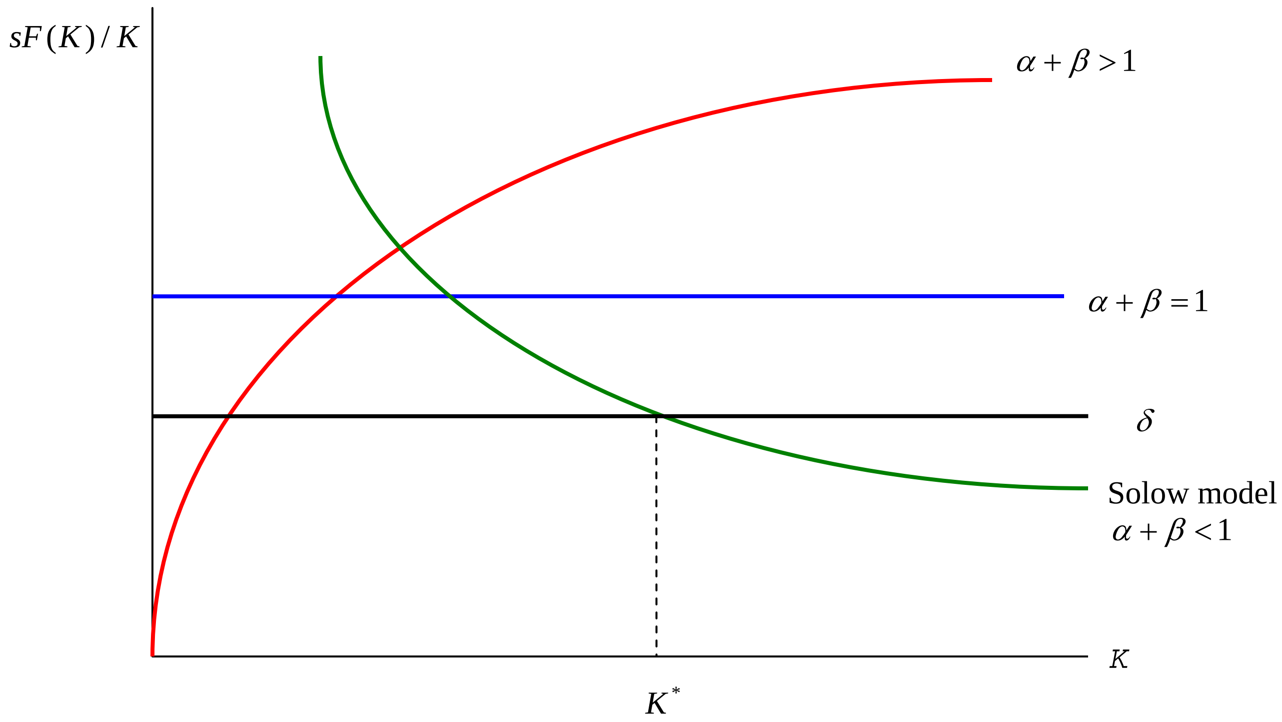
3. $\alpha + \beta > 1$: increasing returns to capital, which imply exploding growth.

Remarks on the AK model

- Constant growth rate
- Savings matter
- No convergence
- Shocks have permanent effects
- Scale effects
- Welfare implications

To compare the three cases, consider the simplest model with exogenous saving rates and depreciation. Let $Y = \overline{A}K^{\alpha}$ and $\overline{A} = AK^{\beta}$. Then

$$\left.\begin{array}{l} Y = AK^{\alpha+\beta} \\ \dot{K} = sY - \delta K \end{array}\right\} g = sAK^{\alpha+\beta-1} - \delta$$



sF(K)/K
α + β > 1
α + β = 1
δ
Solow model
α + β < 1
K
K*

3.3 Infrastructure and Growth

Barro (1990)

Assumption: government expenditures affect the productivity of privately owned factors.

Poduction function:

$$Y(t) = AK(t)^{1-\alpha} \left(\gamma(t)L(t)\right)^{\alpha}$$

$$y(t) = Ak(t)^{1-\alpha}\gamma(t)^{\alpha}$$

Proportional tax on income, τ

Balanced government budget

The competitive economy

$$\max \int_0^{\infty} \frac{c(t)^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt \tag{1}$$

$$\text{s.t.} \quad \dot{k}(t) = (1-\tau)Ak(t)^{1-\alpha}\gamma(t)^{\alpha} - c(t)$$

$$\Rightarrow \quad \frac{\dot{c}}{c} = \frac{(1-\tau)A(1-\alpha)(\gamma/k)^{\alpha} - \rho}{\sigma} \tag{2}$$

Balanced budget condition,

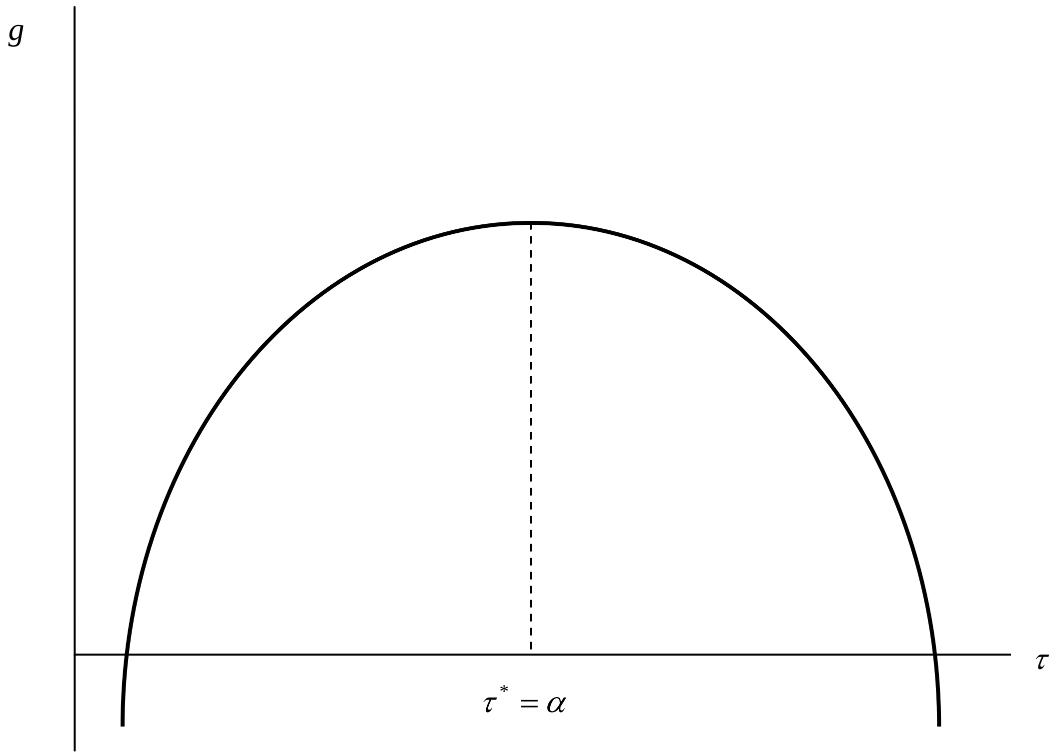
$$\gamma = \tau y \ \Rightarrow \frac{\gamma}{k} = (A\tau)^{1/(1-\alpha)} \tag{3}$$

and the competitive growth rate is

$$g^c = \frac{(1-\alpha)A^{\frac{1}{1-\alpha}}(1-\tau)\tau^{\frac{\alpha}{1-\alpha}} - \rho}{\sigma} \tag{4}$$

The maximum rate of growth in a competitive economy

$$\max_{\tau} g \quad \Rightarrow \quad \max_{\tau} \ (1-\tau)\tau^{\frac{\alpha}{1-\alpha}}$$
$$\Rightarrow \quad \tau^* = \alpha$$



g
τ
$\tau^* = \alpha$

Is the competitive equilibrium socially optimal?
Social planner

$$\max \int_0^\infty \frac{c(t)^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \tag{5}$$

$$\text{s.t.} \quad \dot{k}(t) = A(1-\tau)(A\tau)^{\alpha/(1-\alpha)} k(t) - c(t)$$

$$\Rightarrow \quad g^s = \frac{A^{\frac{1}{1-\alpha}}(1-\tau)\tau^{\frac{\alpha}{1-\alpha}} - \rho}{\sigma} \tag{6}$$

Planner takes externality into account: more capital increases tax revenue and hence raises the productivity of capital.

4 Human capital

Lucas (1988)

Crucial assumptions

* Output depends on human capital (education) of the labour force

$$Y = AK^{\beta}(L^e)^{1-\beta}$$

where

$$L^e = uhL$$

Individuals have 1 unit of time at each instant and $u \in [0,1]$ is time spent at work

* Accumulation of human capital: the main input is "time"

$$\dot{h} = zh(1-u)$$

is the production function for human capital

Write per capita output as

$$y = Ak^{\beta}\left(uh\right)^{1-\beta}$$

Then

$$\begin{aligned} &\max_{c,u} \int_0^{\infty} \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ \text{s.t. } \dot{k} &= Ak^{\beta}(uh)^{1-\beta} - c \\ \dot{h} &= z(1-u)h \end{aligned}$$

Two control variables, c and u, and two state variables, k and h.

The Hamiltonian

$$H(c,u,k,h,\lambda,\mu,t) = \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} + \lambda(Ak^{\beta}(uh)^{1-\beta} - c) + \mu(z(1-u)h)$$

and the agent chooses consumption and the allocation of time between education and production.

First order conditions

$$\frac{\partial H}{\partial c} = 0 \ , \qquad \frac{\partial H}{\partial u} = 0 \ ,$$
$$\frac{\partial H}{\partial k} = -\dot{\lambda} \ , \ \frac{\partial H}{\partial h} = -\dot{\mu}$$

Then

$$\frac{\dot{c}}{c} = \frac{A\beta k^{\beta-1}(uh)^{1-\beta} - \rho}{\sigma}$$

Can show

$$g_c^* = g_k^* = g_h^*$$

and

$$g_c^* = \frac{z - \rho}{\sigma}$$

which implies that the steady state level of $(1 - u)$ is

$$1 - u^* = \frac{z - \rho}{z\sigma}$$

That is

$$g^* = z(1 - u^*)$$

5 Solow versus the AK model

Two crucial differences between the two that could be tested empirically:

1. determinants of long-run growth rates
2. returns to capital

Cross-country evidence

- Barro and Sala-i-Martin (1995)

$$g_{60-85} = f(\text{edu, I/GDP, Gov/GDP})$$

But:

- "some" evidence of "conditional convergence"
- reverse causation, eg. education

What are the returns to capital?

The augmented Solow model
Mankiw, Romer and Weil (1992)

$$Y_t = A_t K_t^{\alpha} H_t^{\beta} L_t^{1-\alpha-\beta}$$

Investments in the two types of capital

$$s_k Y_t$$
$$s_h Y_t$$

Then

$$g_t = \lambda \left(\log y^* - \log y_t \right)$$

where

$$\lambda = (1-\alpha-\beta)(x+n+\delta)$$

$$y^* = \left(\frac{s_k^{\alpha} s_h^{\beta}}{(x+n+\delta)^{\alpha+\beta}} \right)^{1/(1-\alpha-\beta)}$$

and

x	technical change
n	population growth
δ	depreciation

6 Poverty Traps I: The Big Push

Murphy, Shleifer, and Vishny (JPE 1989)

6.1 A Simple Model with a Unique Equilibrium

Utility

$$U = \exp\left[\int_0^N \ln x(q)dq\right] \tag{1}$$

Labour, L, is paid a wage 1
Aggregate income

$$Y(n) = \pi n + L \tag{2}$$

n : number of sectors that have industrialised

Two types of frims for each good
- Cottage production: $c(x) = 1 \cdot x \Rightarrow p = 1$
- Mass production: IRS $c(x) = F + x/\alpha$, $\alpha > 1$

Limit pricing $p = 1$
Expenditure on each good is $y = Y/N$
Profits are

$$\begin{aligned}\pi(n) &= y(n) - \left(F + \frac{y(n)}{\alpha}\right) \\ &= \frac{\alpha - 1}{\alpha} y(n) - F \equiv ay(n) - F\end{aligned}$$

Write profit as a function of aggregate income

$$\pi(n) = a\frac{Y(n)}{N} - F \tag{3}$$

Equations (2) and (3) yield

$$Y(n) = \frac{L - nF}{1 - an/N} \tag{4a}$$

$$\pi(n) = \frac{aL - NF}{N - an} \tag{4b}$$

Multiplier

$$\frac{dY(n)}{dn} = \frac{aL/N - F}{(1 - an/N)^2} = \frac{\pi(n)}{1 - an/N}.$$

Unique Nash Equilibrium
No industrialisation

$$\pi(0) = ay(0) - F = a\frac{L}{N} - F$$

Then

$$\pi(0) < 0 \Leftrightarrow a\frac{L}{N} < F \tag{5a}$$

All industrialise

$$\begin{aligned} \pi(N) &= ay(N) - F \\ &= a\frac{L - NF}{N - aN} - F = \alpha\left(a\frac{L}{N} - F\right) \end{aligned}$$

Then

$$\pi(N) > 0 \Leftrightarrow a\frac{L}{N} > F > 0 \tag{5b}$$

$$\begin{aligned} a\frac{L}{N} &> F \quad \Rightarrow \text{ all firms industrialise at } n = 0 \\ a\frac{L}{N} &< F \quad \Rightarrow \text{ no firm industrialises at } n = N \end{aligned}$$

6.2 A Model with a Factory Wage Premium

Same production structure as above
Crucial assumption: compensate workers to move to industry
Utility in cottage production

$$U_c = \exp\left[\int_0^N \ln x(q)dq\right] \tag{1a}$$

Utility in mass production

$$U_m = \exp\left[\int_0^N \ln x(q)dq\right] - v \tag{1b}$$

Competitive factory wage:

$$w = 1 + v \tag{6}$$

Monopolist's profits

$$\pi = \left(1 - \frac{1+v}{\alpha}\right)y - F(1+v) \tag{7}$$

Assume

$$\alpha - 1 > v \tag{8}$$

No industrialisation equilibrium

$$\pi = \left(1 - \frac{1+v}{\alpha}\right)\frac{L}{N} - F(1+v) < 0 \tag{9}$$

Industrialisation equilibrium
All firms industrialise,

$$y = \pi + \frac{1+v}{N}L$$

and positive profits are earned

$$\pi = \alpha\left(\frac{L}{N} - F\right) - \frac{L}{N}(1+v) > 0 \tag{10}$$

Multiple equilibria
Rexpress above as

$$\begin{aligned} \frac{L}{N}\frac{\alpha-(1+v)}{\alpha} &< F(1+v) && (9') \\ \frac{L}{N}\frac{\alpha-(1+v)}{\alpha} &> F && (10') \end{aligned}$$

If

$$\frac{L}{N}\frac{\alpha-(1+v)}{\alpha(1+v)} < F < \frac{L}{N}\frac{\alpha-(1+v)}{\alpha}$$

then, multiple equilibria exist

6.3 A Dynamic Model of Investment

Two period model of industrialisation (no factory wage premium). No capital

Consumers
Utility

$$U = \left[\int_0^1 x_1^\gamma(q)dq\right]^{\theta/\gamma} + \beta\left[\int_0^1 x_2^\gamma(q)dq\right]^{\theta/\gamma} \tag{11}$$

where we have normalized the number of goods to 1. By symmetry of all goods we can write utility as

$$U = x_1^\theta + \beta x_2^\theta \tag{11'}$$

There are L consumers and hence L units of labour each period
Let aggregate incomes be y_1, y_2 . Since there is no capital, aggregate consumption must be equal to aggregate income $\Rightarrow x_1 = y_1/L$, $x_2 = y_2/L$

Then the

$$\max U = [x_1]^\theta + \beta [x_2]^\theta$$
$$\text{s.t.} \quad \frac{y_1}{L} + \frac{1}{1+r}\frac{y_2}{L} = x_1 + \frac{1}{1+r}x_2$$

Usual condition

$$\frac{U_1(x_1,x_2)}{U_2(x_1,x_2)} = \frac{1}{R} \Rightarrow \left(\frac{x_1}{x_2}\right)^{\theta-1} = \frac{\beta}{R}$$

where $R = 1/(1+r)$ is endogenous

Firms
Cottage production:
CRS, one unit labour $\rightarrow$ one unit output
Mass production:
- invest F units of labour in the first period
- produce in second period: one unit labour $\rightarrow$ $\alpha > 1$ units output

Assume

$$(\alpha - 1)L > F \tag{12}$$

Price of competitive fringe: 1
Monopolist's price 1

Monopolist's profit

$$\pi = \frac{1}{1+r}\left(y_2 - \frac{y_2}{\alpha}\right) - F = Ray_2 - F \tag{13}$$

Low-income equilibrium

$$y_1 = y_2 = L$$

Perfect smoothing: $R = \beta$
For this to be an equilibrium,

$$\pi = \beta aL - F < 0 \tag{14}$$

Present value of income

$$y_c = y_1 + Ry_2 = (1+\beta)L \tag{15}$$

High-income equilibrium
All sectors industrialise

$$y_1 = L - F \tag{16}$$
$$y_2 = \alpha L > L \tag{17}$$

and then $x_1 = 1 - F/L$ and $x_2 = \alpha$
Will the consumer accept the resulting utility level?

$$\left(\frac{x_1}{x_2}\right)^{\theta-1} = \frac{\beta}{R} \Rightarrow R = \beta\left(\frac{\alpha L}{L-F}\right)^{\theta-1} \leq \beta \tag{18}$$

For this to be an equilibrium,

$$\pi = Ra\alpha L - F = a\alpha L\beta\left(\frac{\alpha L}{L-F}\right)^{\theta-1} - F > 0 \tag{19}$$

Present value of income

$$y_m = L - F + R\alpha L = L - F + \beta\left(\frac{\alpha L}{L-F}\right)^{\theta-1}\alpha L \tag{20}$$

Multiple Equilibria
(14) and (19) can be simultaneously satisfied if

$$\beta aL < F < \frac{F}{(1-F/L)^{1-\theta}} < \alpha^{\theta}\beta aL$$

7 Poverty Traps II: Threshold effects

Azariadis-Drazen (1990)

OLG model + human capital accumulation

Continuum of two-period OLG families

Mass 1, and no population growth

Human capital is inhereted from the previous generation

$$h_{1,t} = h_{2,t-1}$$

So individuals of a same generation are identical

Human capital is accumulated during 1st period of life

$$h_{2,t} = (1 + \gamma \cdot v_t^\theta) h_{1,t} \qquad \text{where } \; 0 < \theta < 1$$

Linear preferences,

$$U_t = c_{1,t} + \delta c_{2,t}, \quad \text{where } \delta = \frac{1}{1+r}$$

Output per worker is $y = h$ per unit of time. Then

$$\begin{aligned} \max_v U_t &= (1 - v_t) h_{1,t} + \delta h_{2,t} \\ \text{s.t} \quad h_{2,t} &= \left(1 + \gamma \cdot v_t^\theta\right) h_{1,t} \end{aligned}$$

which gives the unique solution:

$$v^* = [\delta\theta\gamma]^{\frac{1}{1-\theta}}$$
$$g^* = \frac{h_{2,t}}{h_{2,t-1}} - 1 = \gamma {v^*}^\theta$$

Similar to Lucas, as θ and γ matter. But: different welfare effects due to intertemporal externalities

Azariadis-Drazen argue that the education technology has positive *threshold externalities*,

$$\gamma(v_{t-1}) = \begin{cases} \underline{\gamma} & \text{if} \quad v_{t-1} \leq v_0 \\ \overline{\gamma} & \text{if} \quad v_{t-1} > v_0, \end{cases}$$

where $0 < v_0 < 1$ and $\underline{\gamma} < \overline{\gamma}$.

Available education technology depends on education investment of previous generation

Low-growth equilibrium

$$\underline{v} = \arg\max_{v} \left\{ (1-v)h_{1,t} + \delta(1+\underline{\gamma}v^{\theta})h_{1,t} \right\}$$

that is

$$\underline{g} = \underline{\gamma} \cdot \underline{v}^{\theta} = \underline{\gamma}(\delta\theta\underline{\gamma})^{\frac{\theta}{1-\theta}}$$

High-growth equilibrium

$$\overline{v} = \arg\max_{v} \left\{ (1-v)h_{1,t} + \delta(1+\overline{\gamma} \cdot v^{\theta})h_{1,t} \right\}$$

that is

$$\overline{g} = \overline{\gamma} \cdot \overline{v}^{\theta} = \overline{\gamma}(\delta\theta\overline{\gamma})^{\frac{\theta}{1-\theta}}$$

These two equilibria coexist iff

$$\underline{v} < v_0 < \overline{v}$$

Then:
(1) multiple steady-state growth paths,
(2) the outcome is determined by *history,*
(3) policy implications: education subsidies can avoid low-development traps

8 Poverty Traps III: Income Distribution

8.1 Income distribution and macroeconomics

Galor and Zeira (1993)

OLG economy with constant population, L
Single good that can be produced with two CRS technologies, one uses unskilled and the other skilled labor
Agents can work as unskilled both periods, or invest in human capital when young and work as skilled workers when old
One unit of labor is supplied in each period
Investment in human capital is indivisible, $h > 0$

Uutility function

$$U_t = (1-\beta)\log c_t + \beta \log b_t \tag{1}$$

Borrowing rate i, greater than the risk-free rate, r
Due to the cost of monitoring investment in an intangible asset
Notation $x_{jt} = b_{jt-1}$

Production: Two sectors
Unskilled

$$Y_u = w_u L_t^u$$

Skilled

$$Y_s = A K_t^{\alpha} \left(L_t^s\right)^{1-\alpha}$$

Small open economies: constant world interest rate, r. Then

$$\begin{aligned} r &= \alpha A \left(L_t^s / K_t\right)^{1-\alpha} \\ K_t / L_t^s &= (\alpha A / r)^{1/(1-\alpha)} \end{aligned}$$

Also

$$\begin{aligned} w_s &= (1-\alpha) A \left(K_t / L_t^s\right)^{\alpha} \\ K_t / L_t^s &= (\alpha A / r)^{1/(1-\alpha)} \end{aligned}$$

substitute for K_t / L_t^s

$$w_s = (1-\alpha) A (\alpha A / r)^{\alpha/(1-\alpha)}$$

Constant wages

There are three types of agents: those who do not invest in h, those who invest and lend ($x \geq h$), and those who invest and borrow

For an individual who does not invest in education

$$\begin{aligned} U_u(x) &= \log\left[w_u + (x + w_u)(1 + r)\right] + \gamma \ , \\ b_u(x) &= \beta\left[w_u + (x + w_u)(1 + r)\right] \end{aligned}$$

where $\gamma \equiv \beta \log \beta + (1 - \beta) \log(1 - \beta)$

For an individual with $x \geq h$ who invests

$$\begin{aligned} U_s(x) &= \log\left[w_s + (x - h)(1 + r)\right] + \gamma \ , \\ b_s(x) &= \beta\left[w_s + (x - h)(1 + r)\right] \ . \end{aligned}$$

For an agent who borrows and invests

$$\begin{aligned} U_s(x) &= \log\left[w_s + (x - h)(1 + i)\right] + \gamma \ , \\ b_s(x) &= \beta\left[w_s + (x - h)(1 + i)\right] \ . \end{aligned}$$

Assumption

$$w_s - h(1 + r) \geq w_u(2 + r)$$

All those that can afford education without borrowing, invest

Those who have to borrow choose to study if $U_s(x) \geq U_u(x)$,C i.e. if

$$x \geq f \equiv \frac{w_u(2 + r) + h(1 + i) - w_s}{i - r} \ ,$$

that is, if inheritance is "large enough"

What is the level of output?

The distribution of inheritances received by individuals born in period t, D_t, determines the number of skilled and unskilled workers in dynasty t,

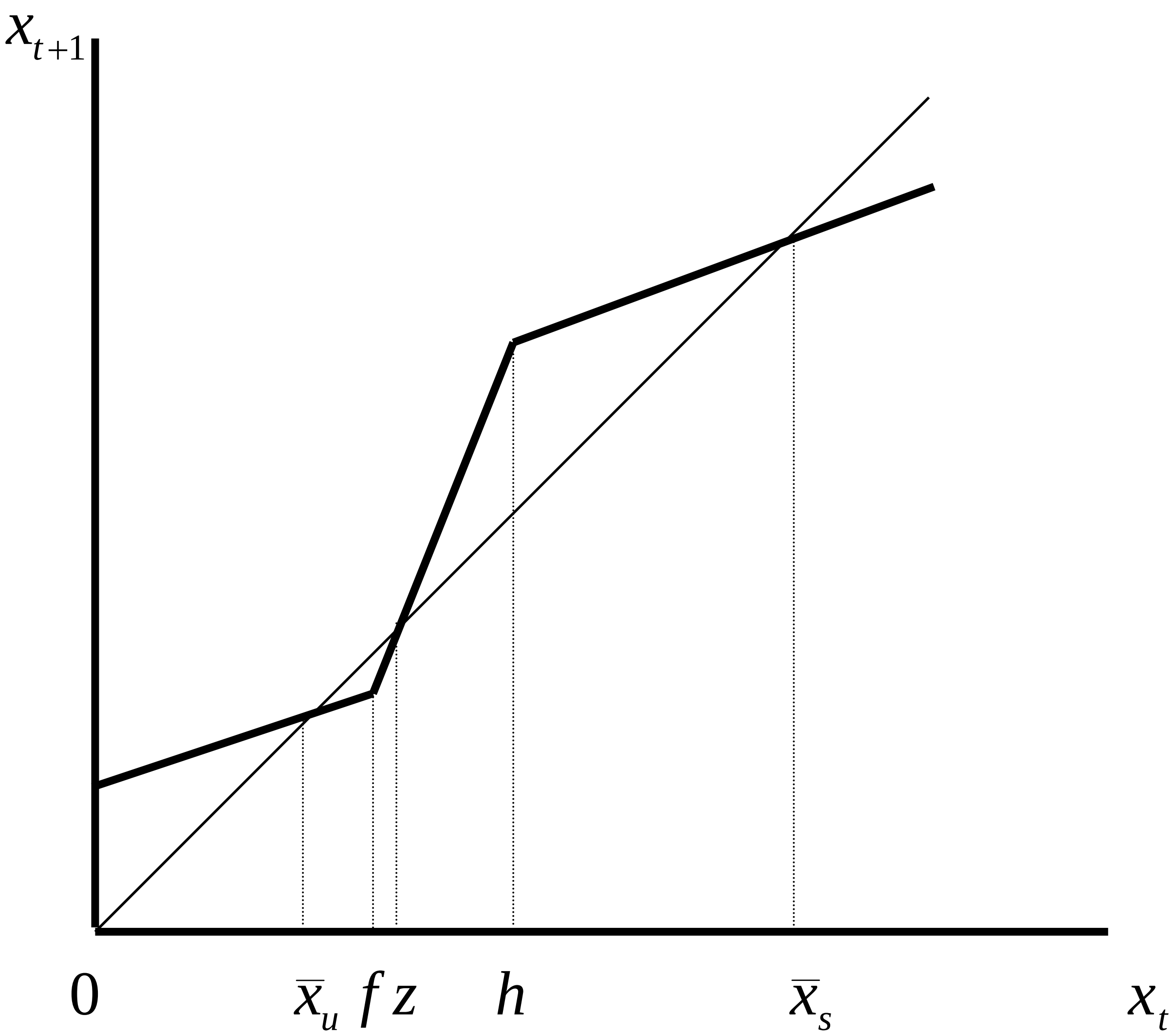
$$L_t^s = \int_f^\infty dD_t(x_t) \quad \text{and} \quad L_t^u = \int_0^f dD_t(x_t) \ ,$$

and hence the level of output

Dynamics:

$$x_{t+1} = \begin{cases} \beta\left[w_u + (x_t + w_u)(1+r)\right] & \text{if } \ x_t < f \\ \beta\left[w_s + (x_t - h)(1+i)\right] & \text{if } \ f \leq x_t < h \\ \beta\left[w_s + (x_t - h)(1+r)\right] & \text{if } \ h \leq x_t \ . \end{cases}$$

Assume $(1+r)\beta < 1$


x_{t+1}
0
$\bar{x}_u$
f
z
h
$\bar{x}_s$
x_t

All dynasties who inherit less than f work as unskilled and converge to

$$\bar{x}_u = \frac{\beta}{1-\beta(1+r)}(2+r)w_u \ .$$

Their descendents will always choose not to invest in education

Dynasties who can study without borrowing converge to

$$\bar{x}_s = \frac{\beta}{1-\beta(1+r)}\left[w_s - h(1+r)\right] \ ,$$

and their children will always study

What about those who borrow?

Point z is given by the intersection of the corresponding bequest function with the 45^0 line,

$$z = \beta\left[w_s + (z-h)(1+i)\right]$$

So

$$z = \frac{\beta\left[h(1+i) - w_s\right]}{(1+i)\beta - 1}$$

If $x_t > z$, then $x_{t+1} > x_t \to$ her child will also study $\to$ converge to $\bar{x}_s$

If $x_t < z$, then $x_{t+1} < x_t \to$ eventually a descendent will receive $x < f$ $\to$ not study $\to$ converge to $\bar{x}_u$

Long-run number of skilled workers

$$L^s_\infty = \int_z^\infty dD_t(x_t) \ .$$

Long-run distribution of income : two point distribution, where the mass of agents at each level is uniquely determined by the initial distribution of wealth

Long-run average level of output

$$y_\infty = 2w_u + (w_s - w_u)\frac{L^s}{L} \ ,$$

which depends on the initial distribution of bequests

9 Technical Change: Expanding Product Variety

Increase in variety of inputs (Young 1920) →growth based on increasing returns due to specialization
Growth due to growth in K but agents are now rewarded
Monopolistic competition under product differentiation (Dixit and Stiglitz)
R&D activity
Internal IRS: imperfect competition
Intermediate goods

9.1 Expanding Product Variety: Romer (1990)

Three sectors:

final good
intermediate goods
R&D

Two types of labour: L and H

Final output sector

$$Y = H_1^{\alpha} L^{\beta} \int_0^A x_i^{1-\alpha-\beta} di \tag{1}$$

Competitive
Symmetry $Y = (AH_1)^{\alpha}(AL)^{\beta}K^{1-\alpha-\beta}$
where $K = xA$

Intermediate goods sector
Sunk cost of producing a new variety, P_A
Fixed cost –> monopolistic competition
Same production function as the final good
Machinery does not depreciate

- flow cost rx_i
- flow revenue $p(x_i)x_i$

Research sector

$$\dot{A} = zH_2A \tag{2}$$

Sell new design for P_A to one intermediate goods producer

Solving the model
Competitive factor pricing in the final goods sector

$$p_i(x) = (1-\alpha-\beta)H_1^{\alpha}L^{\beta}x_i^{-\alpha-\beta}$$

$$w_1 = \alpha H_1^{\alpha-1}L^{\beta}x^{1-\alpha-\beta}A$$

$$w_L = \beta H_1^{\alpha}L^{\beta-1}x^{1-\alpha-\beta}A$$

Profit function for intermediate goods producers

$$\begin{aligned}\pi(x) &= p(x)x - rx \\ &= (1-\alpha-\beta)H_1^{\alpha}L^{\beta}x^{1-\alpha-\beta} \; - rx\end{aligned}$$

Choose x

$$x = \left(\frac{(1-\alpha-\beta)^2}{r}H_1^{\alpha}L^{\beta}\right)^{1/(\alpha+\beta)} \tag{3}$$

Hence $\pi = x(p-r) = xr(\alpha+\beta)/(1-\alpha-\beta)$
The research firm extracts the entire monopoly rents

$$P_A = \frac{\pi}{r} = \frac{\alpha+\beta}{(1-\alpha-\beta)}x \tag{4}$$

Skilled labour is paid its marginal value product

$$w_2 = zAP_A$$

Labour market equilibrium

$$w_1 = w_2 \;\; \Rightarrow \;\; P_A = \frac{\alpha}{z}\frac{x^{1-\alpha-\beta}L^{\beta}}{H_1^{1-\alpha}} \tag{5}$$

Using (3), (4) and (5) we get employment in the production sector

$$H_1 = \theta\frac{r}{z}$$

where $\theta = \alpha/(\alpha+\beta)(1-\alpha-\beta)$
Then

$$H_2 = H - H_1 = H - \theta\frac{r}{z}$$

Steady state growth
Recall

$$\begin{aligned} Y &= A H_1^{\alpha} L^{\beta} x^{1-\alpha-\beta} \\ &\Rightarrow \frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} = z H_2 \end{aligned}$$

The rate of output growth:

$$g = zH_2 = zH - \theta r$$

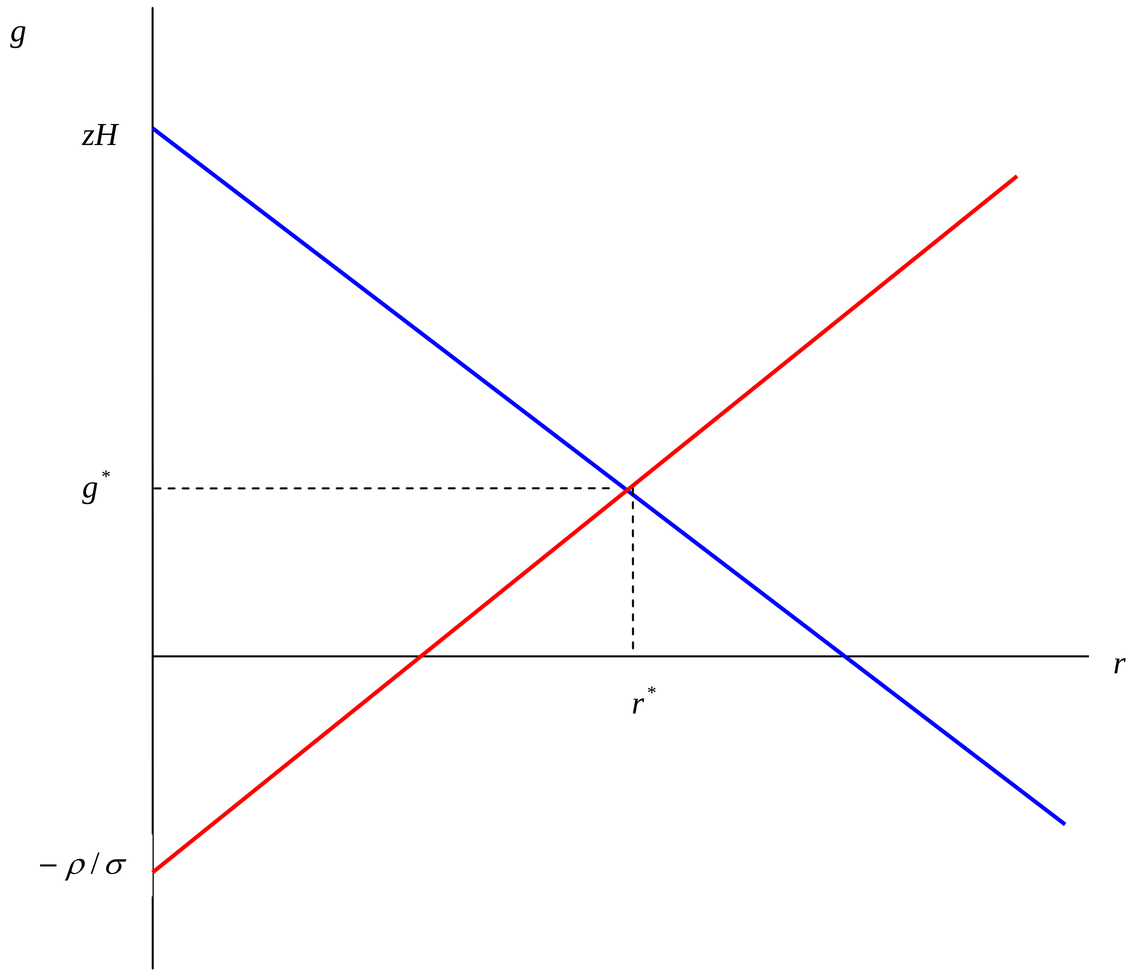
The Ramsey equation :

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma}$$

In steady state $g = \dot{c}/c$

$$g = \frac{zH - \theta\rho}{1 + \theta\sigma}$$

$$r = \frac{z\sigma H + \rho}{1 + \theta\sigma}$$



g
zH
g^*
r
r^*
$-\rho/\sigma$

Welfare analysis

Two sources of market failure

(i) monopoly markup

(ii) research spillovers

The problem faced by the social planner is

$$\begin{aligned} Max \quad U &= \int_0^\infty \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt \\ s.t. \ \dot{K} &= K^{1-\alpha-\beta} A^{\alpha+\beta} H_1^\alpha L^\beta - C \\ \dot{A} &= zAH_2 \\ H &\geq H_1 + H_2 \end{aligned}$$

The Hamiltonian of this problem is

$$\begin{aligned} \mathcal{H} &= \frac{C^{1-\sigma}-1}{1-\sigma} e^{-\rho t} + \mu z A H_2 \\ &\quad + \lambda \left(K^{1-\alpha-\beta} A^{\alpha+\beta} (H-H_2)^\alpha L^\beta - C \right) \end{aligned}$$

first-order conditions

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial C} &= 0 \ , \ \frac{\partial \mathcal{H}}{\partial L_2} = 0 \\ \frac{\partial \mathcal{H}}{\partial K} &= -\dot{\lambda} \ , \ \frac{\partial \mathcal{H}}{\partial A} = -\dot{\mu} \end{aligned}$$

The solution

$$g^* = \frac{zH/\varphi - \rho}{1/\varphi - 1 + \sigma}$$

Recall

$$g = \frac{zH/\theta - \rho}{1/\theta + \sigma}$$

where

$$\varphi \equiv \frac{\alpha}{\alpha+\beta}$$

so that

$$\varphi = \theta(1-\alpha-\beta) < \theta$$

10 Technical Change: Increasing Product Quality

Aghion and Howitt (1992)

Assumptions

Consumers

L ∞ly-lived agents

Utility: $u(y) = \int_0^\infty y_\tau e^{-r\tau} d\tau$,

where τ denotes time

$$\text{Can work in either} \quad \left.\begin{array}{c} \bullet \text{ research, } n \\ \bullet \text{ manufacturing, } m \end{array}\right\} L = n + m$$

Final good sector

Competitive

Output depends upon the input of a single intermediate good, x, and on the quality index, A

$$y = A \cdot x^\alpha \quad 0 < \alpha < 1 \tag{1}$$

Intermediate good sector

Monopolistic: uses only labour –> $x = m$

Labour market equilibrium condition

$$L = x + n$$

Research sector

Innovations increase the quality index

$$A_t = \gamma A_{t-1} \text{ where } \gamma > 1$$

$$A_t = A_0 \gamma^t$$

Let t be the quality level or "vintage".

Competitive (free entry)–> patent race

Random R&Dprocess: Poisson with parameter $\lambda > 0$

Poisson arrival rate $\lambda \cdot n$

Innovations are drastic

The intermediate goods monopolist

The profit flow π_t of the intermediate producer

$$\pi_t = \max_x [p_t(x) \cdot x - w_t \cdot x]$$

Competitive final good sector implies

$$p_t(x) = A_t \cdot \alpha x^{\alpha-1}$$

That is

$$x_t = \arg\max_x \{A_t \alpha x^\alpha - w_t x\}$$

Then

$$x_t = \left(\frac{\alpha^2}{w_t/A_t}\right)^{1/(1-\alpha)}$$

$$\pi_t = \frac{1-\alpha}{\alpha} w_t \cdot x_t$$

Let $\omega_t = w_t/A_t$

$$x_t = \widetilde{x}\left(\omega_t\right) \quad \text{and} \quad \pi_t = A_t \cdot \widetilde{\pi}\left(\omega_t\right)$$

and

$$\frac{\partial \widetilde{x}\left(\omega_t\right)}{\partial \omega_t} < 0 \quad \text{and} \quad \frac{\partial \widetilde{\pi}\left(\omega_t\right)}{\partial \omega_t} < 0$$

The research sector

$$\begin{aligned} \text{cost} &= w_t z d\tau \\ \text{expected benefit} &= \lambda z d\tau V_{t+1} \end{aligned}$$

The free entry gives the *arbitrage condition*:

$$w_t = \lambda \cdot V_{t+1}$$

Asset equation:

$$rV_{t+1} = \pi_{t+1} - \lambda n_{t+1} \cdot V_{t+1}$$

$$V_{t+1} = \frac{\pi_{t+1}}{r + \lambda n_{t+1}} \tag{2}$$

The model is now fully characterized by both:
- the *arbitrage equation*

$$\omega_t = \lambda \cdot \frac{\gamma \widetilde{\pi}(\omega_{t+1})}{r + \lambda n_{t+1}} \tag{A}$$

- the *labour market clearing equation*

$$L = n_t + \widetilde{x}(\omega_t) \tag{L}$$

The Steady State level of Research

Steady state: (A) and (L), with $\omega_t \equiv \omega$ and $n_t \equiv n$.

$$\omega = \lambda \cdot \frac{\gamma \widetilde{\pi}(\omega)}{r + \lambda n} \tag{$\widehat{A}$}$$

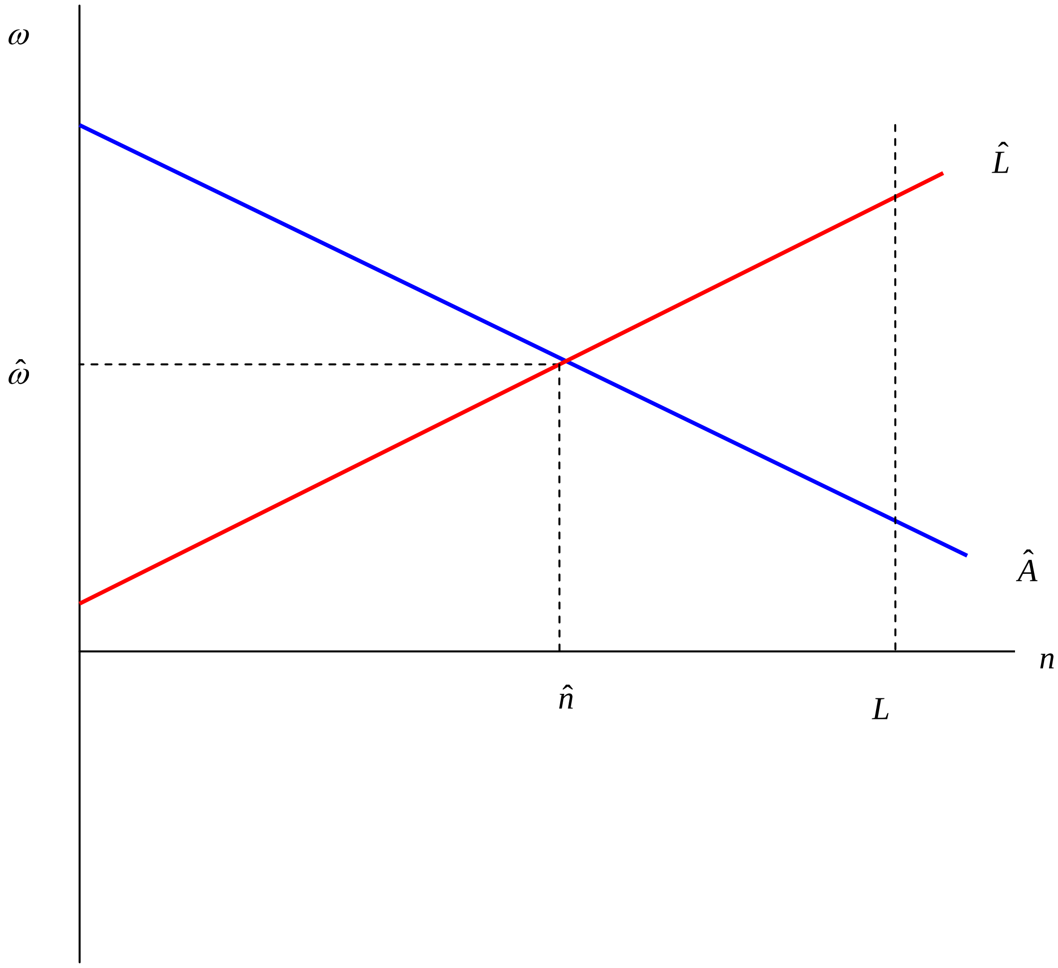
$$n + \widetilde{x}(\omega) = L \tag{$\widehat{L}$}$$

($\widehat{A}$) and ($\widehat{L}$) can be combined as

$$1 = \lambda \frac{\gamma \frac{1-\alpha}{\alpha}(L - \widehat{n})}{r + \lambda \widehat{n}} \tag{3}$$

This yields

$$\widehat{n} = \frac{\lambda \gamma \frac{1-\alpha}{\alpha} L - r}{\lambda \gamma \frac{1-\alpha}{\alpha} + r}$$


ω
$\hat{L}$
$\hat{\omega}$
$\hat{A}$
n
$\hat{n}$
L

The Steady State Rate of Growth

The steady-state flow of consumption goods

$$\begin{aligned} y_t &= A_t \widehat{x}^{\alpha} = A_t (L - \widehat{n})^{\alpha} \\ &\Rightarrow y_{t+1} = \gamma y_t \end{aligned} \tag{4}$$

The path of the log of final output $\ln y(\tau)$ is a random step function, of size $\ln \gamma > 0$.

Taking the time interval between τ and $\tau + 1$, we have

$$\ln y(\tau + 1) = \ln y(\tau) + (\ln \gamma) \cdot \varepsilon(\tau)$$

Given that $\varepsilon(\tau)$ is distributed Poisson with parameter $\lambda \widehat{n}$, we have

$$E(\ln y(\tau + 1) - \ln y(\tau)) = \lambda \widehat{n} \ln \gamma$$

The average growth rate in steady-state is

$$g = \lambda \widehat{n} \ln \gamma \tag{G}$$

Welfare Analysis

A social planner maximizes the discounted flow of income

$$\begin{aligned} U &= \int_0^{\infty} e^{-r\tau} \cdot y(\tau) \cdot d\tau \\ &= \int_0^{\infty} e^{-r\tau} \left(\sum_{t=0}^{\infty} \Pi(t, \tau) A_t x^{\alpha} \right) d\tau \end{aligned} \tag{5}$$

The Poisson process with parameter λn implies

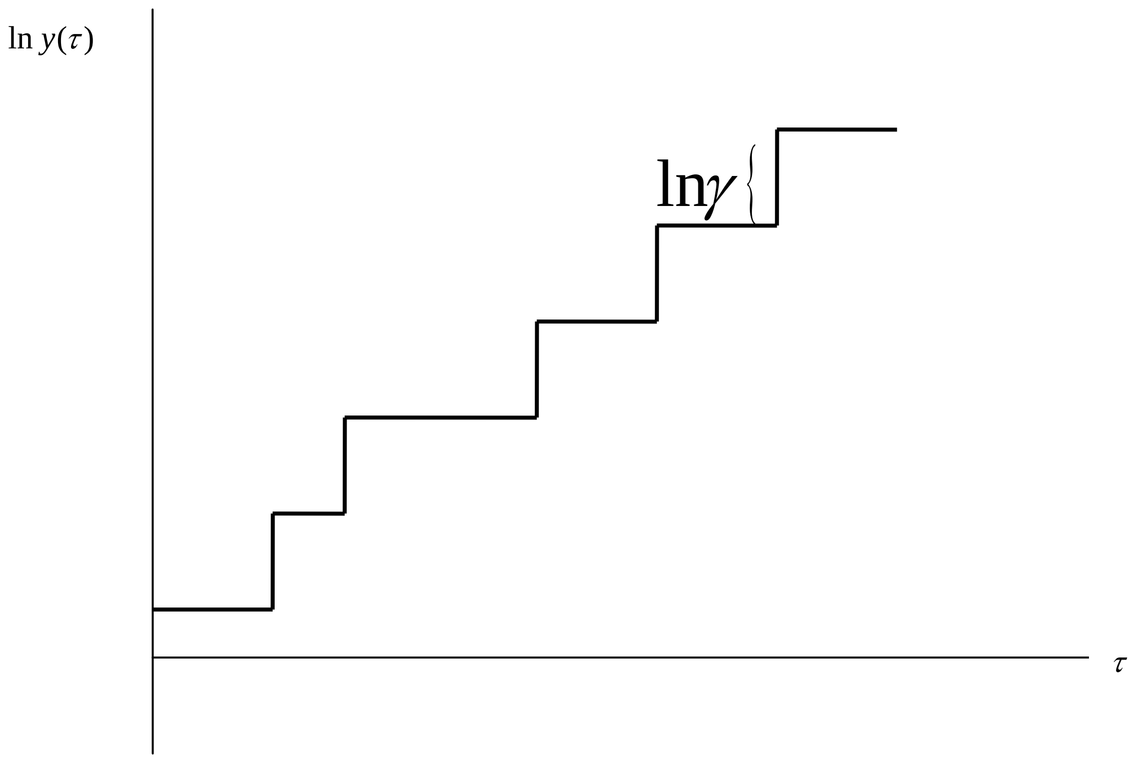
$$\Pi(t, \tau) = \frac{(\lambda n \tau)^t}{t!} \cdot e^{-\lambda n \tau}$$

the constraint is

$$L = x + n$$

Using $A_t = A_0 \gamma^t$ and $L = x + n$ welfare becomes

$$\max_n U(n) = \frac{A_0 (L - n)^{\alpha}}{r - \lambda n (\gamma - 1)}$$



ln y(τ)
lnγ
τ

The socially optimal level of research is

$$1 = \frac{\lambda(\gamma - 1)\left(\frac{1}{\alpha}\right)(L - n^*)}{r - \lambda n^*(\gamma - 1)} \tag{6}$$

and the average growth rate is $g^* = \lambda n^* \ln \gamma$.

Recall

$$1 = \frac{\lambda\gamma\left(\frac{1-\alpha}{\alpha}\right)(L - \widehat{n})}{r + \lambda\widehat{n}} \tag{7}$$

Three differences

1. *Intertemporal spillover effect:* social discount rate $<$ private discount rate
2. *Appropriability effect*
3. *Business-stealing effect*

The business-stealing effect dominates when there is much monopoly power (α close to zero) *and* innovations are not too large

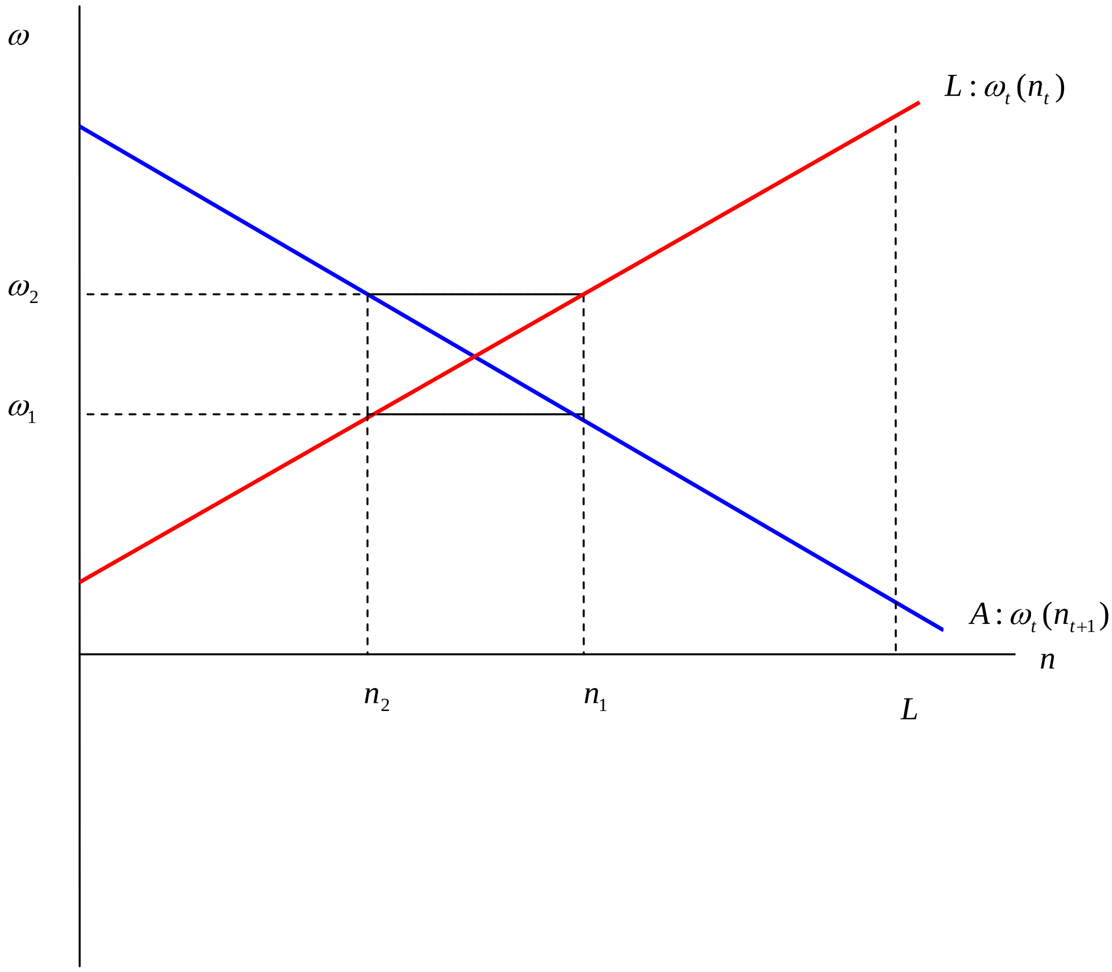
Laissez-faire growth will be excessive!

Uneven Growth

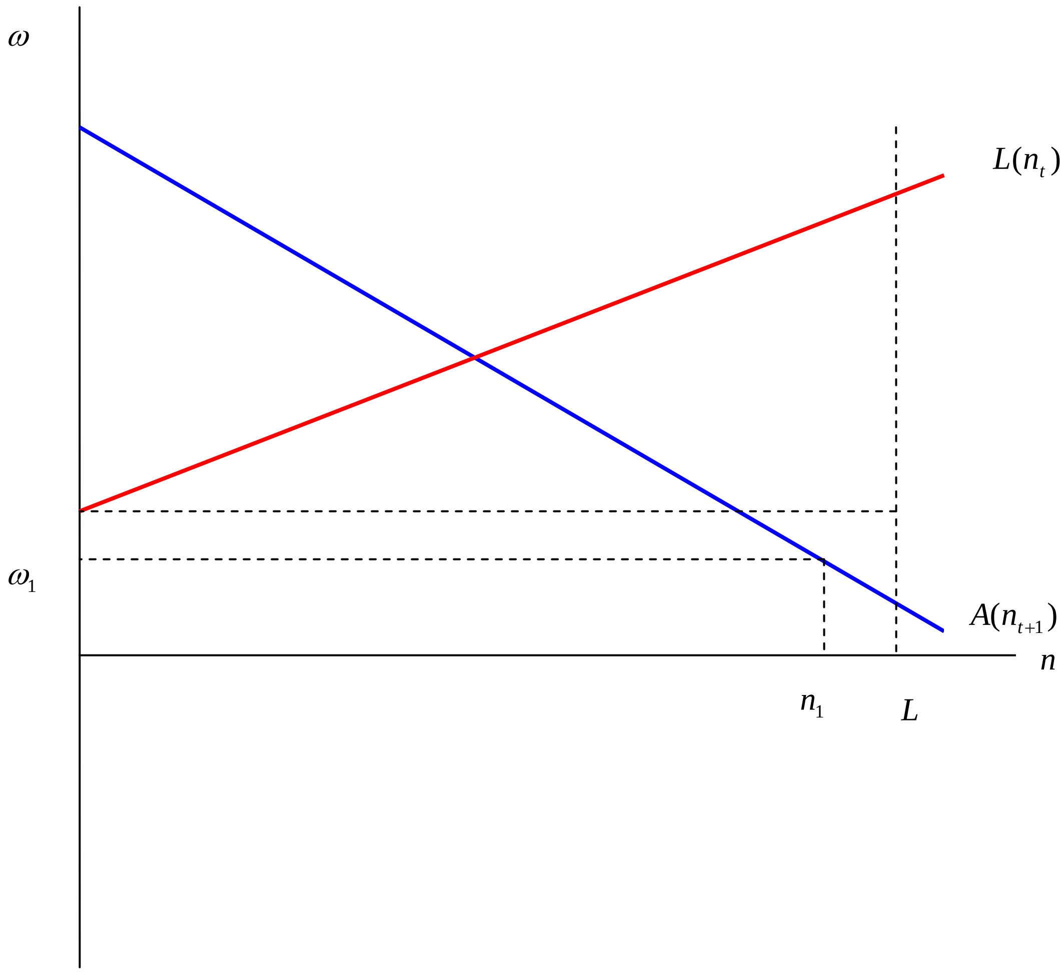
Negative correlation between current and future research: more research tomorrow n_{t+1} implies more creative destruction ($r + \lambda n_{t+1} \uparrow$) and less profits ($\pi_{t+1} = A_{t+1}\widetilde{\pi}(\omega_{t+1}) \downarrow$) after the next innovation ($t+1$) occurs. This discourages current research $\downarrow n_t$

Equations (A) and (L) give

$$n_t = \psi(n_{t+1}), \quad \psi' < 0. \tag{8}$$



ω
ω2
ω1
L : ωt(nt)
A : ωt(nt+1)
n
n2
n1
L



ω
$L(n_t)$
ω_1
$A(n_{t+1})$
n
n_1
L

11 Complementarities between human capital and R&D

11.1 The Nelson-Phelps Approach to Education

Neo-classical approach (MRW) *and* Lucas: human capital is an ordinary input in production
Nelson-Phelps (*AER* 1966): education increases individuals' capacity to *innovate* and to *adapt* to new technologies –> speeds up technological *diffusion*

Predictions of the N-P approach

1. The rate of innovation should increase with the *level* of education.

2. Marginal productivity of education attainment is an increasing function of the rate of technical progress.

3. Complementarity between education and R&D activities:

(i) macro policies which affect innovation → affect the relative labour demand and the skill distribution of employment and earnings

(ii) subsidies to education ↑ profitability of R&D → speed up technological progress.

11.2 Complementarity between R&D and education investments

Redding (1996)

Workers

Continuum of two-period OLG workers
Linear utility $U_t = c_{1,t} + \delta c_{2,t}$
All individuals are born with $h_{1,t} = 1 \ \forall \ t$
Invest a fraction v of time when young in education,

$$h_{2,t} = 1 + \gamma \cdot v^{\theta}$$

where γ is constant and $0 < \theta < 1$.

Entrepreneurs

Continuum of 2-period OLG entrepreneurs with linear utility
Do research when young and produce when old:

$$y^i_{j,t+1} = A^i_{t+1} \cdot h_{j,t+1},$$

(i) A^i_{t+1} : entrepreneur i's productivity at $t+1$
(ii) $h_{j,t+1}$: human capital of *the* $j-$worker

Innovation

By investing a non-monetary cost equal to

$$\alpha\mu A,$$

entrepreneurs can increase productivity from A to λA with

$$\text{probability} \quad \mu$$

where $\lambda > 1$ and $0 \leq \mu \leq 1$ is effort

Employment

Workers are self-employed when young:

$$(1-v)A$$

where A is the current leading-edge technology

When old they are randomly matched with firms, and get a fraction β of output surplus.

Optimal Decisions

Entrepreneurs: choose R&D effort to $\max V(\mu)$

$$\max_{\mu}\{-\mu\alpha A + \delta(1-\beta)(\mu\lambda + 1 - \mu)(1+\gamma v^{\theta})A\}$$

Then

$$\mu^* = \begin{cases} 1 & \text{if } \alpha < \delta(\lambda-1)(1+\gamma v^{\theta})(1-\beta) \\ 0 & \text{if } \alpha > \delta(\lambda-1)(1+\gamma v^{\theta})(1-\beta) \end{cases}$$

effort depends on the worker's education

Workers

$$\max_{v}\{(1-v)A + \delta\cdot\beta\cdot[\mu\lambda + 1 - \mu](1+\gamma v^{\theta})A\}$$

Then

$$v^* = [\beta\delta\theta\gamma(\mu\lambda + 1 - \mu)]^{1/(1-\theta)}$$

increasing in the probability of innovation μ
Symmetric equilibrium: same v^* and same μ^* for all agents

Multiple Steady States

1. Low-development trap: $\mu^* = 0$ and therefore $v^* = \underline{v}$.
Can occur if

$$1 + \gamma(\beta\delta\theta\gamma)^{\frac{\theta}{1-\theta}} < \frac{\alpha}{\delta(1-\beta)(\lambda-1)}$$

The growth rate is $g = \underline{g} = 0$.

2. High-growth equilibrium: $\mu^* = 1$ and therefore: $v^* = \overline{v}$.
Can occur if

$$\frac{\alpha}{\delta(1-\beta)(\lambda-1)} < 1 + \gamma(\lambda\beta\delta\theta\gamma)^{\frac{\theta}{1-\theta}}$$

The growth rate is $g = \overline{g} = \ln\lambda$.