Fiscal Federalism and Soft Budget Constraint: Does the pattern of public spending matter?

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Abstract

This paper analyses the impact of both horizontal tax competition and the pattern of public spending on the softness of the regional budget constraint, the regional incentives to borrow and the regional public good provision. We show that, the regional budget constraint is always soft when the public good is provided to households whereas the result is strongly modified when the public good provided in the first period is devoted to the production process: the local budget constraint can be either soft or hard.

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1 Introduction

This paper analyses the impact of both horizontal tax competition and the nature of the public good on the softness of the regional budget constraint. Following Rodden, Eskeland and Litvack (2003), a soft budget constraint can be defined as “the situation when an entity (say, a province) can manipulate its access to funds in undesirable ways”. Hence, the inability of the rescuer to generate expectations of no bailout entails a soft budget

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constraint. There is a long line of empirical papers and case studies which have dealt with issues arising from soft budget constraints in federations (see for instance Jones, Sanguinetti and Tomassi (1999), Von Hagen (1991), Poterba (1995), Borge and Rattso (1999), Garcia-Mila, Goodspeed and McGuire (2001), Von Hagen and Dahlberg (2002), Rodden, Eskeland and Litvack (2003)). What emerges from all these studies is that the soft budget constraint is in some extent present in all countries but “its severity and the proposed mechanisms to handle it depend on each country’s institutions” (Rodden and alii (2003)). However, three main mechanisms favoring soft budget constraint emerge from these studies (Rodden and alii (2003)). The first one is that regional governments facing an unexpected adverse fiscal shock may be encouraged to claim for additional funds from the federal government when they have a limited fiscal autonomy and are heavily dependent in their financing on federal government transfers. Second, political reasons may explain bailouts, especially when the federal government needs the votes of electors (or some interest groups) of a given region in order to stay in office. Finally, the federal government is more likely to be unable to resist bailout demands if sub-national governments fail to provide public goods which are of high importance from its own point of view. Our paper is in line with this argument.

On the theoretical side, few recent papers deal with both public finance literature and soft budget constraint (see the survey on soft budget constraint by Kornai, Maskin and Roland (2003)). Exceptions are papers by Wildasin (1997), Qian and Roland (1998), Goodspeed (2002) and Köthenbürger (2004). A key-point of almost all these papers is that the soft budget constraint is formulated in the context of a sequential game where the first move is made by regional governments which generally borrow; the federal government has the second move and at that point, the costs to the federal government of no providing additional funds may exceed these of providing one. However, reasons why the soft budget constraint issue arises differ according to the authors. Wildasin (1997) suggests that large sub-national jurisdictions are more likely to be rescued by the federal government than smaller ones because of negative externalities they may cause to other jurisdictions if they were to fail (we find again the well-known “too big to fail” argument of the banking literature). Qian and Roland (1998) show that fiscal decentralization together with tax-base mobility may serve as a commitment device to harden budget constraints of state-owned enterprises in increasing the opportunity cost of bailouts. Nevertheless, in their paper the federal government does not act as a player so that whether public enterprises constraints are hard or soft is derived exogenously. Goodspeed (2002) demonstrates that transfers from higher layers of government to lower layers generally involve a “common pool” effect since a part of the bailout must be paid for through increased taxes and then shared by all the regions. He endogenously derives soft budget constraint bailout behavior on the part of the federal government but ignores tax interactions among governments since transfers are financed through an immobile and exogenous tax base. Conversely, Köthenbürger (2004) explicitly introduces capital mobility among regional governments in a static model ‘a la Wildasin (1988)’. He shows that ex-post federal transfer policy results in internalizing the impact of interregional tax competition on regional tax policy but at the expense of a new source of inefficiency that turns to be welfare deteriorating relative to tax competition.

Our paper is mainly in line with the paper by Goodspeed. However, contrary to him, we assume that (i) horizontal tax competition is at work; (ii) the public good can be
either a consumption or an industrial public good. Indeed, we set up a simple model of federal government transfer decisions with inter-temporal regional spending choices when horizontal tax externalities are at work. The externality results from horizontal tax competition among regions to attract mobile capital. Furthermore, we alternatively assume that the public good provided at the first period can be either a consumption or an industrial public good. Modeling the public good as a consumption good is the most standard choice of modeling. However, there are few papers dealing with both kinds of public goods and the resulting inefficiency with respect to the optimal composition of public expenditure (the papers by Keen and Marchand (1997) and Matsumoto (2000)) are exceptions). This choice of modelling is however relevant for at least two reasons: first, because our model can be connected to a recent literature (see Justman and alii (2002) and (2004)) that shows that regional governments are likely to compete with respect to the quality of their infrastructures, and that it may have an impact on the intensity of tax competition and then on equilibrium tax rates. Second, we assume in our model that the industrial public good (i.e an investment in infrastructures) is financed by borrowing whereas the public good is financed through taxation. This assumption is clearly in line with the so-called “golden rule of public finance” and the reform proposals of the European Growth and Stability Pact. Besides, it is noteworthy that tax competition models do not account for this assumption which however nicely fits with public accountability rules in most countries since they assume that both kinds are financed through taxation.

In our framework, we show that the pattern of public spending crucially influences the softness of the local budget constraint. More precisely, the production of an industrial public can limit or totally canceled the softness of the local budget constraint.

Our paper is organized as follows. Section 2 presents the main features of the model in a context of horizontal tax competition with a consumption public good provided in both periods. Section 3 checks how the introduction of an industrial public good at the first period modifies the previous results.

2 The model

The economy consists in a federation run by a federal government comprising \( n \) identical regions governed by regional governments. In each period, a regional public good is provided to households in each region and financed by both regional tax revenues and a federal transfer once the debt is repaid.

2.1 Consumers

The representative two-period lived consumer of each region derives an utility \( U(c_{i1}, G_{i1}, c_{i2}, G_{i2}) \) from the public good \( (G_{i1}, G_{i2}) \) and the private good \( (c_{i1}, c_{i2}) \) consumptions:

\[
U(c_{i1}, G_{i1}, c_{i2}, G_{i2}) = u(c_{i1}) + v(G_{i1}) + c_{i2} + v(G_{i2})
\]

where the utility functions \( u \) and \( v \) are increasing in every argument, twice differentiable and concave.

In period 1, each representative consumer is endowed with \( \bar{w} \) units of good which are allocated between private consumption and savings:
where $S_i = s_i^1 + \sum_{j \neq i} s_j^1$ with $s_i^1 \geq 0$ and $\sum_{j \neq i} s_j^1 \geq 0$. $s_i^1$ stands for home investments whereas $\sum_{j \neq i} s_j^1$ represents investments made in foreign regions. Savings invested in a region $j$ are remunerated at the before-tax interest rate $r_j$ on which a regional tax $\tau_j$ is levied according to the source principle$^1$. In period 2, the representative agent consumes the proceeds of her savings plus a rent $\Pi_i (r_i)$ earned in her jurisdiction:

$$c_{i2} = \sum_{j=1}^{n} (1 + r_j - \tau_j) s_j^1 + \Pi_i$$

By this way, we implicitly assume that the consumer owns the firm located in her region.

The consumer maximizes her utility (1) under the budget constraints (2) and (3). Integrating the arbitrage condition, the intertemporal budget constraint of the consumer writes $\bar{w} = c_{i1} + c_{i2} - \Pi_i$. The optimality condition is given by $\frac{\partial u}{\partial c_{i1}} = (1 + \rho)$ where $\rho$ is the federation-wide net return on capital determined on the capital market. Saving can then be rewritten as a function $S_i = S_j = S (\rho)$ with $S > 0$.

2.2 Regional governments

Each regional government aims at maximizing the utility function $U_i$ of the representative consumer located in its region. At the first period the regional government decides the level of the regional public good ($G_{i1}$) which is financed by an exogenous federal transfer ($T_{i1}$) and regional borrowing ($B_{i1}$):

$$G_{i1} = T_{i1} + B_{i1}$$

In the second period, the regional government provides a regional public good$^2$ ($G_{i2}$). We assume that the regional government has to maintain a balanced budget. As a result, the repayment of the debt reduces the budgetary room of manoeuvre, ceteris paribus. The regional public good and the repayment of the debt are financed by both federal transfers ($T_{i2}$) and the revenue of regional taxes on capital ($\tau_i K_i$):

$$G_{i2} = T_{i2} + \tau_i K_i - (1 + r_i) B_{i1}$$

where $K_i$ stands for the amount of capital available in region $i$.

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$^1$There are two polar principles of interjurisdictional taxation: the residence (of the taxpayer) principle and the source (of income) principle. The source principle implies that all incomes originating in a region are taxed in this region regardless of the region of residence of the taxpayers.

$^2$Contrary to Wildasin (1997) we assume that no externalities of production or consumption are at work.
2.3 Federal government

The federal government, which is assumed to be benevolent, maximizes the aggregated utility of citizens \( W_i = \sum_{i=1}^{n} U_i \). It has no tax power so that the transfers granted to some regions \( (T_{i2} > 0) \) are balanced by the contributions made by the other regions \( (T_{-i2} < 0) \):

\[
\sum_{i=1}^{n} T_{i2} = 0 \tag{6}
\]

In other words, the federal government proceeds to a budget reallocation.

2.4 Capital market

Output in region \( i \) is a function of capital located in \( i \), denoted by \( K_i \). The production function is assumed to be monotonously increasing in capital \( (\frac{\partial F}{\partial K_i} > 0) \) with decreasing marginal product \( (\frac{\partial^2 F}{\partial K_i^2} < 0) \). Rents arising in region \( i \) are given by

\[
\Pi_i = F(K_i) - r_i K_i
\]

and are assumed to totally accrue to the representative consumer.

For each region \( i \), the amount of capital \( K_i \) equalizes the total amount of savings \( \sum_{j=1}^{n} s_j^i \) invested in the region.

Capital is costless mobile across regions so that the capital relocates until it earns the same post-tax return \( \rho \) in each region. The arbitrage condition on the capital market is given by

\[
\rho = r_i - \tau_i = r_j - \tau_j \quad \forall \ i, j \tag{7}
\]

which implicitly defines the demand for capital in region \( j \) as \( K_j = K(\rho + \tau_j) \) with \( K_j' = \frac{1}{F_{K_j} K_j} < 0 \)

The capital market clearing condition of the federation

\[
\sum_{i=1}^{n} K_i (\rho + \tau_i) = n S(\rho)
\]

implicitly defines the net return \( \rho \) which is a decreasing function of the regional tax rate:

\[
\frac{d\rho}{d \tau_j} = \frac{K_j'}{n S' - \sum_{i=1}^{n} K_i'} \in [-1, 0]
\]

whereas the interest rate is an increasing function of the regional tax rate:

\[
\frac{dr_j}{d \tau_j} = 1 + \frac{n S' - \sum_{i=1, i \neq j}^{n} K_i'}{n S' - \sum_{i=1}^{n} K_i'} > 0 \quad \text{and} \quad \frac{dr_j}{d \tau_i} = \frac{d\rho}{d \tau_i} < 0
\]
Finally, in line with empirical findings, we postulate that the elasticity of the regional tax base with respect to the regional tax rate, denoted by 
\[
e_i \equiv \frac{\partial K_i}{\partial \tau_i} = K_i \frac{d\tau_i}{d\tau_i} \left( K_i \right),
\]
is not too high, so that \( e_i \in [-1, 0] \).

2.5 Sequence of budgetary interactions

Each level of government interacts in a two-period model. In the first period, regional governments play as Nash competitors when choosing their level of the public good \((G_{i1})\) and their level of debt \((B_{i1})\). They also play a Nash game when choosing their tax rate level \((\tau_i)\) and their level of regional public good \((G_{i2})\) in period 2. Conversely, they play as Stackelberg leaders towards the federal government when determining how much they borrow, whereas they play as Nash competitors with the federal government when choosing their tax rate \((\tau_i)\). In other words, the level of the regional debt will depend on the credibility of the federal government not to bail out. Finally, consumers who are supposed to be immobile choose their level of savings and consumption. We solve the model by backward induction.

3 Optimal budgetary decisions

3.1 Federal government program

The federal government maximizes the aggregated utility of citizens located in the federation with respect to the vector of transfers \(T_2\) and subject to the budget constraints of consumers, regions and its own budget constraint:
\[
\max_{T_2} \sum_{i=1}^{n} [u(c_1) + v(G_{i1}) + c_{i2} + v(G_{i2})]
\]

s.t.
\[
c_1 = w - S
\]
\[
c_{i2} = (1 + \rho)S + \Pi_i(r_i)
\]
\[
\sum_{i=1}^{n} T_{i2} = 0
\]
\[
G_{i1} = T_{i1} + B_{i1}
\]
\[
G_{i2} = T_{i2} + \tau_iK - (1 + r_i)B_{i1}
\]

From the first-order conditions, it results that the federal government chooses the vector of optimal transfers so as to equalize the marginal utilities of the regional public good. In other words, the federal government aims at implementing inter-individual equalization across the whole territory:

\[
\frac{\partial v}{\partial G_{i2}} = \frac{\partial v}{\partial G_{j2}} \implies G_{i2} = G_{j2} \quad \forall i, j
\] (8)

A key issue of the paper consists in determining the reaction function of the federal government, i.e. how it will react in terms of transfers granted to regions in second period, following an increase in region \(i\) ’s borrowing in the first period. Differentiating the first-order conditions (8) with respect to \(T_{i2}, T_{j2} \forall j \neq i\) and \(B_{i1}, B_{11}\), and summing them, leads to the following lemma:

**Lemma 1** The federal government’s reaction function to regional borrowing is given by :

\[
\frac{dT_{i2}}{dB_{i1}} = \frac{(n - 1)}{n} (1 + r_i) \quad \text{and} \quad \frac{dT_{j2}}{dB_{i1}} = -\frac{1}{n} (1 + r_i)
\]

Lemma 1 shows how the federal government, aiming at equalizing the public good consumption across the whole territory, will adjust its allocation of transfers to a given region \(i\) in period 2 when that region increases its borrowing. The federal government’s best reaction function (its optimal choice of grants \(\{T_{12}, ..., T_{i2}, ..., T_{n2}\}\) as a function of borrowing) depends on two elements: the additional cost of the debt and the intensity of the reallocation mechanism \(\frac{(n-1)}{n}\). The federal government is inclined to increase a region’s period 2 allocation of grants when that region increases its borrowing in order to allow the regional government to repay its debt in the second period \(((1 + r_i) dB_{i1})\) while maintaining an optimal level of regional public good consumption in period 2. This effect always goes towards softening\(^3\) the regional budget constraint. In other words, an increase of the regional debt does not strengthen the regional budget constraint in the sense that the debt repayment effect tends to increase the transfer granted by the federal government.

\(^3\)The regional budget constraint is known as "soft" when the federal government pays additional transfers to a region \(i\) when that region increases its borrowing \((\frac{dT_{i2}}{dB_{i1}} > 0)\). On the contrary, the regional budget constraint is described as "hard" when the federal government doesn’t modify its scheme of transfers following an increase of borrowing.
Moreover, we can rewrite the federal government’s reaction function to regional borrowing as follows

\[
\frac{dT_{12}}{dB_{11}} = \frac{(1 + r_i)}{(A_i)} - \frac{1}{n} \left(1 + r_i\right) \tag{9}
\]

The region \(i\) receives an additional transfer \((A_i)\) following its increase of borrowing but at the same time bears a part of the financing of this additional transfer \((B_i)\) as all the regions do. In other words, the federal government makes all the regions including \(i\) bear the burden resulting from the extra transfer granted to region \(i\). Then it is clear that the larger the number of regions, the higher the reallocation mechanism and the softer the budget constraint. This result is simply due to the fact that a larger number of regions allow the federal government to finance the bailout on a larger fiscal base and so on to limit the reduction of transfers to each region. The reallocation mechanism goes towards softening the local budget constraint as well.

**Proposition 1** When the local governments aim to equalize the marginal utilities across jurisdictions, the local budget constraint is always soft

**Proof.** Directly from lemma 1 ■

As mentioned in the previous paragraph, both elements which determine the federal government’s reaction function to regional borrowing go towards softening the local budget constraint. The federal government is always inclined to bailout a region \(i\), through lump-sum taxes levied on the other regions, when that region \(i\) increases its borrowing. Indeed, the federal government is unable to commit ex post not to rescue the region because of its aim at equalizing marginal utilities of the public good across the territory as a whole. Once the regional governments have played, the cost of no bailing out is ex post higher from the federal government’s point of view than bailing out. As a matter of fact, when a region increases its borrowing in the first period, it results that, ceteris paribus, regional public good provision in the second period is lower. As a result, the condition regarding the equalization of marginal utilities does not hold any longer in the second period. Ex post, transfers allow the federal government to meet again this condition.

Moreover, the additional transfer granted to the region \(i\) is financed by lump-sum taxes levied on all the regions (included region \(i\) as we will see). As a result, the fact that region \(i\) increases its borrowing generates negative externalities on the other regions as a whole. Since the federal government aims at equalizing marginal utilities of the regional public good across the territory, it proceeds to a reallocation of transfers. Each region contributes in proportion to its relative valuation of one unit of regional public good, in comparison with region \(i\).

### 3.2 Regional government program

The regional policy-maker maximizes the intertemporal utility function of the representative household of its region subject to the constraints on private and public consumptions at each period taking account of the reaction functions of the federal government:
\[
\max_{B_i, \tau_i} u(c_1) + v(G_{i1}) + c_2 + v(G_{i2}) \\
\text{s.t.} \\
\begin{aligned}
c_1 &= \bar{w} - S \\
c_2 &= (1 + \rho)S + \Pi_i(r_i) \\
G_{i1} &= T_{i1} + B_{i1} \\
G_{i2} &= T_{i2} + \tau_iK_i - (1 + r_i)B_{i1}
\end{aligned}
\]
and
\[
\frac{dT_{i2}}{dB_{i1}}, \frac{dT_{j2}}{dB_{i1}} \forall j \neq i
\]

The first-order conditions with respect to \(B_{i1}\) and \(\tau_i\) are:
\[
\frac{\partial u}{\partial G_{i1}} + \frac{\partial v}{\partial G_{i2}} \left[ -(1 + r_i) + \frac{dT_{i2}}{dB_{i1}} \right] = 0
\]
and
\[
\left[ \frac{dp}{d\tau_i} S_i - K_i \frac{dr_i}{d\tau_i} \right] - \frac{\partial v}{\partial G_{i2}} \left( \frac{dr_i}{d\tau_i} B_{i1} - K_i - \tau_iK'_i \frac{dr_i}{d\tau_i} \right) = 0
\]

(11) The optimal tax rate is such that the variation of the interest debt repayment is lower than the variation of the regional fiscal revenues \(\left( \frac{dr_i}{d\tau_i} B_{i1} - K_i - \tau_iK'_i \frac{dr_i}{d\tau_i} > 0 \right)\).

This FOC leads to the following results

**Lemma 2** At the symmetric equilibrium, the opportunity cost of borrowing of each region is given by
\[
\frac{\partial v}{\partial G_{i1}} = \frac{1}{n} (1 + r)
\]
The opportunity cost boils down to the part of the bailout financed by the region itself. The higher the number of the regions in the federation, the lower the burden borne by each of them and so the lower the opportunity cost of borrowing.

And

**Lemma 3** At the symmetric equilibrium, the marginal cost of public funds is given by
\[
\frac{\partial v}{\partial G_{i2}} = \frac{1}{(1 + e) - \frac{dr_{i1}}{dr K_i}} > 1
\]
The distorsive effects of the horizontal tax competition, both on the regional tax base and on the cost of the debt repayment, leads to an underprovision of the second-period regional public good. Moreover, from the well known benchmark case of lump sum tax which would imply \(\frac{\partial v}{\partial G_{i2}} = 1\), both the property tax on capital and the debt as financing the local public good at the first period go towards an underprovision of the public good provided in the second period.
Proposition 2  The soft budget constraint limits the opportunity cost of borrowing

Proof: from equation (10), it is clear that for \( \frac{dT_{i2}}{dB_{i1}} = 0 \) then \( \frac{\partial u_i}{\partial v_i} = (1 + r) \).

The intuition is straightforward. As the budget constraint is soft, the region is incited to borrow since the federal government will bailout and it is not costly for the region to borrow. Moreover, the larger the number of country, the less costly the borrowing since the budget constraint is softer.

Note that at the symmetric equilibrium \( (\tau_i = \tau_j = \tau \) and \( B_{i1} = B_{j1} = B) \), there is no budget reallocation between regions:

\[ T_i^* = T_j^* = T^* = 0 \]

since the federal government has no tax power and can only manipulate transfers. This result is no longer at work when regions are asymmetric because the federal government takes into account the heterogeneity between regions.

4  Provision of an industrial public good

4.1 Consumers

We now assume that in each region \( i \) an industrial public good \( (I_i) \), which is supposed to increase the marginal productivity of the private capital, is produced in the first period. It can be seen as public infrastructures or human capital. This industrial public good is financed with both a regional debt and an exogenous federal transfer granted to the region.

The utility of the representative two-period lived consumer of each region becomes:

\[ U(c_{i1}, c_{i2}, G_{i2}) = u(c_{i1}) + c_{i2} + v(G_{i2}) \] (12)

The expression of the first period private consumption is unchanged whereas in period 2, the representative agent rent depends on the level of the industrial public good:

\[ c_{i2} = \sum_{j=1}^{n} (1 + r_j - \tau_j) s^j_i + \Pi_i \] (13)

The consumer program stays unchanged and saving can then be rewritten as a function \( S_i = S_j = S'(\rho) \) with \( S' > 0 \).

4.2 Governments

In the first period, the regional government decides the level of an industrial public good \( (I_i) \) which production is financed by an exogenous federal transfer \( (T_{i1}) \) and regional borrowing \( (B_{i1}) \):
\[ I_i = T_{i1} + B_{i1} \]  

Note that according to this budget constraint, the decision of borrowing and the decision of producing the industrial public good are strictly linked. By this way, our paper differs from those of Keen and Marchand (1998) or Zodrow and Mieskowki (1986) since the decision of level of the industrial public good will alter the level of the local public good provided in the second period.

The local and federal budget constraints stay unchanged in the second period.

### 4.3 Capital market

In addition to the properties of the production function presented in the previous section, we assume that \( \frac{\partial^2 F}{\partial k_i \partial I_i} > 0 \), i.e. the more industrial public good is provided by the regional government, the higher the marginal product of capital. Rents arising in region \( i \) are given by

\[ \Pi_i = F(K_i, I_i) - r_i K_i \]

One can easily verify that rents are monotonously increasing with the industrial public good (\( \frac{\partial \Pi_i}{\partial I_i} > 0 \)) and monotonously decreasing with the capital cost \( r_i \left( \frac{\partial \Pi_i}{\partial r_i} = -K_i < 0 \right) \).

The demand for capital in region \( j \) is now given by \( K_j(r_j, I_j) = K(\rho + \tau_j, I_j) \) with

\[ \frac{dK_j}{dr_j} = K_j' = \frac{1}{F_{K_j K_j}} < 0 \text{ and } \frac{dK_j}{dI_j} = K_j' = -\frac{F_{K_j I_j}}{F_{K_j K_j}} > 0 \]

The capital market clearing condition allows us to determine

\[ \frac{d\rho}{dI_j} = \frac{K_j'}{nS' - \sum_{i=1}^{n} K_i'} > 0 \]

whereas the net return with respect to the tax rate is unchanged.

The sequence of decision of the game stays unchanged. \( I_1 \) is determined in the first period, together with \( B_{i1} \).

### 5 Optimal budgetary decisions

#### 5.1 Federal government program

The federal government maximizes the aggregated utility of citizens located in the federation with respect to the vector of transfers \( T_2 \) and subject to the budget constraints of consumers, regions and its own budget constraint:

\( ^{4}\text{This assumption is based on a review of the empirical evidence on aggregate production relationships that suggests that public infrastructure has indeed almost always been found to be complementary with private capital (see Sturn and al (1996))} \)
\[
\max_{T_2} \sum_{i=1}^{n} [u(c_1) + c_{i2} + v(G_{i2})]
\]
\[
s.t.
\]
\[
c_1 = w - S
\]
\[
c_{i2} = (1 + \rho)S + \Pi_i
\]
\[
\sum_{i=1}^{n} T_{i2} = 0
\]
\[
G_{i2} = T_{i2} + \tau_i K_i - (1 + r_i) B_{i1}
\]
\[
I_i = T_{i1} + B_{i1}
\]

The first-order conditions are unchanged, that is, the federal government chooses the vector of optimal transfers so as to equalize the marginal utilities of the regional public good: \( \frac{\partial v}{\partial G_{i2}} = \frac{\partial v}{\partial G_{j2}} \Rightarrow G_{i2} = G_{j2} \ \forall i, j \)

Differentiating the first-order conditions with respect to \( T_{i2}, T_{j2} \ \forall j \neq i \) and \( B_{i1} \), and summing them, leads to the following expression:

The federal government’s reaction function to regional borrowing is given by

\[
\frac{dT_{i2}}{dB_{i1}} = \left[ \frac{(n-1)}{n} \left( (1 + r_i) - \tau_i K'_{i1} \right) + \frac{\partial \rho}{\partial I_{i1}} \frac{1}{n} \sum_{j \neq i} (\tau_j K'_{j} - B_{j1}) \right]
\]

**Lemma 4** For the particular case of equal tax rates and equal level of debt (\( \tau_i = \tau_j = \tau \) and \( B_{i1} = B_{j1} = B \)), the federal government’s reaction function to regional borrowing is given by:

\[
\frac{dT_{i2}}{dB_{i1}} = \frac{(n-1)}{n} \left( (1 + r_i) - \tau K'_{i1} \right)
\]

Lemma 4 shows the additional effect at work when an industrial public good is provided in the first period. This effect is due to the degree of complementarity between the production factors \( (K'_i) \). This complementarity between the production factors implies a rise in the capital of the region so that the regional receipts increase. This effect goes toward hardening the local budget constraint.

**Proposition 3** The local budget constraint is soft (resp. hard) if \( (1 + r_i) > (\text{resp. } < \tau_i K'_{i1}) \)

proof: directly from lemma 4

When an industrial public good is provided in the first period, the federal government is not always inclined to bailout a region \( i \) when that region increases its borrowing. Even if the federal government is unable to commit ex post not to rescue the region, it integrates that in addition to the increased cost of the debt, a rise in borrowing increases the receipts of the region through a production factors effect. The behavior of the federal government depends on the relative weight of the increasing cost of the debt and the expected additional tax revenues. If the marginal cost of the debt repayment dominates, the federal government helps the region in providing the regional public good in order to satisfy its aim of equalization. Otherwise, he reallocates the additional revenues of the region among the federation.
5.2 Regional government program

The regional policy-maker maximizes the intertemporal utility function of the representative household of its region subject to the constraints on private and public consumptions at each period taking account of the reaction functions of the federal government:

\[
\max_{B_{i1}, r_i} u_i(c_{i1}) + c_{i2} + v(G_{i2}) \\
\text{s.t.}
\]

\[
c_{i1} = \bar{w} - S \\
c_{i2} = (1 + \rho)S + \Pi_i \\
I_i = T_{i1} + B_{i1} \\
G_{i2} = T_{i2} + \tau_i K_i - (1 + r_i) B_{i1}
\]

and

\[
\frac{dT_{i2}}{d B_{i1}}, \frac{dT_{j2}}{d B_{i1}} \forall j \neq i
\]

The first-order conditions with respect to \( B_{i1} \) and \( \tau_i \) are:

\[
\frac{\partial \Pi_i}{\partial I_{i1}} + \frac{\partial v}{\partial G_{i2}} \left[ \tau_i K_i' + \tau_i K_i' \frac{\partial \rho}{\partial I_{i1}} - \frac{\partial \rho}{\partial I_{i1}} B_{i1} - (1 + r_i) + \frac{dT_{i2}}{d B_{i1}} \right] = 0 \tag{15}
\]

and

\[
\frac{d \rho}{d \tau_i} S_i - K_i \frac{d r_i}{d \tau_i} - \frac{\partial v}{\partial G_{i2}} \left( \frac{d r_i}{d \tau_i} B_{i1} - K_i - \tau_i K_i' \frac{d \rho}{d \tau_i} \right) = 0 \tag{16}
\]

According to FOC (16), we can establish that \( \tau_i \) is determined optimally such that

\[
\frac{d r_i}{d \tau_i} B_{i1} - K_i - \tau_i K_i' \frac{d \rho}{d \tau_i} > 0
\]

A main point of our paper is to analyze the impact of the soft budget constraint phenomenon on the regional budgetary decisions. We first evaluate how the expected reaction of the federal government modifies the regional incentives to borrow. Deriving the first-order condition with respect to the level of debt leads to the expression of the opportunity cost of the private good consumption \( c_{i2} \) in terms of regional public good provision \( G_{i2} \) foregone (hereafter the opportunity cost of borrowing):

The region \( i \)'s opportunity cost of borrowing is given by

\[
\frac{1}{\partial v_i} = \left[ (1 + r_i) - \frac{d T_{i2}}{d B_{i1}} - \tau_i K_i' - \tau_i K_i' \frac{\partial \rho}{\partial I_{i1}} + \frac{\partial \rho}{\partial I_{i1}} B_{i1} \right] = F'_{I_i}
\]

\[
\text{Lemma 5} \quad \text{At the symmetric equilibrium, the opportunity cost of borrowing of each region is given by}
\]

\[
\frac{F'_{I_i}}{\partial v} = \frac{1}{n} \left[ (1 + r) - \tau K_i' \right] - \frac{\partial \rho}{\partial I_1} (\tau K_i' - B_1) \tag{17}
\]
An increase in the industrial public good provision generates via the rise in $r_i$ both a reduction of the tax base and an increase in the debt repayment which tends to discourage the region to borrow in the first period. Moreover the softer (harder) the budget constraint and the lower (higher) the opportunity cost of borrowing.

We now turn to the mechanisms which are at work when the regional government sets its tax rate. The maximization of the regional government’s program with respect to the tax rate leads to the marginal cost of public funds, which expresses the regional incentives to tax:

The region $i$’s marginal cost of public funds is given by

$$\frac{\partial v_i}{\partial G_{i2}} = \frac{d\rho_i}{d\tau_i} S_i - K_i \frac{dr_i}{d\tau_i}$$

and evaluated at the symmetric equilibrium, we obtain $\frac{\partial v_i}{\partial G_{i2}} = \frac{1}{(1+\epsilon)-\frac{d\tau_i}{d\tau_i}} > 1$. Even if the mechanisms which determine the optimal level of the tax rate seem to not be altered by the introduction of the industrial public good, the level of the optimal tax rate is crucially modified compared to the local public good case and in particular because the level of capital $K$ depends on $I$.

6 Conclusion

The aim of this paper is to study how both horizontal and the pattern of public spending play on the budgetary choices. More precisely, it focuses on the regional choices of borrowing and taxing as well as the federal transfers scheme. We are able to state that when a local public good is provided in each period, the local budget constraint is always soft. By this way, the marginal cost of borrowing of the local government is reduced. Conversely, when an industrial public good is produced in the first period, the results are crucially modified since the local budget constraint can be either soft or hard.

Few extensions to this paper can be considered. One of this would consist in wondering how the introduction of vertical tax externalities via a tax base sharing phenomenon will alter our results.

7 References


8 APPENDIX

8.1 Appendix 1: proof of lemma 1

1) Let consider the \((n - 1)\) first-order conditions of the federal government’s program for region \(i\):

\[ G_{i2} = G_{j2} \quad \forall j \neq i \]

Differentiating these first-order conditions (FOCs) with respect to \(T_{i2}, T_{j2} \forall j \neq i\) and \(B_{11}\) leads to

\[ \frac{\partial}{\partial T_{i2}} [G_{i2} - G_{j2}] dT_{i2} + \frac{\partial}{\partial T_{j2}} [G_{i2} - G_{j2}] dT_{j2} + \sum_{l \neq i, j} \frac{\partial}{\partial T_{l2}} [G_{i2} - G_{j2}] dT_{l2} + \frac{\partial}{\partial B_{11}} [G_{i2} - G_{j2}] dB_{11} = 0 \quad \forall j \neq i \]

Summing these expressions gives:

\[ (n - 1) \frac{\partial}{\partial T_{i2}} [G_{i2}] dT_{i2} + \sum_{j \neq i} \frac{\partial}{\partial T_{j2}} [-G_{j2}] dT_{j2} + (n - 1) \frac{\partial}{\partial B_{11}} [G_{i2}] dB_{11} + \sum_{j \neq i} \frac{\partial}{\partial B_{11}} [-G_{j2}] dB_{11} = 0 \]

\[ \Leftrightarrow \]

\[ (n - 1) dT_{i2} - \sum_{j \neq i} dT_{j2} + (n - 1) \left[ \tau_i K'_i + \tau_i K'_i \frac{\partial r_i}{\partial B_{11}} - \frac{\partial r_i}{\partial B_{11}} B_{11} - (1 + r_i) \right] dB_{11} \]

\[ \quad - \sum_{j \neq i} \left[ \tau_j K'_j \frac{\partial r_j}{\partial B_{11}} - \frac{\partial r_j}{\partial B_{11}} B_{11} \right] dB_{11} = 0 \]

\[ \Leftrightarrow \]

\[ dT_{i2} = \left( (1 + r_i) - \tau_i K'_i - \tau_i K'_i \frac{\partial r_i}{\partial B_{11}} + \frac{\partial r_i}{\partial B_{11}} B_{11} + \frac{1}{(n - 1)} \sum_{j \neq i} \left( \tau_j K'_j \frac{\partial \rho}{\partial I_{11}} - \frac{\partial \rho}{\partial I_{11}} B_{j1} \right) \right) dB_{11} \]

\[ + \frac{1}{(n - 1)} \sum_{j \neq i} dT_{j2} \]