**Economic Growth and Development : Exam 2005**

Consider the model by Barro (1990). The production function takes the form

$$Y_t = AK_t^{\alpha}(\gamma_t L_t)^{1-\alpha} \qquad \text{where } \ 0 < \alpha < 1$$

where $K_t$ is the aggregate stock of capital, $L_t$ the labour force, and $\gamma_t$ government expenditure. The utility function of the representative individual is given by

$$U = \int_0^{\infty} \frac{c_t^{1-\sigma} - 1}{1-\sigma} e^{-\rho t} dt$$

where $c_t$ is consumption. The representative individual supplies one unit of labour at each point in time. A proportional tax, $\tau$, is levied on all income and all revenues are spent on the single public good $\gamma$. The government cannot borrow, and hence must hold a balanced budget at each point in time

1. Write the production function in per capita terms and obtain the wage rate and interest rate. (3 points)

2. Normalise population to L=1. Write the government budget constraint and determine $\gamma$ as a function of $\tau$. Can the production function be written as an AK function? What are the equilibrium wage and interest rate? (3 p.)

3. What is the optimal rate of growth of consumption? (2 points)

4. Find the competitive growth rate of this economy, g, and represent it graphically as a function of $\tau$. (3 points)

5. Find the tax rate that maximizes growth, $\tau^*$. What is the intuition for the value of $\tau^*$? (3 points)

6. Now suppose that the government uses two taxes, a proportional tax on capital income, $\tau_K$, and a proportional tax on labour income, $\tau_w$ , so that the new government budget constraint is

$$\gamma = \tau_K rk + \tau_w w$$

(i) Write the consumer's maximization problem and obtain the rate o growth of consumption for a given $\gamma$. From the government budget constraint and factor prices, find the equilibrium level of $\gamma$ as a function of the two tax rates. What is the equilibrium rate of growth of this economy? (3 points)

(ii) Differentiate the growth rate with respect to and with respect to . What is the labour income tax that maximises growth? What is the capital income tax that maximises growth? Provide the intuition for your results. (3 points)

**Solution**

Now government uses two taxes.

$$\gamma = \tau_K rk + \tau_w w$$

Hence

$$\max \int_0^\infty \frac{c^{1-\sigma}-1}{1-\sigma} e^{-\rho t} dt$$

$$\text{s.t.} \quad \dot{k} = (1-\tau_K)rk + (1-\tau_w)w - c$$

and

$$\frac{\dot{c}}{c} = \frac{(1-\tau_K)A(1-\alpha)(\gamma/k)^\alpha - \rho}{\sigma}$$

We can find the ratio of government expenditure to private capital expenditure from the government's budget constraint:

$$\gamma = \tau_K(1-\alpha)k^{1-\alpha}\gamma^\alpha L \; + \tau_w \alpha k^{1-\alpha}\gamma^\alpha L$$

Hence

$$\frac{\gamma}{k} = \left[((1-\alpha)\tau_K + \alpha\tau_w)AL)\right]^{1/(1-\alpha)} \quad .$$

Substituting

$$g = (1-\tau_K)((1-\alpha)\tau_K + \alpha\tau_w)^{\frac{\alpha}{1-\alpha}} \frac{(1-\alpha)(AL^\alpha)^{\frac{1}{1-\alpha}}}{\sigma} - \frac{\rho}{\sigma}$$

Now differentiate with respect to the two growth rates. For the labour tax

$$\frac{dg}{d\tau_w} > 0$$

which implies that growth is maximized at

$$\tau_w = 1$$

For the capital tax

$$\frac{dg}{d\tau_K} \overset{sign}{=} -((1-\alpha)\tau_K + \alpha\tau_w) + (1-\alpha)(1-\tau_K) = \alpha(1-\tau_w) - \tau_K$$

at $\tau_w = 1$, we have

$$\frac{dg}{d\tau_K} = -\tau_K$$

hence, choose

$$\tau_K = 0$$

We than have

$$g = \frac{(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}(AL^{\alpha})^{\frac{1}{1-\alpha}} - \rho}{\sigma}$$

which is greater than

$$g = \frac{(1-\alpha)^2\alpha^{\frac{\alpha}{1-\alpha}}(AL^{\alpha})^{\frac{1}{1-\alpha}} - \rho}{\sigma}$$

Growth is no longer limited by the tradeoff between the positive and the negative impact of taxation, but by the possibility of raising revenue from wages.

**Economic Growth and Development : Exam February 2007**

Consider the model by Big Push model of Murphy, Shleiffer and Vishny.

*Consumers*

There are two sectors in the economy, a cottage or traditional sector, and a modern or industrial sector. We suppose that the utility of a worker employed in cottage production is given by

$$U_c = \exp\left[\int_0^N \ln x(q)dq\right] \tag{1a}$$

where there is a continuum $N$ of different goods available in this economy, indexed by $q$, and $x(q)$ denotes the amount of variety $q$ consumed.

Utility in the modern sector is given

$$U_m = \exp\left[\int_0^N \ln x(q)dq\right] - v \tag{1b}$$

where $v$ denotes the disutility cost of working in the modern sector.

There are $L$ workers in this economy, and we take the wage in the cottage industry as the numeraire, that is $w_c = 1$

*Production*

Each good is produced in its own sector by either of two types of firms:

- Cottage production (traditional sector): there is a competitive fringe of firms that produces with a constant returns technology. One unit of labor is needed to produce one unit of the good.

- Mass production (modern sector): in each sector there is a unique firm with access to an increasing returns technology. The cost of producing x units of the goods is

$$c(x) = F + \frac{x}{\alpha} \tag{2}$$

units of labour, where $F$ is the fixed cost of setting up the factory and $\alpha > 1$. We assume that the productivity gains from industry exceed disutility cost,

$$\alpha - 1 > v$$

1.(2 points) From the utility functions obtain

(i) how much consumers spend in each variety, given aggregate income, $Y$,

(ii) the wage paid to those working in the modern sector

2.(4 points) From the production functions

(i) Obtain the price of goods produced in the cottage sector

(ii) Obtain the price of goods produced by the modern sector

(iii) Write the monopolist's profits as a function of aggregate income, $v, F$ and $\alpha$.

3.(3 points) Suppose that no sector industrializes.

(i) Write aggregate income, $Y$.

(ii) Use your expressions for aggregate income and that for profits obtained in 2 to get the equilibrium level of profits as a function of model parameters.

(iii) When will a no-industralization equilibrium occur?

4.(3 points) Suppose that all sectors industrialize.

(i) Write aggregate income, $Y$.

(ii) Use your expressions for aggregate income and that for profits obtained in 2 to get the equilibrium level of profits and income as a function of model parameters.

(iii) When will a full-industralization equilibrium occur?

5.(3 points) Can multiple equilibria occur? When? Explain why.

6.(5 points) Suppose now that the economy opens up to international trade. There is external demand, and each good faces a demand of $x$. There are no imports.

(i) Write the new equation for the monopolist's profits as a function of domestic income and foreign demand

(ii) How high would foreign demand need to be for the no-industrialization equilibrium to disappear?

(iii) Explain your results

SOLUTION

1.Unit prices and symmetry across goods, imply that expenditure on each good is $y = Y/N$

Competitive factory wage:

$$w_m = 1 + v \tag{3}$$

2.Cottage sector. The are constant marginal cost and the wage is 1, hence $p_c$=1.

Modern sector. The monopolist engages in limit pricing, so $p = p_c = 1$.

The monopolist in each sector decides whether to industrialize or abstain from production. She maximizes profits taking the demand curve as given. Monopolist's profits are

$$\pi = \left(1 - \frac{1+v}{\alpha}\right) y - F(1+v) \tag{4}$$

3.No industrialization. Aggregate income is

$$Y = L \tag{5}$$

For this to be an equilibrium, given the wage bill, no profits are earned

$$\pi(0) = \left(1 - \frac{1+v}{\alpha}\right) \frac{L}{N} - F(1+v) < 0 \tag{6}$$

4.Full industrialization. Aggregate income is

$$Y = N\pi + (1+v)L \tag{7}$$

so a sector's demand is

$$y = \pi + (1+v)L/N \tag{8}$$

Substitute in the profit equation to get

$$\pi(N) = \alpha\left(\frac{L}{N} - F\right) - \frac{L}{N}(1+v) > 0 \tag{9}$$

and income is

$$Y = \alpha\,(L - NF) \tag{10}$$

For this to be an equilibrium, given the wage bill, no profits are earned

5.Equations Note that (6) and (9) can be expressed as

$$\frac{L}{N}\frac{\alpha-(1+v)}{\alpha} < F(1+v) \tag{9'}$$

$$\frac{L}{N}\frac{\alpha-(1+v)}{\alpha} > F \tag{10'}$$

If

$$\frac{L}{N}\frac{\alpha-(1+v)}{\alpha(1+v)} < F < \frac{L}{N}\frac{\alpha-(1+v)}{\alpha}$$

then, multiple equilibria exist.

6.As in lecture

7.Now

$$\pi = \left(1-\frac{1+v}{\alpha}\right)(y+x) - F(1+v)$$

Under no industrialization, demand in a particular sector is now

$$y = L/N + x$$

1.Profits earned are

$$\pi(0) = \left(1-\frac{1+v}{\alpha}\right)\left(\frac{L}{N}+x\right) - F(1+v)$$

This equilibrium disappears if

$$x \geq \frac{F(1+v)}{1-\frac{1+v}{\alpha}} - \frac{L}{N}$$

**Economic Growth and Development : Exam January 2008**

Consider the model by Redding (1996)

*Workers*

There is a continuum of two-period generations of workers, with utility

$$U_t = c_{1,t} + \delta c_{2,t}$$

where $c_{1,t}$ is consumption when young, $c_{2,t}$ is consumption when old, and $\delta$ is the discount rate. They are endowed with one unit of time each period.

All individuals are born with one unit of human capital, $h_{1,t} = 1$ for all $t$. When young they invest a fraction $v$ of time in education, and get

$$h_{2,t} = 1 + v^{\theta}$$

units of human capita, where $\theta = 1/2$.

Workers remain self-employed when young, producing output $(1 - v)A_t$ where $(1 - v)$ is production time and $A_t$ denotes the current freely available technology. When old, worker $j$ is randomly matched with firm $i$. Together they will produce $y^i_{j,t+1}$, and the worker will receive fraction $\beta$ of output.

*Entrepreneurs*

There is a continuum of two-period generations of entrepreneurs. They do research when young and can produce only when old. The utility of the entrepreneur is

$$V_t = d_{1,t} - \alpha A_t \mu + \delta d_{2,t}$$

where $d_{1,t}$ is consumption when young, $d_{2,t}$ is consumption when old, $\mu$ is the effort put in by the entrepreneur (with $0 \leq \mu \leq 1$), and $\alpha A_t \mu$ is the cost of effort.

*Innovation*

The current freely available technology is $A_t$. If an entrepreneur does not innovate at $t$, then at $t+1$ he will produce with technology $A_t$. If entrepreneur $i$ innovates at $t$, then he will produce at $t + 1$ with technology $A^i_t = \lambda A_t$ where $\lambda > 1$. The probability of innovating is equal to the effort exerted, $\mu$.

After one period, the best technology becomes available to all, so that $A_{t+2} = \lambda A_t$.

*Production*

Workers and entrepreneurs are randomly matched. The consumption good is produced according to the linear technology:

$$y^i_{j,t+1} = A^i_{t+1} \cdot h_{j,t+1},$$

where $A^i_{t+1}$ denotes entrepreneur $i$'s productivity at date $t+1$, and $h_{j,t+1}$ is the human capital of *the* $j-$worker employed by the entrepreneur at date $t+1$. They share output so that the worker gets a fraction $\beta$ of output and the entrepreneur a fraction $(1-\beta)$.

There is no capital good in this economy.

1.(2 points) What does the absence of a capital good imply for the consumption patterns of entrepreneurs and workers?

2.(3 points) Write down the expected utility of an entrepreneur and maximize it with respect to effort. Discuss how it depends on the expected level of education.

3.(2 points) Write down the expected utility of a worker and maximize it with respect to the education investment. Discuss how it depends on the expected level of technology.

4.(8 points) Equilibrium

(i) Define the symmetric Nash equilibrium of this model.

(ii) When will a low-development trap exist? What will the growth rate of output be?

(iii) When will a high growth equilibrium exist? What will the growth rate of output be?

(iv) Can multiple equilibria occur? When? Explain why.

(v) What is the effect of the technological parameter $\lambda$ on the equilibria?

5.(5 points) Policy analysis.

Suppose now that the government subsidizes education. If a worker spends a fraction $v$ of time in education, she receives a subsidy $svA$.

(i) Write the new problem for the worker and find her education decision.

(ii) How large does the subsidy have to be to get an economy out of a poverty trap?

(iii) Explain

SOLUTION

1.(2 points) What does the absence of a capita good imply for the consumption patterns of entrepreneurs and workers?

2.(3 points) Write down the expected utility of an entrepreneur and maximize it with respect to effort. Discuss how it depends on the expected level of education.

The entrepreneur chooses R&D effort (i.e. $\mu$) to

$$\max_{\mu} V(\mu) = \{-\mu\alpha A + \delta(1-\beta)(\mu\lambda + 1 - \mu) \cdot (1 + v^{\theta})A\}.$$

Hence

$$\mu^* = \begin{cases} 1 & \text{if } \alpha < \delta(\lambda - 1)(1 + v^{\theta})(1 - \beta) \\ 0 & \text{otherwise,} \end{cases}$$

thus the more workers invest in education (i.e. the higher $v$) the more will entrepreneurs invest in R&D.

3.(2 points) Write down the expected utility of a worker and maximize it with respect to the education investment. Discuss how it depends on the expected level of technology.

Allocation of working time between current production and education

$$\max_{v}\{(1-v)A + \beta\delta[\mu\lambda + 1 - \mu](1 + v^{\theta})A\}$$

This yields the optimal education time:

$$v^* = [\beta\delta\theta(\mu\lambda + 1 - \mu)]^2$$

which is an increasing function of the probability of innovation $\mu$.

4.(8 points: 1+2+2+2+1) Equilibrium

(i) Define the symmetric Nash equilibrium of this model

All workers are the same –> same $v^*$.All entrepreneurs are the same –> same $\mu^*$.All expectations are fulfilled.

(ii) When will a low-development trap exist? What will the growth rate be?

Low-development trap: $\mu^* = 0$ and therefore $v^* = \underline{v} = (\beta\delta\theta\gamma)^{\frac{1}{1-\theta}}$. For it to exist we then simply need that:

$$1 + \beta\delta\theta < \frac{\alpha}{\delta(1-\beta)(\lambda - 1)}$$

The growth rate is

$$g = \underline{g} = 0$$

(iii) When will a high growth equilibrium exist?

High-growth equilibrium: $\mu^* = 1$ and therefore: $v^* = \overline{v} = (\lambda\beta\delta\theta)^{\frac{1}{1-\theta}}$. In order for a high-growth steady-state path to exist, we need that:

$$\frac{\alpha}{\delta(1-\beta)(\lambda-1)} < 1 + \lambda\beta\delta\theta$$

The growth rate is

$$g = \overline{g} = \ln \lambda$$

(iv) Can multiple equilibria occur? When? Explain why.

When these two restrictions are simultaneously satisfied, there are multiple equilibria

$$\begin{aligned} 1 + \beta\delta\theta &< \frac{\alpha}{\delta(1-\beta)(\lambda-1)} < 1 + \lambda\beta\delta\theta \\ 1 &< \frac{2}{\beta\delta}\left(\frac{\alpha}{\delta(1-\beta)(\lambda-1)} - 1\right) < \lambda \end{aligned}$$

Which is attained depends on expectations. Because of the strategic complementarity between R&D and education

(v) Which technological parameter would imply that the poverty trap does not exist? Why?

Lambda large enough

5.Policy analysis (5 points)

$$\max_v \{(1-v)A + svA + \beta\delta[\mu\lambda + 1 - \mu](1 + v^\theta)A\}$$

This yields the optimal education time:

$$v^* = \left[\frac{\beta\delta\theta(\mu\lambda + 1 - \mu)}{1-s}\right]^{\frac{1}{1-\theta}} = \left[\frac{\beta\delta(\mu\lambda + 1 - \mu)}{2(1-s)}\right]^2$$

Need

$$1 + \frac{\beta\delta}{2(1-s)} < \frac{\alpha}{\delta(1-\beta)(\lambda-1)}$$

ie

$$s > 1 - \frac{\beta\delta}{2}\left(\frac{\alpha}{\delta(1-\beta)(\lambda-1)} - 1\right)^{-1}$$