

Redistribution and Entrepreneurship with Schumpeterian Growth¹

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Abstract

We examine the effects of redistributive taxation on growth and inequality in a Schumpeterian model with risk-averse agents. There are skilled and unskilled workers, and the growth rate is determined by the occupational choice of skilled agents between entrepreneurship and employment. We show that redistribution provides insurance to entrepreneurs and increases the growth rate. The effects on inequality are such that low tax rates increase inequality relative to laissez-faire due to changes in wages, but higher tax rates can simultaneously raise growth and reduce inequality. We contrast the optimal linear income tax with alternative policies for promoting R&D and find that it is preferable on both equity and efficiency grounds.

JEL Classification: H21-O3 - O4

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1 Introduction

The revival of interest in the relationship between inequality and growth has led economists to raise new questions about the effects of redistribution. The traditional incentive argument that redistribution reduces physical capital accumulation has been emphasized in models such as those of Alesina and Rodrik (1994) and Persson and Tabellini (1994). However, a number of authors have stressed alternative mechanisms that may reverse these results. In the presence of imperfect credit markets, redistribution can be growth-enhancing either through an ‘opportunity creation effect’ that allows more agents to invest in education, as in Galor and Zeira (1993), or through an ‘incentive mechanism’ in the presence of moral hazard that increases work-effort, as in Aghion and Bolton (1997). Even when credit is readily available, redistributive taxation that is used to finance public education expenditures may raise the return to private educational investments, increase the average level of education in an economy and promote growth; see Saint-Paul and Verdier (1993).¹

A possible reading of this literature is that redistribution fosters growth only in developing countries, where credit markets are highly imperfect and growth is driven by factor accumulation. Meanwhile, in industrial economies, which have well-functioning financial institutions, and where growth is due to private R&D activities, the above mechanisms may not apply and the reduction in the returns to entrepreneurs due to redistributive taxation is likely to hurt growth. Such argument ignores the fact that even in the industrial economies there is a capital market imperfec-

¹See Aghion, Caroli, and García-Peñalosa (1999) and Bertola, Foellmi, and Zweimüller (2006) for reviews of this literature.

tion, namely the absence of private insurance for those who engage in risky income-generating activities, as argued by Mayshar (1977) and Sinn (1996).

There is plenty of evidence that entrepreneurs face large risks. In the United States, 61.5 per cent of businesses exit within five years, and the founder of a private company faces a risk of about 10 per cent of losing all his/her investment in the first ten years (Moskowitz and Vissing-Jorgensen, 2002); and the cross-sectional standard deviation of self-employment earnings is substantially higher than wages from paid employment (Hamilton, 2000). This evidence contrasts with the standard approach in the growth literature, where, despite major steps to provide microfoundations for the innovation process, the role of risk-aversion and insurance has received little attention.

In this paper, we examine the effects of redistribution on growth and inequality in a Schumpeterian setting by adding risk aversion to the model of Aghion and Howitt (1992). In this context there are two sources of inequality—that between skilled and unskilled agents who are different *ex ante*, and that among skilled agents who choose different occupations and become different *ex post*. In particular, some skilled workers will choose to work in production for fixed wages, while others will become entrepreneurs/researchers and receive random profits. It is this endogenous choice of occupation that determines the growth rate. The presence of unskilled workers implies that the social planner will have equity considerations as well as the pure insurance ones derived from the riskiness of research.

We consider the effects of a linear progressive income tax. We show first that the tax provides a certain degree of insurance to entrepreneurs, which stimulates research and growth.² This effect is reinforced when skill acqui-

²One can ask why Europe has relatively more social spending but lower entrepreneur-

sition is endogenous. We then explore the implication of redistribution for equilibrium wages and income inequality. There are two elements at play. First, there is a direct effect whereby redistribution increases the income (and utility) of the unskilled and reduces that of skilled agents *for given wages*. Second, redistribution shifts workers from skilled manufacturing employment into entrepreneurship, thus raising the skilled wage and reducing the unskilled wage. This implies that skilled workers can experience an increase in their net incomes despite being net fiscal contributors. A small amount of redistribution promotes growth but increases income inequality, as the wage-effect dominates. A greater degree of redistribution can, however, simultaneously reduce inequality and promote growth. Furthermore, it is possible that the net effect of redistributive taxation is such that both the skilled and the unskilled are better off in every period.

We compare the effect of the linear progressive tax to some of the alternatives that have been proposed in the R&D literature, such as research subsidies or research joint ventures, and find that the former is preferable on both equity and efficiency grounds. Lastly, we consider how the optimal linear income tax rate varies with the size of intertemporal knowledge spillovers. A stronger spillover has two effects. On the one hand it implies a larger return to innovation; on the other, it increases the income of a successful entrepreneur and hence raises inequality. Both of these effects imply that the socially optimal tax rate is an increasing function of the degree of

ship rates than the United States. There are two likely reasons for this. First, the generous discharge provisions in US bankruptcy law may encourage innovation. As Skeel (2001) puts it, "In the United States, bankruptcy has long served as a partial substitute for the more generous social protections provided by other nations." Second, the evidence from OECD countries provided by Ilmakunnas and Kanninen (2001) suggests that, "The Welfare State does not provide insurance to share the entrepreneur's risk of failure." Instead, the existing programs mainly provide unemployment insurance and are hence designed to reduce the risk faced by workers.

intertemporal spillovers.

The idea that redistribution can act as social insurance when private risk-pooling arrangements are absent was first noted by Eaton and Rosen (1980) and Varian (1980). Kanbur (1981) and Boadway, Marchand and Pestieau (1991) examined how redistribution affects the occupational choice between risky entrepreneurship and paid employment.³ A central concern in these papers is the implication of occupational choice for optimal taxation. The models used, however, are static as entry into entrepreneurship is assumed to have no impact on innovation or growth. As a result the social planner faces a tradeoff: greater redistribution reduces ex post inequality, but, because it provides more insurance against business failures, it encourages excessive risk taking (i.e. leads to too many entrepreneurs). Allowing for the effect of entrepreneurship on growth adds an important new dimension to the optimal tax problem.⁴

A crucial assumption in our analysis is that we rule out the possibility of private insurance for researchers. Two caveats are in order. First, an important fraction of R&D takes place within large manufacturing firms, and in this case it is the firm rather than the researcher that bears the risk. However, existing evidence indicates that small firms account for a disproportionate number of patents and innovations relative to their size, and that these patents are important, being twice as likely as large-firm patents to be among the one percent most cited; see Acs and Audretsch (1988) and U.S.B.C Small Business Administration (2003). Moreover, small and new

³Bird (2001) finds that, in OECD countries, the dispersion of individual market income is positively correlated with measures of the size of the welfare state.

⁴There are few papers focusing on the role of risk-aversion in the growth process. Zeira (2005) explains the concentration of R&D in monopolistic firms as a risk-sharing method. Saint-Paul (1992) and Acemoglu and Zilibotti (1997) emphasize the importance of risk-sharing arrangements in countries with capital shortages.

firms play an increasingly important role in industrial economies and have been shown to be positively correlated with growth rates in OECD countries; see Acs and Audretsch (1988), Carree and Thurik (1998), Greenwood and Jovanovic (1999). Second, although small research firms have some access to external finance which can provide insurance, this is at best partial, and the flow of internal finance is the principal determinant of the rate at which small high-tech firms acquire technology through R&D, as documented by Himmelberg and Petersen (1994). Extensive theoretical work explains why this is so, as moral hazard and adverse selection result in no (or limited) finance for highly risky activities such as innovation; see Arrow (1962), Myers and Majluf (1984), and Aghion and Tirole (1994). All this indicates that although R&D is partly financed by large firms and venture capital, small firms in which the entrepreneur bears substantial risks account for an important fraction of innovative activity in industrial economies.

Our paper is closely related to Zeira (1988), who shows how greater progressivity induces risk-averse individuals to acquire more human capital.⁵ Caucutt, Imrohoroglu and Kumar (2003, 2006) argue that progressive taxation increases education through a liquidity effect. Finally, Chou and Talmain (1996) examine an R&D model in which redistribution can be both growth-enhancing and Pareto-improving. Their mechanism relies on the elasticity of individual labor supplies. The wealth effect from redistribution may raise the aggregate labor supply and hence the growth rate. With infinitely lived agents, faster growth offsets the static loss imposed by redistribution on rich individuals, and a Pareto-improvement is possible.

⁵However, some authors have argued that remaining unskilled actually entails a greater risk due to the greater probability of being unemployed, and that education is precisely a way to reduce this risk; see Gould, Moav, and Weinberg (2001).

Our analysis differs from these approaches in that we focus on occupational choice.

The paper is organised as follows. The next section presents the model. Section 3 describes the equilibrium and the laissez-faire solution. Section 4 constructs the social welfare function and derives the first-best allocation. Section 5 analyzes the impacts of redistribution on innovation, on the utilities of skilled and unskilled agents, and on inequality and social welfare. The model is then extended to allow for endogenous skill acquisition. Section 6 provides numerical simulations of the model. These show the effect of the optimal tax rate on growth, welfare, and the Gini coefficient, as well as the relationship between the size of the intertemporal spillover and the optimal tax rate. We also compare the simulations of three alternative policies for promoting research. Section 7 concludes.

2 The model

2.1 Population

The population consists of non-overlapping generations, each living for one period. Each generation is of size $N \in \mathbb{N}$, and consists of $L \geq 1$ unskilled and $H \geq 2$ skilled workers. Unskilled workers are employed in the production of the final good. Skilled workers make an occupational choice at the beginning of their lives, choosing between being an entrepreneur or a manufacturing worker in the intermediate-good sector.

Researchers (also called ‘entrepreneurs’) undertake R&D in order to invent a higher quality intermediate good and obtain a patent for it. The number of entrepreneurs in period t is denoted R_t . The remaining $M_t = H - R_t$ skilled workers are hired for fixed wages to produce the intermediate good.

All individuals are risk-averse and have identical utility functions, which depend only on consumption, C . The utility of someone born in period t is assumed to be given by

$$U(C_t) = (C_t)^\alpha, \quad 0 < \alpha < 1. \quad (1)$$

2.2 Production technologies

We consider a small open economy that produces a single homogeneous final good and a single intermediate good. The final good is produced by a competitive sector according to the production function

$$Y_t = A_t x_t^\theta L^{1-\theta} \quad (2)$$

where $0 < \theta < 1$, A_t is the index of total factor productivity, which depends on the ‘quality’ or vintage of the intermediate good used, and x_t is the amount of intermediate good employed. The price of the final good is the numeraire.

Each innovation increases the value of A_t by a factor $\gamma > 1$, with $A_{t+1} = \gamma A_t$ if an innovation occurs in t and $A_{t+1} = A_t$ if no innovation occurs. We assume that innovations are drastic in the presentation of the model. The case of non-drastic innovations yields the same qualitative results.

There is a large number of risk-neutral firms willing to produce intermediate goods. After an innovation occurs, one of these firms purchases the patent for the new intermediate good and becomes a monopolist. The intermediate good is produced using skilled labor alone according to the linear

technology

$$x_t = M_t, \tag{3}$$

and is not traded.

2.3 Research

R_t individuals engage in R&D in order to invent the ‘next’ vintage of the intermediate good and receive a patent for it. Each researcher has a probability λ of making the discovery. If multiple researchers make the discovery the patent is randomly assigned to one of them. Supposing that R&D activities are mutually-exclusive, then duplication of research is avoided and the aggregate probability of an innovation in period t is $\Pr(\text{innovation}) = \lambda R_t$.⁶ We assume that $\lambda < 1/H$, which implies that even if all the skilled work as researchers the probability of innovation is less than one.

A patent is infinitely lived and enters into operation in the period following the discovery.⁷ That is, an intermediate good invented at t will start generating profits at $t + 1$. The researcher who has been awarded the patent can sell it to a risk-neutral intermediate-good firm which will pay at time t

⁶Note that researchers in each period have a private incentive to avoid duplication. If, however, duplication of R&D activities cannot be avoided, then each potential innovator must take into account that there could be ‘ties’ with any number $i \leq R_t - 1$ of other innovators, whereby the chance the particular innovator obtains the patent is $1/(1 + i)$. Then the unconditional probability of a given entrepreneur obtaining the patent is

$$\lambda(R_t) \equiv \lambda \sum_{i=0}^{R_t-1} \left(\frac{(R_t - 1)!}{i!(R_t - 1 - i)!} \frac{\lambda^i (1 - \lambda)^{R_t - 1 - i}}{1 + i} \right).$$

Note that $\lambda'(R_t) < 0$ and that $\lambda(R_t) \approx \lambda$ when λ is small. Using $\lambda(R_t)$ in place of a fixed λ in the occupational choice equation does not change qualitatively our main results. A fixed λ can be viewed as an approximation to the case of $\lambda(R_t)$ for small λ and has the benefit of permitting closed form solutions.

⁷For an analysis of the impact of patent-design on growth see O’Donoghue and Zweimüller (2004).

the expected value of the innovation, denoted V_{t+1} . We suppose that none of these risk-neutral firms engage directly in research, as discussed in the introduction.

2.4 Profits, wages, and the value of a patent

Since the final-good sector is competitive, all factors are paid the marginal product. Let p_t denote the price of the intermediate good, and w_t the wage of skilled labor used to produce the intermediate good. Differentiating (2) to obtain the inverse demand function for intermediate goods, we can write the monopolist's problem as

$$\max_{x_t} \Pi = p_t x_t - w_t x_t \quad (4)$$

$$\text{subject to } p_t = A_t \theta x_t^{\theta-1} L^{1-\theta}. \quad (5)$$

The profit maximizing production of x_t is therefore, $x_t = (A_t \theta^2 / w_t)^{\frac{1}{1-\theta}} L$ and the price $p_t = w_t / \theta$. This expression, together with the market-clearing condition for skilled workers, $x_t = M_t$, gives the skilled wage w_t , the unskilled wage v_t , and the monopolist's profit Π_t as

$$w_t = \theta^2 \frac{Y_t}{M_t} \quad (6)$$

$$v_t = (1 - \theta) \frac{Y_t}{L} \quad (7)$$

$$\Pi_t = (1 - \theta) \theta Y_t. \quad (8)$$

We shall impose the condition, $L/H > (1 - \theta)/\theta^2$, which ensures that skilled wages exceed unskilled wages for any level of employment.

The value of an innovation to the intermediate-good firm is then determined by the familiar asset condition $rV_{t+1} = \Pi_{t+1} - \lambda R_{t+1} V_{t+1}$, where r is

the (exogenously given) interest rate. This implies

$$V_{t+1} = \frac{\Pi_{t+1}}{r + \lambda R_{t+1}}, \quad (9)$$

indicating that the value of an innovation to a risk-neutral firm is equal to the stream of profits generated by the innovation, discounted by the interest rate plus the probability of being replaced if a new vintage is invented next period.⁸ Note that $w_{t+1} = w_t$, $v_{t+1} = v_t$, and $\Pi_{t+1} = \Pi_t$ if an innovation does not occur at t . If there is an innovation, wages and profits increase by a factor of γ , that is $w_{t+1} = \gamma w_t$, etc.

2.5 Taxation

We consider a linear tax system having the general form $T(I) = -B + \tau I$, where I is individual income, τ the tax rate, and B a demogrant. An individual pays taxes on her income and receives a transfer B . A researcher who obtains a patent at t and sells it for V_{t+1} faces a tax bill of τV_{t+1} . We suppose that instead of the researcher paying the entire amount in the period in which she sells the innovation, the intermediate-good firm pays the researcher the *net* value of the innovation $(1 - \tau)V_{t+1}$ and then pays the taxes due as a proportion of its profits each period. Assuming that the government sets a constant tax rate and holds a balanced budget in each

⁸An alternative way to model the intermediate goods sector is to assume that patents last only one period and after that the goods can be produced by a large number of competitive firms. This type of assumption is used by Cooper et al. (2001) and Lambson and Phillips (2007). It implies that, depending on whether or not there has been an innovation, the intermediate goods sector will produce under monopoly or perfect competition. We studied this case in an earlier version of the paper and found equivalent results (García-Peñalosa and Wen, 2004).

period, its budget constraint is

$$NB_t = \tau (\Pi_t + w_t M_t + v_t L) = \tau Y_t. \quad (10)$$

Our assumption on the timing of the tax payment is made in order to avoid having the transfer B_t vary depending on whether or not an innovation has occurred. If all the taxes were paid at t , then tax revenue would be $\tau (V_{t+1} + w_t M_t + v_t L)$ in the periods in which there is an innovation and $\tau (w_t M_t + v_t L)$ if there is none. Our assumption avoids the resulting variation in B_t . Note that from the researcher's point of view the net value of the innovation is always $(1 - \tau)V_{t+1}$ irrespective of when the tax is paid.

3 Equilibrium

3.1 Occupational choice

An unskilled worker faces no occupational choice and obtains utility $U_{ut} = ((1 - \tau)v_t + B_t)^\alpha$, which can be written as

$$U_{ut} = Y_t^\alpha \left((1 - \tau) \frac{(1 - \theta)}{L} + \frac{\tau}{N} \right)^\alpha. \quad (11)$$

Skilled workers make an occupational choice between working in manufacturing (m) or entrepreneurship (e). The income of an unsuccessful entrepreneur is simply the transfer B_t . The expected utility of being an entrepreneur is then

$$U_{et} = \lambda (C_{\pi t})^\alpha + (1 - \lambda) (B_t)^\alpha, \quad (12)$$

where $C_{\pi t} \equiv (1 - \tau)V_{t+1} + B_t$ is the consumption of a period- t patent winner. Using the fact that, since there was an innovation at t , then $Y_{t+1} = \gamma Y_t$, we

can express $C_{\pi t}$ as

$$C_{\pi t} = \left((1 - \tau) \frac{\theta(1 - \theta)\gamma Y_t}{r + \lambda R_{t+1}} + \frac{\tau Y_t}{N} \right). \quad (13)$$

The utility of an entrepreneur is decreasing in R_{t+1} because more future research shortens the expected life of an innovation and hence lowers the value of a patent.

Skilled manufacturing workers obtain utility $U_{mt} = ((1 - \tau)w_t + B_t)^\alpha$, which can be expressed as

$$U_{mt} = Y_t^\alpha \left((1 - \tau) \frac{\theta^2}{M_t} + \frac{\tau}{N} \right)^\alpha. \quad (14)$$

The utility of a skilled worker is increasing in the number of entrepreneurs, because a reduction in manufacturing employment raises the skilled wage. Let $\theta^2 N > H$ and $\gamma\theta(1 - \theta)N > r + \lambda H$ to ensure, respectively, that skilled workers and the successful entrepreneurs are net fiscal contributors.

Arbitrage determines the equilibrium allocation of skilled individuals across occupations, which is given by the equal-utilities condition, $U_{mt} = U_{et}$, that is⁹

$$\left((1 - \tau) \frac{\theta^2}{H - R_t} + \frac{\tau}{N} \right)^\alpha = \lambda \left((1 - \tau) \frac{\theta(1 - \theta)\gamma}{r + \lambda R_{t+1}} + \frac{\tau}{N} \right)^\alpha + (1 - \lambda) \left(\frac{\tau}{N} \right)^\alpha. \quad (15)$$

Note from (15) that the occupational choice at period t depends on expectations about future research, as this determines the expected lifetime

⁹Since R_t must be an integer, the equal-utilities condition may not hold exactly. A more precise, but awkward, statement of the equilibrium value of R_t is that, $U_m(R_t) \geq U_e(R_{t+1})$ and $U_m(R_t + 1) < U_e(R_{t+1})$.

of a patent and hence its value, i.e. $R_t = R(R_{t+1})$. The stationary perfect foresight equilibrium is given by $R^* = R_t = R_{t+1}$ for all t . Since the equilibrium condition defined by (15) is forward-looking, and assuming that the stationary equilibrium is stable, the economy jumps to R^* .¹⁰ The steady state equal-utilities condition determining occupational choices can then be written as

$$\left(\frac{(1-\tau)\theta^2}{H-R^*} + \frac{\tau}{N}\right)^\alpha = \lambda \left((1-\tau)\frac{\theta(1-\theta)\gamma}{r+\lambda R^*} + \frac{\tau}{N} \right)^\alpha + (1-\lambda) \left(\frac{\tau}{N}\right)^\alpha \quad (16)$$

which defines the equilibrium level of research, R^* , as a function of the tax rate and model parameters. Since the left-hand side of (16) is strictly increasing in R and right-hand side is strictly decreasing, there is a unique solution to the arbitrage condition. Note that if agents were risk-neutral (i.e. $\alpha = 1$), redistribution would have no impact on occupational choice, and hence would not affect the growth rate.

3.2 Laissez-faire equilibrium

We start by considering laissez-faire. In this case, entrepreneurs who fail receive no income. The equilibrium the number of researchers is given by

$$R^* = \frac{\lambda^{(1-\alpha)/\alpha}\gamma(1/\theta-1)H - r/\lambda}{1 + \lambda^{(1-\alpha)/\alpha}\gamma(1/\theta-1)}. \quad (17)$$

The comparative statics are easily established.

Lemma 1 *The laissez-faire equilibrium value of R^* is increasing in H , λ , γ , and α , and decreasing in r and θ .*

¹⁰The stability condition is simply $|dR_t/dR_{t+1}| < 1$, which for $\tau = 0$ is equivalent to $\lambda^{(\alpha-1)/\alpha} < \gamma(1/\theta-1)$. When there is taxation, no explicit condition for $|dR_t/dR_{t+1}| < 1$ can be obtained.

These are analogous to results in Aghion and Howitt (1992), except for the effect of risk-aversion. As expected, greater risk-aversion (lower α) decreases entrepreneurship.

3.3 Innovation and long-run growth

The change in output between periods t and $t + 1$ can be written as

$$\ln \left(\frac{Y_{t+1}}{Y_t} \right) = \ln \left(\frac{A_{t+1}}{A_t} \right). \quad (18)$$

Because innovations are stochastic, so will be the realized growth rate. We define the growth rate g_t as the expected rate of technological change, $g_t = E(\ln A_{t+1} - \ln A_t)$. In steady state, there is a constant probability $q \equiv \lambda R$ that an innovation occurs, and the expected long-run growth rate is then

$$g = \lambda R \ln \gamma. \quad (19)$$

The long-run probability of innovation, and hence the growth rate, is increasing in the number of researchers.

4 Social Welfare

4.1 Welfare Function

We consider a utilitarian welfare function, where the social planner weights the utilities of the various types of individuals by their population share. Note that there are two possible states of the world, depending on whether or not an innovation has occurred at time t . We suppose that the planner weights these two states according to the probability of being in one or the

other, that is λR and $(1 - \lambda R)$. Social welfare at period t , denoted W_t , is then given by

$$W_t = LU_{ut} + (H - R)U_{mt} + \lambda R(C_{\pi t}^\alpha + (R - 1)B_t^\alpha) + (1 - \lambda R)RB_t^\alpha, \quad (20)$$

which, using the arbitrage condition and letting $u_m \equiv U_{mt}/Y_t^\alpha$ and $u_u \equiv U_{ut}/Y_t^\alpha$ denote the output-adjusted utilities of skilled and unskilled workers, implies

$$W_t = (A_t M^\theta L^{1-\theta})^\alpha (Lu_u + Hu_m). \quad (21)$$

All the terms in this expression are time-invariant except the technology index A_t , which increases by a factor γ with each innovation. Letting δ denote the social discount factor, the discounted value of expected social welfare over an infinite horizon can then be written as

$$W = \sum_{t=0}^{\infty} \delta^t \sum_{s=0}^t \left(\frac{t!}{s!(t-s)!} \right) q^s (1-q)^{t-s} (\gamma^\alpha)^s W_0, \quad (22)$$

where W_0 is welfare at time $t = 0$. Using the binomial theorem and the definition $q = \lambda R$, this becomes

$$W = \frac{(A_0(H - R)^\theta L^{1-\theta})^\alpha}{1 - \delta [1 + (\gamma^\alpha - 1)\lambda R]} [Lu_u + Hu_m]. \quad (23)$$

This expression has an intuitive interpretation. The numerator of the first term in (23) is simply initial output, which is then discounted at the social discount rate modified to take into account the expected rate of innovation. The number of researchers has a negative effect on welfare via the level of output and a positive effect because it accelerates innovation. The second term in (23) captures pure redistributive considerations. Since the utility

function is concave in consumption, a more equal distribution of post-tax income will result in a higher value of the term $[Lu_u + Hu_m]$. Taxation has a direct redistributive effect, as well as an indirect effect through the changes in the equilibrium skilled and unskilled wages.

4.2 First-best

It is useful to consider the benchmark case of the first-best allocation of labor. Suppose the planner can allocate workers to sectors, and use lump-sum taxation in order to redistribute within a generation. Then the planner's problem is simply to choose the allocation of skilled workers that maximizes the expected discounted flow of output, that is,

$$W^{FB} = \frac{(A_0(H - R)^\theta L^{1-\theta})^\alpha}{1 - \delta[1 + (\gamma^\alpha - 1)\lambda R]} \times N^{1-\alpha}. \quad (24)$$

Maximizing (24) with respect to R implies that the first-best allocation of skilled labor is given by

$$R^{FB} = \frac{(\gamma^\alpha - 1)H - \alpha\theta(1/\delta - 1)/\lambda}{(1 - \alpha\theta)(\gamma^\alpha - 1)}. \quad (25)$$

The differences between first-best research, R^{FB} , and laissez-faire research, R^* as given by (17), reflect the three margins discussed by Aghion and Howitt (1992): the intertemporal spillover effect, the appropriability effect, and the business stealing effect. A new effect is due to risk-aversion.

One can see that the R^{FB} is likely to exceed R^* if δ is high (i.e. the planner cares a lot about dynamic gains), and if H is large or θ is small. If the number of skilled workers is large and/or the elasticity of output with respect to skilled manufacturing employment is small (θ small) the

static output loss occurring when skilled workers move into research is low, hence the planner would choose a higher growth rate. The effect of all other variables is ambiguous. In particular, less risk aversion (larger α) or a greater technological spillover (higher γ) increase both the laissez-faire and the first-best level of research.

5 The impact of redistribution

5.1 Redistribution and research

If we examine the right-hand side of (16) we can distinguish two effects of redistributive taxation on the utility of an entrepreneur. First, a higher marginal tax rate, holding constant the demogrant (represented by the term τ/N), reduces a successful entrepreneur's net income. This captures the traditional 'incentive' argument that taxes discourage entrepreneurship. Second, there is an insurance effect due to the transfer, which protects entrepreneurs to some extent against the risk of failure.

Proposition 1 *Redistributive taxation has the following effects on research:*

(i) *For an economy in the laissez-faire equilibrium, introducing a small amount of redistribution increases the number of researchers:*

$$\lim_{\tau \rightarrow 0^+} dR/d\tau = \infty;$$

(ii) *There exists a value $\bar{\lambda} > 0$ such that for all $\lambda \leq \bar{\lambda}$ the number of researchers is strictly increasing in the tax rate;*

(iii) *The equilibrium number of researchers when $\tau \rightarrow 1$ is given by*

$$\hat{R} = \frac{\gamma(1/\theta - 1)H - r/\lambda}{1 + \gamma(1/\theta - 1)},$$

which is greater than the laissez-faire number of researchers R^* .

Proof: See Appendix.

The first part of the proposition says that some amount of redistributive taxation can increase the number of researchers. This occurs because in the laissez-faire equilibrium unsuccessful entrepreneurs have no consumption.¹¹ Their marginal utility is then infinitely large, and the insurance effect always dominates any incentive effect. The second part says that a sufficiently low probability of success ensures that the insurance effect of redistribution always dominates the incentive effect, making the level of research a monotonically increasing function of the tax rate.¹² Lastly, providing full insurance results in a larger number of researchers than under laissez faire. In fact, \widehat{R} is also the laissez-faire number of entrepreneurs when individuals are risk neutral.

To better understand these results it is convenient to represent (16) graphically, as in figure 1. The curve labelled u_m is the left-hand side of (16), which is the utility of skilled workers divided by Y_t^α . Clearly u_m decreases with τ since the skilled wage is above average income, implying that the introduction of a tax shifts u_m downwards. The curve labelled u_e is the right-hand side of (16), i.e. the utility of entrepreneurs divided by Y_t^α . The schedule u_e may shift upwards or downwards with τ depending on whether the insurance effect or the incentive effect dominates. Whenever a higher value of τ shifts u_e upwards the equilibrium number of researchers increases to R' , as depicted. If u_e shifts downwards, that is, if the incentive

¹¹In reality, entrepreneurs may have other forms of wealth to consume in the event of failure, but this merely underscores the point that, in the absence of social insurance, potential entrepreneurs may delay their entry into business until they have funds to fall back on.

¹²Numerical simulations suggest that R is an increasing function of $\tau \in [0, 1)$, but we are unable to prove this as a general result.

effect were stronger, then the number of researchers would increase if u_e shifted down by less than u_m .

Figure 1 around here

5.2 Wages, utility and the distribution of income

Redistribution also affects the utility of the current generation through general equilibrium changes in the wage rates of skilled and unskilled agents. To see this write the utility of a skilled individual working in manufacturing as $U_{mt} = ((1 - \tau)w_t + \tau Y_t/N)^\alpha$. The direct impact of redistribution is to reduce her post-tax income for a given wage and hence would tend to reduce her utility; but there is also an indirect impact due to the fact that, if the tax increases the number of researchers, this will reduce the supply of skilled labor in manufacturing and hence increase her wage and utility. The opposite occurs for an unskilled individual, with $U_{ut} = ((1 - \tau)v_t + \tau Y_t/N)^\alpha$, as the direct impact of redistribution would tend to increase her utility given v_t , while the reduction in the number of skilled workers producing the intermediate good would tend to reduce the productivity of unskilled workers and hence their wage.

Similarly, taxation has both a direct and an indirect effect on income inequality. Income inequality in a given period depends on whether or not there has been an innovation in the period. If there has been one, an individual receives the value of the patent and inequality is greater than if there were no innovation. In the appendix we show that the expected value of the Gini coefficient of the incomes of a given generation can be expressed as

$$\begin{aligned}
Gini &= \frac{1-\tau}{2N} \left[\theta^2 L - (1-\theta)H + [2(1-\theta)(1-\lambda) + \theta^2(1-2\lambda)] R \right. \\
&\quad \left. + \theta(1-\theta) \frac{\gamma\lambda R(N-1)}{r+\lambda R} \right]. \tag{26}
\end{aligned}$$

Clearly, a larger tax rate tends to reduce inequality for given gross wages, but can increase it through its impact on R . On the one hand, a larger number of researchers would increase the wage of the skilled and reduce that of the unskilled, thus tending to raise inequality.¹³ On the other hand, it affects the expected income of a successful entrepreneur, which is captured by the last term in the square brackets. A higher value of R reduces the value of a patent (as it shortens its expected lifetime) but increases the probability that there is a successful entrepreneur. This second effect dominates and hence a higher R raises the Gini coefficient. The net effects of taxation on utilities and inequality can be derived for a tax rate close to zero.

Proposition 2 *For an economy in the laissez-faire equilibrium, introducing a small amount of redistribution ($\tau \rightarrow 0^+$)*

(i) *Increases (decreases) the wage and utility of skilled (unskilled) workers in the current period;*

(ii) *Increases income inequality as measured by the expected value of the Gini coefficient of income.*

Proof: See Appendix.

Proposition 2 implies that for tax rates close to zero, the general equilibrium effects dominate the direct redistributive impact of taxes and transfers.

¹³A similar effect is obtained by García-Peñalosa and Turnovsky (2007) who find, in a model where growth is driven by physical capital accumulation, that redistributive taxation has ambiguous effects on inequality due to the response of factor prices to taxes.

As a result, the initial impact of a small redistribution is to make the skilled better off and the unskilled worse off, and the expected value of the Gini coefficient rises, indicating greater inequality. The numerical examples show that, for large tax rates, inequality declines with the degree of redistribution. Moreover, there may be some levels of taxation where the combination of the direct and indirect effects of redistribution can increase the rate of innovation and raise the utility of *both* skilled and unskilled workers in the current period. An example of this is provided in Section 6.

5.3 Welfare effects of taxation

Using the equation (24) to substitute W^{FB} into (23), the welfare function can be written as

$$W(R, \tau) = W^{FB}(R) \times \frac{[Lu_u(R, \tau) + Hu_m(R, \tau)]}{N^{1-\alpha}}. \quad (27)$$

The expression for first-best welfare, $W^{FB}(R)$, captures efficiency considerations, while the second term is the weighted sum of the utilities of the skilled and unskilled and captures equity considerations. Differentiating the welfare function yields

$$\begin{aligned} \frac{dW(R, \tau)}{d\tau} \times (N^{1-\alpha}) &= [Lu_u + Hu_m] \frac{dW^{FB}}{dR} \frac{dR}{d\tau} \\ &+ W^{FB} \left[L \frac{\partial u_u}{\partial \tau} + H \frac{\partial u_m}{\partial \tau} \right] + W^{FB} H \frac{\partial u_m}{\partial R} \frac{dR}{d\tau}. \end{aligned} \quad (28)$$

There are three effects of the tax rate on welfare. The first term on the right-hand side of (28) captures efficiency considerations. More research has a positive growth effect, but a negative level effect, since it reduces the level

of current output. The net effect depends on the sign of dW^{FB}/dR which, in general, may be positive or negative, and is zero at R^{FB} . The second term embodies redistributive considerations, with a higher tax redistributing income from the skilled to the unskilled, for given wages. The last term is the general equilibrium effect operating through the share of income received by skilled workers, which increases as the number of researchers rises. This last effect can also be viewed as the impact of social insurance on the utility of the skilled.

In choosing the tax rate the social planner then needs to consider (i) the effect of τ on R , (ii) the static loss and dynamic gain of a higher R and (iii) the distributive impact of the tax for a given generation, adding to the standard static optimal taxation problem the fact that wages change with the tax. Analytical solutions for the optimal tax are impossible to derive, but we can establish the following.

Proposition 3 *If the first-best number of entrepreneurs exceeds the laissez-faire level, then introducing a small amount of redistribution ($\tau \rightarrow 0^+$) increases social welfare.*

Proof: See Appendix.

Small tax rates increase social welfare for two reasons. The first is the efficiency gain. Since we are supposing that the first-best level of research is greater than the laissez-faire, the increase in research increases the discounted flow of output. The second is the insurance effect provided by redistribution. This effect is positive because of the very high marginal utility of unsuccessful entrepreneurs who, under laissez-faire, have zero consumption.

5.4 Endogenous Human Capital

The model so far takes the stock of human capital, H , as given and constant. We can extend our analysis to allow for endogenous skill acquisition. Each individual is endowed with 1 unit of time. Assume that education requires spending a fraction e of time studying, where $e < 1$ is fixed. Only educated individuals are capable of becoming skilled. We suppose that the probability of becoming skilled depends on the level of skills of the individual's parent. Specifically, an individual with a skilled parent who invests in education become skilled with probability 1, while the offspring of an unskilled individual who studies becomes skilled with probability s , and remains unskilled with probability $1 - s$. Individuals who chose not to study become unskilled with certainty.

Let H_t denote the number of skilled agents in period t and $L_t = N - H_t$ denote the number of unskilled. Let L_t^e denote the number of offsprings of unskilled individuals from generation- t who choose to study. Supposing that all the offspring of the skilled invest in education (see below), the expected number of skilled agents at time $t+1$ is $H_{t+1} = H_t + sL_t^e$. Since the population is large, we suppose that this is also the actual number of skilled individuals. Hence, there is no uncertainty about the aggregate number of skilled agents.

Since individuals may now spend time not working, the stocks of skilled and unskilled labor are, respectively, $\widehat{H}_{t+1} = (1 - e)H_{t+1}$, and $\widehat{L}_{t+1} = N - H_{t+1} - e(1 - s)L_t^e$, which can also be expressed as $\widehat{L}_{t+1} = N - H_{t+1} - \widehat{s}(H_{t+1} - H_t)$, where $\widehat{s} \equiv e(1 - s)/s$. The probability of innovation also has to be redefined, with $\widehat{\lambda}$ being the probability that a researcher innovates when she spends $(1 - e)$ units of time doing R&D.

There are now three levels of utility: that obtained by a skilled individ-

ual, $U_{mt}(H_t, \tau)$, that obtained by an individual who invests in education but fails to become skilled and hence works only for a fraction of time $(1 - e)$, denoted $U_{ut}(H_t, \tau, 1 - e)$, and that obtained by an individual who does not invest in education, $U_{ut}(H_t, \tau, 1)$. They are given by

$$\begin{aligned} U_{mt}(H_t, \tau) &= Y_t^\alpha \left(\frac{(1 - \tau)\theta^2}{H_t - R_t} + \frac{\tau}{N} \right)^\alpha, \\ U_{ut}(H_t, \tau, 1) &= Y_t^\alpha \left(\frac{(1 - \tau)(1 - \theta)}{N - (1 + \widehat{s})H_t + \widehat{s}H_{t-1}} + \frac{\tau}{N} \right)^\alpha, \\ U_{ut}(H_t, \tau, 1 - e) &= Y_t^\alpha \left(\frac{(1 - \tau)(1 - \theta)(1 - e)}{N - (1 + \widehat{s})H_t + \widehat{s}H_{t-1}} + \frac{\tau}{N} \right)^\alpha, \end{aligned}$$

where $R_t = R_t(H_t, \tau, R_{t+1})$ is given by equation (15).

Consider now the education decision. The child of a skilled worker will invest in education if and only if $U_{mt}(H_t, \tau) > U_{ut}(H_t, \tau, 1)$, that is if

$$\frac{\theta^2}{H_t - R_t} > \frac{(1 - \theta)}{N - (1 + \widehat{s})H_t + \widehat{s}H_{t-1}}. \quad (29)$$

We suppose that at time zero this inequality holds.

For the offspring of an unskilled worker, there is uncertainty associated with education. She will invest if and only if

$$sU_{mt}(H_t, \tau) + (1 - s)U_{ut}(H_t, \tau, 1 - e) \geq U_{ut}(H_t, \tau, 1).$$

Thus the arbitrage condition determining the number of children of the unskilled investing in education (and hence the size of H) is

$$\begin{aligned} & s \left(\frac{(1 - \tau)\theta^2}{H_t^* - R_t(H_t^*, \tau, R_{t+1})} + \frac{\tau}{N} \right)^\alpha + (1 - s) \left(\frac{(1 - \tau)(1 - e)(1 - \theta)}{N - (1 + \widehat{s})H_t^* + \widehat{s}H_{t-1}} + \frac{\tau}{N} \right)^\alpha \\ &= \left(\frac{(1 - \tau)(1 - \theta)}{N - (1 + \widehat{s})H_t^* + \widehat{s}H_{t-1}} + \frac{\tau}{N} \right)^\alpha, \end{aligned} \quad (30)$$

where H_t^* is the equilibrium stock of human capital at time t . Equation (30), together with the arbitrage condition for the allocation of skilled workers between research and manufacturing, (16), determines the level of human capital and its allocation to the two activities.

It is possible to show that the dynamic equation (30) converges to a steady state, such that $H_t^* = H_{t-1}^* = H^*$ for all t , implying that there is a constant stock of human capital and that only the children of the skilled invest in education (see Appendix).¹⁴ In steady state, equations (29) and (30) become

$$\frac{\theta^2}{H^* - R(H^*, \tau)} > \frac{(1 - \theta)}{N - H^*}, \quad (31)$$

and

$$\begin{aligned} \left(\frac{(1 - \tau)(1 - \theta)}{N - H^*} + \frac{\tau}{N} \right)^\alpha &= s \left(\frac{(1 - \tau)\theta^2}{H^* - R(H^*, \tau)} + \frac{\tau}{N} \right)^\alpha \\ &+ (1 - s) \left(\frac{(1 - \tau)(1 - \theta)}{N - H^*} + \frac{\tau}{N} \right)^\alpha, \end{aligned} \quad (32)$$

where $R(H^*, \tau)$ is the steady state level of research, defined by equation (16). We can now establish the following.

Proposition 4 *For an economy in the laissez-faire equilibrium, introducing a small amount of redistribution ($\tau \rightarrow 0^+$)*

- (i) *Increases the stock of human capital;*
- (ii) *Increases the number of researchers, and hence growth.*

Proof: See Appendix.

¹⁴It is straightforward to show that if the children of the skilled who study become educated with a probability s' , such that $s < s' < 1$, all our results hold, although H converges to a steady state in which some of the offspring of the unskilled study.

Proposition 4 implies that income redistribution, by reducing the risk associated with education, increases the stock of human capital, as in Zeira (1988). In our model, this results in a stronger effect of redistribution on growth than when the stock of human capital is given. As well as the direct effect of taxation on entrepreneurial incomes, redistribution now also has an indirect effect on R through H^* . Taxation increases the stock of human capital and hence reduces the marginal product of skilled labor in production, resulting in a flow of skilled workers into research.

6 Optimal Redistribution

6.1 Benchmarks

We illustrate the implications of the model for growth and inequality with some numerical examples (with H fixed). It is important to emphasize that the model is highly stylized and abstracts from capital accumulation and labor supply effects. The latter is particularly important, as it implies unrealistically large tax rates. Scale effects are also present. Nevertheless, these numerical examples illustrate the various mechanisms in operation.

Table 1 gives the baseline parameter values. We assume that the social and private discount factors are identical, that is, $\delta = (1 + r)^{-1}$. The results are presented in table 2. The first column reports the tax rate, followed by the equilibrium number of researchers and the rate of growth. The tax rate and the growth rate are given in percentage points, and the latter has been annualized under the assumption that each time period in the model corresponds to 10 years, about the lifetime of a research project. The headings U_u and U_m denote, respectively, the utilities of unskilled and skilled workers of the first generation. The utility of the unskilled in the

laissez-faire equilibrium is normalized to 1, and all other utilities are given relative to this one. The term ΔW is the percentage change in social welfare relative to the laissez-faire value, and $Gini(in)$ denotes the Gini coefficient of income in the periods in which an innovation occurs (i.e., the more unequal state of the world; see Appendix).

Tables 1 and 2 around here

Row (1) reports the laissez-faire equilibrium. The economy exhibits relatively low research activity and an annual growth rate of 0.42%. The expected utility of skilled workers is 12% higher than that of unskilled workers and the Gini coefficient of about 12% indicates a moderate degree of income inequality. Row (2) gives the first-best solution. In this case, about 4% of the labor force is engaged in research and the growth rate is 1.23%. Welfare is almost 23% higher than under laissez-faire and income is identical for all workers, implying no inequality. These two benchmarks are to be compared with the effects of linear income taxation in rows three to five, and with three alternative policies in rows six to eight. We discuss these in turn.

6.2 Linear Income Tax

Rows (3) to (5) consider the effect of the linear income tax we have examined in the model. Row (3) provides an example of a Pareto-improving income tax. A tax rate of 20% raises the growth rate and increases the utility of both the skilled and the unskilled of the first generation. Recall that this is due to the fact that the increase in R induced by taxation reduces skilled employment and raises the skilled wage, thus offsetting the direct impact

of redistribution on skilled workers. Row (4) gives the optimal tax rate. The very high optimal tax rate arises from the planner's desire to encourage growth; as well as a preference for income equality and because uncertainty generates a substantial utility loss that the social planner wants to neutralize by providing insurance; see Eaton and Rosen (1980). The optimal tax rate would, of course, be lower if we allowed for an elastic labor supply or if the discount factor were smaller. At the optimal tax rate, the initial generation of skilled workers is worse off than under *laissez-faire*, while unskilled workers are better off. Research activity is nearly five times the *laissez-faire* level and is only slightly higher than the first-best level of research. The increase in welfare is also close to that obtained in the first-best equilibrium.

Despite the planner's concern for redistribution, the optimal tax rate is not 100%. The reason for this is that such a tax induces *too much research*, lowering current output and increasing the rate at which vintages are replaced by the next quality. To illustrate these losses, row (5) reports the effect of a tax rate of 100%. Although the rate difference with the optimal tax is small, there is a considerable increase in the number of researchers as we move from 99.8% to 100%. In this case, the growth rate is maximized but welfare is lower than with the optimal tax.

6.3 Alternative R&D policies

There are three policies for promoting R&D that may be contrasted with the linear income tax that has been the focus of our paper. These are policies that have been proposed in the literature on R&D-driven growth in order to encourage research but which have no equity aspects. The first consists of a subsidy to researchers that is financed through a proportional income tax

on the entire population. In this case, the subsidy received by researchers is $B = \tau Y/R$ and the arbitrage equation becomes¹⁵

$$\left(\frac{(1-\tau)\theta^2}{H-R}\right)^\alpha = \lambda \left((1-\tau) \frac{\theta(1-\theta)\gamma}{r+\lambda R} + \frac{\tau}{R} \right)^\alpha + (1-\lambda) \left(\frac{\tau}{R}\right)^\alpha. \quad (33)$$

Under this policy the level of taxation required to finance a given demogrant is lower because the subsidy specifically targets entrepreneurs. This case is illustrated in row (6) of table 2, which reports the tax rate required to maximize social welfare when only researchers are subsidized. The resulting tax rate is only 4.1%. The number of researchers exceeds substantially the first-best and social welfare is much lower than with the optimal income tax. The reason why the number of researchers is large is the combination of two effects. First, since the subsidy is targeted, even a small tax rate provides a large transfer and hence a substantial degree of insurance to unsuccessful entrepreneurs. Second, because the tax rate is small, the net income of a successful entrepreneur, $(1-\tau)V_t$, is large. The result is a larger shift of the U_e function than with the linear income tax of row (4), and hence a greater value of R . As there is no redistribution towards the unskilled, income inequality is only slightly less than under *laissez-faire*.

The second policy consists of providing full insurance to entrepreneurs. We suppose that when an innovation occurs the government appropriates the value of the patent, V_t , and that it pays a transfer to researchers. With a transfer of λV_t to each entrepreneur, the government maintains, on average, a balanced budget. Row (7) reports the effect of this policy. It results in the same allocation of researchers as the growth-maximizing tax rate

¹⁵We consider subsidies as an alternative to redistribution. Mayshar (1977) examines the simultaneous impact of redistributive taxation and government subsidies in a static economy with risky investments.

but generates substantially more inequality. The reason for this is that the unskilled do not benefit from redistribution. In fact, they experience a substantial utility loss (due to the lower wage) while the utility of the skilled rises considerably. Consequently, social welfare is lower and inequality higher than with the optimal income tax.

The third policy we examine consists of all researchers at a particular point in time sharing the value of the innovation, no matter who discovers it. This setup resembles a ‘research joint venture’.¹⁶ Under this scenario there is only partial insurance, as researchers receive no income when an innovation does not occur. Row (8) reports the results in this case. The number of researchers is only slightly higher than with the optimal income tax, but inequality is worse and welfare is lower as, once again, this policy does not compensate unskilled workers for their loss of income due to the general equilibrium effect on unskilled wages.

These numerical examples of alternative policies for promoting research identify some of the efficiency and equity drawbacks of alternative R&D policies. The optimal linear income tax does remarkably well at inducing a level of research that is close to the first-best while reducing income inequality.

6.4 Intertemporal spillovers and optimal tax rates

Our framework allows us to address the question of how the presence of intertemporal knowledge spillovers affects the optimal tax. On the one hand, stronger intertemporal spillovers, that is higher values of γ , tend to increase the optimal tax rate because a higher return to research makes the planner more willing to forgo current output in order to accelerate growth. On the

¹⁶ Although cooperation among researchers can be undertaken privately, it is often initiated by governments. It is for this reason that we consider it a potential policy.

other, a greater spillover increases the value of an innovation and hence the income of a successful entrepreneur, thus raising income inequality. As a result, we would expect the optimal tax rate to be increasing in γ . However, a higher γ also increases the competitive level of research through various channels, including the business stealing effect that tends to make competitive growth too high. Consequently, it is unclear how γ will affect the optimal tax rate. It is not possible to determine analytically the effect of spillovers on the optimal tax rate, hence we have resorted to numerical simulations. Figure 2 illustrates that, for our baseline parameters, the optimal tax rate is an increasing function of γ .

Figure 2 around here

7 Conclusion

We have examined a neglected implication of the Schumpeterian growth model: namely, that redistribution can increase the incentives for individuals to undertake R&D. If agents are risk-averse, the social insurance effect may be strong enough to offset the standard disincentive effects of taxation, resulting in faster growth. Furthermore, these effects of redistributive taxation can complement the reduction in the inequality between the incomes of skilled and unskilled workers achieved at the optimal tax rate. As the size of the intertemporal spillover increases the optimal tax rate rises. In this way, we have added intertemporal considerations to the more standard optimal tax problem with occupational choice and differential abilities.

A large part of the literature on taxation and growth focuses on infinitely-lived agents. In this case, policies that increase the growth rate tend to also increase the welfare of all individuals, since the dynamic effect of faster growth offsets static losses. Our approach, in contrast, has examined the utility gains and losses of short-lived agents, and shown that even in this case Pareto-improvements are possible. The reason for this is that redistribution affects occupational choices, increasing the number of researchers and reducing skilled employment in manufacturing, which in turn raises the (pre-tax) manufacturing wage. As a result, it is possible that the after-tax income of skilled workers increases despite them being net fiscal contributors. All members of the current and future generations may thus have a higher welfare than in the absence of redistributive taxation.

We have focused on the impact of a linear progressive income tax. However, an important and well-known implication of the type of model we analyze is that R&D subsidies can affect the growth rate. In our simulations, R&D subsidies improved social welfare by less than the linear income tax. More importantly, such subsidies have been the object of substantial criticism from economists, because of the scope for diversion of expenditures and manipulation to which they are subject (Katz and Ordovery, 1990). This manipulation can take place at the firm level, or even at the level of the government which can use them in order to engage in tax competition with other countries. By focusing on redistributive taxation, we are not arguing that R&D subsidies should not be used, but rather we have stressed the value of redistribution to foster risk-taking, when R&D subsidies are deemed problematic and when equity is a policy objective.

Two caveats are in order. First, we have focused on a small open economy but have not allowed for imports of technology. Clearly, the possibility

of such imports would limit the effect of fiscal policy on growth, although the mechanism we have described would nevertheless be important if the process of adoption and implementation of foreign technologies were costly and uncertain. Second, an important limitation of our model is the fact that the labor supply is inelastic. As is well known from the literature on static optimal taxation, allowing for an endogenous labor supply would considerably reduce the optimal tax rate. Such considerations in the context of a growth model are avenues for further research.

8 Appendix

This appendix derives a number of the results and propositions in the text.

8.1 The Gini Coefficient

When the population is divided into G groups the Gini coefficient is given by

$$Gini = \frac{N}{2Y} \sum_{i=1}^G \sum_{j=1}^G |Y_i - Y_j| n_i n_j$$

where Y is aggregate income, Y_i and Y_j the post-tax incomes of individuals in group i, j and n_i and n_j the proportion of the populations in each group. When there is an innovation there are four types of agents and the Gini coefficient is given by

$$Gini(in) = \frac{1-\tau}{2N} \left[\theta^2 L - (1-\theta)M + \theta(1-\theta) \left(\frac{\gamma(N-1)}{r+\lambda R} - (R-2) \right) + R - 2 \right].$$

When there is no innovation there are only three types of agents, as nobody receives monopoly profits, and hence

$$Gini(no) = \frac{1-\tau}{2N} [\theta^2(L+R) - (1-\theta)(M-R)].$$

The probability-weighted Gini coefficient is defined as

$$Gini = \lambda R \times Gini(in) + (1-\lambda R) \times Gini(no).$$

This is the expected value of the Gini coefficient and can be expressed as equation (26).

8.2 Proofs of propositions

Proof of Proposition 1. Suppose an interior solution to the arbitrage equation exists.

(i) Evaluation of $dR/d\tau$ at $\tau = 0$.

Write the steady state equal-utilities condition (16) as

$$u_m(R, \tau) = u_e(R, \tau) \tag{A.1}$$

where

$$\begin{aligned} u_m(R, \tau) &\equiv \left(\theta^2 \frac{1-\tau}{H-R} + \frac{\tau}{N} \right)^\alpha \quad \text{and} \\ u_e(R, \tau) &\equiv \lambda \left(\frac{\gamma\theta(1-\theta)}{r+\lambda R} (1-\tau) + \frac{\tau}{N} \right)^\alpha + (1-\lambda) \left(\frac{\tau}{N} \right)^\alpha. \end{aligned}$$

are the output-adjusted utilities. Totally differentiating (A.1) yields¹⁷

$$\frac{dR}{d\tau} = \frac{\partial u_e / \partial \tau - \partial u_m / \partial \tau}{\partial u_m / \partial R - \partial u_e / \partial R}. \tag{A.2}$$

Setting $\tau = 0$ it is easy to show that the denominator of (A.2) is positive and finite, $\partial u_m / \partial \tau$ is finite, and $\partial u_e / \partial \tau$ tends to infinity as τ approaches zero. Thus $dR/d\tau$ approaches $+\infty$ as $\tau \rightarrow 0$ from above.

(ii) Sign of $dR/d\tau$ for $\lambda < \bar{\lambda}$.

The sign of $dR/d\tau$ is given by (A.2). The denominator is positive and, under the assumption that skilled workers are net fiscal contributors, $\theta^2 N > H$, we have $\partial u_m / \partial \tau < 0$. Then a sufficient condition for $dR/d\tau > 0$ is

¹⁷The derivatives should be interpreted as so-called q-derivatives, which apply to interger-valued variables.

$\partial u_e / \partial \tau > 0$. Differentiating we obtain

$$\frac{\partial u_e}{\partial \tau} = \alpha \lambda \left(\frac{\gamma \theta (1 - \theta)}{r + \lambda R} (1 - \tau) + \frac{\tau}{N} \right)^{\alpha - 1} \left(\frac{1}{N} - \frac{\gamma \theta (1 - \theta)}{r + \lambda R} \right) + \alpha (1 - \lambda) \frac{\tau^{\alpha - 1}}{N^\alpha}. \quad (\text{A.3})$$

A successful innovator is a net fiscal contributor if

$$\frac{\gamma \theta (1 - \theta)}{r + \lambda R} > \frac{1}{N}$$

which implies that the two terms in (A.3) have opposite signs.

Now consider the second derivative of u_e ,

$$\frac{\partial^2 u_e}{\partial \tau^2} = -\alpha (1 - \alpha) \left[\lambda \left(\frac{\gamma \theta (1 - \theta)}{r + \lambda R} (1 - \tau) + \frac{\tau}{N} \right)^{\alpha - 2} \left(\frac{1}{N} - \frac{\gamma \theta (1 - \theta)}{r + \lambda R} \right)^2 + (1 - \lambda) \frac{\tau^{\alpha - 2}}{N^\alpha} \right] < 0,$$

implying that $\partial u_e / \partial \tau$ is decreasing in τ . Note also that

$$\frac{\partial u_e}{\partial \tau} \Big|_{\tau=0} = \infty \quad \text{and} \quad \frac{\partial u_e}{\partial \tau} \Big|_{\tau=1} = \frac{\alpha}{N^\alpha} \left[1 - \lambda N \frac{\gamma \theta (1 - \theta)}{r + \lambda R} \right].$$

Using the value of $\lim_{\tau \rightarrow 1} R$ obtained below, we have

$$\frac{\partial u_e}{\partial \tau} \Big|_{\tau=1} = \frac{\alpha}{N^\alpha} \left[1 - (\theta + \gamma(1 - \theta)) \frac{\theta N}{r/\lambda + H} \right].$$

This derivative is decreasing in λ . Let $\bar{\lambda}$ be the critical value for which

$\frac{\partial u_e}{\partial \tau} \Big|_{\tau=1} = 0$, defined by

$$\bar{\lambda} \equiv \frac{r}{\theta (\theta + \gamma(1 - \theta)) N - H}.$$

For $\lambda \leq \bar{\lambda}$, we have $\partial u_e / \partial \tau \geq 0$ at $\tau = 1$, implying $\partial u_e / \partial \tau \geq 0$ and hence $dR/d\tau > 0$ for all τ . For $\lambda > \bar{\lambda}$, we have $\partial u_e / \partial \tau < 0$ at $\tau = 1$,

implying that $u_e(\tau)$ is first increasing and then decreasing in the tax rate.

Then, there exists a value $\bar{\tau}$ defined by

$$\left. \frac{\partial u_e(R(\tau), \tau)}{\partial \tau} \right|_{\tau=\bar{\tau}} = 0$$

such that $dR/d\tau > 0$ for all $\tau < \bar{\tau}$, but $dR/d\tau$ could be negative for higher tax rates.

(iii) Value of R at $\tau = 1$.

Define $f(R, \tau) \equiv u_m(R, \tau) - (\frac{1}{N})^\alpha$ and $g(R, \tau) \equiv u_e(R, \tau) - (\frac{1}{N})^\alpha$ and then we can write the steady state equal-utilities condition (A.1) as

$$\frac{f(R, \tau)}{g(R, \tau)} = 1. \tag{A.4}$$

Substituting $\tau = 1$ into (A.4) gives the indeterminate expression $0/0$.

Applying L'Hopital's Rule, we have

$$\lim_{\tau \rightarrow 1} \frac{f(R, \tau)}{g(R, \tau)} = \lim_{\tau \rightarrow 1} \frac{df(R, \tau)/d\tau}{dg(R, \tau)/d\tau} = \frac{1/N - \theta^2/(H - R)}{1/N - \lambda\gamma\theta(1 - \theta)/(r + \lambda R)}.$$

Setting this expression equal to 1 yields

$$\lim_{\tau \rightarrow 1} R = \frac{\gamma(1 - \theta)H - \theta r/\lambda}{\theta + \gamma(1 - \theta)}.$$

■

Proof of Proposition 2. For the effect of an infinitesimal tax rate on equilibrium wages, differentiate (6) and (7) with respect to τ , using (3) to substitute for x in the production function. The signs of the wage derivatives then follow immediately from proposition 1(i). For the effect of the tax on the equilibrium utilities of skilled workers, differentiate (14) and rearrange

terms to obtain

$$\left. \frac{dU_m}{d\tau} \right|_{\tau=0} = \frac{\alpha Y^\alpha}{M^{1+\alpha}} \theta^{2\alpha} \left[\frac{dR}{d\tau} - M \left(1 - \frac{M}{\theta^2 N} \right) \right] = \infty$$

using proposition 1(i) for $dR/d\tau$. A similar calculation is applied to the utility of unskilled workers, equation (11), to show that $dU_u/d\tau = -\infty$ at $\tau = 0$ to complete the proof.

To examine the impact on inequality note that

$$\begin{aligned} \frac{dGini}{d\tau} &= \frac{\partial Gini}{\partial \tau} + \frac{\partial Gini}{\partial R} \frac{dR}{d\tau} \\ &= -\frac{Gini}{1-\tau} + \frac{1-\tau}{2N} \Omega \frac{dR}{d\tau} \end{aligned}$$

where

$$\Omega \equiv 2(1-\theta)(1-\lambda) + \theta^2(1-2\lambda) + \theta(1-\theta) \frac{\gamma(N-1)\lambda r}{(r+\lambda R)^2}.$$

To evaluate $dGini/d\tau$ at $\tau = 0$ recall from Proposition 1(i) that $dR/d\tau$ tends to ∞ , implying that the sign of $dGini/d\tau$ is given by the sign of Ω . Recall that we have assumed $H \geq 2$ and $\lambda H < 1$, which implies that $\lambda < 1/2$ and thus $\Omega > 0$. Therefore $dGini/d\tau = \infty$ at $\tau = 0$. ■

Proof of Proposition 3. Differentiating welfare we have

$$\frac{dW}{d\tau} = W^{FB} \left[L \frac{\partial u_u}{\partial \tau} + H \frac{\partial u_m}{\partial \tau} \right] + \left([Lu_u + Hu_m] \frac{dW^{FB}}{dR} + W^{FB} H \frac{\partial u_m}{\partial R} \right) \frac{dR}{d\tau}.$$

For an infinitesimal tax rate $\partial u_u/\partial \tau$ and $\partial u_m/\partial \tau$ are finite while $dR/d\tau = \infty$ (from proposition 1). Hence the sign of the derivative is given by the sign of the term in brackets that multiplies $dR/d\tau$. Note that $\partial u_m/\partial R > 0$.

Moreover, if $R^* < R^{FB}$ then $dW^{FB}/dR > 0$ since W^{FB} is strictly increasing and concave and attains its maximum at R^{FB} . Thus $dW/d\tau = \infty$ at $\tau = 0$. ■

Proof of Convergence of H_t . Differentiating the arbitrage equation (30) and evaluating it at $\tau = 0$, we get

$$\frac{dH_t^*}{dH_{t-1}} = \frac{\widehat{s}(a_3 - a_2)}{a_1 + (1 + \widehat{s})(a_3 - a_2)},$$

where

$$\begin{aligned} a_1 &\equiv s \left(\frac{\theta^2}{H_t^* - R_t(H_t^*, \tau, R_{t+1})} \right)^{\alpha-1} \frac{\theta^2}{H_t^* - R_t(H_t^*, \tau, R_{t+1})^2} \left(1 - \frac{\partial R_t}{\partial H_t} \right), \\ a_2 &\equiv (1-s) \left(\frac{(1-e)(1-\theta)}{N - (1+\widehat{s})H_t^* + \widehat{s}H_{t-1}} \right)^{\alpha-1} \frac{(1-e)(1-\theta)}{(N - (1+\widehat{s})H_t^* + \widehat{s}H_{t-1})^2}, \\ a_3 &\equiv \left(\frac{(1-\theta)}{N - (1+\widehat{s})H_t^* + \widehat{s}H_{t-1}} \right)^{\alpha-1} \frac{(1-\theta)}{(N - (1+\widehat{s})H_t^* + \widehat{s}H_{t-1})^2}. \end{aligned}$$

The terms a_1, a_2, a_3 are all positive. A sufficient condition for convergence is then $a_3 - a_2 > 0$. Using the expressions for a_2 and a_3 , this inequality is simply $1 > (1-s)(1-e)^\alpha$, which always holds, implying that the stock of human capital converges for small tax rates. ■

Proof of Proposition 4. Differentiating equation (32) and evaluating it at $\tau = 0$ yields

$$\frac{dH}{d\tau} = \frac{b_3 + b_1 \partial R / \partial \tau}{b_1 (1 - \partial R / \partial H) + b_2} \quad (\text{A.5})$$

where

$$\begin{aligned} b_1 &\equiv s \left(\frac{\theta^2}{H - R} \right)^\alpha \frac{1}{H - R} > 0 \\ b_2 &\equiv \left(\frac{1-\theta}{N-H} \right)^\alpha \frac{1}{N-H} (1 - (1-s)(1-e)^\alpha) > 0 \end{aligned}$$

and

$$b_3 \equiv s \left(\frac{\theta^2}{H-R} \right)^\alpha \left(\frac{H-R}{\theta^2 N} - 1 \right) \left(\frac{1-\theta}{N-H} \right)^\alpha \left(1 - (1-s)(1-e)^\alpha - \frac{N-H}{(1-\theta)N} \left(1 - \frac{(1-s)}{(1-e)^{1-\alpha}} \right) \right).$$

At the laissez-faire solution for R , we have

$$\frac{\partial R}{\partial H} = \frac{1}{1 + \left(\lambda^{(1-\alpha)/\alpha} \gamma (1/\theta - 1) \right)^{-1}} < 1,$$

implying that $(1 - \partial R / \partial H)$, and hence the denominator in (A.5), are positive and finite. From proposition 1, $dR/d\tau = +\infty$ at $\tau = 0$, implying that $dH/d\tau = +\infty$ at $\tau = 0$.

Lastly, since $\partial R / \partial H > 0$ and $dR/d\tau > 0$ at $\tau = 0$, then $dR(H(\tau), \tau)/d\tau > 0$ at $\tau = 0$. ■

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Table 1: Baseline Parameter Values

Production technology	$\theta = 0.25$	$\gamma = 1.8$
Research sector	$\lambda = 0.0005$	$r = 1.5$
Preferences	$\alpha = 0.9$	$\delta = 0.40$
Population	$H = 1,500$	$L = 18,500$

Table 2: Optimal Tax Rates and Pareto-Improvements

Type of tax	τ	R	g	U_u	U_m	ΔW	$Gini(in)$
(1) Laissez-faire	0	145	0.42	1	1.12	-	11.86
(2) First-best	-	686	1.23	0.99	0.99	22.67	0.00
(3) Pareto-improvement	20.0	215	0.61	1.03	1.17	4.82	9.48
(4) Optimal tax	99.8	711	1.91	1.07	1.07	22.63	0.02
(5) Growth-maximizing tax	100	797	2.13	1.04	1.04	22.45	0.00
(6) Research subsidy	4.1	844	2.24	0.82	1.77	5.90	10.45
(7) Full insurance	-	797	2.13	0.86	1.75	9.19	3.35
(8) Research joint venture	-	727	1.96	0.88	1.64	8.11	9.39

Figure 1
Equilibrium Number of Researchers

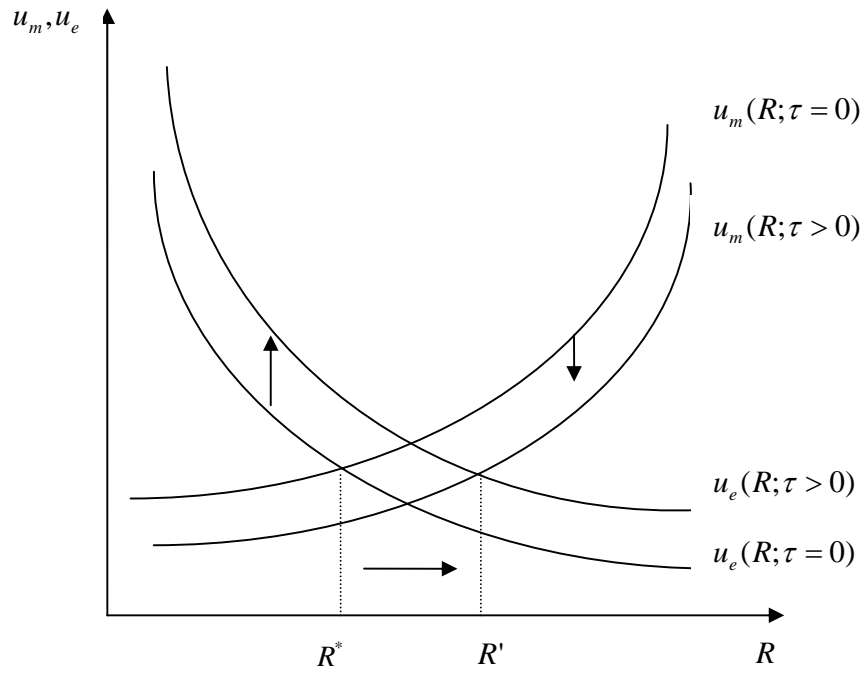


Figure 2

Spillovers and optimal tax rates

