Consumption Externalities:

A Representative Consumer Model when Agents are Heterogeneous[*]

Cecilia García-Peñalosa
CNRS and GREQAM
2 rue de la Charité
F-13002 Marseille, France

Stephen J. Turnovsky
University of Washington, Seattle
UW Economics
Box 353330, Savery 302
Seattle, WA 98195

Revised version
July 2007

Abstract: We examine a growth model with consumption externalities where agents differ in their initial capital endowment and their reference group. We show under which conditions the aggregate equilibrium with heterogeneous agents replicates that obtained with a representative consumer, despite the fact that different individuals have different consumption levels. Next we consider the implications of the presence of consumption externalities for the long-run distributions of income and wealth. We find that, in a growing economy, "keeping up with the Joneses" results in less inequality than would prevail in an economy with no consumption externalities.

JEL Classification Numbers: D31, O41

Key words: consumption externalities; growth; wealth distribution; income distribution.

---

[*] García-Peñalosa's research was supported in part by the Institut d'Economie Publique (IDEP) at GREQAM. Turnovsky's research was supported in part by the Castor endowment at the University of Washington. Much of the research for this paper was undertaken while Turnovsky was visiting GREQAM. The paper has benefited from a seminar presentation at Bilkent University, Ankara.

1. Introduction

The role of consumption externalities has a long history in economics, dating back to early work by Veblen (1899), and first introduced as a determinant of aggregate consumption by Duesenberry (1949) in his formulation of the "relative income hypothesis." Recently, there has been a revival of interest in the role of consumption externalities in the context of models of "jealousy" and "keeping up with the Joneses." Their implications for a range of important issues have been investigated. These include: asset pricing (Abel, 1990, 1999; Constantinides, 1990; Gali, 1994, Campbell and Cochrane, 1999), short-run macroeconomic stabilization policy (Ljungqvist and Uhlig, 2000), consumption (Dupor and Liu, 2003), and capital accumulation and growth (Fisher and Hof, 2000; Cooper, García-Peñalosa and Funk, 2001; Liu and Turnovsky, 2005). Empirical evidence supporting the role of consumption externalities has been provided by Easterlin (1995), Clark and Oswald (1996), and Frank (1997), among others. The generalization of utility to incorporate these externalities has helped our understanding of a number of important phenomena pertaining to portfolio choice, aggregate savings, and capital accumulation.

A key feature of virtually all models incorporating consumption externalities is that they adopt the representative agent framework. That is, the standard approach is to assume that the economy consists of a large number of identical agents, each of whom takes some reference (external) consumption level as given, and maximizes his own utility conditional on the given externality. The macroeconomic equilibrium is then obtained by imposing symmetry, and equating all individuals to the average consumption level. This procedure raises the following fundamental question. If all agents are identical, why does a rational agent not realize that in equilibrium everyone's consumption will be equal, in which case the rationale for comparing one's own consumption level with that of others should disappear? Put another way, it would seem that heterogeneity across agents is a fundamental component of any analysis of consumption externalities, such as the effect of jealousy or keeping up with the Joneses, for the behavior of the aggregate magnitudes.

In this paper we analyze the role of consumption externalities in a neoclassical growth model and consider two forms of heterogeneity across agents, different initial wealth endowments and different reference consumption levels. We address two issues. First, we examine the conditions under which a heterogeneous agent economy with consumption externalities, behaves as does the representative agent model. [1] This issue has been addressed by Caselli and Ventura (2000) under conventional preferences and we investigate the extent to which similar results hold in the presence of consumption externalities. We identify two circumstances under which the macroeconomic equilibrium generates that of the representative consumer model. First, when all agents share a common reference consumption level, namely the average economy-wide level of consumption, utility must be homogeneous of any arbitrary degree in own consumption and leisure. Second, if agents have different reference consumption levels, then utility must be separable into a homogeneous function of own consumption and leisure, on the one hand, and a homogeneous function of the agent's reference consumption level, on the other.

Under these conditions we find that the aggregate equilibrium mimics that obtained by imposing symmetry. This provides a justification for the standard practice of assuming identical agents. Yet, there is no such symmetry, as agents with different wealth endowments have different levels of consumption. Viewed in this way, the symmetric equilibrium can be seen as being a short-hand computational device for characterizing the average behavior of a heterogeneous economy, rather than being any stringent restriction requiring all agents to be identical.

The second issue concerns the implications of consumption externalities for the distributions of income and wealth. With agents differing in their endowments, they will experience different time paths for wealth and income. For simplicity, we consider the case in which utility depends on own and aggregate consumption but labor supply is inelastic. This setup has the analytical advantage that the steady-state levels of average output and capital are unaffected by the externality, rendering the role of the externality much more transparent. In this case the crucial mechanism whereby the consumption externality influences the distribution of wealth and income is through its impact on the

[1] See Sorger (2000) for a related model in which this equivalence does not hold, and García-Peñalosa and Turnovsky (2006) for a study of the distributional implications of an AK-model.

economy-wide speed of convergence to the steady state. The key result is that keeping up with the Joneses accelerates the speed of convergence, thus reducing wealth inequality following an expansion in the aggregate capital stock. The effect of the externality on income distribution is less uniform in the short run, although it too will likely decline over time.

The remainder of the paper is structured as follows. Section 2 sets out the model and derives the macroeconomic equilibrium. Section 3 then identifies the two cases in which the aggregate economy evolves independent of the distribution. Sections 4 and 5 discuss the distributional consequences of the consumption externality. Section 6 supplements this with a numerical example and Section 7 concludes. Where possible, technical details are relegated to an appendix.

2. The Model

We begin by setting out the components of the model.

2.1 Technology and factor payments

We assume that there is a representative firm that produces output using a standard neoclassical production function[2]

$$Y = F\left(K, L\right) \tag{1}$$

where K and L denote the aggregate capital stock and employment of labor, respectively. The wage rate, w, and the return to capital, r, are determined by their respective marginal physical products,

$$w(K,L) = \frac{\partial Y}{\partial L} = F_L(K,L) \tag{2a}$$

$$r(K,L) = \frac{\partial Y}{\partial K} = F_K(K,L) \tag{2b}$$

where $w_K = F_{KL} > 0;\ w_L = F_{LL} < 0;\ r_K = F_{KK} < 0;\ r_L = F_{KL} > 0$.[3]

[2] That is both factors of production have positive, but diminishing, marginal physical products and the production function exhibits constant returns to scale (linear homogeneity).

[3] The signs $F_{LL} < 0, F_{KK} < 0$ are a consequence of diminishing marginal product, while $F_{KL} > 0$ is a consequence of the assumption of linear homogeneity, an implication of which is $rK + wL = F_K K + F_L L = Y$.

2.2 Consumers

The economy is populated by *N* individuals, represented as a continuum, each indexed by *i*. We normalize the population so that *N*=1, hence *K* and *L* are the average capital stock and labor supply. Individuals are identical in all respects except for their initial endowments of capital, $K_{i,0}$. We define the share of capital owned by agent *i* as $k_i(t) \equiv K_i(t)/K(t)$, so that relative capital has mean 1. We denote its initial distribution function by $H_0(k)$, the initial density function by $h_0(k)$, and the initial (given) standard deviation of relative (cross-sectional) capital by $\sigma_{k,0}$.

Each such agent is endowed with a unit of time that can be allocated either to leisure, l_i, or to work, $1 - l_i \equiv L_i$. Agent *i* maximizes lifetime utility, assumed to be a function of his own consumption C_i, leisure l_i, and an external reference level of consumption $\bar{C}_i$. We suppose that $\bar{C}_i$, the reference consumption of agent *i*, is the average current consumption of a subset of the population, the peer group of individual *i*, which may, or may not, comprise the entire population.[4] We allow reference consumption to differ across agents. There are many reasons for this, the most persuasive being that within a society, individuals tend to sort themselves into peer groups, with their reference consumption level being the average consumption of their peer group.[5] We hence suppose that agent *i* maximizes

$$\max \int_0^\infty U\left(C_i(t), l_i(t), \bar{C}_i(t)\right) e^{-\beta t} dt, \qquad (3)$$

The utility function is assumed to be increasing and concave in own consumption and leisure, C_i and l_i, and in addition, homogeneous of degree *R* in these same variables.

The key issue concerns the externalities imposed by the reference consumption level on the wellbeing of the individual household. In considering the externality it is useful to contrast two

[4] The formulation we shall be discussing specifies the reference consumption level to be *contemporaneous* consumption of the reference group. A related literature formulates the reference consumption level in terms of past consumption, either of some external reference group or of the agent's own past behavior. This generates a time non-separable utility function. This class of preferences is often referred to as "habits", with the two cases being characterized as "external" versus "internal" habits; see e.g. Carroll, Overland and Weil (1997, 2000), Alvarez-Cuadradro, Monteiro, and Turnovsky (2004), Alonso-Carrera, Caballé, and Raurich (2005a). The same issues we are addressing in the present paper arise in this context as well.

[5] For example, Clark and Oswald (1996) show that individuals' wellbeing tends to depend on their wage relative to that of their co-workers rather than relative to that of the average wage. See also Akerlof (1997).

different ways it may impinge on the individual. First, it may impact in a purely passive way, simply raising or lowering the welfare of the household, without necessarily generating any response in behavior. Thus, following Dupor and Liu (2003), we may say that the household feels either *jealous* $(U_3 < 0)$ or *altruistic* $(U_3 > 0)$ when other agents' consumption increases. Alternatively, the externality may induce some kind of *direct* response by the household, as a consequence of its impact on the interaction of aggregate consumption with other arguments in the household's utility function. This is often referred to as *keeping up with the Joneses,* for which different formulations can be found in the literature. For example, Gali (1994), in a model abstracting from labor supply, describes it in terms of the cross partial derivative $U_{13}(C_i, \bar{C}_i) / U_1(C_i, \bar{C}_i) > 0$. However, this definition is not invariant with respect to the ordinal utility measure. For this reason, using a model with endogenous labor supply, Dupor and Liu (2003) specify it in terms of the effect of the average per capita consumption on the household's marginal rate of substitution between his own consumption and leisure. [6]

It is clear that the first type of externality is necessary for the existence of the second, but not vice versa. Thus, if $U_3 \equiv 0$ there is no externality to which the agent can respond. On the other hand, if the utility function is additively separable in reference consumption, any feelings of jealousy, for example, would have no impact on individual behavior even though they affect utilities. However, most widely adopted utility functions are non-separable, in which case the two types of externalities are not independent. Examples of this are given in Sections 4 and 5 below.

We shall impose the following restrictions on the consumption externalities:

(i) $U_1 + U_3 \geq 0$

(ii) $U_{11} + U_{13} < 0$

(iii) $U_{21} + U_{23} \geq 0$

These conditions assert that either the externality augments the direct effect of own consumption, or, if it is offsetting, it is dominated by the own effect. Thus, for example, if an individual along with

[6] That is, they define keeping up with the Joneses as $d\left(U_1/U_2\right)/d\bar{C}_i > 0$ and what they call "running away from the Joneses" as $d\left(U_1/U_2\right)/d\bar{C}_i < 0$.

the entire economy receives an additional unit of consumption, the direct positive effect of having more consumption is not dominated by any negative effect due to jealousy $(U_3 < 0)$.[7] Likewise, (ii) imposes an upper limit on the "keeping up with the Joneses" effect. As we will illustrate in Section 4 below, this is essentially a stability condition.

The maximization of (3) is subject to the agent's capital accumulation constraint

$$\dot{K}_i(t) = r(t)K_i(t) + w(t)(1 - l_i(t)) - C_i(t) \tag{4}$$

and yields the corresponding first-order conditions [8]

$$U_1\left(C_i, l_i, \bar{C}_i\right) = \lambda_i \tag{5a}$$

$$U_2\left(C_i, l_i, \bar{C}_i\right) = \lambda_i w \tag{5b}$$

$$r = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{5c}$$

where λ_i is agent *i*'s shadow value of capital, together with the transversality condition

$$\lim_{t\to\infty} \lambda_i K_i e^{-\beta t} = 0 \tag{5d}$$

These optimality conditions are standard, except that the marginal utility of consumption and leisure of agent *i* depends on reference consumption. Taken together with the individual's accumulation equation and the corresponding conditions for the aggregate economy, we can derive the macroeconomic equilibrium and the dynamics of the aggregate economy.

2.3. The macroeconomic equilibrium

Summing over the agents, equilibrium in the capital and labor markets is described by

$$K(t) = \int_0^1 K_i(t)\,di \tag{6a}$$

[7] When the reference consumption level is average consumption, $U_1 + U_3$ equals the social shadow value of capital in equilibrium and is presumably positive; see Liu and Turnovsky (2005). The limiting case $U_1 + U_3 = 0$ corresponds to a utility function which depends upon the agent's relative consumption.
[8] Time dependence of variables will be omitted whenever it causes no confusion.

$$L(t) = \int_0^1 L_i(t)di = \int_0^1 (1 - l_i(t))di = 1 - l(t) \tag{6b}$$

Equation (6b) gives the relationship between aggregate leisure and the aggregate labor supply. Note that in equations (2) we have defined the wage and the interest rate, *w*, *r*, and expressed them as functions of average employment, L. From (6b), we can equally well write them as functions of aggregate leisure time, $(1 - l)$, namely, $w = w(K, l)$ and $r = r(K, l)$.

To obtain the macroeconomic equilibrium we begin by dividing (5b) by (5a) to obtain

$$\frac{U_2\left(C_i, l_i, \bar{C}_i\right)}{U_1\left(C_i, l_i, \bar{C}_i\right)} = w(K, l) \tag{7}$$

The homogeneity of the utility function implies that we can express the utility function in the form $U\left(C_i, l_i, \bar{C}_i\right) = l_i^R u\left(c_i, \bar{C}_i\right)$ where $c_i \equiv C_i / l_i$ is agent *i*'s consumption to leisure ratio. The ratio of the marginal utility of leisure to that of consumption is then given by

$$\frac{U_2\left(C_i, l_i, \bar{C}_i\right)}{U_1\left(C_i, l_i, \bar{C}_i\right)} = \frac{Ru\left(c_i, \bar{C}_i\right) - u_1\left(c_i, \bar{C}_i\right)c_i}{u_1\left(c_i, \bar{C}_i\right)} \tag{8}$$

This expression implies that the ratio of marginal utilities is a function of the agent's consumption-leisure ratio and his reference consumption $\bar{C}_i$. Using (8), the first-order condition (7) can be expressed as

$$c_i = \frac{C_i}{l_i} = \phi\left(w(K, l), \bar{C}_i\right) \tag{9}$$

Equation (9) implies that an individual's optimal consumption-leisure ratio now depends both on the wage rate, which is common across all agents, and on his own reference consumption. As a result only agents with the same reference group will choose the same consumption-leisure ratio, c_i.

Using (2a) and (2b) we may write the individual's accumulation equation, (4), in the form

$$\dot{K}_i = r(K, l)K_i + w(K, l)(1 - l_i) - C_i \tag{10}$$

Taking the time derivative of (5a) and combining with (5c) implies

$$\beta - r = \frac{U_{11}C_i}{U_1}\frac{\dot{C}_i}{C_i} + \frac{U_{12}l_i}{U_1}\frac{\dot{l}_i}{l_i} + \frac{U_{13}\bar{C}_i}{U_1}\frac{\dot{\bar{C}}_i}{\bar{C}_i} \text{ for each } i \tag{11}$$

This is the Euler equation modified to take into account the fact that leisure and reference consumption change over time. Equations (9), (10) and (11) then describe the dynamic behavior of agent *i*'s consumption. We next want to examine the conditions under which the aggregate behavior of the model is independent of the distribution of capital and reference consumption across agents, so that the economy in fact behaves as a representative agent.

3. The independence of aggregate behavior from distribution

There are two types of preferences for which aggregate behavior can be shown to be independent of distribution across agents. The first is when all agents have the same reference consumption which is equal to average economy-wide consumption; the second is when the reference consumption level enters utility as a separable homogeneous function.

3.1 Homogeneous reference groups

Suppose all individuals adopt the same reference consumption level, which is equal to average per capita consumption, $C \equiv \int_0^1 C_i di$. The maximization problem of agent *i* is then

$$\max \int_0^\infty U\left(C_i, l_i, C\right) e^{-\beta t} dt, \tag{3'}$$

subject to his capital accumulation constraint (4), and yields the corresponding first-order conditions

$$U_1\left(C_i, l_i, C\right) = \lambda_i \tag{5a'}$$

$$U_2\left(C_i, l_i, C\right) = \lambda_i w \tag{5b'}$$

$$r = \beta - \frac{\dot{\lambda}_i}{\lambda_i} \tag{5c}$$

As before, we can use (5a') and (5b') together with the homogeneity of the utility function to get

$$c_i = \frac{C_i}{l_i} = \phi\left(w(K,l), C\right) \tag{9'}$$

Equation (9') implies that an individual's optimal consumption-leisure ratio depends only on variables common to all individuals, the wage rate and per capita consumption. As a result, all agents will choose the same consumption-leisure ratio, c_i .

Taking the time derivative of (5a') and combining with (5c) implies

$$\left(\frac{U_{11}C_i}{U_1}\right)\frac{\dot{C}_i}{C_i}+\left(\frac{U_{12}l_i}{U_1}\right)\frac{\dot{l}_i}{l_i}=\beta-r-\left(\frac{U_{13}C}{U_1}\right)\frac{\dot{C}}{C} \quad \text{for each } i \tag{11'}$$

Homothetic preferences imply that the three elasticities of the marginal utility of consumption in (11') depend only on c_i and C, and hence, by (9') are the same for all agents (see Appendix A.1).

Differentiating equation (9') we get

$$\frac{\dot{C}_i}{C_i}-\frac{\dot{l}_i}{l_i}=\left(\frac{\phi_1 w}{\phi}\right)\frac{\dot{w}}{w}+\left(\frac{\phi_2 w}{\phi}\right)\frac{\dot{C}}{C} \qquad \text{for each } i \tag{12}$$

Equations (11') and (12) can be viewed as a pair of equations in $\dot{C}_i/C_i$ and $\dot{l}_i/l_i$, and since all agents confront the same wage and reference consumption they imply that

$$\frac{\dot{C}_i}{C_i}=\frac{\dot{C}}{C};\frac{\dot{l}_i}{l_i}=\frac{\dot{l}}{l} \qquad \text{for all } i \tag{13}$$

That is, at all points of time, each agent will choose the same growth rates for consumption and leisure, which therefore equal the economy-wide average growth rate for consumption and leisure.

Now turn to the aggregates. Summing (9') over individuals implies that the aggregate economy-wide consumption-leisure ratio also equals $\phi(.)$, namely

$$c=\frac{C}{l}=\phi\big(w(K,l),C\big) \tag{9''}$$

It then follows from (13), together with (9') and (9") that relative consumption equals relative leisure, i.e.

$$\frac{C_i}{C}=\frac{l_i}{l}=\theta_i \tag{14}$$

where θ_i is a constant and $\int_0^1 \theta_i di=1$. The homogeneity of the utility function, together with equation (14), implies

$$U_1\left(C_i, l_i, C\right) = U_1\left(\theta_i C, \theta_i l, C\right) = \theta_i^{R-1} U_1\left(C, l, C\right) \tag{15a}$$

$$U_2\left(C_i, l_i, C\right) = U_2\left(\theta_i C, \theta_i l, C\right) = \theta_i^{R-1} U_2\left(C, l, C\right) \tag{15b}$$

while summing over (10) yields the aggregate accumulation equation

$$\dot{K} = r(K,l)K + w(K,l)(1-l) - C \tag{10'}$$

Recalling the definitions of $r(K,L)$ and $w(K,L)$, and the homogeneity of the production function, we can summarize the aggregate macroeconomic equilibrium dynamics as follows:

$$U_1\left(C, l, C\right) = \zeta_i \tag{16a}$$

$$U_2\left(C, l, C\right) = \zeta_i F_L(K,L) \tag{16b}$$

$$L + l = 1 \tag{16c}$$

$$\dot{K} = F(K,L) - C \tag{16d}$$

$$F_K(K,L) = \beta - \frac{\dot{\zeta}_i}{\zeta_i} \tag{16e}$$

where for notational ease we redefine the shadow value, $\zeta_i \equiv \lambda_i \theta_i^{1-R}$.

Equations (16a)-(16e) are autonomous equilibrium dynamic equations in the economy-wide average quantities of capital, *K*, consumption, *C*, leisure, *l*, labor supply *L*, and the modified shadow value, ζ_i. These equilibrium relationships are standard. The first two equations reflect the static optimality conditions for consumers, while (16c) is simply the labor market clearing condition. Equation (16d) describes product market clearing, while equation (16e) is the Keynes-Ramsey consumption rule. Since ζ_i is a co-state variable, which is determined by the evolution of the aggregate state variable (capital) in accordance with (16), all individuals choose the same value, so that $\zeta_i = \zeta$, say, and the *i* subscript can be dropped.

The system (16) has two key implications. First, the aggregate dynamics are clearly independent of the distribution of wealth across agents; second, these equilibrium relationships are identical to those obtained in the model with homogeneous agents when a symmetric equilibrium is imposed (i.e. all agents are *assumed* to chose the same level of consumption); see Liu and Turnovsky (2005, equations (2)). Our analysis indicates that there is no need to impose symmetry; even if consumption levels differ across agents, the optimality conditions imply that all individuals choose the same consumption-leisure ratio, which, given homotheticity, is sufficient for the aggregate magnitudes to behave *as if* we were in a symmetric equilibrium.

Assuming that the economy is stable, these aggregate quantities converge to a steady state characterized by a constant average per capita capital stock, labor supply, and leisure time, which we denote by $\tilde{K}$, $\tilde{C}$, $\tilde{L}$ and $\tilde{l}$, respectively. Setting $\dot{K} = \dot{\zeta} = 0$ in (16), yields the steady state conditions:

$$\frac{U_2\left(\tilde{C},\tilde{l},\tilde{C}\right)}{U_1\left(\tilde{C},\tilde{l},\tilde{C}\right)} = F_L\left(\tilde{K},\tilde{L}\right) \tag{17a}$$

$$\tilde{L} + \tilde{l} = 1 \tag{17b}$$

$$F(\tilde{K},\tilde{L}) = \tilde{C} \tag{17c}$$

$$F_K(\tilde{K},\tilde{L}) = \beta \tag{17d}$$

Again, these steady-state conditions are standard and coincide with (6) of Liu and Turnovsky (2005).[9]

We can summarize these results as follows

Proposition 1: Consider an economy in which agents care about the average level of consumption and differ in their initial stock of capital. If the instantaneous utility function is homogeneous of degree R in own consumption and leisure, the macroeconomic equilibrium is the one obtained when individuals are identical and symmetry is imposed. Individual consumption levels and labor supplies will nevertheless differ across agents.

[9] As Liu and Turnovsky (2005) discuss, the stability and steady-state equilibrium depend upon keeping up with the Joneses; and are independent of pure jealousy, which affects only the welfare level associated with any equilibrium path.

Many of the results obtained with identical agents can be replicated. For example, it is possible to show that the first-best solution with identical agents is the same as the one that maximizes the welfare of the average agent in the heterogeneous-endowments economy. The results obtained in the literature about over/under consumption and over/under accumulation apply (see Liu and Turnovsky, 2005, proposition 2).[10]

3.2 Heterogeneous reference groups and separable consumption

When agents have different reference consumption levels, the aggregate behavior depends on the distribution of reference groups, for the general utility function. Because c_i and $\overline{C}_i$ vary across agents, the elasticities $U_{11}(C_i, l_i, \overline{C}_i)C_i / U_1(C_i, l_i, \overline{C}_i)$ are no longer constant. As a result, (11) implies that agents with different reference groups chose different growth rates for the shadow value of capital, and hence for consumption. The aggregate behavior of the model then depends on the distribution of the reference consumption levels across agents.

Are there preferences for which heterogeneity in the reference group can nevertheless lead to an aggregate behavior which is independent of the distribution of preferences? This occurs for multiplicatively separable utility functions of the form

$$U\left(C_i, l_i, \overline{C}_i\right) \equiv V\left(C_i, l_i\right) G\left(\overline{C}_i\right) \qquad (18)$$

where $V(C_i, l_i)$ is homogeneous of degree *R*, and $G(\overline{C}_i)$ is homogeneous of degree *S*. In this case, the first order condition (7) becomes

$$\frac{V_2(C_i, l_i)}{V_1(C_i, l_i)} = w(K, l) \qquad (7')$$

which implies

$$c_i = \frac{C_i}{l_i} = \phi(w(K, l)) \qquad (19)$$

The consumption-leisure ratio is hence independent of reference consumption and a function of the

[10] However, the implications concerning the tax structure required in order to implement the first best solution are not necessarily the same, as the taxation of capital and labor income will have distributive implications.

wage only, implying a common consumption-leisure ratio across all agents irrespective of their endowment or preferences.[11] Summing over the agents yields the corresponding aggregate relationship

$$c = \frac{C}{l} = \phi\left(w(K,l)\right) \tag{19'}$$

Differentiating equation (19') we get

$$\frac{\dot{C}_i}{C_i} - \frac{\dot{l}_i}{l_i} = \frac{\phi_1 w}{\phi}\frac{\dot{w}}{w} \qquad \text{for each } i \tag{12'}$$

Equations (11) and (12') then imply

$$\frac{\dot{C}_i}{C_i} = \frac{\dot{\overline{C}}_i}{\overline{C}_i} = \frac{\dot{C}}{C}; \frac{\dot{l}_i}{l_i} = \frac{\dot{l}}{l} \quad \text{for all } i \tag{13'}$$

In addition to (14), which holds as before, agent *i*'s reference consumption level satisfies a similar relationship, $\overline{C}_i = \upsilon_i C$. The homogeneity conditions imply that the marginal utilities satisfy

$$U_j\left(C_i, l_i, \overline{C}_i\right) \equiv V_j\left(C_i, l_i\right) G\left(\overline{C}_i\right) = V_j\left(\theta_i C, \theta_i l\right) G\left(\upsilon_i C\right) = \theta_i^{R-1}\upsilon_i^S V_j\left(C, l\right) G\left(C\right) \qquad j = 1, 2$$

The previous analysis then applies and the dynamic behavior of the aggregate magnitudes is then given by

$$V_1(C,l)G(C) = \pi_i \tag{20a}$$

$$V_2(C,l)G(C) = \pi_i F(K,L) \tag{20b}$$

$$L + l = 1 \tag{20c}$$

$$\dot{K} = F(K,L) - C \tag{20d}$$

[11] Note that equation (19) also holds when preferences are additively separable, i.e. $U\left(C_i, l_i, \overline{C}_i\right) = V\left(C_i, l_i\right) + G\left(\overline{C}_i\right)$, so that the consumption externality has only a pure jealousy or altruistic effect. In this case, the externality does not appear in the first-order conditions of the consumer's optimality problem and hence has no effect on either the dynamic behavior or the steady state of the economy.

$$F_K(K,L) = \beta - \frac{\dot{\pi}_i}{\pi_i} \tag{20e}$$

where $\pi_i \equiv \lambda_i \theta_i^{1-R} \upsilon_i^{-S}$ is the modified shadow value of capital, and is the same for all agents, i.e. $\pi_i = \pi, \ \forall i$. Assuming stability, the system converges to the steady state

$$\frac{V_2(\tilde{C},\tilde{l})}{V_1(\tilde{C},\tilde{l})} = F_L(\tilde{K},\tilde{L}) \tag{21a}$$

$$\tilde{L} + \tilde{l} = 1 \tag{17b}$$

$$F(\tilde{K},\tilde{L}) = \tilde{C} \tag{17c}$$

$$F_K(\tilde{K},\tilde{L}) = \beta \tag{17d}$$

In this case, both the transitional aggregate dynamics and the long-run behavior are independent of the distribution of wealth, and mimic those obtained by imposing a symmetric equilibrium. Furthermore, the long-run equilibrium is unaffected by the consumption externality, although this will still influence the transitional dynamics.

We can summarize these results as follows

Proposition 2: Consider an economy where agents differ in their initial stock of capital and in their reference consumption level. If the instantaneous utility function is multiplicatively separable and is homogeneous of degree R in own consumption and leisure, on the one hand, and homogeneous of degree S in the consumption externality, on the other, the macroeconomic equilibrium is identical to that obtained when individuals are identical and symmetry is imposed. Individual consumption levels and labor supplies will nevertheless differ across agents.

A remark is in order. Note that there is no need for the externalities to be symmetric. The reason for this is that all that is required for the equilibrium is that reference consumption grows at the same rate as own consumption, which with homothetic preferences is always the case irrespective of who is the reference group of agent *i*. It is then possible for a subgroup of the population, say those with low wealth, to care about the consumption of the whole population, while another subgroup, say those with high wealth, care only about the consumption of those who also have high wealth.

4. Inelastic Labor Supply

In Section 3 we have identified conditions, under which the aggregate behavior of the economy proceeds independently of the distribution of wealth across agents. Under these conditions, the characterization of the aggregate economy as if it were described by a single representative consumer, due to Caselli and Ventura (2000), extends to the situation where agents are subject to consumption externalities. With individual agents in fact being heterogeneous, the aggregate equilibrium is associated with a non-degenerate distribution of wealth and income, thus raising the question of how the presence of consumption externalities affects the distribution of wealth and income. In particular, is wealth more or less unequally distributed when such externalities are present than when they are absent?

Unfortunately, an analytical comparison of the distributions of wealth and income is somewhat intractable for the general forms of the utility functions employed above. In what follows we examine a widely employed and important case: an economy with exogenous labor supply, in which the externality stems from the average economy-wide consumption level. The only source of heterogeneity is then the initial capital endowment. In this case, the steady state is unaffected by the consumption externality (see Liu and Turnovsky, 2005), while the long-run distributions of wealth, income, and consumption will depend on the magnitude and sign of the externality. This is an attractive feature of the model for our purposes, as it implies that the welfare of the average individual is independent of the externality, and that the latter affects only distribution. In more general setups this dichotomy need not apply.

Suppose that all individuals supply one unit of labor inelastically, and that they maximize the following intertemporal isoelastic utility function[12]

[12] The constant elasticity utility function is frequently used in the literature; see, for example, Gali (1994) for an early example. It is also prevalent in the related "habits" literature; see e.g. Carroll et al (1997), Alvarez-Cuadrado, et al (2004).

$$U \equiv \max \int_0^\infty \frac{1}{\gamma}\left[C_i C^{-\varepsilon}\right]^\gamma e^{-\beta t} dt, \qquad \text{with} \quad -\infty < \gamma < 1, \gamma(1+\varepsilon) < 1 \tag{22}$$

where $\kappa \equiv 1/(1-\gamma)$ equals the intertemporal elasticity of substitution. This utility function has the property

$$\operatorname{sgn}\left(\partial U/\partial C\right) = -\operatorname{sgn}(\varepsilon); \quad \operatorname{sgn}\left(\partial^2 U/\partial (C_i C)^2\right) = -\operatorname{sgn}(\gamma\varepsilon)$$

and we shall identify the consumption externality as being negative or positive according to whether $\varepsilon > 0$ (jealous) or $\varepsilon < 0$ (altruistic). The following relationship between "jealousy" and "keeping up with the Joneses" [as defined by Gali, 1994] holds: the agent will keep up with the Joneses if either (a) he is jealous $(\varepsilon > 0)$ and the intertemporal elasticity of substitution is less than one $(\gamma < 0)$ or (b) he is altruistic $(\varepsilon < 0)$ and the intertemporal elasticity of substitution is greater than one $(\gamma > 0)$. Since the overwhelming preponderance of empirical evidence suggests that the intertemporal elasticity of substitution is relatively small, certainly below unity, we shall be mainly concerned with the former case.[13] We are also assuming that all agents treat the economy-wide average consumption level as the common reference group. Rewriting (22) as

$$U \equiv \max \int_0^\infty \frac{1}{\gamma}\left[C_i^{1-\varepsilon}\left(\frac{C_i}{C}\right)^\varepsilon\right]^\gamma e^{-\beta t} dt \tag{22'}$$

we see that utility is a weighted average of absolute and relative consumptions, the respective weights being $(1-\varepsilon, \varepsilon)$. Thus $\varepsilon = 0$ corresponds to the conventional utility measure based only on own consumption, while if $\varepsilon = 1$, utility depends only upon the agent's consumption relative to that of his peers. In addition, imposing conditions (i) and (ii) on utility leads to the following two restrictions on the externality: (i) $1 \geq \varepsilon$, (ii) $1 - \gamma(1-\varepsilon) > 0$. This maximization is subject to the agent's capital accumulation constraint

$$\dot{K}_i(t) = r(t)K_i(t) + w(t) - C_i(t). \tag{4'}$$

In addition we assume that the production function is of the CES form

$$Y = F(K, L) = \left(\alpha K^{-\rho} + (1-\alpha)L^{-\rho}\right)^{-1/\rho} \tag{1'}$$

[13] See e.g. the discussion of the empirical evidence summarized and reconciled by Guvenen (2006).

where $\omega \equiv (F_K F_L)/(F_{KL}F) = 1/(1+\rho)$ and $s \equiv F_K K/F$ denote the elasticity of substitution and the capital share, respectively.

4.1 Evolution of the macroeconomic aggregates

With the labor supply fixed inelastically, and dropping L for notational simplicity, the dynamic equations governing the aggregate behavior can be conveniently expressed as

$$\dot{K} = F(K) - C \tag{23a}$$

$$\frac{\dot{C}}{C} = \frac{F_K(K) - \beta}{1-(1-\varepsilon)\gamma} \tag{23b}$$

It is readily apparent that given the conventional neoclassical assumptions on production, (23) is a saddlepoint if and only if the restriction (ii) on the externality holds. Rewriting this as $-\gamma\varepsilon < 1-\gamma$, we see that it imposes an upper bound on the extent of keeping up with the Joneses for the aggregate system as represented by (23) to be locally stable. Assuming this holds, the aggregate economy then converges to the steady-state capital stock and consumption levels, $\tilde{K}$ and $\tilde{C}$, respectively, determined by[14]

$$F_K(\tilde{K}) = \beta \tag{24a}$$

$$\tilde{C} = F(\tilde{K}) \tag{24b}$$

Linearizing equations (23a) and (23b) around the steady state (24a) and (24b), the stable paths for K and C can be expressed as

$$K(t) = \tilde{K} + (K_0 - \tilde{K})e^{\mu t} \tag{25a}$$

$$C(t) = \tilde{C} + \left(\beta - \mu\right)\left(K(t) - \tilde{K}\right) \tag{25b}$$

where $\mu < 0$, the stable eigenvalue, is the solution to:

[14] We are also imposing the transversality conditions, $\lim_{t\to\infty} \lambda_i K_i(t) e^{-\beta t} = 0$ for each *i*.

$$\mu^2 - \beta\mu + \frac{F_{KK}F}{1-\gamma(1-\varepsilon)} = 0 \tag{26}$$

and summarizes the convergence speed of the aggregate economy. [15]

Clearly, while the steady state conditions (24a) - (24b) are independent of the consumption externality, as reflected in ε, the dynamic behavior of the aggregate economy depends on the value of ε, through its impact on μ. The slope of the stable arm, (25b), is positive; accumulating capital is therefore associated with increasing consumption, a standard property of the Ramsey model. Equation (26) implies $\mathrm{sgn}(d\mu/d\varepsilon) = \mathrm{sgn}(\gamma)$. When $\gamma < 0$, a negative consumption externality, causing keeping up with the Joneses behavior, raises the marginal utility of wealth, inducing faster capital accumulation met by an immediate reduction in consumption. Over time, the more rapid capital accumulation leads to a subsequent faster increase in consumption, and thus increases the speed of convergence of the aggregate economy. As we will see below, this channel is the crucial mechanism through which consumption externalities exert distributional effects.[16]

4.2 The dynamics of the relative capital stock

To derive the dynamics of individual *i's* capital stock, relative to the economy-wide average, $k_i(t) \equiv K_i(t)/(K(t)/L)$, we combine (4') and the corresponding aggregate relation to obtain

$$\dot{k}_i(t) = \frac{w(K,L)L}{K}\left(1-k_i(t)\right) - \frac{C}{K}\left(\theta_i - k_i(t)\right) \tag{27}$$

where *K, C* evolve in accordance with (23a, 23b) and the initial relative capital, $k_{i,0}$, is given from the initial endowments. Linearizing this equation around steady state and solving, and imposing the condition that the relative share of capital remains bounded, (for each i), we can show that the stable solution for the time path of the relative stock of capital of agent *i* is (see Appendix)

$$k_i(t) - \tilde{k}_i = \left(\frac{\delta(t)-1}{\delta(0)-1}\right)(k_{i,0} - \tilde{k}_i) = \left(\frac{K(t)-\tilde{K}}{K_0 - \tilde{K}}\right)(k_{i,0} - \tilde{k}_i) = e^{\mu t}(k_{i,0} - \tilde{k}_i) \tag{28}$$

[15] Eicher and Turnovsky (1999) define the speed of convergence of a variable $x(t)$ to its stationary value $\bar{x}$, to be $\dot{x}(t)/(\bar{x} - x(t))$. In the present case, where the stable transitional path is a first-order, this reduces to $-\mu$.

[16] If the keeping up with the Joneses behavior is too intense, leading to a violation of the condition (ii), then it will induce instability. See Alonso-Carrera, Caballé, and Raurich (2005b)

where $\tilde{k}_i$ is the steady-state endowment of individual *i*, defined by

$$k_{i,0} - 1 = \delta(0)\left(\tilde{k}_i - 1\right) \tag{29}$$

and

$$\delta(t) \equiv 1 + (1-\tilde{s})\frac{\beta(1-\omega)+\mu\omega}{(\beta-\mu)\omega}\left(\frac{K(t)}{\tilde{K}} - 1\right). \tag{30}$$

From (28) we see that $k_i(t)$ also converges to its steady state value at the rate μ. Then, as in Caselli and Ventura (2000), the cross-section of wealth converges to a long-run distribution in which all agents hold positive amounts of wealth, wealth is unequally distributed, and the ranking of agents according to wealth is the same as in the initial distribution. As we will see, the eigenvalue μ, and hence ε, plays a crucial role in determining the dynamics of wealth distribution, and consequently, income distribution.

5. Distributional Consequences

5.1 Distribution of wealth

Having established the existence of a long-run distribution of wealth, we can compare it to the initial distribution and examine the effect that the presence of the consumption externality has on inequality. It is convenient to measure distribution by the standard deviation of the cross-sectional distribution of capital (wealth) across agents, although it can be shown that the same analysis applies in terms of more conventional Gini coefficients.[17] Because of the linearity of (28) and (29), we can immediately transform these equations into corresponding expressions for the standard deviation of the cross-sectional distribution of capital. Specifically,

$$\sigma_k(t) = \frac{\delta(t)}{\delta(0)}\sigma_{k,0} \tag{31}$$

where $\sigma_{k,0}$ is the initial distribution of capital. The change in the steady-state distribution of capital can thus be expressed as

[17] See Turnovsky and García-Peñalosa (2007).

$$\Delta\tilde{\sigma}_k \equiv \tilde{\sigma}_k - \sigma_{k,0} = (1-\delta(0))\tilde{\sigma}_k = \frac{h(\tilde{K})}{\beta-\mu}\frac{\tilde{K}-K_0}{\tilde{K}}\tilde{\sigma}_k \tag{32}$$

where

$$h(\tilde{K}) \equiv (1-\tilde{s})\left(\frac{1-\omega}{\omega}\beta + \mu\right) \tag{33}$$

Wealth inequality then increases during the transition to the steady state if and only if [18]

$$h(\tilde{K})\left(\tilde{K}-K_0\right) > 0 \tag{34}$$

There are then two factors that determine whether inequality increases or decreases during the transition: the initial capital (relative to the long run) and the sign of $\left(\beta(1-\omega)+\mu\omega\right)$.[19] Suppose the economy is accumulating capital, so that $\left(\tilde{K}-K_0\right) > 0$. It is immediately evident that $h(\tilde{K})$ is certainly negative for values of the elasticity of substitution $\omega \geq 1$, but could be positive for $\omega < 1$. Indeed, $h(\tilde{K}) > 0$ if and only if $\omega < 1$ and

$$\frac{1-\omega}{\omega}\beta > -\mu \tag{35}$$

The key issue concerns how the presence of the consumption externality impinges on (33) and hence on (32). With fixed labor supply and the steady state independent of ε the answer is that the effect of the consumption externality operates entirely through its impact on μ. In the Appendix we show that using (26) we can express (35) as

$$\frac{1}{1-\omega} - \gamma + \gamma\varepsilon > \frac{\omega}{1-\omega}\frac{1}{\beta}\left(\frac{\beta}{\alpha}\right)^{\omega} \tag{36}$$

The striking feature of (36) is that it provides a simple criterion for whether wealth inequality increases or decreases with an accumulation of the aggregate capital stock, expressed in terms of the four structural parameters, $\alpha, \beta, \gamma, \omega$ as well as the consumption externality, ε. Note that what

---

[18] For example, (24a) implies that an increase in the rate of time preference leads to an increase in $\tilde{K}$ and hence, if the economy starts from an initial steady state, implies $\tilde{K} - K_0 > 0$.

[19] The role played by the initial condition has been established by Caselli and Ventura (2000) in the Ramsey model with standard preferences. See also García-Peñalosa and Turnovsky (2007) for a discussion of the intuition for this result.

matters is not the sign of ε but rather that of $\gamma\varepsilon$; keeping up with the Joneses, i.e. $\gamma\varepsilon < 0$, implies that (36) is less likely to be satisfied. For the plausible case, $\gamma < 0$, when $d\mu/d\varepsilon < 0$, a negative consumption externality, by reducing $h(\tilde{K})$, is more likely to render (32) negative, so that an increase in capital is associated with a decline in wealth inequality.

Furthermore,

$$\frac{d\Delta\tilde{\sigma}_k}{d\varepsilon} = \frac{(1-\tilde{s})\beta\tilde{\sigma}_k}{\omega(\beta-\mu)^2}\left(\frac{\tilde{K}-K_0}{\tilde{K}}\right)\frac{d\mu}{d\varepsilon} < 0 \tag{37}$$

so that a negative consumption externality reduces wealth inequality in the sense of reducing the increase associated with the accumulating capital.

We may summarize the implications of consumption externalities for wealth inequality in the following proposition. To be concrete we shall express it in terms of an economy converging from below.

Proposition 3: (i) If $\omega > 1$, an economy that starts below its steady-state capital stock, i.e. $K_0 < \tilde{K}$, will always experience a reduction in wealth inequality during the transition, irrespective of the nature of the consumption externality.

(ii) If $\omega < 1$, an economy that starts out below its steady-state capital stock will experience a reduction in wealth inequality during the transition if and only if (36) holds. Keeping up with the Joneses, i.e. $\gamma\varepsilon < 0$, decreases the likelihood of (36) being met, while $\gamma\varepsilon > 0$ increases it.

(iii) Keeping up with the Joneses ($\gamma\varepsilon > 0$) decreases (increases) long-run wealth inequality, either absolutely, or in the sense of reducing the increase (decrease) in inequality associated with the increase in capital.

The main general message to emerge is that keeping up with the Joneses is likely to lead to less wealth inequality in a growing economy. With the steady-state equilibrium being independent of the externality, the key mechanism for this is via the impact on the adjustment speed of the economy. The following intuition applies. As capital is being accumulated in response to a prior stimulus its marginal product declines, thus reducing the return to capital. By increasing the speed of adjustment, μ, a negative consumption externality accelerates this process and hence reduces the relative amount of income being earned by capital over time. Since wealthier people own more capital, this narrows the long-run distribution of wealth.

5.2 Distribution of income

We turn now to the distribution of income. The income of individual *i* at time *t* is given by $Y_i(t) = r(t)K_i(t) + w(t)$, while average economy-wide income is $Y(t) = r(t)K(t) + w(t)$. We are interested in the evolution of relative income, defined as $y_i(t) \equiv Y_i(t)/Y(t)$, and which recalling the definition of the capital share, $s(t)$, may be expressed as

$$y_i(t) - 1 = s(t)(k_i(t) - 1) \tag{38}$$

Analogous to (31) we may write

$$\sigma_y(t) = s(t)\sigma_k(t) = s(t)\frac{\delta(t)}{\delta(0)}\sigma_{k,0} \tag{39}$$

The relative income of agent *i* depends on two components, the share of capital and his relative capital endowment. The capital share determines the relative contributions of capital and labor to overall income, and hence to income inequality, for given individual endowments. Since capital is more unequally distributed than is labor a higher capital share implies greater income inequality.[20]

From equation (39) it is possible to obtain the asymptotic distribution of income, $\tilde{\sigma}_y$. Moreover, it is possible to show that the distributions of relative consumption and welfare equal the asymptotic distribution of income (see Appendix A.3). The effect of the consumption externality on these distributions is then the same as its long-run effect on the income distribution. The change in the steady-state distribution of income can be expressed as

[20] Equation (39) also implies that since $s(t) < 1$, $\sigma_y(t) < \sigma_k(t)$, consistent with empirical evidence.

$$\Delta\tilde{\sigma}_y \equiv \tilde{\sigma}_y - \sigma_{y,0} = \left[(1-\delta(0))s_0 + (\tilde{s}-s_0)\right]\tilde{\sigma}_k = \left(\tilde{s} - s_0\delta(0)\right)\tilde{\sigma}_k \qquad (40)$$

This comprises two components, the first due to the change in wealth discussed in Section 5.1, and the second due to the change in factor returns, reflected in the capital share. In the case of the Cobb-Douglas production function, we have $\tilde{s} = s_0$, and the behavior of the long-run income distribution will simply mirror that of wealth. Otherwise, it is straightforward to establish:

$$\Delta\tilde{\sigma}_y = \tilde{s}\tilde{\sigma}_k\left(\frac{\tilde{K}-K_0}{\tilde{K}}\right)\left(\frac{h(\tilde{K})}{\beta-\mu} + (1-\tilde{s})\frac{\omega-1}{\omega}\left(1-\frac{h(\tilde{K})}{\beta-\mu}\frac{\tilde{K}-K_0}{\tilde{K}}\right)\right) \qquad (41)$$

where $h(\tilde{K})$ is defined in (33). The first term in brackets is the wealth effect, while the second is the effect due to factor returns. In both cases, consumption externalities operate through μ. The factor returns effect depends critically upon $(\omega-1)$, accentuating the wealth effect if $\omega<1$ and offsetting it if $\omega>1$. In the latter case it is possible for the long-run wealth and income distributions to respond in opposite ways; see Fig. 1. From (41), and using (33), we derive

$$\frac{d\Delta\tilde{\sigma}_y}{d\mu} = \tilde{s}\tilde{\sigma}_k\frac{\beta}{\beta-\mu}\left(1-\frac{(\omega-1)(1-\tilde{s})}{\omega}\left[\frac{\tilde{K}-K_0}{\tilde{K}}\right]\right)\left(\frac{\tilde{K}-K_0}{\tilde{K}}\right)\frac{d\mu}{d\varepsilon} \qquad (42)$$

Thus we may deduce the following proposition:

Proposition 4: *The dynamics of the distribution of income and the effect of consumption externalities on these dynamics depends crucially on the size of the long-run change in the aggregate capital stock. If the economy is accumulating capital and* $\omega<1$, *then a negative (positive) consumption externality will reduce (increase) income inequality. This will continue to hold if* $\omega>1$, *provided the steady-state change in capital is not too large.*[21]

6. Numerical Simulations

To obtain further insights into the dynamics of wealth and income distribution we simulate

[21] The formal condition is that $\Delta\tilde{K}/\tilde{K} < \omega/[(\omega-1)(1-s)]$, a simple sufficient condition for which is that the increase in the aggregate capital stock be less than 100%.

the economy in response to an increase in productivity. The simulations are based on the CES production function and the constant elasticity utility function used in sections 4 and 5, and the following parameters, characterizing the benchmark economy:

Basic parameters:	$A=1, \alpha=1/3$
	$\rho=1,\ 0,\ -0.2$ ($\omega\ =0.5,\ 1,\ 1.25$)
	$\gamma=-9,-1.5,-0.25$ ($\kappa=0.1, 0.4, 0.8$)
	$\varepsilon=-1, 0, 1$

These parameters are standard and non-controversial.[22] Our objective is to focus on the sensitivity of the effects of the consumption externality on wealth and income inequality across variations in the elasticity of substitution in production between 0.5 (low) and 1.25 (high) and the intertemporal elasticity of substitution between 0.1 (low) and 0.8 (high). The consumption externality ranges between 1 (negative externality) and -1 (positive externality).[23] The distributional parameter α is held fixed at 0.4, while $A=1$ scales the initial level of productivity. These parameter values are standard and non-controversial.

We assume that the economy is initially in a steady state and we introduce an increase in the level of technology in the form of an increase in *A* from 1 to 1.5. The resulting changes in the long-run inequality of wealth and income are reported in Table 1 in the form of grids, for three panels corresponding to low, medium, and high values for ω. One advantage of abstracting from labor is that the steady state equilibria are independent of ε, normalizing many of the comparisons. Figures 1 illustrates some of the sample dynamic paths, focusing on their sensitivity to key parameters. All such paths illustrated are relative to their initial equilibrium.

To understand the table and figure, it is helpful to first identify the Cobb-Douglas production function ($\omega=1$), an intertemporal elasticity of substitution κ=0.4 (i.e. $\gamma=-1.5$) and no consumption externality ($\varepsilon=0$) as a benchmark case. This is identified in bold in Table 1 and leads

[22] For example, the intertemporal elasticity of substitution 0.4 is well within the range of empirical estimates summarized by Guvenen (2006). Allowing the elasticity of substitution in production to vary between 0. 5 and 1.25 covers the range of most of the empirical estimates, while $\alpha=1/3$ implies that one third of output goes to capital in a Cobb-Douglas world, also broadly consistent with empirical evidence.

[23] We are unaware of any direct empirical evidence on the magnitude of ε. The values $\varepsilon=1$, $\varepsilon=-1$ correspond to two plausible extreme forms of utility. The former is based on relative consumption, while the latter implies symmetry in that the agent derives identical utility from an increase in reference consumption as he does from his own consumption.

to a long-run reduction in both wealth and income inequality of 9.46%. Moreover, with the labor supply being inelastic and factor shares fixed, the time paths for the change in wealth and income inequality are identical, as illustrated in Figure 1, panel (ii).

A negative consumption externality reduces both wealth and income inequality by identical amounts, 13.19% for $\varepsilon = 1$, and is therefore stabilizing. In contrast, positive consumption externalities, $\varepsilon = -1$, reduce these inequalities by only 7.51%, and is therefore destabilizing, relative to the benchmark.

Figure 1 illustrates how the responses of wealth inequality and income inequality to the consumption externality depend upon the production function. As soon as we deviate from the Cobb-Douglas technology, the time paths and long-run responses of wealth inequality and income inequality diverge. Panel (i) illustrates the case of a low elasticity of substitution ($\omega = 0.5$). In this case wealth inequality in the absence of the consumption externality is reduced and displays slightly more sensitivity to the consumption externality than in the Cobb-Douglas case. The reduction in ω causes income inequality to fall dramatically and its sensitivity to the consumption externality to decline as well.

Panel (iii) illustrates the case of a high elasticity of substitution ($\omega = 1.25$). In this case the higher capital reduces wealth inequality by 8.84% and is also fairly sensitive to variations in the consumption externality. The most striking aspect is that the high elasticity of substitution causes the long-run income inequality to increase by 0.88% and that a positive consumption externality exacerbates this. A negative consumption externality leads to a decrease in long-run instability of nearly 2%, Both $\varepsilon = 0, \varepsilon = 1$ demonstrates aspects of a Kuznets curve in that inequality increases during the initial phase of the transition and then ultimately declines.

All of these long-run adjustments are consistent with Propositions 3 and 4. They also illustrate the significance of the speed of convergence in determining the long-run impact on wealth and income inequality. Two things stand out. First, the speed of convergence is very sensitive to the elasticity of substitution in production. For the benchmark value $\omega = 1$ it is 2.1%.[24] Reducing ω to

[24] This speed of convergence confirms to early benchmark estimates obtained by Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil (1992). Subsequent estimates have shown more variability, being sensitive to the time period, the sample set of countries, and estimation methods. For example, Islam (1995) estimates the rate of convergence to be 4.7% for non-oil countries and 9.7% for OECD countries. Caselli, Esquivel, and Lefort (1996) use a

0.5 raises the convergence speed to 9.27%, while increasing ω to 1.25 reduces it to 0.65%. The inverse relationship between the speed of convergence and the elasticity of substitution in production has been discussed previously; see Turnovsky (2002). The significance here is its consequences for wealth and income inequality. The second aspect is the strong positive relationship between the consumption externality, ε, and the speed of convergence, the strength of which varies inversely with ω.

7. Conclusions

The role of consumption externalities has been extensively studied recently in the context of "keeping up with the Joneses". The standard approach has been to adopt the representative agent framework and to consider a symmetric equilibrium in which all agents are identical. This procedure begs the question why rational agents, knowing that in equilibrium they are all identical, are concerned about the consumption of their peers in the first place? Heterogeneity seems to be at the heart of the issue and this paper has introduced consumption externalities into an economy in which agents are heterogeneous with respect to their initial endowments of capital.

It has addressed two issues. First it has shown that the aggregate behavior of this heterogeneous economy mimics that of the representative agent economy under fairly general conditions. In the case that all agents use the same reference consumption level, all that is required is that each agent's utility function is homogeneous in own consumption and leisure. If they have different reference consumption levels, the utility function must be separable into a homogeneous function of own consumption and leisure, together with a homogeneous function of the consumption externality. Under these conditions the representative agent equilibrium can be viewed as a short-cut to deriving the aggregate equilibrium of a heterogeneous-agent economy.

The second issue we have addressed is the consequences of the consumption externality for various measures of inequality, particularly wealth and income inequality. Focusing on the case where labor is supplied inelastically, we have shown that the consumption externality influences

---

GMM estimator to correct for sources of inconsistency due to correlated country-specific effects and endogenous explanatory variables and obtain a convergence rate of around 10%. Evans (1997) using an alternative method to generate consistent estimates of convergence finds them to be around 6%.

these distributional measures through its impact on the speed of convergence.

While most of the literature has focused on the role of consumption externalities in utility, other sources of externality have also been discussed. Kurz (1968), and more recently Zou (1995), have introduced aggregate capital as a potential externality into utility, while Alesina, Glaeser, and Sacerdote (2005) have argued in favor of leisure externalities as a possible explanation for differences in working hours between Europe and the United States. Introducing either of these externalities has no effect on the basic structure of our equilibrium as set out in Sections 2 and 3. Other authors, such as Corneo and Jeanne (1997, 2001), Futagami and Shibata (1998) and Fisher (2004) introduce status into utility, measured by relative wealth. This form of externality implies that each agent's return to capital is individual specific, in which case the dichotomy characteristic of our solution ceases to hold. Aggregate behavior and distribution become jointly determined. This case merits further study.

Appendix

This Appendix is devoted to the derivation of several technical details

A.1 Properties of the homogeneous utility function

We start by obtaining a number of results for the utility function. Let the utility function be $U\left(C_i, l_i, \bar{C}_i\right)$, where $\bar{C}_i$ denotes the consumption of the reference group of agent *i*. The homogeneity of the utility function implies that we can express the utility function as $U(C_i, l_i, C) = l_i^R u(c_i, C)$ where $c_i \equiv C_i / l_i$ is agent *i*'s consumption to leisure ratio. The ratio of the marginal utility of leisure to that of consumption is then given by

$$\frac{U_2\left(C_i, l_i, \bar{C}_i\right)}{U_1\left(C_i, l_i, \bar{C}_i\right)} = \frac{Ru\left(c_i, \bar{C}_i\right) - u_1\left(c_i, \bar{C}_i\right)c_i}{u_1\left(c_i, \bar{C}_i\right)} \tag{A.1}$$

Since the right-hand side of this expression depends only on the consumption-leisure ratio, we can invert equation (7) and express the consumption-leisure ratio as a function of the wage and reference-group consumption,

$$c_i = \phi\left(w(K, l), \bar{C}_i\right). \tag{A.2}$$

Furthermore, consider the following elasticities of the marginal utility of consumption

$$\frac{U_{11}\left(C_i, l_i, \bar{C}_i\right)C_i}{U_1\left(C_i, l_i, \bar{C}_i\right)} = \frac{u_{11}\left(c_i, \bar{C}_i\right)c_i}{u_1\left(c_i, \bar{C}_i\right)} \tag{A.3a}$$

$$\frac{U_{12}\left(C_i, l_i, \bar{C}_i\right)l_i}{U_1\left(C_i, l_i, \bar{C}_i\right)} = \frac{(R-1)u_1\left(c_i, \bar{C}_i\right) - u_{11}\left(c_i, \bar{C}_i\right)c_i}{u_1\left(c_i, \bar{C}_i\right)} \tag{A.3b}$$

$$\frac{U_{13}\left(C_i, l_i, \bar{C}_i\right)\bar{C}_i}{U_1\left(C_i, l_i, \bar{C}_i\right)} = \frac{u_{12}\left(c_i, \bar{C}_i\right)\bar{C}_i}{u_1\left(c_i, \bar{C}_i\right)} \tag{A.3c}$$

From (A.2) all these expressions imply that the elasticities are the same for individuals with the same reference consumption, $\bar{C}_i$. Hence, if all individuals have the same reference group so that $\bar{C}_i$ is common for all *i*, then these elasticities are constant across agents.

A. 2 The dynamics of the relative capital stock

To solve for the time path of individual *i's* relative capital stock, we first note from (27) that agent *i's* steady-state share of capital satisfies

$$w(\tilde{K})\left(1-\tilde{k}_i\right)=\tilde{C}\left(\theta_i-\tilde{k}_i\right) \qquad \text{for each } i \qquad \text{(A.4)}$$

where $\theta_i \equiv C_i / C$. To analyze the evolution of the relative capital stock, we linearize equation (27) around the steady-state (A.4). This yields

$$\dot{k}_i(t)=\tilde{r}\left(k_i(t)-\tilde{k}_i\right)-\left(1-\tilde{k}_i\right)\frac{\tilde{w}F_{KK}}{\mu(1-\gamma)}\frac{K(t)-\tilde{K}}{\tilde{K}}+\frac{F_{KL}}{\tilde{K}}\left(1-\tilde{k}_i\right)\left(K(t)-\tilde{K}\right) \qquad \text{(A.5)}$$

which we can write as

$$\dot{k}_i(t)=\beta\left(k_i(t)-\tilde{k}_i\right)+h(\tilde{K})\left(1-\tilde{k}_i\right)\frac{K_0-\tilde{K}}{\tilde{K}}e^{\mu t} \qquad \text{(A.6)}$$

where

$$h(\tilde{K})\equiv LF_{KL}(\tilde{K},L)-\frac{LF_L(\tilde{K},L)}{F(\tilde{K},L)}(\beta-\mu) \qquad \text{(A.7)}$$

Using $\omega \equiv (F_K F_L)/(F_{KL}F)$ and $s \equiv F_K K/F$, (A.7) can be rewritten as (33). Solving (A.6) and using the transversality condition yields

$$k_i(t)-1=\delta(t)(\tilde{k}_i-1) \qquad \text{(A.8)}$$

where

$$\delta(t)\equiv 1+\frac{h(\tilde{K})}{\beta-\mu}\left(\frac{K(t)}{\tilde{K}}-1\right), \qquad \text{(A.9)}$$

Setting $t=0$ in (A.8) and (A.9), we have

$$k_{i,0} - 1 = \delta(0)(\tilde{k}_i - 1) = \left(1 + \frac{h(\tilde{K})}{\beta - \mu}\left(\frac{K_0}{\tilde{K}} - 1\right)\right)(\tilde{k}_i - 1) \tag{A.10}$$

enabling us to express the evolution of agent *i*'s relative stock of capital from its initial level in the form:

$$k_i(t) - 1 = \frac{\delta(t)}{\delta(0)}(k_{i,0} - 1) \tag{A.11}$$

Using (A.11), and the equations describing the evolution of the aggregate economy, (25), we can express the time path for $k_i(t)$ as (28).

To derive the condition (36), note that (26) implies that the eigenvalue is given by

$$\mu = \frac{\beta}{2}\left[1 - \sqrt{1 - \frac{4F(\tilde{K})F_{KK}}{(1-\gamma(1-\varepsilon))\beta^2}}\right] \tag{A.12}$$

With the CES production function, $F_{KK} / F_K = -(1-s)/(\omega K)$ and

$$\frac{1-s}{s} = \frac{1-\alpha}{\alpha}\left(\frac{L}{K}\right)^{-\rho} \tag{A.13}$$

Also, the steady state condition (24a) implies that the steady state capital-labor ratio is defined by

$$(1-\alpha)\left(\frac{L}{\tilde{K}}\right)^{-\rho} = \left[\left(\frac{\beta}{\alpha}\right)^{-(1-\varepsilon)} - \alpha\right] \tag{A.14}$$

These expressions together with (A.12) allow us to write (35) in the form (36).

A.3 The cross-section of consumption and welfare

We can determine the individual's consumption path. First, having determined $(\tilde{k}_i - 1)$, (A.4) yields agent *i*'s relative consumption θ_i, namely

$$\theta_i - 1 = \left(1 - \frac{\tilde{w}}{\tilde{C}}\right)\left(\tilde{k}_i - 1\right) = \frac{\tilde{s}}{\delta(0)}\left(k_{i,0} - 1\right) \tag{A.15}$$

Since the rate of growth of consumption is the same for all agents, the distribution of consumption remains unchanged over time. From (A.15) we have

$$\sigma_\theta = \frac{\tilde{s}}{\delta(0)}\sigma_{k,0} = \tilde{\sigma}_y \tag{A.16}$$

and the constant distribution of consumption equals the asymptotic distribution of income.

To compute the distribution of welfare note that the level of utility of individual *i* as the economy traverses its equilibrium path is given by

$$\int_0^\infty \frac{1}{\gamma}\left[C_i C^{-\varepsilon}\right]^\gamma e^{-\beta t}dt = (\theta_i)^\gamma \int_0^\infty \frac{1}{\gamma}\left[C^{1-\varepsilon}\right]^\gamma e^{-\beta t}dt = \frac{1}{\gamma}\left[1-\tilde{s}+\tilde{s}\tilde{k}_i\right]^\gamma \int_0^\infty \frac{1}{\gamma}\left[C^{1-\varepsilon}\right]^\gamma e^{-\beta t}dt \equiv X(\tilde{k}_i) \tag{A.17}$$

where we have used (14) and (A.15) to substitute for individual consumption. The welfare of the individual with an average capital stock, and hence, from (A.15), a level of consumption equal to the average is

$$\int_0^\infty \frac{1}{\gamma}\left[C^{1-\varepsilon}\right]^\gamma e^{-\beta t}dt \equiv X(1) \tag{A.18}$$

The welfare of individual *i* relative to that of the individual with average consumption is then

$$x(\tilde{k}_i) \equiv X(\tilde{k}_i)/X(1) = \left(\theta_i\right)^\gamma = \left[1-\tilde{s}+\tilde{s}\tilde{k}_i\right]^\gamma \tag{A.19}$$

We can now compute a measure of welfare inequality. A natural metric for this is obtained by applying the following monotonic transformation of relative lifetime utility, enabling us to express the relative utility of individual *i* as

$$u(\tilde{k}_i) \equiv x(\tilde{k}_i)^{1/\gamma} = 1+\tilde{s}(\tilde{k}_i - 1) \tag{A.20}$$

which implies a standard deviation of relative welfare

$$\sigma_u = \frac{\tilde{s}}{\delta(0)}\sigma_{k,0} = \tilde{\sigma}_y \tag{A.21}$$

Again, welfare inequality will vary in the same way as long-term wealth inequality.

References

Abel, A.: Asset prices under habit formation and catching up with the Joneses, American Economic Review (Papers and Proceedings) 80, 38-42 (1990).

Abel, A.: Risk premia and term premia in general equilibrium, Journal of Monetary Economics 43, 3-33, (1999).

Akerlof, G.A.: Social distance and social decisions, Econometrica 65, 1005-27 (1997).

Alesina, A., Glaeser, E. , Sacerdote, B.: Work and leisure in the US and Europe: why so different? NBER Macroeconomics Annual 2005, 1-64, (2005).

Alonso-Carrera J., Caballé, J., Raurich, X.: Growth, habit formation and catching-up with the Joneses, European Economic Review 49, 1665-1691, (2005a).

Alonso-Carrera J., Caballé, J., Raurich, X.: Can consumption spillovers be a source of equilibrium indeterminacy?, working paper CREA-Barcelona Economics 154 (2005b).

Alvarez-Cuadrado, F., Monteiro, G., Turnovsky, S.J.: Habit formation, catching up with the Joneses, and non-scale growth, Journal of Economic Growth 9, 47-80, (2004).

Barro, R.J., Sala-i-Martín, X.: Convergence, Journal of Political Economy 100, 223-251, (1992).

Campbell, J.Y., Cochrane, J.N.,: By force of habit: A consumption-based explanation of aggregate stock behavior, Journal of Political Economy 107, 205-251, (1999).

Carroll, C., Overland, J., Weil, D.: Comparison utility in a growth model, Journal of Economic Growth 2, 339-367, (1997).

Carroll, C., Overland, J., Weil, D.: Saving and growth with habit formation. American Economic Review 90, 341-355, (2000).

Caselli, F., Esquivel, G., Lefort, F.: Reopening the convergence debate: A new look at cross-country empirics, Journal of Economic Growth 1, 363-390, (1996).

Caselli, F., Ventura, J.: A representative consumer theory of distribution, American Economic Review 90, 909-926, (2000).

Clark, A.E., Oswald, A.J.: Satisfaction and comparison income, Journal of Public Economics 61, 359-381, (1996).

Constantinides, G.M.: Habit formation: A resolution of the equity premium puzzle, Journal of Political Economy 98, 519–543, (1990).

Cooper, B., García-Peñalosa, C., Funk, P.: Status effects and negative utility growth, Economic Journal 111, 642-665(2001).

Corneo, C., Jeanne, O.: On relative wealth effects and the optimality of growth, Economics Letters 54, 87-92, (1997).

Corneo, C., Jeanne, O.: Status, the distribution of wealth, and growth, Scandinavian Journal of Economics 103, 283-293 (2001).

Duesenberry, J.S.: Income, Saving and the Theory of Consumer Behavior, Harvard University Press, Cambridge, Mass. (1949).

Dupor, B., Liu, W.F.: Jealousy and equilibrium overconsumption, American Economic Review 93, 423-428, (2003).

Easterlin, R.A.: Will raising the incomes of all increase the happiness of all? Journal of Economic Behavior and Organization 27, 35-47, (1995).

Eicher, T.S., Turnovsky, S.J.: Convergence speeds and transition dynamics in non-scale growth models, Journal of Economic Growth 4, 413–428, (1999).

Evans, P.: How fast do countries converge? Review of Economics and Statistics 79, 219-225, (1997).

Fisher, W.H.: “Status preference, wealth, and dynamics in an open economy,” German Economic Review 5, 335-356, (2004).

Fisher, W.H., Hof, F.X.: Relative consumption, economic growth, and taxation, Journal of Economics (Zeitschrift für Nationalökonomie) 72, 241-262, (2000).

Frank, R.: Happiness and economic performance, Economic Journal, 107, 1815-1831, (1997).

Futagami, K., Shibata, A.: Keeping one step ahead of the Joneses: status, the distribution of wealth, and long-run growth, Journal of Economic Behavior and Organization 36, 93-111, (1998).

Gali, J.: Keeping up with the Joneses: Consumption externalities, portfolio choice, and asset prices, Journal of Money, Credit, and Banking 26, 1–8, (1994).

García-Peñalosa, C., Turnovsky, S.J.: Growth and income inequality: A canonical model. Economic Theory 28, 25-49, (2006).

García-Peñalosa, C. , Turnovsky, S.J.: The Dynamics of Wealth Inequality in a Simple Ramsey Model: A Note on the Role of Production Flexibility. Working Paper University of Washington, (2007).

Guvenen, F.: Reconciling conflicting evidence on the elasticity of intertemporal substitution: A macroeconomic perspective, Journal of Monetary Economics 53, 1451-1472, (2006).

Islam, N.: Growth empirics: A panel data approach, Quarterly Journal of Economics 110, 1127-1170, (1995).

Kurz, M.: Optimal economic growth and wealth effects. International Economic Review 9, 348-357, (1968).

Liu, W.F., Turnovsky, S.J.: Consumption externalities, production externalities, and long-run macroeconomic efficiency, Journal of Public Economics 89, 1097-1029, (2005).

Ljungqvist, L., Uhlig, H.: Tax policy and aggregate demand management under catching up with the Joneses, American Economic Review 90, 356-366, (2000).

Mankiw, N.G., Romer, D., Weil, D.: A contribution to the empirics of economic growth, Quarterly Journal of Economics 107, 407-38, (1992).

Sorger, G.: Income and wealth distribution in a simple model of growth. Economic Theory 16, 23-42 (2000).

Turnovsky, S.J.: Intertemporal and intratemporal substitution, and the speed of convergence in the neoclassical growth model, Journal of Economic Dynamics and Control 26, 1765-1785, (2002).

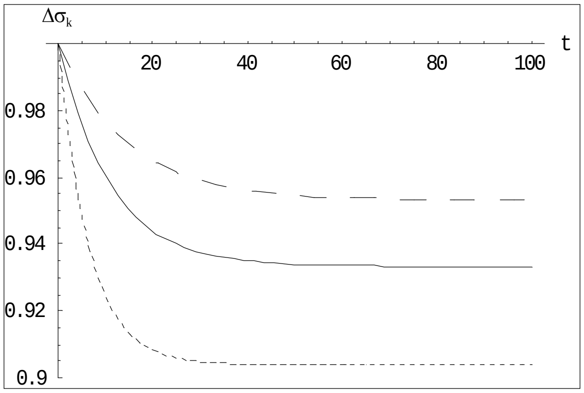
Turnovsky, S.J., García-Peñalosa, C.: Distributional Dynamics in a Neoclassical Growth Model: The Role of Elastic Labor Supply, forthcoming Journal of Economic Dynamics and Control (2007).

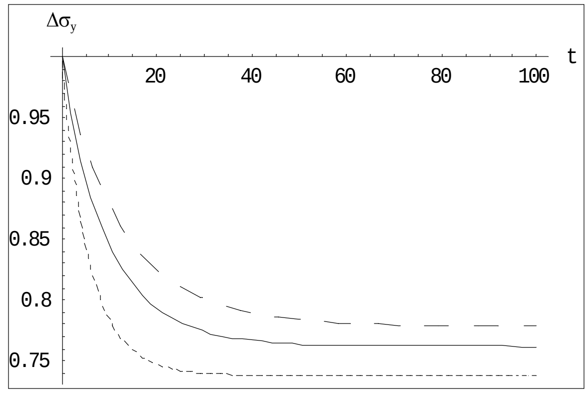
Veblen, T.B.: The Theory of the Leisure Class: An Economic Study of Institutions, Modern Library, New York, (1899).

Zou, H.F.: The spirit of capitalism and long-run growth, European Journal of Political Economy 10, 279-293, (1995).

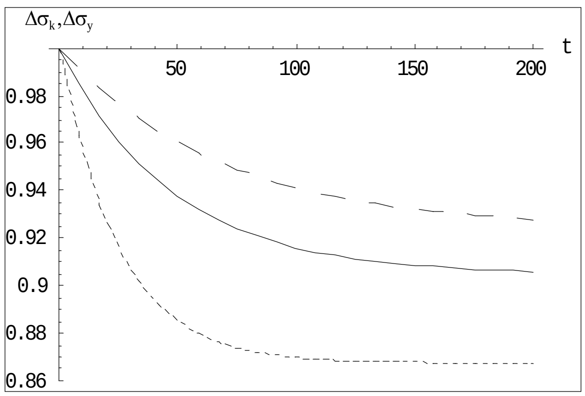
Fig 1: Increase in A from 1 to 1.5

Elast of subst = 0.5

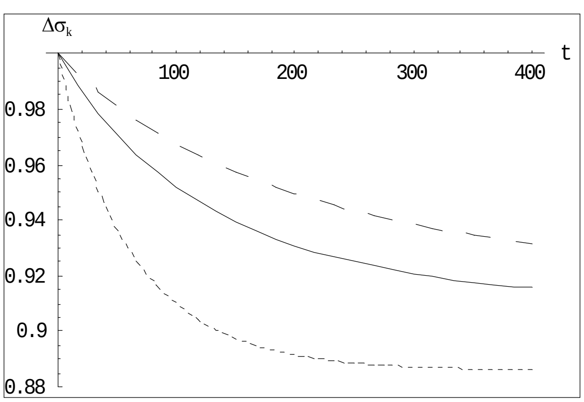


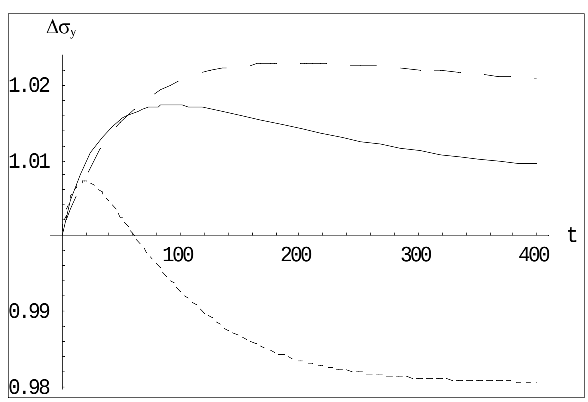


Elast of subst =1



Elast of subst =1.25





———— $\varepsilon=0$ - - - - - $\varepsilon=1$ — — $\varepsilon=-1$