Bayesian Learning and Gender Segregation

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We present an explanation for the persistence of gender segregation in occupations and for the observed cross-country differences in its extent. Agents have imperfect information about their probability of success in different occupations and base their career choices on prior beliefs about these probabilities. Beliefs are updated according to Bayes’s rule, implying that past differences in preferences over occupations across genders affect the beliefs of the current generation. Consequently, even when men and women become identical in their preferences, their career choices differ. Moreover, the way in which preferences change is shown to affect the degree of segregation.

I. Introduction

In industrialized countries, men and women tend to work in different occupations. For example, in 1985 over two-thirds of active women in the United States worked in occupations that were at least 70% female (Jacobs 1989; Charles 1992). Furthermore, such gender segregation has proved largely resistant to change (see Sec. II). In this article we develop a theory to account for the persistence of occupational gender segregation and for the observed cross-country differences. One approach to explain-

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ing segregation sees it as a result of statistical discrimination (Phelps 1972; Arrow 1973). An alternative literature seeks to link occupational segregation to differences in wider male and female roles (see Becker 1985, inter alia). Our model falls into the latter category, but we depart from previous treatments of this kind by showing that when there is imperfect information about labor market parameters, patterns of behavior can continue to differ even if male-female differences in preferences about combining child care and work narrow, or even disappear.

The analysis is an application of the learning model developed by Piketty (1995). Agents have imperfect information about the extent to which the probability of success in various occupations is affected by effort and by gender characteristics, and they base their career choices on prior beliefs about these probabilities. Beliefs are updated according to Bayes’s rule and transmitted from mother to daughter and from father to son, so determining the career decisions of the next generation. We exploit two implications of Bayesian updating models: that the distribution of beliefs converges, although not necessarily to the true parameter values, and that the equilibrium distribution of beliefs depends on the initial distribution. In terms of the particular problem we want to address, if, in the past, differences in preferences induced different learning paths for men and women, then once preferences become the same, equilibrium beliefs for the two groups of agents will still differ. Past circumstances—in this case sex-differentiated preferences about adult responsibilities—will continue to exert an influence even when they have disappeared or substantially changed.

The way in which preferences change is a major determinant of inter-country differences in persistence. Countries where “women become more like men,” in the sense that their preferences about income and child care are close to those traditionally associated with male workers, will generate a lot of experimentation, and thus learning, by women who are now entering traditional male occupations. As a result, the beliefs of women will be close to those of men and occupational segregation will decline. On the other hand, societies where “men become more like women” give rise to less experimentation and consequently there will be greater long-run persistence of segregation.1

The article is organized as follows. In Section II, we review the evidence about gender segregation, and in Section III, we present a model of occupational choice in which agents are trying to learn about the parameters governing labor market outcomes. Section IV addresses the question of

1 The issue of persistence in the absence of discrimination is also addressed by Lundberg and Startz (1998). In their model, differences in labor market performance between groups persist because of community externalities in the accumulation of human capital when these communities are segregated. While such a model is relevant in explaining racial inequalities, it is not applicable to the case we deal with here.
### Table 1
**Occupational Sex Segregation in 10 Countries**

<table>
<thead>
<tr>
<th>Country</th>
<th>Index of Dissimilarity</th>
<th>Ratio Index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>.630</td>
<td>.96</td>
</tr>
<tr>
<td>Finland</td>
<td>.616</td>
<td>.98</td>
</tr>
<tr>
<td>Norway</td>
<td>.573</td>
<td>.99</td>
</tr>
<tr>
<td>Netherlands</td>
<td>.567</td>
<td>1.10</td>
</tr>
<tr>
<td>Great Britain</td>
<td>.567</td>
<td>.92</td>
</tr>
<tr>
<td>France</td>
<td>.556</td>
<td>.82</td>
</tr>
<tr>
<td>Germany</td>
<td>.523</td>
<td>.79</td>
</tr>
<tr>
<td>Japan</td>
<td>.502</td>
<td>.72</td>
</tr>
<tr>
<td>United States</td>
<td>.463</td>
<td>.65</td>
</tr>
<tr>
<td>Italy</td>
<td>.449</td>
<td>.41</td>
</tr>
<tr>
<td>Average of the 10 countries</td>
<td>.548</td>
<td>.82</td>
</tr>
<tr>
<td>OECD average</td>
<td>.563</td>
<td></td>
</tr>
</tbody>
</table>


**Note.**—The index of dissimilarity represents the number of women that would have to change occupations in order to bring about an equal proportion by sex across all occupations. It is defined as

\[
D = \frac{1}{2} \sum_{i=1}^{I} |p_m - p_f|,
\]

where \(p_m\) is the proportion of males in the labor force employed in occupation \(i\), \(p_f\) is the proportion of females, and \(I\) is the number of occupational categories. The ratio index is defined as

\[
R = \frac{1}{I} \sum_{i=1}^{I} \left| \ln(F/M) - \frac{1}{I} \sum_{i=1}^{I} \ln(F/M) \right|,
\]

where \(F\) is the number of women in occupation \(i\) and \(M\) is the number of men in occupation \(i\).

whether gender segregation would disappear once traditional differences in preferences were eroded and examines the factors that determine the extent of segregation. In Section V, we discuss the implications of this analysis including those concerning policy. Section VI concludes.

**II. Occupational Segregation**

The model we present is built around some stylized facts concerning women’s labor force participation. Women began to enter the labor force of the industrialized societies in large numbers at a time when assumptions about the division of domestic responsibilities between spouses required that women’s occupational ambitions be subordinate to such responsibilities, or, at the very least, that their ambitions be compatible with their responsibilities. Nowadays segregation is still evident despite the introduction, in many countries, of equal opportunities and antidiscrimination legislation, the increased provision of child care by employers and the state, and the shift in median views about the division of domestic and work responsibilities among spouses or partners.

Table 1 presents two different measures of occupational segregation for 10 countries. Both show the same picture of substantial cross-national...
variation. Sweden, Finland, and Norway display high levels of gender segregation, while countries such as the United States, Italy, and Japan have rather lower—but nevertheless substantial—levels.

Evidence from the United States (Jacobs 1989; Reskin 1993) suggests some reduction in occupational segregation by gender during this century, with most of the change occurring in the 1970s, as seen in figure 1. For Britain, Blackburn, Siltanen, and Jarman (1995) report fluctuations in occupational segregation by gender over the period 1951–91 but no clear evidence of any downward trend. In a study of 58 countries between 1960 and 1980, Jacobs and Lim (1995) found some indications of a gradual though rather modest decline in gender segregation. The persistence of occupational segregation by gender stands in marked contrast to the changes that have occurred both in women’s rates of labor force participation and in their occupational aspirations. Female rates of labor force participation have increased in the postwar period in all developed countries (though at different rates). One of the most striking features of women’s labor force participation is the disappearance of their hitherto characteristic “M-shaped” pattern of participation. Whereas in the 1950s female participation was high in their early 20s and 50s but low in the middle years in which most women bore children, this pattern has changed, and women’s participation rates are now roughly constant throughout the working years (Anker 1998, p. 142). This is reflected in increasing rates of labor force participation among women with children under age 18. In the United States and the United Kingdom, the rate for this group more than doubled between 1960 and 1990, while for Germany the increase was of the order of 50% (Alwin, Scott, and Braun 1999).

Fig. 1.—Gender segregation in the United States
At the same time, women’s aspirations and expectations have also altered. In the United States, data from the General Social Survey shows a steady increase in the proportion of individuals who approve of “a married woman earning money in business or industry if she has a husband capable of supporting her” and also a convergence in the views of men and women. In 1972, 68% of women and 63% of men approved; in 1994, 81% of both sexes approved (see Alwin et al. 1999, table 2). Similarly, in response to the statement “it is much better for everyone if the man is the achiever and the woman takes care of the home and the family,” in 1977, 37% of women and 31% of men disagreed. In 1994, these proportions were 67% and 62%, respectively. For Britain, survey data shows a steady decline in the proportion of women who believe that “it is a wife’s job to look after the home and family.” In 1980, only 33% of women disagreed with this statement, while in 1989, this had doubled to 69% (Scott 1990, p. 53). At the later date, differences in responses by age were substantial among women, while differences between men and women were relatively small (see Scott 1990, pp. 54–55).

Two main methodological problems arise in the measurement of occupational segregation. The first is that such measures will not be invariant to the choice of occupational unit. The more disaggregated the data, the greater is the degree of measured segregation as we start to capture vertical segregation (i.e., hierarchical differentiation within the same occupation) as well as horizontal segregation (segregation by gender across different occupations). The second problem concerns the construction of an index of segregation: such indexes tend to be influenced not simply by the extent of segregation but also by changes in labor force participation and in the sectoral composition of the economy. However, both of these issues, and the resulting difficulties, are well known and widely discussed in the empirical literature (for recent discussions, see Blackburn et al. [1995]; Watts [1995]). In particular, Anker (1998) shows how the broad time trends and cross-country patterns discussed above are robust when we correct for occupational classification differences and when we use a variety of segregation indexes.

The characteristics of male and female occupations are well documented (Anker 1997, p. 329). Female-dominated jobs tend to offer lower wages but may be more flexible in either the short or long run. This may be due to flexibility in working hours, part-time employment, or the possibility of returning to work after a period outside the labor force. Examples of male occupations are skilled manual workers, sailors, engineers, and architects; examples of female occupations are nurses, care attendants, child-care workers, teachers, and secretaries (see Rubery and Fagan 1993, pp. 23–23).

In developing our argument, we draw on another widely observed empirical regularity. Despite the fact that, in many industrialized societies,
women’s level of educational attainment has, over the past 20 years, come to equal or exceed that of men, there still remain pronounced differences in the subjects and forms of education that the two genders pursue (Blau and Ferber 1986; Reskin and Roos 1990; Brown and Corcoran 1997; Jonsson 1999). Most notably, women are markedly underrepresented in mathematics and in most of the scientific and technical disciplines. We, hence, explain segregated labor market outcomes as, in large part, a consequence of such differentiated educational choices.

III. A Model of Occupational Choice

The economy is populated by gender dynasties. A dynasty consists of a sequence of either women or men within a family. Each member of a dynasty lives for two periods, is educated in the first one, and works in the second. In the second period, individuals also form men-women couples and raise children, the responsibilities for which may or may not be divided between them. Each couple is assumed to have a son and a daughter; hence, the population and its gender division stays constant over time. All agents are educated and all work.

Individuals differ in their “ability” level, denoted $I$, which includes both natural ability and other social and economic advantages and disadvantages that can be passed from parents to children. Ability is inherited by a daughter from her mother and by a son from his father. It is therefore constant over time for each gender dynasty. For simplicity, we consider only two ability levels: high and low, denoted $IH$ and $IL$, respectively, where $IL < IH \leq 1$. The distribution of abilities is assumed to be the same for men as for women.

A. Career Paths

There are two career paths in the economy, denoted A and B. Type A jobs are characterized by high wages (and also prestige or personal satisfaction derived from the job) but do not permit a satisfactory combination of work and domestic duties (which we henceforth label “child care”). Type B jobs offer lower wages, although these jobs can be combined with child care, as they are more flexible in the short or long-run or both. Crucially, we assume that type B jobs include professional child-care workers. Education of type A leads to type A jobs and similarly for B.

2 In 1990, the average number of years of education in Organization for Economic Cooperation and Development (OECD) countries was 10.6 for adult men and 10.3 for adult women (Anker 1998, p. 141).

3 It would be possible to let ability be an average of that of both parents. Clearly, if individuals marry within their ability level, nothing is changed. If agents can form couples with individuals of different ability, our results would be unaffected since, as we will see, the equilibria of the learning model do not depend on $I$. In this case, there would, however, be more than two ability groups.
Agents who choose career path A may end up performing one of two jobs. If they get “high” grades in education, they perform job AH; if they get “low” grades, they perform job AL. The utility obtained by any agent from an AH job is greater than that from an AL job. Hence, we say that an individual who has chosen career path A has “succeeded” if at the end of the education period he or she gets an AH job, while we say that the agent has “failed” if he or she gets an AL job. All agents who choose career path B perform jobs with the same rewards, which we denote simply B.

The probability of success in education of type A is given by

\[ P_{ig} = \pi_g I_i + \theta_g e_i, \]

where \( g = m, w \) denotes gender (men and women) and \( \pi_g \) and \( \theta_g \) are gender-specific parameters, \( I_i \) is the ability of individual \( i \), and \( e_i \) is the (costly) effort he or she chooses to exert during the education period. Effort is assumed to be private information, and is normalized so that \( e \in [0, 1] \). We impose the parameter restriction \( \pi + \theta \leq 1 \) in order to ensure that the probability of success in A is less than or equal to one.

B. Preferences

Agents’ preferences over jobs differ depending on the weight given to child-care opportunities and to income in their utility function. The utility of the \( j \)th job for agent \( i \) is given by \( U(j, r) \), where \( r \) denotes the weight given to child-care opportunities. The larger is \( r \), the greater the desire to bear child-care responsibilities. All men are assumed to have the same child-care preferences, denoted \( r_m \), and all women have preferences \( r_w \), where \( r_{m} \geq r_{w} \).

We make three assumptions about the utility of different jobs. First, all agents prefer AH jobs to AL jobs, that is, \( U(AH, r) \geq U(AL, r) \) \( \forall r \). Second, the greater the desire to bear child-care responsibilities, the higher the utility of B jobs relative to A jobs, that is, \( \partial[U(B, r) - U(AH, r)] / \partial r > 0 \) and \( \partial[U(B, r) - U(AL, r)] / \partial r > 0 \). Third, the more child-care opportunities are valued by an individual, the greater is the difference in utility between failing and succeeding in A. Define \( \Delta U(A, r) \) as the difference in utility between succeeding and failing in A, that is,

\[ \Delta U(A, r) = U(AH, r) - U(AL, r). \]

Then, the above assumption implies that \( \partial \Delta U(A, r) / \partial r > 0 \). That is, the utility of the two jobs is more similar for those with low than for those with high values of \( r \). This will occur whenever the difference in the child-

4 This “education process” at the end of which the job type is determined could also be interpreted as the degree of success in a job in the early stages of a career, which subsequently affects promotion.
care possibilities provided by AH and AL is greater than the difference in income. This third assumption is crucial for the results derived below.

Exerting effort during the education period involves a utility cost. We assume that studying has a lower cost the more able individual $i$ is and that the cost of exerting an amount of effort $e$ is given by $C(e, I_i) = e^2/(2cI_i)$, where $c > 0$. For simplicity, there is no effort cost associated with education of type B. Agent $i$’s expected utility from career path A is then given by the expression

$$EU(A) = P_u U(AH, r_i) + (1 - P_u) U(AL, r_i) - C(e, I_i),$$  \hspace{1cm} (2)

while the expected utility from career path B is simply $U(B, r_i)$.

C. The Information Structure

Agents have imperfect information about the parameters that determine the probability of success in A, $\pi$ and $\theta$. Agents are therefore uncertain as to the returns to effort vis-à-vis those to ability in determining educational success. There are two possible values that these parameters can take, $(\theta, \pi)$ and $(\theta', \pi')$. Let $\pi + \theta = \pi' + \theta'$, and assume that $\pi > \pi'$ and $\theta < \theta'$. The first set of values thus corresponds to the inherited component having a large effect on the probability of success and effort having little impact, while $(\theta', \pi')$ corresponds to a situation in which effort has a stronger impact on the probability of success. There is no correlation between the values of the parameters for men and for women. This may be because men and women may differ in their capacity to perform different types of jobs, or because of discrimination.

Denote by $z$ the subjective probability attached by an agent to $(\theta', \pi')$ being the true state of the world. The beliefs of the individual are then $\tilde{\theta} = z\theta' + (1 - z)\theta$ and $\tilde{\pi} = z\pi' + (1 - z)\pi$, and the subjective probability of success in educational path A is $\tilde{P}(z) = \tilde{\pi}I + \tilde{\theta}e$. Thus the agent’s beliefs are wholly captured by the parameter $z$.

Following Piketty (1995), we assume that dynastic learning takes the form of Bayesian updating, with beliefs being transmitted from parent to child. A newborn agent holds a prior belief $\tilde{z}$. The individual bases his or her effort and education decisions on this prior. After the outcome of the education process is realized, the individual updates the prior according to Bayes’s rule and transmits the new belief to his or her offspring.$^5$

$^5$ In this set-up, learning from the aggregate is not possible because, although job outcomes are observed, effort is private information. See Piketty (1995).
D. The Choice of Career Path

Suppose that an agent with beliefs \( \bar{\theta} \) and \( \bar{\pi} \) decides to take career path A. Effort will be chosen so as to maximize the expected utility from A,

\[
e(z, r) = \arg \max_{e\geq 0} (\bar{\pi} I - \bar{\theta} e) \cdot U(AH, r) + (1 - \bar{\pi} I - \bar{\theta} e) \cdot U(AL, r) - \frac{e^2}{2cI},
\]

that is,

\[
e(z, r, I) = c\bar{\theta} U(A, r) I.
\]

We assume that the cost of effort is high enough (\( c \) low enough) to ensure that \( e \leq 1 \). Substituting for \( e \) in equation (2), the expected utility from career path A becomes

\[
EU(A, z, r, I) = U(AL, r) + \bar{\pi} \Delta U(A, r) I + \frac{cI}{2} [\bar{\theta} \Delta U(A, r)]^2.
\]

Agent \( i \) chooses A only if it yields a higher expected utility than career path B, \( EU(A, r, z, I) > U(B, r) \); that is, if his or her ability is greater than

\[
\bar{I}(z, r) = \frac{U(B, r) - U(AL, r)}{\bar{\pi}(z) \Delta U(A, r) + \frac{cI}{2} [\bar{\theta}(z) \Delta U(A, r)]}.
\]

Differentiating equation (4), we find that \( \partial \bar{I}/\partial r > 0 \) and \( \partial \bar{I}/\partial z < 0 \). When a greater weight is placed on child care, B becomes relatively more attractive. A higher probability of success, that is, greater ability, is hence required in order to choose A. A high value of \( z \) implies that the agent attaches a great weight to effort as compared with ability. He or she exerts high effort (see eq. [3]) and will therefore require a lower threshold ability in order to choose A.

It is important to note that all child care is done by individuals in B jobs. There are circumstances in which women who are in A jobs will contract out child care, either because their partner is also in an A job or because men may not be willing to undertake child care, even if they are in B jobs. Some of the women employed in type B jobs will thus be engaged both in paid child care as professional child-care workers caring for the children of A-type women and in unpaid caretaking for their own child.\(^6\)

\(^6\) For an analysis of possible equilibria when child care must be performed by one parent, see Engineer and Welling (1999).
IV. The Persistence of Gender Segregation

We want to examine whether gender segregation is eroded as societies move from a “traditional” gender division of child care or domestic responsibilities to a situation in which such differences disappear. The above exposition implies that there are two reasons why individuals will make different career choices: differences in preferences and differences in beliefs about the probability of success in a particular career. The question is then whether past preferences can affect current beliefs and therefore result in different behavior of men and of women even when these preferences have changed.

As a benchmark, consider a situation in which there is perfect information. Since all parameters are equal for men and women except that the latter have a higher value of \( r \), the threshold level of ability will initially be greater for women than for men. This yields an equilibrium under traditional preferences in which there are more men than women in A. As \( r \) becomes the same for men and women, all differences between the two groups are eroded. The threshold ability would be the same for men and for women, and their labor market choices would become identical. Under imperfect information, on the other hand, there is hysteresis. The reason for this is that, when beliefs evolve according to Bayes’s rule, equilibrium beliefs depend on the initial distribution of beliefs and hence on past actions. Past differences in preferences across genders thus determine what men and women have learned about the labor market and will exert a long-lasting influence on patterns of occupational segregation.

In this section, we first examine how a dynasty’s beliefs evolve if agents update them according to Bayes’s rule and then show that the beliefs of men and women differ even when their preferences have become identical.

A. Dynastic Learning

The sequence of events is as follows. The prior \( z \) determines an agent’s ability threshold \( \bar{I}(z, r) \), as well as the effort \( e(z, r) \) exerted. Individuals whose ability is above their threshold choose A and, by observing whether they succeed or not, they update their prior, \( z \), according to Bayes’s rule. The posterior belief of a man (woman) in the \( t \)th generation is inherited by his son (her daughter) as his (her) prior belief. For agents who choose A, beliefs are updated in each generation. Individuals whose ability is below the threshold choose B, and their beliefs about \( \pi \) and \( \theta \) remain unchanged because their own experiences provide no new information about the process governing success in A.
The posterior belief of an agent who chooses career path A at \( t \) and succeeds is then given by Bayes’s rule,

\[
\frac{\Pr(\text{AH} | \theta') \Pr(\theta')}{\Pr(\text{AH} | \theta') \Pr(\theta') + \Pr(\text{AH} | \theta) \Pr(\theta)},
\]

where \( \Pr(\cdot) \) denotes the probability of the event in brackets occurring. A similar expression holds for the posterior belief of an agent who fails, denoted \( z_i^{\text{AH}} \). Substituting for the various probabilities, we can express the posterior beliefs of an agent who succeeds and one who fails as

\[
z_i^{\text{AH}}(z_i, e) = \frac{z_i(\pi' I + \theta'e)}{z_i(\pi' I + \theta'e) + (1 - z_i)(\pi I + \theta e)}, \tag{5}
\]

\[
z_i^{\text{AL}}(z_i, e) = \frac{z_i(1 - \pi' I - \theta'e)}{z_i(1 - \pi' I - \theta'e) + (1 - z_i)(1 - \pi I - \theta e)}. \tag{6}
\]

The functions in (5) and (6) describe the evolution over time of a dynasty’s beliefs. Whether the posterior weight placed on \( \theta' \) is greater than the prior depends on whether, for the level of effort chosen by the agent, the actual outcome is more likely to have occurred for \( \theta' \) than for \( \theta \). That is, from equations (5) and (6), we can write

\[
z_i^{\text{AH}}(z_i, e) > z_i \iff \pi' I + \theta'e > \pi I + \theta e, \tag{7}
\]

\[
z_i^{\text{AL}}(z_i, e) > z_i \iff \pi' I + \theta'e < \pi I + \theta e. \tag{8}
\]

If an individual experiences a success and the probability of success (conditional on her effort) is greater for \( \theta' \) than for \( \theta \), she will place a higher weight on \( \theta' \) being the true state of the world. That is, \( z_i^{\text{AH}} > z_i \). The opposite holds for the case of a failure. Bayesian learning consists of interpreting actual outcomes in the light of one’s prior beliefs. Consequently, the same outcome can give rise to different posteriors.

A general property of Bayesian learning is that the stochastic process \( \{z_i\} \) describes a martingale: what today’s member of dynasty \( i \) expects her offspring to know next period is what the individual knows today. Moreover, the likelihood ratio, defined as \( l_i \equiv (1 - z_i)/z_i \), follows a stochastic process \( \{l_i\} \), which describes a martingale conditional on the true state of the world. Hence, standard martingale convergence results can be applied.

Assume, without loss of generality, that the true parameter values are \( (\theta', \pi') \) for both men and women, so that the belief \( z = 1 \) is equivalent to placing full weight on the truth. Then, we can state the following proposition:

**Proposition 1.** Consider the stochastic process \( \{z_i\} \):
i) For any initial belief, $z_0$, and for every dynasty, beliefs converge toward some stationary belief, $z_\infty$, with probability one.

ii) There are two possible stable stationary beliefs: one places full weight on the truth, $z_\infty = 1$, while the other places a strictly positive weight on the alternative state of the world, $z_\infty = z^\circ$, where $z^\circ \in (0, 1)$.

iii) Given the true state of the world: (a) initial beliefs such that $z_0 \leq z^\circ$ converge to $z^\circ$ with probability one, and (b) initial beliefs such that $z_0 > z^\circ$ converge to 1 with probability $p(z_0, z^\circ)$ and to $z^\circ$ with probability $[1 - p(z_0, z^\circ)]$, where

$$p(z_0, z^\circ) = \frac{z_0 - z^\circ}{z_0(1 - z^\circ)}. \tag{9}$$

Proof. See appendix A.

The first two parts of proposition 1 tell us that Bayesian learning processes converge to a stationary belief, although not necessarily to the truth. Any stable stationary belief must place a positive weight on the true state of the world. Intuitively, this means that, when an outcome is observed, the truth is “to some extent” recognized. The third part says that Bayesian updating implies that, for any given dynasty, the probability of converging to one or another equilibrium belief depends on its initial belief, the reason being that an outcome can give rise to different posterior beliefs depending on the probabilities initially attributed to the various parameter values. Consequently, the equilibrium distribution of beliefs is determined by the initial distribution.

To understand these results, figure 2 depicts the Bayesian updating functions $z_{t+1}[z_0, e(z_t)]$ and $z_{t+1}^{*}\{z_0, e(z_t)\}$ as a function of $z_t$. The Bayesian updating functions have three fixed points at which $z_{t+1}^{*}\{z_0, e(z_t)\} = z_t$. Two of these fixed points correspond to placing full weight on either of the two possible parameter values, $z = 0$ and $z = 1$. The third, is an interior solution $z^\circ$, such that

$$\pi I + \theta e^* = \pi' I + \theta' e^*. \tag{10}$$

where $e^* = e(z^\circ, r)$. When an agent holds the prior $z^\circ$, the resulting effort level $e^\circ$ makes the expected probability of success the same under the two sets of parameters. Consequently, nothing can be learned from a success or a failure, and the posterior belief is the same as the prior. We call this interior fixed point the “confounded learning belief” because it attaches a strictly positive probability to a set of parameter values other than the truth.\footnote{See Smith and Sørensen (2000).}
Bayesian learning implies that if men and women differ in their preferences, they will hold different equilibrium beliefs. From equations (3) and (10), the interior fixed point is given by

\[ z^*_i = \frac{1}{\theta' - \theta} \left( \frac{\pi - \pi'}{\theta' - \theta} c \Delta U(A, r) - \theta \right). \]  

Individual preferences thus affect the confounded learning belief, as a higher \( r \) increases the difference between succeeding and failing in \( A, \Delta U(A, r) \) and reduces \( z^* \). Hence, the larger the weight given to child care, the further the confounded learning belief is from the true belief, \( z = 1 \). The equilibrium distribution of beliefs thus depends on the preference parameter, as \( r \) affects both \( z^* \) and, from equation (9), the proportion of agents that learn the truth.

Successive learning across generations may, however, be incomplete. If, at some point, the agent’s threshold ability, \( I \), exceeds his or her ability, he or she will choose \( B \) rather than \( A \) and so will acquire no additional information on which to update the inherited prior. Let \( IH \) be such that high-ability agents always choose \( A \) (i.e., \( I(z = 0, r) < IH \) for all \( r \)). Their equilibrium distribution of beliefs thus converges to a two-point distri-
bution with men holding either $z = 1$ or $z^* > z_0$, and women holding either $z = 1$ or $z^* < z_0$.

For low-ability agents, suppose that $\tilde{I}(z = 1, r) < IL < \tilde{I}(z^*, r)$, so that they choose A only if their beliefs about the role of effort are high enough. There is thus a threshold belief, defined by $\tilde{I}(\tilde{z}, r) = IL$, such that low-ability agents with beliefs less than $\tilde{z}$ will choose B. This threshold belief is greater than the confounded learning belief (since $\partial I/\partial z < 0$) and is larger the greater the weight placed on the child-care opportunities provided by the job because $d\tilde{z}/dr = - (\partial I/\partial z)/(\partial I/\partial z) > 0$.

The equilibrium distribution of beliefs for low-ability individuals will then be as follows. Those agents with initial beliefs in the interval $(0, \tilde{z})$ choose B and maintain their initial belief. Those individuals with $z_0 > \tilde{z}$ choose A. Their beliefs will converge to either $z = 1$ or “toward” the confounded belief. However, this convergence is halted by the threshold at $\tilde{z}$, at which value they stop choosing A. For them, the probability of converging to the true belief is given by

\[ p(z_0, \tilde{z}) = \frac{z_0 - \tilde{z}}{z_0(1 - \tilde{z})}, \] (12)

which is lower than $p(z_0, z^*)$. Because $\tilde{z}$ depends on the value of $r$, men and women will hold different equilibrium beliefs.

C. Persistence

Consider now the evolution of beliefs as the economy passes through two highly stylized eras. In what we might term the “traditional era,” men and women had an identical initial distribution of beliefs but different preferences, with women placing more weight on child care than men, $r_w > r_m$. As a result, men and women differed in the amount of effort they exerted and in their choice of careers. Their learning paths and the beliefs to which they converged were therefore not the same. We then take the long-run distribution of beliefs under traditional preferences as the starting distribution of beliefs in the second or “current era,” in which both men and women have the same preference parameter, $r$, where $r_w \geq r \geq r_m$. The beliefs of both men and women will now converge to the same values. However, because at the start of the current era beliefs were not the same, the fractions of men and women holding each equilibrium belief, and hence their career choices, will differ.

To illustrate this argument we show how the mass of agents who converge to either of the equilibrium values differs according to the two ascriptive features that distinguish the agents in our model—gender and

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8 The probability is obtained from the martingale condition, $z_0 = (1 - z_0) \cdot \tilde{z} + z_0p \cdot 1 + (1 - p) \cdot \tilde{z}$. It is lower than $p(z_0, z^*)$ as the paths converging to $z = 1$ that imply holding, at some point, a belief $z < \tilde{z}$ have been excluded.
ability level—and how it depends on the way in which preferences change over time.

Let the initial distribution of priors be given by the density function $f(z)$, with support $(0, 1)$, and suppose that it is the same for all four types of agents. Consider, first, the equilibrium beliefs of high-ability men. Since they always choose A, the fraction of high-ability male dynasties who converge to the true belief in the first era is given by

$$MH_1 = \int_{z_m^*}^{1} p(z, z_m^*) f(z) dz.$$  

(13)

As we can see, this fraction depends both on $z_m^*$ and on the initial distribution of beliefs. The remaining $(1 - MH_1)$ dynasties converge to the confounded learning belief $z_m^*$. Similar expressions can be obtained for the fractions of high-ability women, low-ability men, and low-ability women who converge to the true belief in the first era, denoted $WH_1$, $ML_1$, and $HL_1$, respectively (see app. B).

The change in the preference parameter $r$ at the start of the current era leads to a new confounded learning value $z^*$ and the threshold belief $z^*$, so that $z_m^* \geq z^* \geq z_w^*$ and $z_m^* \leq z \leq z_w^*$. In this era, those who begin with the true beliefs keep them, as they are stable. All other beliefs are no longer an equilibrium, and, hence, individuals with such beliefs start learning again.

The possible equilibrium beliefs are now the same for men and women: the beliefs of high-ability individuals converge to either the true value or the—now common—$z^*$, while those of low-ability agents go to 1 or $z$. However, the proportion of men and women that learn the truth differs. We can therefore characterize the distribution of beliefs of men and women in each ability group by these proportions, denoted, respectively, $MH_2$, $WH_2$, $ML_2$, and $HL_2$.

Since the only difference between men and women is these proportions, we define the extent of gender segregation in the current era as the difference between the proportions of men and women in each ability group that hold the belief $z = 1$. For the high-ability group segregation is then given by

$$SH(r) = MH_2 [z^*(r)] - WH_2 [z^*(r)].$$  

(14)

Gender segregation is a function of the new confounded learning belief, $z^*$, and, therefore, of the new preference parameter, $r$.

Whether or not the number of men who hold the true belief is greater than the number of women depends on how new preferences relate to the initial ones. The function $SH(r)$ is depicted in figure 3. It is strictly

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*Low-ability agents can also hold their initial belief if they never chose A.
increasing in $r$, and it is negative for $r = r_m$ and positive for $r = r_w$ (see app. B). If, in the second era, preferences change so that “men become similar to women,” that is, $r$ is close to $r_w$, more men than women will hold the true belief, $MH_2 > WH_2$. Conversely, suppose that, in the second era, both men and women have a value of $r$ close to the one that men had in the first era. In this case, women will hold a higher average belief than men, $MH_2 < WH_2$. Thus, whether a greater proportion of male or female high-ability agents hold the true belief at the end of the second era depends on the manner in which traditional gender differences in responsibilities for child care narrow.

The extent of gender segregation among low-ability individuals is given by the function

$$SL(r) = ML_z[\mathcal{Z}(r)] - WL_z[\mathcal{Z}(r)],$$

(15)

which is strictly increasing and positive for all $r \in [r_m, r_w]$, as depicted in figure 3 (see app. B). Hence, $ML_z \geq WL_z$, which implies that the pro-

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10 It is straightforward to check that, if the true parameter values were those associated with $z = 0$, the opposite would hold: more women would hold the true belief. However, this would not affect our conclusions about gender segregation, as will be seen in the discussion in Sec. V, since what generates the observed segregation patterns is the fact that men hold beliefs that lead them to exert more effort than women.
portion of low-ability men who hold the true belief is always at least as large as the proportion of low-ability women. The difference between the two depends, as among high-ability agents, on how preferences change. The closer the second era preferences are to those of men in the first era, the smaller is the difference between the number of men and the number of women who learn the truth.

V. Discussion

There are two central implications of the foregoing analysis: gender segregation in occupations persists even when all differences in preferences have disappeared, and the extent of segregation depends on the way in which preferences change. Persistence is captured by two features of the model: the number of women choosing A is smaller than the number of men, and women are likely to succeed in A less often than men. In our highly stylized analysis with only two ability groups, each of these groups’ behavior captures one of these aspects. In the low-ability group, only those agents who hold the true belief choose career path A. Since the number of women who hold the belief $z = 1$ is lower than the number of men, fewer women choose A—that is, we see horizontal segregation. In the case of high-ability individuals, all men and all women choose career path A. However, if the new set of common preferences is not too close to the traditional value of $r$ for men, men hold, on average, a higher belief about the role of effort than women. Consequently, men exert, on average, more effort than women, and the proportion of men who succeed will also be greater. Thus, in occupations of type A, there will be more men than women in the best jobs—in other words, there exists vertical segregation within these occupations. To the extent that, despite the documented convergence in male and female preferences, differences between them still exist, the observed extent of segregation will be due to the remaining differences in preferences as well as in beliefs.

The model also provides a possible explanation for cross-national variations in segregation. As we have seen in Section IV, the extent of gender segregation that persists once men and women have the same preferences depends on whether the preferences about income and child-care possibilities are closer to those traditionally associated with male workers or to those associated with female workers. Countries in which women have become “more like men” will exhibit less gender segregation, while those where men have become “more like women” would exhibit a greater persistence of segregation. The reason for this is that, in the former, the change in preferences will generate a lot of experimentation, and thus learning, by women who are now entering traditional male occupations. As a result, the beliefs of women will be close to those of men and so will their career and effort choices. On the other hand, societies where
Table 2
Occupational Segregation and Relative Wages

<table>
<thead>
<tr>
<th>Country</th>
<th>Index of Dissimilarity</th>
<th>Female/Male Earnings Ratio</th>
<th>Female Participation Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>.630</td>
<td>77.3</td>
<td>.85</td>
</tr>
<tr>
<td>Austria</td>
<td>.607</td>
<td>72.7</td>
<td>.44</td>
</tr>
<tr>
<td>Australia</td>
<td>.581</td>
<td>73.3</td>
<td>.59</td>
</tr>
<tr>
<td>Switzerland</td>
<td>.581</td>
<td>64.6</td>
<td>.60</td>
</tr>
<tr>
<td>Norway</td>
<td>.573</td>
<td>71.4</td>
<td>.59</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>.567</td>
<td>61.4</td>
<td>.59</td>
</tr>
<tr>
<td>Hungary</td>
<td>.558</td>
<td>64.9</td>
<td>.66</td>
</tr>
<tr>
<td>Germany</td>
<td>.523</td>
<td>70.2</td>
<td>.45</td>
</tr>
<tr>
<td>United States</td>
<td>.463</td>
<td>65.4</td>
<td>.66</td>
</tr>
</tbody>
</table>

Sources.—The dissimilarity index is from Anker (1998). Earnings ratios and participation rates are taken from Blau and Kahn (1996, pp. S32 and S39). The gender earnings ratios are adjusted for hours of work only and are for the period 1985–89.

Note.—The correlation between dissimilarity and wages is .52.

men assume some of the traditional female responsibilities give rise to less experimentation by women, and consequently there will be greater long-run differences in the beliefs held by the two gender groups.

We suggest that whether men become more like women or vice versa depends, in large part, on the extent to which gender differences decline through mechanisms promoted by the state or by the market. In the absence of welfare state provisions, market pressures mean that women can only hope to attain gender equality if their preferences come to resemble those of men. But welfare state provisions frequently seek to encourage greater gender equity through legislation that, among other things, allows men to devote more time to child care without having a negative effect on their careers. Thus we should expect that, to the extent that men and women come to share the same preferences, these should be nearer to men’s original preferences in market societies than is the case in welfare state societies. As a result, the latter—exemplified by Norway and Sweden—would exhibit a greater degree of occupational segregation than the former—notably the United States. This is precisely what empirical studies have found, as we can see from tables 1 and 2.

Our model also sheds light on another aspect of gender inequality in the labor market. Studies of the gap between female and male wages have found that it correlates strongly with the extent of gender segregation in occupations as well as with the overall level of wage inequality (Blau and Kahn 1992, 1996). The former relationship is evident in table 2, where the correlation between the wage gap and the degree of segregation is 0.52. Where wage dispersion is large, as in the United States, small differences in the jobs performed by men and women result in a large difference in the remuneration they receive. Blau and Kahn point out that the institutional setting becomes crucial, with countries with centralized
Bayesian Learning and Gender Segregation

wage setting exhibiting more pay equality across gender than those with a decentralized wage-determination process.

As we have already argued, the institutional setting is also crucial in determining the extent of occupational segregation. These two findings together imply that a small wage gap and a large level of segregation can both be explained by the same set of institutions. Welfare state economies tend to be characterized both by centralized wage setting and by greater child-care provision, and thus they give rise to the labor market outcomes observed in Sweden and Norway. Countries where wage determination and parental care provision are left to the market would generate a greater wage gap but fewer differences in the career choices of men and women.

Our analysis suggests that policies that might be used to counter gender segregation should modify the learning trajectories that dynasties follow. A direct approach might then be to help women learn from the experiences of successful “role models.” However, it is not enough merely to know whether or not such role models were successful: women cannot infer anything about the underlying parameters without knowledge of their effort and ability. What is required is that successful “role models” transmit their beliefs. If the educational system could provide a means by which such role models could transmit to young women beliefs which were, on average, higher than the beliefs they inherited from their mothers, this would lead to higher prior beliefs and to an increased proportion of dynasties converging to the true belief.

A less direct way in which learning trajectories might be modified is through affirmative action. In the context of a model of statistical discrimination, Coate and Loury (1993) argue that affirmative action may have a negative impact on gender segregation once the policy is removed. Positive discrimination would make it easier for women to succeed, and, hence, it might reduce their incentives to invest. If this is the case, employers’ beliefs that women invest less than men would be supported. Hence, once the policy is removed, discrimination against women, and therefore gender segregation, will persist.

Our very different framework has, not surprisingly, very different implications. Because a Bayesian learning mechanism implies that history is important in determining equilibrium beliefs, temporary policies can have long-run effects. Suppose a temporary affirmative action policy induced more women to choose “male” occupations. The number of women who, as a result, hold the true belief will be greater than it would have been had the policy not been implemented, because those dynasties that would never have chosen male occupations are given a chance to learn about the probability of success in them. Although some will revert to choosing female-dominated careers, others will learn the truth and maintain their belief even after the policy has been removed. Thus temporary affirmative action policies can permanently eliminate negative stereotypes held by
(some) women about their probability of success in male-dominated occupations.

VI. Conclusions

It is a common observation that circumstances prevailing in the past can continue to shape behavior long after they have changed. A Bayesian learning mechanism provides one explanation for this phenomenon. Because equilibrium beliefs depend on initial beliefs, history is important. In the case of gender segregation, past differences between men and women in preferences for child care have persistent effects, because they lead men and women to hold different beliefs about the role of effort in determining success in traditionally “male” occupations. As a result, members of each gender make different career choices even if, currently, they have the same preferences.

The learning model we have presented also points to a possible reason why the degree of segregation varies from one country to another. Changes in preferences that make women willing to choose male-dominated careers will generate more information about the probability of success of female workers in these occupations and, hence, will result in a stronger reduction in the extent of segregation. This mechanism can help us understand the somewhat surprising finding that the Scandinavian countries, where social attitudes have resulted in men sharing—to a relatively large extent—family responsibilities, have a particularly high degree of segregation.

Appendix A

Proof of Proposition 1

Proofs of the various parts of proposition 1 are found in McLennan (1984), Piketty (1995), and Smith and Sørensen (2000). Here we provide an intuition of these proofs.

i) From Piketty (1995, proposition 1) and Smith and Sørensen (2000, lemma 5). Under Bayesian updating, the likelihood ratio $l_t = (1 - z_t)/z_t$ follows a martingale conditional on the true state of the world, that is, $E(l_{t+1} | l_t) = l_t$ (see Piketty 1995, pp. 578–80). The martingale convergence theorem then implies that the likelihood ratio, and hence beliefs, converge.

ii) From Piketty (1995, proposition 2) and Smith and Sørensen (2000, lemma 5). The Bayesian updating functions given by equations (5) and (6) have three fixed points: $z = 0$, $z = 1$, and an interior fixed point $z^*$. However, $z = 0$ is not stable. The martingale convergence theorem implies that the likelihood ratio converges to a finite value, $l_\infty < \infty$; thus $z_\infty > 0$. Hence, only 1 and $z^*$ are stable equilibrium beliefs.

iii) From McLennan (1984, pp. 343–44). Taking the martingale con-
dition \( E(z_{i+1} \mid z_i) = z \) to the limit implies that the unconditional expectation of the long-run beliefs equals the initial beliefs; that is, \( E(z_\infty \mid z_0) = z_0 \). Let \( p \) be the probability that the sequence of beliefs converges to \( z_\infty = 1 \) conditional on \((\pi', \theta')\) being the true state of the world. Then, if the initial belief is such that \( z_0 > z^* \), we have \( z_0 = (1 - z_0)z^* + z_0[p \cdot 1 + (1 - p) \cdot z^*] \). This yields \( p(z_\infty, z^*) \), as given by equation (9). If the initial belief is such that \( 0 < z_0 < z^* \), \( z^* \) is reached with probability one, from ii.

Appendix B

The Proportions of Men and Women Who Learn the Truth

Consider the group of high-ability men. In the first era, a fraction \( MH_1 \), given by equation (13), learns the truth. The rest converge to \( z_m^* \). In the second era, the latter are not in equilibrium, as their belief is above the new confounded-learning belief, \( z^* \). Thus \( z_m \) converges to 1 with probability \( p(z_m^*, z^*) \) and to \( z^* \) with probability \( [1 - p(z_m^*, z^*)] \). The fraction that holds the true belief in the second era is then given by \( MH_2(z^*) = MH_1 + (1 - MH_1)p(z_m^*, z^*) \). Using equation (13), we have

\[
MH_2(z^*) = \int_{z_m^*}^{1} p(z, z_m^*)f(z)dz + p(z_m^*, z^*) \left[ 1 - \int_{z_m^*}^{1} p(z, z_m^*)f(z)dz \right], \tag{B1}
\]

which we can express as

\[
MH_2(z^*) = \int_{z_m^*}^{1} p(z, z_m^*)f(z)dz - \int_{z_m^*}^{1} p(z_m^*, z^*)f(z)dz + \int_{z_m^*}^{1} p(z_m^*, z^*)f(z)dz. \tag{B2}
\]

The proportion of high-ability women who converge to the true belief in the first era is \( WH_1 = \int_{z_w^*}^{1} p(z, z_w^*)f(z)dz \). In the second era, those who place full weight on the truth keep their belief. Those who had converged to \( z_w^* \) now hold a belief lower than the new confounded-learning equilibrium, and therefore converge to \( z^* \) with probability one. The fraction that holds \( z = 1 \) in the second era is then \( WH_2 = WH_1 \), which we can write as

\[
WH_2 = \int_{z_w^*}^{1} p(z, z_w^*)f(z)dz + \int_{z_m^*}^{1} p(z, z_m^*)f(z)dz. \tag{B3}
\]

Let the extent of gender segregation for high-ability agents be given by \( SH(z^*) = MH_2(z^*) - WH_2(z^*) \). Recall that \( z^* \) is a decreasing function of \( r \). We can then examine how the extent of segregation varies with
When \( r = r_m \), that is, \( z_{m}^{a} = z_{m}^{a} \), \( MH_2 = MH_1 \), and \( WH_2 = WH_1 \); that is,

\[
SH(r_m) = \int_{z_h}^{1} p(z, z_m^{a}) f(z) dz - \int_{z_h}^{1} p(z, z_m^{a}) f(z) dz - \int_{z_h}^{1} p(z, z_m^{a}) f(z) dz,
\]

which is negative as \( p(z, z_m^{a}) > p(z, z_m^{a}) \). For \( r = r_w \), that is, \( z_{w}^{a} = z_{w}^{a} \), we have

\[
SH(r_w) = \int_{z_h}^{1} p(z, z_m^{a}) f(z) dz - \int_{z_h}^{1} p(z, z_m^{a}) f(z) dz + \int_{0}^{z_{h}} p(z_{m}^{a}, z_{w}^{a}) f(z) dz,
\]

which is positive. Using (B2) and (B3) to differentiate, we find that \( \frac{\partial SH(z_{m}^{a})}{\partial z_{m}^{a}} < 0 \). That is, \( SH(r) \) is a strictly increasing function of \( r \), which is negative at \( r = r_m \) and positive at \( r = r_w \).

Consider now low-ability individuals. In the first era, male dynasties converge to either \( z = 1 \) or \( z_{m}^{a} \) or maintain their initial belief if it was below \( z_{m}^{a} \). Since the belief \( z = 1 \) is stable and \( z \geq z_{m}^{a} \), the fraction of low-ability men who hold \( z = 1 \) is not altered when preferences change. That is, \( ML_2 = ML_1 \), which we can express as

\[
ML_2(z) = \int_{z_w}^{1} p(z, z_m^{a}) f(z) dz + \int_{z_w}^{1} p(z, z_m^{a}) f(z) dz + \int_{z_w}^{1} p(z, z_m^{a}) f(z) dz.
\]

In the first era, low-ability female dynasties either maintain their initial belief or converge to \( z = 1 \) or \( z_{m}^{a} \), where \( z < z_{m}^{a} \). When preferences change, those women who have a belief greater than \( z \) now find themselves above the threshold value. They choose A and continue learning. The fraction that holds the true belief is then

\[
WL_2(z) = WL_1 + p(z, z_m^{a}, z) \int_{z_w}^{1} [1 - p(z, z_m^{a})] f(z) dz + \int_{z_w}^{1} p(z, z) f(z) dz
\]

\[
= \int_{z_w}^{1} p(z, z_m^{a}) f(z) dz + \int_{z_w}^{1} [1 - p(z, z_m^{a})] f(z) dz + \int_{z_w}^{1} p(z, z) f(z) dz,
\]

where the first term is the proportion who converged to \( z = 1 \) in the first era, the second term is the proportion of those who converged to \( z_{m}^{a} \) in the first era and learn the truth in the second one, and the third
term is the proportion of those who now choose A for the first time and learn the true parameters.

The extent of gender segregation is given by \( SL(z) = ML_{1,2}(z) - WL_{1,2}(z) \). Recall that \( z \) is strictly increasing in \( r \). For \( r = r_m \), that is, \( z = z_m \), we have \( SL(r_m) = 0 \). For \( r > r_m \), the proportion of women who choose A in the second era is equal to the one in the first era. Since \( z_m < z_w \), it follows that \( SL(r) > 0 \). Differentiating, we find that \( \partial SL(r) / \partial r > 0 \). Hence, \( SL(r) \) is nonnegative and strictly increasing in \( r \) for all \( r > r_m \).

References


