Growth, Income Inequality, and Fiscal Policy:

What are the Relevant Tradeoffs?*

Cecilia García-Peñalosa
GREQAM and CNRS

Stephen J. Turnovsky
University of Washington

Revised version
October 2005

Abstract:

We develop an endogenous growth model with elastic labor supply, in which agents differ in their initial endowments of physical capital. In this context, the growth rate and the distribution of income are jointly determined. We then examine the distributional impact of different ways of financing an investment subsidy. We find that policies aimed at increasing the growth rate tend to result in a more unequal distribution of pre-tax income, consistent with the positive correlation between income inequality and growth observed in the recent empirical literature. However, there seems to be no conflict between efficiency and equity if inequality is measured in terms of the distribution of welfare.

JEL Classification : E62, O17, O40.

Keywords: endogenous growth, income inequality, fiscal policy.

*García-Peñalosa would like to acknowledge the financial support received from the Institut d'Economie Publique (IDEP), Marseille. Turnovsky's research was supported in part by the Castor endowment at the University of Washington. The paper has benefited from seminar presentations at the Central European University, Erasmus University, the University of California, Riverside, the University of Kansas, the University of Munich, and the University of Oregon. The constructive comments of two anonymous referees are also gratefully acknowledged.

1. Introduction

The last decade has seen a revival of interest in the relationship between income inequality and growth. This research has employed a range of theoretical frameworks and yielded conflicting empirical results. While earlier evidence suggested a negative tradeoff between growth and inequality, more recent studies have tended to support a positive relationship; see e.g. Barro (2000), Forbes (2000), and Lundberg and Squire (2003). Virtually all of this debate has ignored the role of distortionary taxes. But once their presence is acknowledged, it becomes evident that policymakers may face two potential tradeoffs, in that growth-enhancing policies may have conflicting effects on the *pre-tax* and *post-tax* distributions of income. Of these two measures, it would seem that the latter is in fact the more relevant as a guide to policy. Indeed, one can go even further. Since presumably, one is ultimately interested in the effect of fiscal policy on economic welfare, a further potential tradeoff – one between growth and *welfare* inequality itself – naturally arises.

In this paper we analyze the effects of fiscal policy on these various growth-inequality tradeoffs. To do so we develop an endogenous growth model with elastic labor supply and agents who differ in their initial endowments of physical capital. In this framework the equilibrium growth rate and the distribution of income are jointly determined, the key mechanism generating the latter being the positive equilibrium relationship we derive between agents' relative wealth (capital) and their relative leisure. This relationship has a very simple intuition. Wealthier agents have a lower marginal utility of wealth. They therefore choose to work less and to enjoy more leisure, and given their relative capital endowments, this generates an equilibrium income distribution.

This role played by endogenous labor supply is analogous to its role in other models of capital accumulation and growth, where it provides the crucial mechanism by which demand shocks, such as government consumption expenditure, will stimulate capital accumulation. The key factor is the wealth effect and the impact this has on the labor-leisure choice. This mechanism is also central to empirical models of labor supply based on intertemporal optimization; see e.g. MaCurdy (1981).

There is substantial empirical evidence documenting this negative relationship between wealth and labor supply. Holtz-Eakin, Joulfaian, and Rosen (1993) find evidence to support the view that large inheritances decrease labor participation. Cheng and French (2000) and Coronado and Perozek (2003) use data from the stock market boom of the 1990s to study the effects of wealth on labor supply and retirement, finding a substantial negative effect on labor participation. Algan, Chéron, Hairault, and Langot (2003) use French data to analyze the effect of wealth on labor market transitions, and find a significant wealth effect on the extensive margin of labor supply.

Assigning such a central role to the adjustment in labor supply relates our analysis to another recent body of literature. The widening gap between working hours in the United States and Europe has recently sparked a debate about the causes and effects of differences in labor supply; see Prescott (2004) and Alesina et al. (2005). This literature has largely focused on whether taxes have driven these differences, and on the impact of labor supply on growth. However, little attention has been paid to the distributional implications of an endogenous labor supply. Our analysis can therefore be viewed as extending this discussion to focus on this important, but neglected, aspect. The central role of the labor-leisure tradeoff in our model, whereby policies that increase labor supply also tend to increase inequality is consistent with the positive correlation between average hours worked in a country and the Gini coefficient of income reported by Alesina et al. (2005) for OECD economies.

As is well-known, the AK model laissez-faire implies a sub-optimally low equilibrium growth rate so that some form of stimulus to investment becomes desirable. We therefore introduce a direct investment subsidy and compare the growth and distributional consequences of financing this subsidy by either a tax on capital income, on labor income, or on consumption. Changes in tax rates are shown to have a substantial effect on the supply of labor, in line with recent empirical evidence; see Cardia, Kozhaya, and Ruge-Murcia (2003), as well as Prescott (2004). Our results highlight the sharply contrasting effects of these three different modes of finance.

Two essential results emerge. First, we find that policies that enhance the growth rate are most frequently associated with greater pre-tax income inequality. This is because growth is fostered by policies that increase the return to capital, and since capital is more unequally distributed than is labor, higher returns to capital translate into greater income inequality. The positive correlation between growth and income inequality, and the fact that both variables are jointly determined, are consistent with the recent empirical findings of Barro (2000), Forbes (2000), and Lundberg and Squire (2003). However, our analysis also indicates that such policies tend to reduce welfare inequality, thus suggesting that (gross) income inequality is a poor proxy for the assessment of the effects of policy on the distribution of welfare. Second, because some policies tend to have opposite effects on the pre-tax (gross) and post-tax gross (net) distributions of income, it is possible to induce faster growth in conjunction with a more equal distribution of disposable income.

Despite the fact that increased growth is compatible with lower welfare inequality, no policy dominates in both dimensions. Tradeoffs therefore still exist among these three modes of finance, and the policy maker needs to weigh these carefully in evaluating the consequences for growth and distribution. Thus, while all three are growth enhancing, consumption tax-financing is superior from a growth perspective, capital income tax-financing is superior from the standpoint of reducing welfare inequality, while wage income tax-financing may actually exacerbate welfare inequality. Overall, the analysis provides support for the use of either a tax on capital income or a tax on consumption to finance a subsidy on investment, in that both policies increase the growth rate and reduce inequality in post-tax income and welfare. But an even more attractive policy consists of adopting a consumption tax together with an equal-in-magnitude wage subsidy to finance the investment subsidy, since this does not distort the labor-leisure choice.

The paper contributes to the recent literature on the relationship between income distribution and growth.[1] It is close to Bertola (1993), who also examines how policies directed at increasing the growth rate affect the distribution of consumption, although his assumption of a constant labor supply implies that the distribution of income is independent of policy choices. It is also related to Bénabou (2002), where the tradeoffs engendered by different fiscal polices are examined. Bénabou considers a model with risky human capital investment, and compares direct income redistribution with redistributive education finance. He finds that the latter is preferable in terms of growth, but inferior from an insurance point of view. We propose an alternative scenario, but share the conclusion that even if it is possible to enhance both growth and equity, different policies affect these two objectives to different extents, and hence a careful analysis of policy options is necessary.

Our approach has two main limitations. First, the assumption that agents differ only in their

[1] See, for example, Galor and Zeira (1993), Alesina and Rodrik (1994), Persson and Tabellini (1994), and Aghion and Bolton (1997), as well as the overview in Aghion, Caroli, and García-Peñalosa (1999).

initial endowments of capital coupled with an AK technology implies that there are no income dynamics. While this is restrictive, it has the compensating advantage of enabling us to examine the distributional consequences of fiscal policy analytically.[2] The other is that we are ignoring other important elements central to the growth-income inequality relationship, most notably human capital and education. These aspects are emphasized by Galor and Zeira (1993) and Viaene and Zilcha (2003), among others. We choose to focus on a different source of income differences, the role of the return to capital, which has been largely ignored by the recent literature. The argument that the behavior of capital returns is essential to understanding distributional differences has, however, been emphasized by Atkinson (2003) and is supported by recent empirical evidence for the OECD (see Checchi and García-Peñalosa, 2005).

The paper is organized as follows. Sections 2 and 3 present the structure of the model and derive the macroeconomic equilibrium. Section 4 employs the framework to address the impact of taxation on both growth and inequality. Section 5 supplements our theoretical analysis with some numerical simulations, used to illustrate some of the distributional implications of the various policies. Section 6 concludes, while technical details are provided in the appendix.

2. The Model

2.1 Description of the decentralized economy

Technology and factor payments

Firms shall be indexed by *j*. We assume that the representative firm produces output in accordance with the Cobb-Douglas production function[3]

$$Y_j = A(L_j K)^{\alpha} K_j^{1-\alpha} \qquad 0<\alpha<1 \tag{1a}$$

[2] This assumption is common to a large part of the literature on distribution and growth, eg. Bertola (1993), Alesina and Rodrick (1994), Persson and Tabellini (1994). However, in García-Peñalosa and Turnovsky (2005) we examine the simultaneous determination of growth and the distribution of income when there are diminishing returns to capital, in which case both the distribution of wealth and income exhibit transitional dynamics. See also Caselli and Ventura (2000) for a more general study of heterogeneity and the dynamics of distribution in growth models.

[3] We note that the use of the Cobb-Douglas production function, although convenient, is actually less restrictive than may appear. All of our analytical results continue to hold if we generalize the production function (1a) to $Y_j = F(L_j K, K_j)$ where F is homogeneous of degree one in the two arguments, $L_j K$ and K_j.

where K_j denotes the individual firm's capital stock, L_j denotes the individual firm's employment of labor, K is the average stock of capital in the economy, so that $L_j K$ measures the efficiency units of labor employed by the firm. The production function exhibits constant returns to scale in the private factors -- labor and the private capital stock.

All firms face identical production conditions. Hence they will all choose the same level of employment and capital stock. That is, $K_j = K$ and $L_j = L$ for all *j*, where L is the average economy-wide level of employment. Furthermore, we assume that the aggregate labor market clears, so that $L = 1 - l$, where l is the average leisure time. The economy-wide capital stock yields an externality such that in equilibrium the aggregate (average) production function is linear in the aggregate capital stock, as in Romer (1986), namely

$$Y = AL^{\alpha}K \equiv \Omega(L)K \tag{1b}$$

where $\Omega(L) \equiv AL^{\alpha}$ and $\partial\Omega / \partial L > 0$.

We assume that the wage rate, ω, and the return to capital, r, are determined by their respective marginal physical products. Differentiating the production function and given that firms are identical, we derive equilibrium factor prices as a function of leisure time,

$$\omega = \alpha\Omega L^{-1}K = \alpha A(1-l)^{\alpha-1}K \equiv w(l)K \tag{2a}$$

$$r = (1-\alpha)\Omega = (1-\alpha)A(1-l)^{\alpha} \tag{2b}$$

These expressions imply that the equilibrium return to capital is independent of the stock of capital while the wage rate is proportional to the average stock of capital, and therefore grows with the economy.[4] In addition, $\partial r / \partial l < 0$ and $\partial w / \partial l > 0$, reflecting the fact that more employment (less leisure) raises the productivity of capital but lowers that of labor.

Consumers

There is a mass 1 of infinitely-lived agents in the economy. Consumers are indexed by *i* and

[4]Intuitively, in a growing economy, with the labor supply fixed, the higher income earned by labor is reflected in higher returns, whereas with capital growing at the same rate as output, returns to capital remain constant.

are identical in all respects except for their initial endowments of capital, K_{i0}. Since the economy is growing, we are interested in the share of individual i in the total stock of capital, k_i, defined as $k_i \equiv K_i/K$. Relative capital has a distribution function $G(k_i)$, mean $\sum_i k_i = 1$, and variance σ_k^2.

All agents are endowed with a unit of time that can be allocated either to leisure, l_i or to work, $1-l_i \equiv L_i$. A typical consumer maximizes expected lifetime utility, assumed to be a function of both consumption and the amount of leisure time, in accordance with the isoelastic utility function

$$\max \int_0^\infty \frac{1}{\gamma}\left(C_i(t)l_i^\eta\right)^\gamma e^{-\beta t}dt, \quad \text{with } -\infty < \gamma < 1, \eta > 0, 1 > \gamma(1+\eta) \tag{3}$$

where $\varepsilon \equiv 1/(1-\gamma)$ equals the intertemporal elasticity of substitution.[5] The preponderance of empirical evidence suggests that this is relatively small, certainly well below unity, so that we shall assume $\gamma < 0$. The parameter η represents the elasticity of leisure in utility. This maximization is subject to the agent's capital accumulation constraint

$$(1-s)\dot{K}_i = (1-\tau_k)rK_i + (1-\tau_w)(1-l_i)wK - (1+\tau_c)C_i \tag{4}$$

where s denotes a subsidy to investment in physical capital, while τ_k, τ_w, and τ_c denote the tax rates on capital income, labor income, and consumption, respectively. With the equilibrium wage rate being tied to the aggregate capital stock, we observe from (4) that the individual's rate of capital accumulation depends on the aggregate stock of capital, which the individual takes as given.

Government policy

The objective of this paper is to examine the various trade-offs faced by the policymaker. The macroeconomic equilibrium therefore needs to take explicit account of the constraints to which the government is subject. Given the four policy instruments defined above, the government balances the public budget at each instant t in accordance with the constraint

$$s\dot{K} = \tau_c C + \tau_k rK + \tau_w(1-l)wK \tag{5}$$

where C denotes aggregate consumption, and l is the average economy-wide average leisure time, so

[5]The restrictions in (3) are required to ensure the concavity of the utility function in its two arguments.

that $(1-l)wK$ denotes the aggregate wage bill.

2.2 Consumer optimization

The consumer's formal optimization problem is to choose her rate of consumption, leisure, and rate of capital accumulation to maximize (3) subject to the accumulation equation (4). The corresponding first-order conditions are

$$C_i^{\gamma-1} l_i^{\eta\gamma} = \frac{1+\tau_c}{1-s}\lambda_i \tag{6a}$$

$$\eta C_i^{\gamma} l_i^{\eta\gamma-1} = \frac{1-\tau_w}{1-s} wK\lambda_i \tag{6b}$$

$$r\left(\frac{1-\tau_k}{1-s}\right) = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{6c}$$

where λ_i is agent *i*'s shadow value of capital, together with the transversality condition

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{6d}$$

These optimality conditions are standard. Together with the individual's accumulation equation (4), they yield the individual saving and leisure decisions. In the Appendix we show that the economy is always on its balanced growth path.[6] Two key relationships that we establish include

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} = \frac{\dot{K}_i}{K_i} = \frac{\dot{K}}{K} \equiv \psi = \frac{r(l)\left(\frac{1-\tau_k}{1-s}\right) - \beta}{1-\gamma} \tag{7}$$

$$l_i - l = \left(l - \frac{\eta}{1+\eta}\right)(k_i - 1) \tag{8}$$

The first equation gives the rate of consumption growth of the individual, while the second yields the individual's leisure choice. Equation (7) asserts that in equilibrium the rates of growth of consumption and capital are equal and the same for all individuals, being equal to the tax-adjusted return to capital less the rate of time preference all multiplied by the intertemporal elasticity of

[6] This is also the case in the representative agent model; see Turnovsky (2000).

substitution. Observe that the return to capital depends upon aggregate leisure, which as we show below, is jointly determined in conjunction with the growth rate by the rate of return equilibrium condition (11a) [i.e. (7)] and the product market equilibrium condition (11b). The macroeconomic equilibrium, including both the growth rate and aggregate labor supply (leisure), is therefore *independent* of the distribution of wealth. Furthermore, the capital stock of all agents grows at the same rate, implying that at any point in time, the share of agent *i*, k_i, remains equal to her initial share $k_{i,0}$, say. That is, the relative wealth position of agents, k_i, is unchanging over time.

Equation (8) represents the "relative labor supply" function and is the crucial mechanism that equates growth rates across individuals. In (A.10b) in the Appendix we show that the transversality condition (6d) implies

$$l > \frac{\eta}{1+\eta}, \tag{9}$$

so that (8) yields a positive equilibrium relationship between relative wealth and leisure, such that the relative wealth position of agent *i*, is unchanging over time. This relationship provides the link between the agent's initial relative endowment of capital and the equilibrium distribution of income. Wealthier agents have a lower marginal utility of wealth. They therefore choose to supply less labor and to "buy" more leisure. In effect, they compensate for their larger capital endowment, and the higher growth rate it would support, by providing less labor, thereby having an exactly offsetting effect on the growth rate.

This role that the elasticity of labor supply is playing in the determination of income distribution is analogous to the role it plays in other similar growth models. For example, government consumption expenditure will stimulate capital accumulation in the Ramsey model, and growth in the Romer model, if and only if labor is supplied elastically. In both cases the underlying responses are driven by wealth effects.

2.3 Macroeconomic equilibrium

With the economy always being on its balanced growth path, the key aggregate equilibrium relationships can be summarized by the following equations:

Equilibrium growth rate

$$\psi = \frac{r\left(\frac{1-\tau_k}{1-s}\right) - \beta}{1-\gamma} \tag{10a}$$

Aggregate consumption-capital ratio

$$\frac{C}{K} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c} l \tag{10b}$$

Goods market equilibrium

$$\psi = \Omega(l) - \frac{C}{K} \tag{10c}$$

Government budget constraint

$$\tau_k r + \tau_w w(1-l) + \tau_c \frac{C}{K} = s\psi \tag{10d}$$

Recalling the definitions of $r(l)$, $w(l)$, and $\Omega(l)$, and given k_i, these equations jointly determine the aggregate consumption-capital ratio, C/K, the average leisure time, l, the average (common) growth rate, ψ, and one of the fiscal instruments given the other three policy parameters. Given l, (8) determines the individual leisure time, l_i, while the individual consumption-capital ratio can be derived by dividing (6a) by (6b) and expressed as

$$\frac{C_i}{K_i} = \frac{w}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l_i}{k_i} \tag{10b'}$$

Substituting (10b) into (10c), and recalling (2a) and (2b), the macroeconomic equilibrium of the economy can be summarized by the following pair of equations that jointly determine the equilibrium mean growth rate, ψ, and average leisure time, l:

RR $$\psi = \frac{(1-\alpha)\Omega(l)(1-\tau_k)/(1-s) - \beta}{1-\gamma}, \tag{11a}$$

PP $$\psi = \Omega(l)\left[1 - \frac{\alpha}{\eta}\frac{1-\tau_w}{1+\tau_c}\frac{l}{1-l}\right]. \tag{11b}$$

The first equation describes the relationship between ψ and l that ensures the equality between the risk-adjusted rate of return to capital and return to consumption. The second describes the combinations of the mean growth and leisure that ensure product market equilibrium holds. We shall focus our attention on solutions that are not only viable, in the sense of satisfying the transversality condition, but also generate positive equilibrium growth. From (11b) and (9), the equilibrium solution for l must therefore lie within the range:

$$\frac{\eta(1+\tau_c)}{\alpha(1-\tau_w)+\eta(1+\tau_c)} > l > \frac{\eta}{1+\eta} \tag{12}$$

2.4 The laissez-faire economy

Setting the tax rates and the subsidy to zero, the equilibrium mean growth rate and leisure in the laissez-faire economy are determined by the following pair of equations:

RR: $$\psi = \frac{(1-\alpha)\Omega(l)-\beta}{1-\gamma}, \tag{11a'}$$

PP: $$\psi = \Omega(l)\left(1-\frac{\alpha}{\eta}\frac{l}{1-l}\right), \tag{11b'}$$

These RR and PP locuses are depicted in Figure 1. First, note that equation PP is always decreasing in l, reflecting the fact that more leisure time reduces output, thus increasing the consumption-output ratio and having an adverse effect on the growth rate of capital. In addition, the RR curve is also decreasing in l. Intuitively, a higher fraction of time devoted to leisure reduces the productivity of capital, requiring a fall in the return to consumption. This is obtained if the growth of the marginal utility of consumption rises, that is, if the balanced growth rate falls. Both schedules are concave, and sufficient conditions for a unique equilibrium to exist, at point Q, say, are

$$\alpha-\gamma+\frac{\beta}{A}>0\,; \quad 1+\frac{1-\alpha l}{\eta(1-l)} > \frac{1-\alpha}{1-\gamma} \tag{13}$$

which are certainly met if $\gamma \le 0$, and hold under much weaker conditions as well.

3. The Distribution of Income and Welfare

We now consider the relative income of agent *i*, having capital stock K_i. Her gross income is $Y_i = rK_i + wK(1-l_i)$, while average economy-wide income is $Y = rK + wK(1-l)$. Using equation (8) to substitute for the indiviual's labor supply, we can write her relative income, $y_i \equiv Y_i/Y$, as

$$y_i(l,k_i) = k_i + \frac{w}{(1+\eta)\Omega}(1-k_i) = k_i + \frac{\alpha}{(1+\eta)(1-l)}(1-k_i) \tag{14}$$

which we may express more compactly as:

$$y_i(l,k_i) - 1 = \rho(l)(k_i - 1), \quad \text{where} \quad \rho(l) \equiv 1 - \frac{\alpha}{(1+\eta)(1-l)}, \tag{14'}$$

Equation (14') emphasizes that the distribution of income depends upon *two* factors, the initial (unchanging) distribution of capital, and the equilibrium allocation of time between labor and leisure, insofar as this determines factor rewards. The net effect of an increase in initial wealth on the relative income of agent *i* is given by $\rho(l)$. As long as the laissez-faire equilibrium is one of positive growth, it is straightforward to show that[7]

$$0 < \rho(l) < 1 \tag{15}$$

Thus relative income is strictly increasing in k_i, indicating that although richer individuals choose a lower supply of labor, this effect is insufficiently strong to offset the impact of their higher capital income. Consequently, the standard deviation of income across the agents, σ_y, which provides a convenient measure of income inequality, is less than their (unchanging) variability of capital, σ_k. To see this more intuitively, note that the relative labor supply function, equation (8), implies that the standard deviation of labor supplies can be expressed as $\sigma_L = \sigma_l = \left[l - \left(\eta/(1+\eta)\right)\right]\sigma_k$. From (9), the term $\left(l - \eta/(1+\eta)\right)$ lies between 0 and $1/(1+\eta)$, indicating that labor supplies are less unequally distributed than are capital endowments, thus reducing the variability of income across agents

[7] The fact that $\rho(l) < 1$ is immediate from its definition in (14'). It is straightforward to show that if the equilibrium is one of positive growth, (11b) suffices to ensure that $\rho(l) > 0$. We should also note that the policy maker could set tax rates so drive $\rho(l)$ to zero, if he wishes to ensure that all agents have the same pre-tax income. But this would require a negative equilibrium growth rate to offset the differential in the initial capital endowments.

relative to that of their underlying capital endowments.

The standard deviation of relative income, σ_y, serves as an analytically convenient measure of (gross) income inequality. The DD locus in the lower panel of Fig. 1 illustrates the relationship between σ_y and the standard deviation of capital endowments, σ_k, namely

$$\textbf{DD} \qquad \sigma_y = \rho(l)\sigma_k \tag{16}$$

Given σ_k, σ_y is a decreasing and concave function of aggregate leisure time. This is because as leisure increases (and labor supply declines) the wage rate rises and the return to capital falls, compressing the range of income flows between the wealthy with large endowments of capital and the less well endowed. Thus, having determined the equilibrium allocation of labor, denoted by Q, from the upper panels in Fig. 1, (16) determines the corresponding unique equilibrium variability of income across agents, denoted by the point M.

Since taxes also have direct redistributive effects, we must distinguish between the *before-tax* and *after-tax* income distributions. We therefore define the agent's after-tax relative income as

$$y_i^N(l,k_i,\tau_k,\tau_w) \equiv \frac{r(1-\tau_k)k_i + w(1-\tau_w)(1-l_i)}{r(1-\tau_k) + w(1-\tau_w)(1-l)} = 1-\rho^N(l,\tau_w,\tau_k)(1-k_i) \tag{17a}$$

where ρ^N summarizes the distribution of after-tax (net) income and is related to the corresponding before-tax (gross) measure, $\rho(l)$, by

$$\rho^N(l,\tau_w,\tau_k) = \rho(l) + \left(1-\rho(l)\right)(1-\alpha)\frac{(\tau_w - \tau_k)}{\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)} \tag{17b}$$

with the standard deviation of after-tax income given by[8]

$$\sigma_y^N = \rho^N(l,\tau_w,\tau_k)\sigma_k \tag{17c}$$

The dispersion of pre-tax income across agents exceeds post-tax dispersion if and only if $\tau_k > \tau_w$.[9]

[8]Again, we rule out $\rho^N < 0$ as a perverse case implying a negative relationship between wealth and after tax income. It can effectively be ruled out for any plausible tax configuration.

[9] In affluent OECD countries, such as US and Canada, pre-tax income inequality typically exceeds post-tax income inequality (as measured by the Gini coefficients) by about 2-4 percentage points, reflecting the progressivity of the tax structure characteristic of such economies.

From (17a) and (17b) we see that the income tax rates, τ_k and τ_w, both exert two effects on the after-tax income distribution. First, by influencing the equilibrium supply of labor, *l*, they influence *gross* factor returns, and therefore the before-tax distribution of income, as summarized by $\rho(l)$. In addition, they have direct redistributive effects, which are summarized by the second term on the right hand side of (17b): a higher tax on capital income reduces this component, while a higher tax on labor income raises it. From (14) and (17) we establish the following partial effects:

$$\frac{\partial \rho^N}{\partial \tau_k} = \left(\frac{1-\rho^N}{1-\rho}\right)\frac{\partial \rho}{\partial l}\frac{\partial l}{\tau_k} - \frac{(1-\rho)(1-\alpha)(1-\tau_w)}{\left[\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)\right]} \tag{18a}$$

$$\frac{\partial \rho^N}{\partial \tau_w} = \left(\frac{1-\rho^N}{1-\rho}\right)\frac{\partial \rho}{\partial l}\frac{\partial l}{\tau_w} + \frac{(1-\rho)(1-\alpha)(1-\tau_k)}{\left[\alpha(1-\tau_w)+(1-\alpha)(1-\tau_k)\right]} \tag{18b}$$

where the first term in each of these expressions captures the indirect effect of taxation on net income inequality, and the second term the direct effect. In contrast, an investment subsidy, *s*, or a consumption tax, τ_c, affect after-tax inequality only indirectly, through their impact on labor supply.

Finally, we compute individual welfare. By definition, this equals the value of the intertemporal utility function (3) evaluated along the equilibrium growth path. Thus, the optimized level of utility for an agent starting from an initial stock of capital, $K_{i,0}$, can be expressed as

$$X(K_{i0}) = \frac{1}{\gamma}\frac{\left((C_i/K_i)l_i^{\eta}\right)^{\gamma}}{\beta-\gamma\psi}K_{i,0}^{\gamma} \tag{19a}$$

The welfare of individual *i* relative to that of the individual with average wealth is then

$$x(k_i) = \frac{\left(C_i/K_i\right)^{\gamma}}{\left(C/K\right)^{\gamma}}\frac{l_i^{\eta\gamma}}{l^{\eta\gamma}}k_i^{\gamma} = \left(\frac{l_i}{l}\right)^{(1+\eta)\gamma} \tag{19b}$$

where the second term is obtained by substituting for (10b) and (10b') and using equation (8) yields

$$x(k_i) = \left[1+\left(1-\frac{\eta}{1+\eta}\frac{1}{l}\right)\left(k_i-1\right)\right]^{\gamma(1+\eta)}. \tag{19c}$$

Consider now two individuals having relative endowments $k_2 > k_1$. Individual 2 will have a higher mean income. The transversality condition (9) implies that if $\gamma > 0$, then their relative welfares satisfy $x(k_2) > x(k_1) > 0$, while if $\gamma < 0$, $x(k_1) > x(k_2) > 0$. However, in the latter case absolute welfare, as expressed by (19a) is negative. Thus in either case, the better endowed agent will have the higher absolute level of welfare.

We can now compute a measure of welfare inequality. A natural metric for this is obtained by applying the following monotonic transformation of relative lifetime utility, enabling us to express the relative utility of individual *i* as

$$x(k_i)^{1/\gamma(1+\eta)} = u(k_i) = 1 + \varphi(l)(k_i - 1) \quad \text{where} \quad \varphi(l) \equiv 1 - \frac{\eta}{1+\eta}\frac{1}{l} \tag{20a}$$

From (9), $0 < \varphi(l) < 1$, and is an increasing, concave function in *l*. Welfare inequality, expressed in terms of equivalent units of capital, can then be measured by the standard deviation of relative utility

$$\sigma_u = \varphi(l)\sigma_k \tag{20b}$$

It is straightforward to show that in the absence of taxes, and assuming positive growth

$$\sigma_y > \sigma_u > \sigma_k \tag{20c}$$

so that welfare inequality exceeds the underlying wealth inequality, but is less than (gross) income inequality. Welfare inequality is plotted as UU in the lower part of Fig. 1.[10] Having determined the equilibrium leisure at Q, (20b) yields the corresponding degree of welfare inequality, denoted by N, which by virtue of (20c) lies above M. From this figure, it is evident that any structural shift or policy change that causes a change in the equilibrium leisure affects equilibrium gross income inequality, M, and welfare inequality, N, in conflicting ways.

We are interested in the impact of taxes and subsidies on the following three measures of inequality: pre-tax income inequality, post-tax income inequality, and welfare inequality.[11] Before examining the impact of specific policies, we summarize how fiscal policy will affect these distributional measures with the following proposition:

[10] The transversality condition (9) implies that the point U lying on the *l* axis lies to the left of the equilibrium point Q.
[11] Note that consumption inequality, $C_i / C - 1 = k_i c_i / c - 1 = (1 - \eta/(1+\eta)(1/l))(k_i - 1)$ and is identical to that of welfare.

Proposition 1: Given the initial distribution of capital across agents:

(i) Fiscal policy influences the before-tax (gross) distribution of income and the distribution of welfare through its effect on the labor supply.

(ii) Any fiscal policy that increases (decreases) the supply of labor increases (decreases) before-tax income inequality and decreases (increases) welfare inequality.

(iii) A labor income tax and a capital income tax both influence the after-tax (net) distribution of capital in two ways; each has a direct redistributive impact, in addition to an indirect one through changes in the labor supply. A consumption tax or investment subsidy has only the latter effects.

Proposition 1 highlights how, when labor supply is endogenous, income inequality is a poor measure of welfare inequality. In fact, changes in pre-tax inequality are inversely related with those of welfare inequality. Is post-tax income inequality therefore a better measure of welfare inequality? The answer is not necessarily. As we have seen, fiscal policy has two effects on the distribution of post-tax income, and net income inequality need not move together with welfare inequality.

To see this, note that welfare inequality is a weighted average of consumption inequality and leisure inequality, both of which will fall as the labor supply increases. A higher labor supply (lower l) raises the return to capital, increasing the incentives to save, and reducing the consumption/capital ratio. This effect is stronger for those with greater wealth holdings, so that the distribution of consumption becomes less unequal. The distribution of leisure also becomes less unequal, [as can be observed from σ_l], and thus the overall effect is to render the distribution of welfare more equal.

4. Fiscal Policy and the Relationship between Inequality and Growth

A familiar feature of the Romer (1986) model is that by ignoring the externality associated with the aggregate capital stock, the decentralized economy generates a sub-optimally low growth rate. This suggests that by increasing the growth rate, an investment subsidy will move the equilibrium closer to the social optimum. With heterogeneous agents, two questions arise. First, how to finance this subsidy if the government is concerned with both average welfare and welfare inequality. An investment subsidy raises the return to capital and will thus favor those with large capital holdings. Are there ways in which this reverse redistribution can be avoided? Second, we want to know the consequences of different policies for the growth-inequality relationship.

4.1 Financing an investment subsidy

In this section we investigate these questions in some detail, by considering the effect of financing an investment subsidy, *s*, using one of the three distortionary taxes.

Subsidy to investment financed by a tax on capital income

Suppose that the fiscal authority decides to finance the subsidy to investment by imposing a tax on capital income, alone. The impact of fiscal changes on the equilibrium growth rate and labor supply (leisure) can be illustrated graphically in Fig. 1. Holding all other taxes constant, an increase in the subsidy rate, *s*, shifts the RR schedule upwards, moving the equilibrium Q to the left along the PP curve, and increasing the growth rate and reducing leisure. An increase in the tax on capital, τ_k, has the opposite effect, shifting the RR schedule downwards, so that the overall effect of this mode of financing depends upon the size of the capital income tax needed to finance the subsidy.

Setting $\tau_w = \tau_c = 0$ in the government budget constraint, (10d), the necessary capital income tax is:

$$\tau_k = \frac{s}{(1-\alpha)}\left(1 - \frac{\alpha}{\eta}\frac{l}{1-l}\right) \tag{21a}$$

From equation (11a) we find that the direct effect of the subsidy dominates, so that the RR schedule shifts upwards, moving the equilibrium Q to the left, increasing the growth rate and reducing leisure. As a result M and N move to the left along the DD and UU curves, respectively, so that before-tax income inequality, $\sigma_y(l)$, increases, while welfare inequality, $\sigma_u(l)$, declines. Recalling the net distribution of income as characterized by (17b), we see that taxing capital income ensures that $\rho^N(l, \tau_w \tau_k) < \rho(l)$. If the redistributive effect dominates, as our simulations below suggest may plausibly occur, the after-tax inequality actually declines, relative to the laissez-faire distribution.

Subsidy to investment financed by a tax on wage income

Alternatively, the subsidy may be fully financed by a wage tax

$$\tau_w = \frac{s}{\alpha}\frac{\left(1-(\alpha/\eta)\left(l/(1-l)\right)\right)}{\left(1-(s/\eta)\left(l/(1-l)\right)\right)} \tag{21b}$$

In this case, both the RR and PP schedules shift up, resulting in a higher growth rate and greater or lower leisure, depending on the relative shifts. The ambiguous impact on leisure arises because the wage tax tends to reduce the supply of labor, while the higher growth rate induced by the subsidy tends to increase it.

The ambiguous response of labor complicates the impact on the inequality of income. First, the increase (decrease) in leisure time will reduce (increase) the dispersion of gross incomes, as seen from (14'). However, the required (positive) wage tax implies taxing the factor that is more equally distributed, and for any given distribution of gross incomes this raises the variability of net incomes (see (17b) above). If the policy reduces leisure it would then unambiguously increase pre-tax and post-tax income inequality. But when leisure increases the two effects work in opposite directions: there will be a reduction in the variability of gross income, while net income inequality may increase or decrease as compared to the equilibrium without taxes.[12]

Subsidy to investment financed by a tax on consumption

As a third example, the subsidy may be financed by setting the consumption tax equal to

$$\tau_c = s\frac{\left(1-(\alpha/\eta)\left(l/(1-l)\right)\right)}{\left((\alpha/\eta)\left(l/(1-l)\right)-s\right)} \tag{21c}$$

in which case $\rho^N(l,\tau_w,\tau_k)=\rho(l)$. Again both schedules shift upwards, increasing the growth rate. In this case it can be shown that leisure declines, so that gross income inequality increases. Since the consumption tax has no direct redistributive effect, the gross and the net distributions of income are identical and hence net income inequality increases as well.

[12] We can, however, see that when the subsidy rate matches the externality, $s=\alpha$, $\tau_w=1$ and $\rho^N=1$ implying that the net income inequality is increased to that of the initial endowment of capital.

We summarize our results with

Proposition 2: Consider the effects of an investment subsidy.

(i) An investment subsidy financed by a capital income tax increases the growth rate, labor supply, and before-tax income inequality. It reduces welfare inequality and has an ambiguous effect on after-tax income inequality.

(ii) An investment subsidy financed by a labor income tax increases the growth rate. Although it has an ambiguous effect on labor supply, and therefore on before-tax income inequality and welfare inequality, it increases after-tax income inequality.

(iii) An investment subsidy financed by a consumption tax increases the growth rate, labor supply, and both before-tax and after-tax income inequality, while it reduces welfare inequality.

Table 1 provides formal expressions for the effects of the investment subsidy on growth and distribution, under the different financing modes. We see that for each of the three methods of finance there are two effects that may either reinforce or offset each other. All three taxes tend to reduce the growth rate. However, if used to finance an investment subsidy, the positive effect of the latter will always dominate and the growth rate will rise.

Proposition 2 highlights the different tradeoffs between inequality and growth generated by this policy. First, changes in fiscal policy that increase the growth rate will also increase pre-tax income inequality, in line with the empirical evidence in Forbes (2000) and others. The reason for this is simply that faster growth induces a greater supply of labor, reducing the wage rate, and raising the return to capital, thus making the distribution of income more unequal.[13] However, this apparent conflict between efficiency and equity disappears when inequality is measured in terms of utility.

Second, pre-tax and post-tax inequality need not move together. While the indirect effect of the subsidy is always the same (increasing the growth rate, labor supply, and hence pre-tax

[13] One can also show that structural changes such as an increase in productivity or a change in the rate of time preference will generate a positive correlation between inequality and growth; see García-Peñalosa and Turnovsky (2006).

inequality), the direct effect varies, depending upon the tax used to finance it. The consumption tax has no direct redistributive impact, the labor income tax redistributes toward those with higher incomes, while the capital income tax redistributes in the reverse direction. These direct effects can be large enough to dominate the indirect effect and in Section 5, below, we present numerical examples where pre-tax and post-tax income respond in opposite ways to changes in τ_k and τ_w.

4.2 Policy rankings

The three modes of financing the subsidy can be ranked in terms of their relative impacts on the growth rate and various inequality measures. These rankings are reported in Table 2, where we see how they can be sharpened by mildly strengthening the restrictions, as indicated, but consistent with our simulation results. This table offers an alternative perspective on the tradeoffs involved for the different modes of financing the investment subsidy.

While, as noted, all methods of financing the subsidy raise the growth rate, consumption tax financing is the most effective, and capital income tax financing the least so. At the same time, capital income tax financing has the most adverse effect on gross income inequality, followed by the consumption tax. Both these cases yield a positive relationship between growth and gross income inequality, consistent with the recent empirical evidence. By contrast, the labor income tax may quite plausibly reduce gross income inequality, leading to a negative growth-gross income inequality relationship in that case. Second, the relative rankings of the three modes of finance are precisely reversed insofar as net income inequality is concerned, being essentially the same as those of welfare inequality. This suggests that while gross income inequality is a poor indicator, net income inequality is a good indicator of the *relative* rankings of welfare inequality.[14] Third, while each mode of financing may be superior in terms of some criterion, it is dominated in others. The table thus highlights the need for a policy maker wishing to stimulate investment in this way to take careful account of the tradeoffs along these various dimensions. In particular, capital income tax financing, although inferior from the standpoint of stimulating growth, may in fact be desirable if the

[14] However, the effect on net income inequality is not necessarily a good indicator of the effect on welfare inequality in all cases, and example of where these two measures move in opposite ways is given in Table 3 below.

policymaker attributes sufficient weight to reducing post-tax or welfare inequality.

We summarize these results with

Proposition 3: Suppose a policy maker wishes to stimulate investment through a subsidy.

(i) From a growth perspective, financing the subsidy using a consumption tax is superior to a wage tax, and in turn to a capital income tax, although all are growth-enhancing.

(ii) In terms of welfare inequality, the capital income tax is superior to the consumption tax, which in turn dominates the wage income tax. These rankings also apply to their impact on *net* income inequality, but are reversed in terms of their impact on *gross* income inequality.

5. Numerical Examples

To obtain further insights into the growth-income inequality relationship we provide some numerical examples. To do so we use the following, mostly conventional, parameter values:

Parameter Values

Production	$A = 0.75,\ \alpha = 0.60$
Preferences	$\beta = 0.04,\ \gamma = -2,\ \eta = 1.75$

The choice of production elasticity of labor measured in efficiency units implies that 60% of output accrues to labor. One consequence of the Romer technology, is that whereas this value is realistic in terms of the labor share of output, it implies an implausibly large externality from aggregate capital which implies extreme solutions for the first-best fiscal policy, discussed below. The choice of the scale parameter A = 0.75, is set to yield a plausible value for the equilibrium capital-output ratio.

Turning to the preference parameters, the rate of time preference of 4% is standard, while the choice of the elasticity on leisure, $\eta = 1.75$, is standard in the real business cycle literature, implying that about 72% of time is devoted to leisure, consistent with empirical evidence. Estimates of the intertemporal elasticity of substitution are more variable throughout the literature. Our choice 0.33 is well in the range of the empirical evidence, which with few exceptions lies in the range (0,1).

Our measures of income distributions are reported in terms of standard Gini coefficients, as well as the standard deviation measures employed in our theoretical discussion. This involves choosing an initial distribution of wealth, which is less straightforward, as data on the distribution of wealth are difficult to obtain.[15] The choice we have made yields a Gini coefficient of income around 33.3%. This is the value observed for net income in Italy in 2000, and compares with 36.8% and 25.2% for the US and Sweden, respectively, in the same year (Luxembourg Income Study, 2005).

The first line of Table 2 reports the benchmark equilibrium in the laissez-faire economy for our base parameters. We see that 72.5% of time is allocated to leisure, yielding a growth rate of 3.3%, a standard deviation of income inequality of 0.206, and a Gini coefficient of income of 33.3%. This is a plausible benchmark and $l = 0.725$ lies in the range [0.636, 0.745], consistent with (12).

The next three rows report the numerical effects of financing a fixed (arbitrary) investment subsidy of 30% through a capital income, wage tax, or a consumption tax, respectively. The results illustrate the analytical results obtained above. Table 3 also confirms some of the more ambiguous effects discussed. For example, it confirms the theoretical possibility that the redistributive effects in both capital income tax and wage income tax financing dominate, so that post-tax inequality in the former declines, while in the latter it rises. In addition, the table reports the welfare gains for the average individual, expressed as equivalent variations in the initial capital stock.[16] One interesting result of these simulations is that financing the subsidy by a tax on capital income is attractive in that it is the only policy that increases growth, while significantly reducing both welfare inequality and net-income inequality. Although using a consumption tax generates more growth and a higher welfare gain for the average agent, the distributions of post-tax income and welfare need not move

[15] We have assumed that distribution of wealth among the 5 quintiles is 0, 0, 1.2%, 12%, 86.8%, which are consistent with the data. For example, in the US in 1992 the bottom 40% of the population held 0.4% of total wealth, while the top 20% owed 83.8% of the total; see Wolff (1998). Also, we have normalized $\sigma_k = 1$.

[16] We have also evaluated the social benefits of policy in terms of a utilitarian welfare function, defined as the sum of the welfares over the heterogeneous agents in the economy. In the case that the distribution of endowments is uniform across agents we find that utilitarian welfare comprises two components: the utility of the median individual, adjusted by a term that takes account of the dispersion across agents. Since the welfare comparisons for this function are qualitatively similar to those obtained by considering just the average individual, we do not report the numerical results in this case. In general, the welfare level of the median agent overstates the utilitarian level of utility.

together in this case.

The last row of the table considers financing the subsidy using a combination of wage and consumption taxes. In particular, we set $\tau_w = -\tau_c$; that is, these two taxes are optimally set, although the subsidy is below the first-best level.[17] The effect of this policy on the growth rate is stronger than in the previous three cases, the reason being that this policy does not distort the allocation of time between labor and leisure. Employing only a wage or a consumption tax tends to reduce the supply of labor, partially offsetting the effect of the subsidy. When both are used, this effect is absent. Since setting $\tau_w = -\tau_c$ results in faster growth than using only one tax, this policy generates larger welfare gains than any of the pure policies. The effect on distribution is quite significant, as the policy implies taxing consumption and subsidizing and wage incomes, which reduces substantially post-tax income inequality and welfare inequality.

6. Concluding Comments

The literature on the relationship between growth and inequality has raised new questions about the impact of fiscal policy, and in particular about whether or not there is a tradeoff between redistribution and growth. The existing literature has focused on the impact of fiscal policy when capital markets are imperfect. In this paper we have argued that if the labor supply is endogenous and agents differ in their initial capital endowments, growth and the distribution of income are simultaneously determined. As a result, macroeconomic policies aimed at increasing the growth rate will have distributional implications even in the absence of capital market imperfections.

The key mechanism whereby the initial distribution of capital endowments influences the distribution of income is through the wealth effect, which implies that wealthier agents supply less labor, although the resulting distribution of labor supplies is less unequal than that of the capital endowments. As a result any policy that tends to increase the supply of labor and raise the relative return to capital raises the return to the factor that is the source of the inequality, causing the

[17] It is straightforward to demonstrate that in the absence of externalities in consumption or in the labor market, the first-best combination of the taxes on labor income and consumption must satisfy $(1-\tau_w)/(1+\tau_c) = 1$ so as not to distort the corresponding optimality conditions (6a) and (6b). This relationship, familiar from the representative agent model [see Turnovsky, 2000] extends to the form of heterogeneity introduced here.

distribution of income to become more unequal.

We have illustrated the effect of policy by examining the distributional consequences of financing an investment subsidy through a variety of taxes. We consider the impacts of the various policies on four key variables: the rate of growth, the pre-tax distribution or income, the post-tax distribution of income, and the distribution of welfare. A number of results emerge. First, we find that policies that increase the growth rate tend to make the distribution of gross income more unequal, but that of welfare more equal. Second, it is often the case that fiscal policy has opposite effects on the distribution of gross and net income. As a result, it is possible to increase the growth rate and to reduce net income inequality, although gross income inequality would be exacerbated. Moreover, net income inequality and welfare inequality need not move together in response to a policy change. These results highlight the fact that using gross income inequality to assess the distributional implications of fiscal policy may be misleading when the labor supply is endogenous.

Finally, we conclude with a caveat. While the simple AK model has the advantage of providing a tractable framework for investigating the growth-inequality relationship and its policy implications, it also has the limitation that the economy is always on its balanced growth path. It therefore cannot address issues pertaining to the dynamics of wealth and income distribution. In current work we find that the essential structure of our analysis carries over to a neoclassical technology having a diminishing marginal product of capital; see García-Peñalosa and Turnovsky, (2005). Specifically, the key relationship linking relative leisure to relative capital continues to hold and plays the same crucial role. In this case, however, the relationship evolves over time, thereby generating dynamic time paths for the distributions of wealth and income. However, further work is needed to determine the effect of fiscal policy on these transitional dynamic time paths. Moreover, extending the present model to include human capital, thereby generating transitional dynamics as in Bond, Wang, and Yip (1996),would not only be important in its own right, but would also be a significant step in determining the robustness of the results with respect to alternative frameworks.

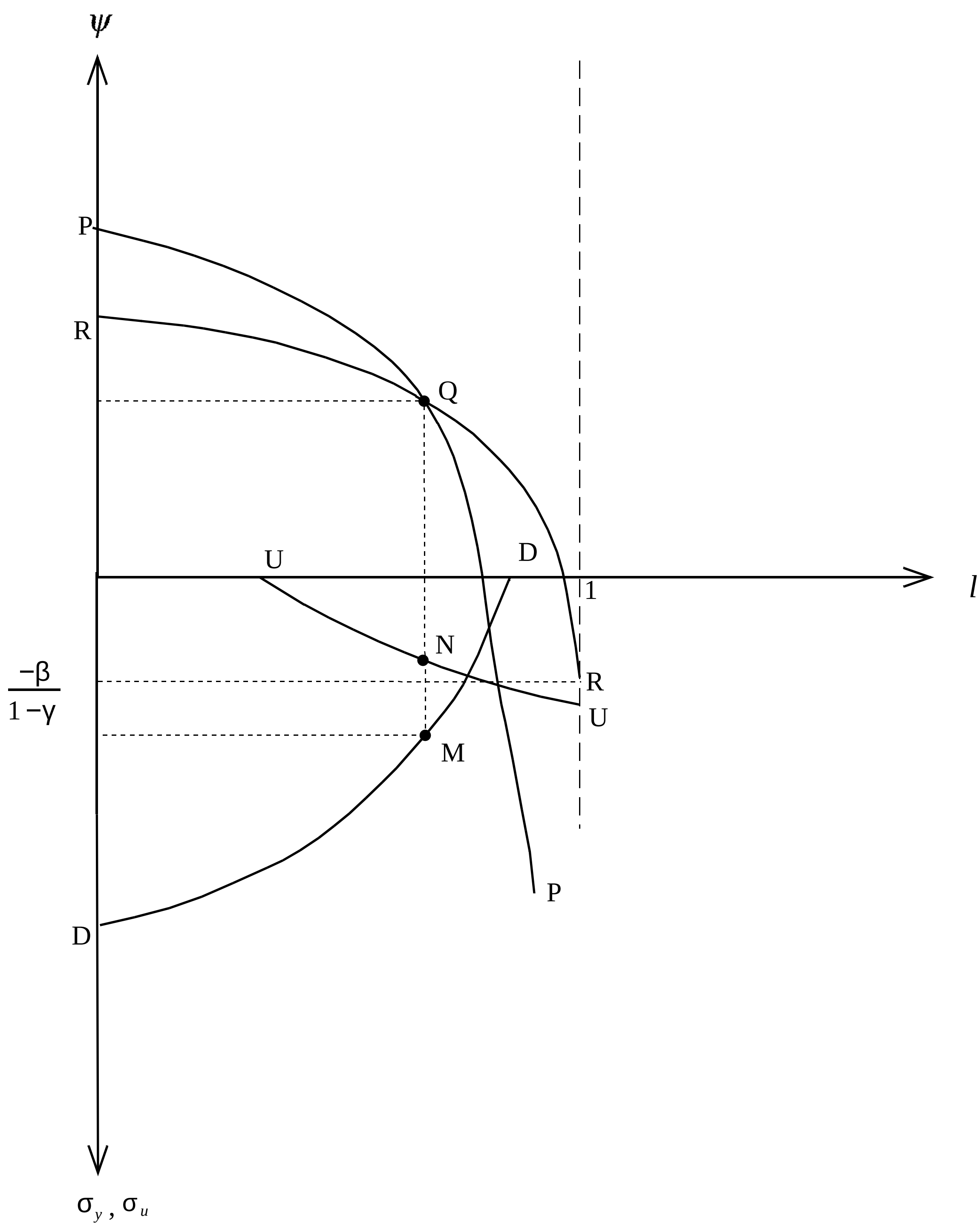

Fig 1: Equilibrium Growth, Employment, and Income Distribution

Table 1

Effects of introducing investment subsidy under alternative modes of finance

	τ_k	τ_w	τ_c
ψ	$-\frac{\alpha^2\Omega^2}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]\left[1+\frac{1-\alpha l}{1-l}\right]>0$	$\frac{\alpha\Omega^2}{D(1-l)}\left(\frac{1-\alpha}{1-\gamma}\right)\left(1+\frac{1}{\eta}-\frac{\alpha l}{\eta(1-l)}\left[1-\frac{l}{\eta(1-l)}\right]\right)>0$	$\frac{\alpha\Omega^2}{D(1-l)}\left(\frac{1-\alpha}{1-\gamma}\right)\frac{1}{\eta}>0$
l	$\frac{\alpha\Omega}{D(1-\gamma)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]<0$	$\frac{\Omega}{D}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$	$\frac{\Omega}{D}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]=-\frac{\beta}{D}<0$
ρ	$-\frac{\alpha\Omega(1-\rho)}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]>0$	$-\frac{\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$	$-\frac{\alpha\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]>0$
ρ^N	$-\frac{\alpha\Omega(1-\rho)}{D(1-\gamma)(1-l)}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]$ $-(1-\rho)\left[1-\frac{\alpha}{\eta}\frac{l}{1-l}\right]$	$-\frac{\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$ $+\frac{(1-\alpha)(1-\rho)}{\alpha}\left[1-\frac{\alpha}{\eta}\frac{l}{1-l}\right]>0$	$-\frac{\alpha\Omega(1-\rho)}{D(1-l)}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]>0$
φ	$\frac{\alpha\Omega(1-\varphi)}{D(1-\gamma)l}\left[1-\frac{1}{\eta}\frac{l}{1-l}\right]<0$	$\frac{\Omega(1-\varphi)}{Dl}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)\frac{l}{\eta(1-l)}-\left(\frac{1-\alpha}{1-\gamma}\right)\right]$	$\frac{\alpha\Omega(1-\varphi)}{Dl}\left[\left(1-\frac{\alpha l}{\eta(1-l)}\right)-\left(\frac{1-\alpha}{1-\gamma}\right)\right]<0$

$D\equiv\frac{\alpha\Omega}{(1-l)}\left(1+\frac{1-\alpha l}{\eta(1-l)}-\frac{1-\alpha}{1-\gamma}\right)>0$ from (13)

Table 2

Relative Rankings of Alternative Modes of Financing Investment Subsidy

Growth rate: $\left.\frac{\partial \psi}{\partial s}\right|_{\tau_c} > \left.\frac{\partial \psi}{\partial s}\right|_{\tau_w} > \left.\frac{\partial \psi}{\partial s}\right|_{\tau_k} > 0$

Gross income inequality $\left.\frac{\partial \rho}{\partial s}\right|_{\tau_k} > \left.\frac{\partial \rho}{\partial s}\right|_{\tau_c} > \max\left[\left.\frac{\partial \rho}{\partial s}\right|_{\tau_w}, 0\right]$

sufficient condition for $\left.\frac{\partial \rho}{\partial s}\right|_{\tau_w} < 0$ is $\tilde{l} < \frac{\eta'}{\alpha + \eta'}$ where $\eta' \equiv \eta\left(\frac{\alpha - \gamma}{1 - \gamma}\right)$

Net income inequality $\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_w} > \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_c} > \max\left[\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_k}, 0\right]$

sufficient condition for $\left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_c} > \left.\frac{\partial \rho^N}{\partial s}\right|_{\tau_k}$ is $\eta < \frac{\alpha}{(1-\alpha)^2}$

Welfare Inequality $\left.\frac{\partial \varphi}{\partial s}\right|_{\tau_k} < \left.\frac{\partial \varphi}{\partial s}\right|_{\tau_c} < \min\left[\left.\frac{\partial \varphi}{\partial s}\right|_{\tau_w}, 0\right]$

sufficient condition for $\left.\frac{\partial \varphi}{\partial s}\right|_{\tau_w} > 0$ is $\tilde{l} < \frac{\eta'}{\alpha + \eta'}$ where $\eta' \equiv \eta\left(\frac{\alpha - \gamma}{1 - \gamma}\right)$

Table 3
Alternative Financing of Fixed Investment Subsidy

s	τ_k	τ_w	τ_c	l	ψ	ρ	ρ^N	φ	Gini Pre-tax	Gini Post-tax	$\Delta(X)$
0	0	0	0	72.5	3.27	0.206	0.206	0.122	33.27	33.27	--
30	10.01	0	0	71.6	4.70	0.230	0.198	0.111	34.77	32.81	7.63
30	0	7.58	0	72.8	5.21	0.198	0.223	0.126	32.77	34.33	9.60
30	0	0	5.38	72.3	5.28	0.213	0.213	0.120	33.72	33.72	10.00
30	0	-18.58	18.58	71.2	5.44	0.243	0.192	0.106	35.52	32.43	10.68

Appendix: Derivation of macroeconomic equilibrium

In this appendix we derive the macroeconomic equilibrium, showing that the economy is always on its balanced growth path. We begin by dividing (6b) by (6a) to obtain

$$\eta \frac{C_i}{K_i} = \left(\frac{1-\tau_w}{1+\tau_c}\right) w(l) l_i \frac{K}{K_i} \tag{A.1}$$

while we may write the individual's accumulation equation (4) in the form

$$\psi_i \equiv \frac{\dot{K}_i}{K_i} = r(l)\left(\frac{1-\tau_k}{1-s}\right) + \left(\frac{1-\tau_w}{1-s}\right) w(l) \frac{K}{K_i}\left((1-l_i) - \frac{l_i}{\eta}\right) \tag{A.2}$$

Taking the time derivative of (6a) and combining with (6c) implies

$$(\gamma-1)\frac{\dot{C}_i}{C_i} + \eta\gamma \frac{\dot{l}_i}{l_i} = \frac{\dot{\lambda}_i}{\lambda_i} = \beta - r(l)\left(\frac{1-\tau_k}{1-s}\right) \qquad \text{for each } i \tag{A.3}$$

indicating that each agent, irrespective of her capital endowment, chooses the same growth rate for her shadow value of capital. Taking the time derivative of (A.1) implies

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{w'(l)l}{w(l)}\frac{\dot{l}}{l} + \frac{\dot{K}}{K} \tag{A.4}$$

Considering equations (A.3) and (A.4) for individuals *i* and *k*, immediately implies that *all* agents will choose the same growth rate for consumption and leisure. This in turn implies that average consumption, *C*, and leisure, *l*, also grow at their respective common growth rates, namely

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}_k}{C_k} = \frac{\dot{C}}{C};\ \frac{\dot{l}_i}{l_i} = \frac{\dot{l}_k}{l_k} = \frac{\dot{l}}{l} \qquad \text{for all } i, k \tag{A.5}$$

Now turn to the aggregates. Summing (A.1) over all agents and noting that $\sum_i k_i = 1$, $\sum_i l_i = l$, the aggregate economy-wide consumption-capital ratio is

$$\eta \frac{C}{K} = \left(\frac{1-\tau_w}{1+\tau_c}\right) w(l) l \tag{A.1'}$$

while summing over (A.2) yields the aggregate accumulation equation

$$\psi \equiv \frac{\dot{K}}{K} = r(l)\left(\frac{1-\tau_k}{1-s}\right) + \left(\frac{1-\tau_w}{1-s}\right) w(l)\left((1-l) - \frac{l}{\eta}\right) \tag{A.2'}$$

The remainder of our derivation is to show that in equilibrium $l(t),\ l_i(t),\ k_i(t) \equiv K_i(t)/K(t)$ are constant through time, so that the economy is always on its balanced growth path. To show this substitute (A.2'), (A.4), and (A.5) into (A.3) expressing it by the following differential equation in *l*:

$$\frac{dl(t)}{dt} = \frac{G(l)}{F(l)} \tag{A.6}$$

where $F(l) \equiv \left[1-\gamma(1+\eta)\right] l^{-1} + (1-\gamma)(1-\alpha)(1-l)^{-1} > 0$

$$G(l) \equiv \left[\left(\frac{1-\tau_k}{1-s}\right)(1-\alpha) - (1-\gamma)\left(\left(\frac{1-\tau_k}{1-s}\right)(1-\alpha) + \left(\frac{1-\tau_w}{1-s}\right)\alpha\left(1-\frac{l}{(1-l)\eta}\right)\right)\frac{Y}{K} - \beta\right]$$

Because time is bounded, in steady-state equilibrium $\dot{l} = 0$, with the corresponding stationary level of *l* being determined where $G(\bar{l}) = 0$. Thus the linearized dynamics of *l* about that point are

$$\frac{dl(t)}{dt} = \frac{G'(\bar{l})}{F(\bar{l})}(l(t) - \bar{l}) \tag{A.7}$$

It is straightforward to establish that $G'(\bar{l}) > 0$ under any plausible conditions, in which case (A.7) is an unstable differential equation. [1] The only solution consistent with the eventual attainment of steady state is for $l(t)$, and therefore for $l_i(t)$, to be constant over time.

The next step is to combine (A.2') and (A.2) to yield the following differential equation in the relative capital stock, $k_i(t) \equiv K_i(t)/K(t)$, namely

$$\dot{k}_i(t) = w(l)\left(\frac{1-\tau_w}{1-s}\right)\left[\left(1 - l_i - \frac{l_i}{\eta}\right) - \left(1 - l - \frac{l}{\eta}\right)k_i(t)\right] \tag{A.8}$$

This equation describes the potential evolution of the relative wealth (capital), starting from the initial endowment k_0. With $l_i,\ l$ both constants this is a simple linear equation, the properties of which will depend the coefficient of $k_i(t)$, which we can determine from the transversality

[1] For example, a uniform tax rate, $\tau_k = \tau_w$, $\gamma < 0$, together with the transversality condition suffice to ensure $G' > 0$.

condition. If (6d) holds for all individuals it implies the aggregate condition

$$\lim_{t\to\infty} \lambda K e^{-\beta t} = 0 \qquad \text{(A.9)}$$

With l constant, (A.2') and (A.3) imply that λ and K both grow at constant rates. It is then straightforward to show that (A.9) will be met if and only if $r\left((1-\tau_k)/(1-s)\right) > \psi$, i.e. the tax-adjusted equilibrium return on capital must exceed the equilibrium growth rate. It then follows from (A.2') that the transversality condition can be further written in the following two equivalent ways

$$(1+\tau_c)\frac{C}{K} > (1-\tau_w)w(1-l) \qquad \text{(A.10a)}$$

$$l > \frac{\eta}{1+\eta}. \qquad \text{(A.10b)}$$

The first equation asserts that part of income is consumed, while the latter imposes the restriction on leisure that ensures that this will be the case.

Now returning to (A.8) we see from (A.11b) that the coefficient of k_i is positive implying that the only solution consistent with long-run stability and the transversality condition is that the right hand side of (A.8) be zero, so that $\dot{k}_i = 0$ for all time. Since k_i reflects capital stocks that evolve gradually over time, this is accomplished by agents selecting their respective leisure, l_i, in accordance with the "relative labor supply" function

$$l_i - l = \left(l - \frac{\eta}{1+\eta}\right)(k_i - 1) \qquad \text{(A.11)}$$

Thus (A.10b) implies a positive relationship between relative wealth and leisure. Setting l_i in accordance with (A.11) implies $\dot{k}_i \equiv 0$, so that the relative wealth position of agents, k_i, is constant over time. The capital stock of all agents grows at the same rate, so that at any point in time, the share of agent i, k_i, remains equal to her initial share $k_{i,0}$, say. Moreover, it follows from (A.3), (A.4), and (A.5) that individual and aggregate consumption also grow at the same common rate:

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{C}}{C} = \frac{\dot{K}_i}{K_i} = \frac{\dot{K}}{K} \equiv \psi = \frac{r(l)\left(\frac{1-\tau_k}{1-s}\right) - \beta}{1-\gamma} \qquad \text{(A.12)}$$

References

Aghion, P. and P. Bolton, 1997, "A Trickle-Down Theory of Growth and Development with Debt Overhang," *Review of Economic Studies* 64, 151-72.

Aghion, P, E. Caroli, and C. García-Peñalosa, 1999, "Inequality and Economic Growth: The Perspective of the New Growth Theories," *Journal of Economic Literature* 37, 1615-1660.

Alesina, A., E. Glasser and B. Sacerdote, 2005, "Work and Leisure in the US and Europe: Why so different?," mimeo.

Alesina, A. and D. Rodrik, 1994, "Distributive Politics and Economic Growth," *Quarterly Journal of Economics* 109, 465-490.

Algan, Y., A. Cheron, J-O Hairault, and F. Langot, 2003, "Wealth Effect on Labor Market Transitions," *Review of Economic Dynamics* 6, 156-178.

Atkinson, A.B. 2003. "Income Inequality in OECD Countries: Data and Explanations," *CESifo Economic Studies* 49, 479-513.

Barro, R.J., 2000, "Inequality and Growth in a Panel of Countries," *Journal of Economic Growth* 5, 5-32.

Bénabou, R., 2002, "Tax and Education Policy in a Heterogeneous-Agent Economy: What Levels of Redistribution Maximize Growth and Efficiency?" *Econometrica* 70, 481-518.

Bertola, G., 1993, "Factor Shares and Savings in Endogenous Growth," *American Economic Review* 83, 1184-98.

Bond, E.W., P. Wang, and C.K. Yip, 1996, "A General Two-Sector Model of Endogenous Growth with Human and Physical Capital: Balanced Growth and Transitional Dynamics," *Journal of Economic Theory* 68, 149-173.

Cardia, E., N. Kozhaya, and F.J. Ruge-Murcia, 2003, "Distortionary Taxation and Labor Supply," *Journal of Money, Credit and Banking* 35, 351-73.

Caselli, F., and J. Ventura, 2000, "A Representative Consumer Theory of Distribution," *American Economic Review* 90, 909-926.

Checchi, D. and C. García-Peñalosa, 2005, "Labor Market Institutions and the Personal Distribution of Income in the OECD", mimeo.

Cheng, I-H. and E. French, 2000, “The Effect of the Run-up in the Stock Market on Labor Supply,” *Federal Reserve Bank of Chicago Economic Perspectives* 24, 48-65.

Coronado, J.L. and M. Perozek, 2003, “Wealth Effects and the Consumption of Leisure: Retirement Decisions During the Stock Market Boom of the 1990s,” *Board of Governors of the Federal Reserve System*, Finance and Economics Discussion Series, 2003-20.

Forbes, K., 2000, “A Reassessment of the Relationship between Inequality and Growth,” *American Economic Review*, 90, 869-887.

Galor, O. and J. Zeira, 1993, “Income Distribution and Macroeconomics,” *Review of Economic Studies* 60, 35-52.

García-Peñalosa, C. and S.J. Turnovsky, 2005, “The Dynamics of Income and Wealth Inequality in a Ramsey Model”, mimeo.

García-Peñalosa, C. and S.J. Turnovsky, 2006, “Growth and Income Inequality: A Canonical Model,” *Economic Theory* 28, 25-49.

Holtz-Eakin, D., D. Joulfaian, and H.S. Rosen, 1993, “The Carnegie Conjecture: Some Empirical Evidence,” *Quarterly Journal of Economics* 108, 413-435.

Lundberg, M. and L. Squire, 2003, “The Simultaneous Evolution of Growth and Inequality,” *Economic Journal*, 113, 2003, 326-344.

Luxembourg Income Study, 2005. *Income Inequality Measures,* http://www.lisproject.org/keyfigures/ineqtable.htm , date: July 11, 2005.

MaCurdy, T., 1981, “An Empirical Model of Labor Supply in a Life Cycle Setting,” *Journal of Political Economy* 89, 1059-1085.

Persson, T. and G. Tabellini, 1994, “Is Inequality Harmful for Growth?” *American Economic Review* 84, 600-621.

Prescott, E.C., 2004, “Why Do Americans Work So Much More Than Europeans?”, *Federal Reserve Bank of Minneapolis Quarterly Review*, 28, 2-13.

Romer, P.M., 1986, “Increasing Returns and Long-Run Growth,” *Journal of Political Economy* 94, 1002-37.

Turnovsky, S.J., 2000, “Fiscal Policy, Elastic Labor Supply, and Endogenous Growth,” *Journal of Monetary Economics* 45, 185-210.

Viane, J.-M. and I. Zitcha, 2003, “Human Capital Formation, Income Inequality, and Growth,” in *Inequality and Growth: Theory and Policy Implications*: T.S. Eicher and S.J. Turnovsky (eds.), Cambridge MA: MIT Press.

Wolff, E.N., 1998, “Recent Trends in the Size Distribution of Household Wealth,” *Journal of Economic Perpectives* 12, 131-50.