Production Sharing Agreements

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June 1, 2011

(Preliminary Version)

Abstract

We study how the contract signed between a country that owns a non renewable resource and a firm in charge of the resource management affects the exploration and production levels. This paper focuses on a specific contract: the production sharing agreement (PSA). Under a PSA, the firm in charge of the exploration and production activities receives to recover its costs (only if a discovery occurs) a share of the production called the cost oil. The remaining production (the profit oil) is shared between the country and the firm. We compare the PSA, to a standard taxation scheme and to the situation without any taxation. We find that by setting a really low cost oil, the government is able to replicate the exploration and production levels induced by a standard taxation scheme. Thus, the optimal PSA can do at least as well as a taxation scheme. Furthermore, by setting the cost oil to 100%, the government is able to induce the same extraction as without taxation but with a lower exploration effort. Moreover, there exists a threshold of the cost oil beyond which increasing the cost oil has no effect on the exploration effort. Finally, under a PSA, the firm may on the one hand, produce less than if it owns the resource (as in a standard taxation scheme) and explore more or less than without taxation. On the other hand, the firm may have the incentive to produce and explore such as the costs equal to the cost oil.

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1 Introduction

Most of hydrocarbons reserves are located in developing countries without technical skills (seismic surveys, drilling wells, production techniques) nor financial means (low access to capital markets) to efficiently exploit natural resources. Hence, governments often delegate the exploration and production activities to international extractive companies. Different types of agreements may be used: concession contracts, production sharing agreements, joint operating agreements or contracts services. There are a priori no optimal contracts, but each contract should take the oilfield characteristics into account: the probability to discover, the size and quality of the deposit, the investment needed and the quality of political institutions. The most studied type of contracts is the concession contracts (see for example Dasgupta et al. (1981), Bergstrom (1982) and Daubanes and Grimaud (2010)). They are usually simplified to a taxation/subsidy scheme. The aim of this literature is to define the optimal taxation scheme under some constraints: oligopolistic markets, pollution, uncertainty or asymmetric information.

Only few papers study the other types of contracts. For example, Van Groenendaal and Mazraati (2006) study the firm’s incentive to invest and its revenue under the Iran’s buy back contracts. The present paper focuses on production sharing agreements (PSA). The PSA is a common type of contracts signed between a government and an oil company whereby the government delegates the exploration and production activities to a firm (see Bindemann (1999) for an overview). In such an agreement, the government remains the owner of the resource and the oil company only acts as a contractor. They were first implemented in Indonesia in the 1960s and they are often used in the Middle East and Central Asia. Today, about 10% of the oil production is done through such agreements. The PSA allows the government to influence the firm’s strategy (investment, exploration, extraction...) using a set of instruments: the cost oil, the profit oil, production and signature bonuses or royalties.

What differentiate the PSA from the other contracts are the cost and profit oil variables. The firm commits to undertake and finance at its own risk the exploration development and the production activity. In compensation, it receives a part of the production (cost oil) whose sale should reimburse the cost up to the limit defined. The costs that are not recoverable in a year are recovered the year after using the same principle. The remaining production (profit oil) is shared with the government.

1 See Babusiaux et al. (2007) for a complete distribution of the oil production depending on the contract.
according to a rule defined by the contract. The firm bears all the risk as its investment is only reimbursed if a deposit is discovered. The profit oil allows the firm to recover its costs if the cost oil is not high enough and to get a reward for its expertise and the risk undertaken. Furthermore, some other instruments (bonuses and royalties) common in concessionary contracts may be used. According to Johnston (1994),

...the cost recovery limit is the only true distinction between concessionary systems and PSA.

To our knowledge this paper is one of the first attempts to formalize the PSA properties without using simulations. Yusgiantoro and Hsiao (1993) study the investment decision in Indonesia under a PSA using 3 scenarios\(^ 2\). All the PSA components are included and they determine the expected monetary value of the project depending on a set of parameters. Several proposals are made to encourage oil exploration and development. For example, the tax credit should be high enough to compensate the negative profit in the initial stage and the firm may be allowed to recover its cost with an unlimited reimbursement. Blake and Roberts (2006) use Monte Carlo simulations to show that PSA using a profit oil sliding scale may have more distortionary effects in the investing decisions (and thus in production) than a taxation system or a joint development zone. Finally Hampson et al. (1991) also using simulations show that the optimal sharing rule should leave the firm with a much larger share of the small discoveries than the actual contract does. The firm’s share of a discovery should decrease as the size of the discovery increases. Those three papers highlight some basic properties of the PSA but are limited by the value used in their simulations.

We propose a general static model and simplify the PSA to a two variables contract. We do not incorporate bonuses and royalties to the PSA (that are common in the other contracts). We focus on the instruments specific to the PSA: the profit and cost oil. Indeed, the effects of royalties and bonuses are well known whereas, the effects of the profit and cost oil have not been studied in a theoretical model. The aim of this paper is to study the effect of the profit oil and cost oil on the firm’s behaviour (exploration and extraction). Moreover, we would like to define the contract that maximizes the government’s revenue taking into account the firm’s strategy. We compare the extraction and exploration levels induced by a PSA and the revenue captured by the government to those under a taxation scheme. Furthermore,

\(^{2}\)The first scenario is the most favourable for a firm, the second one is the least favourable and the last one assumes uncertainty on the present value of the net cash flow (NCF)
we compare the PSA to the hypothetical case where the firm owns the resource. In the latter case, there is no taxation and the firm captures all the revenue from the field exploitation.

We find that by setting a really low cost oil, the government is able to replicate the exploration and production levels induced by a taxation scheme. Thus, the optimal PSA can do at least as well as a taxation scheme. Furthermore, by setting the cost oil to 100%, the government is able to induce the same extraction level as without taxation. However, this PSA induces a lower exploration level as the firm only gets a share of the resource. Furthermore, there exists a threshold of the cost oil beyond which increasing the cost oil has no effect on the exploration effort. Moreover, under a PSA, the firm may on the one hand, produce less than if it owns the resource (as in a standard taxation scheme) and explore more or less than without taxation. On the other hand, the firm may have the incentive to produce and explore such as the costs equal to the cost oil.

The paper is organized as follows. Section 2 defines the optimal exploration and production levels if there is no taxation and if a standard taxation scheme is used. Section 3 studies the situation where a PSA is used. Finally, some concluding remarks are given in section 4.

2 Benchmark

This part studies the optimal exploration and extraction levels under two scenario used as benchmarks. First, we assume that the firm owns the resource and there is no taxation system. Second, we introduce a government that owns the resource and delegates the petroleum activities to a firm. In the next session, we compare those two scenarios to the situation where the government delegates the petroleum activities using a production sharing agreement.

The general problem is the following: an agent is endowed with a potential stock of resources in ground. To exploit this resource, he has to explore. The exploration process can be successful or not. If a discovery occurs, he chooses the amount \( q \) to extract among this discovery \( R \in [R, \bar{R}] \) where \( f(R) \) denote the density function. Before the exploration process, there is no stock of resources and thus the production only starts if a discovery occurs. The probability to discover increases with the exploration effort: \( \rho(e) \). The size of the discovery is independent from the level
of effort. Thus, the probability to discover is endogenous but the discovery size is exogenous.

We denote $p$ the price of the resource.
The production cost $c_p(q, R)$ increases with the quantity extracted $q$ and decreases with the size of the discovery.
The exploration cost $c_e(e)$ increases with the exploration level $e$.
We denote $C = c_p(q) + c_e(e)$

Assumptions: $c_p(q, R)$, $c_e(e)$ and $\rho(e)$ are such as:

$$
\frac{\partial c_p(q, R)}{\partial q} > 0, \quad \frac{\partial^2 c_p(q, R)}{\partial q^2} > 0, \quad \frac{\partial c_p(q, R)}{\partial R} > 0, \quad \frac{\partial c_e(e)}{\partial e} > 0, \quad \frac{\partial^2 c_e(e)}{\partial e^2} > 0, \quad \frac{\partial \rho(e)}{\partial e} > 0 \quad \text{and} \quad \frac{\partial^2 \rho(e)}{\partial e^2} < 0
$$

The agent chooses its exploration level for a given expected production outcome and its extraction level for a given discovery. The agent uses backward induction to choose the optimal levels of exploration and production.

2.1 The firm owns the resource and there is no taxation

A firm is endowed with a potential stock of resource in ground and decides its optimal exploration and production levels. As there is no taxation, it keeps all the revenue from the extraction.

Second step:

$$
Max_{q} U_F(e, q) = -c_e(e) + \rho(e) \left[ pq - c_p(q) \right]
$$

If the stock is sufficient, the production is such as the price equals the marginal production cost: $q^*$ such as $p - c'_p(q^*) = 0$. Because the model is static, the optimal production’s rule is similar to the one of any consumption good. If the stock is lower than $q^*$, it is optimal to exhaust the stock as the price is higher than the marginal cost.

First step:

$$
Max_{e} U_F(e, q^*) = -c_e(e) + \rho(e) \left[ \int_{R}^{q^*} [pR - c_p(R)] f(R) dR + \int_{q^*}^{R} [pq^* - c_p(q^*)] f(R) dR \right]
$$
The optimal exploration effort is such as the marginal cost equals the marginal expected gains.

2.2 The country delegates the petroleum activities using a standard taxation

We now introduce a government in the analysis, indeed, natural resources are usually owned by the country (principle of state permanent sovereignty over natural resources). However, the country may not have the technical skills to efficiently exploit its natural resource and thus delegates the exploration and production activities to a firm.

For now on, we will consider that the country owns the resource and delegates the petroleum activities. Contrary to a benevolent social planner who is all knowing, all powerful, and well-intentioned (see Mankiw’s Principles of Microeconomics textbook), the government is not omnipotent. The country represented by a government is not a social planner and thus may not impose, the exploration, the production and the taxation rate. It is limited and can only impose a taxation scheme, that will induce a level of exploration and extraction. As the resource owner, the government is entitled with a share of the revenue generated by the production of the resource. To collect this revenue, it has to design the optimal taxation scheme. The government represents its citizens’ interests and collects a revenue that will be redistributed among the population. We first assume that the government delegates the petroleum activities using a standard taxation scheme and then a PSA.

The government delegates the petroleum activities using a standard taxation scheme. The firm gets all the revenue from the extraction, bears the exploration and production costs and pays a tax \( \tau \in [0,1] \) on the quantity produced (\textit{ad valorem}). The firm chooses its exploration and production levels knowing that it pays \( \tau \) to the government. The government chooses \( \tau \) that maximizes its fiscal revenue and \( \tau \) must be such as the firm accepts the contract.

Third step:

\[
\max_{e,q} U_F(e,q) = -c_e(e) + \rho(e) [p(1 - \tau)q - c_p(q)]
\]

If the stock is sufficient, the optimal production is \( q_\tau \) such as \( p(1 - \tau) - c_p'(q_\tau) = 0 \).
If not, the stock is exhausted. The higher the taxation rate is, the lower the production is. From the first order condition, \( q_\tau < q^* \). The production level is lower than without taxation.

Second step:

\[
\max_e U_F(e, q_\tau) = -c_e(e) + \rho(e) \left[ \int_R^{q_\tau} [pR(1 - \tau) - cp(R)] f(R) \, dR + \int_R^{q_\tau} [pq_\tau(1 - \tau) - cp_\tau(R)] f(R) \, dR \right]
\]

The optimal exploration effort is such as the marginal cost equals the marginal expected gains. The higher the taxation rate is, the lower the exploration effort is. Indeed the taxation rate decreases the production outcome and thus the expected gains. From the first order condition, \( e_\tau < e^* \). The exploration level is lower than without taxation as the production outcome is lower with a taxation system.

First step:

\[
\max_\tau U_G(e_\tau, q_\tau, \tau) = \rho(e_\tau) \left[ \int_R^{q_\tau} pR f(R) \, dR + \int_R^{q_\tau} pq_\tau f(R) \, dR \right] \quad \text{s.t. } U_F \geq 0 (\lambda)
\]

\[
\frac{\partial L}{\partial \tau} = \rho'(e_\tau) \frac{de_\tau}{d\tau} \tau \left[ \int_R^{q_\tau} pR f(R) \, dR + \int_R^{q_\tau} pq_\tau f(R) \, dR \right] + \rho(e_\tau) \int_R^{q_\tau} p \frac{dq_\tau}{d\tau} f(R) \, dR
\]

\[
+ (1 - \lambda) \rho(e_\tau) \left[ \int_R^{q_\tau} pR f(R) \, dR + \int_R^{q_\tau} pq_\tau f(R) \, dR \right] = 0
\]

Under a standard taxation scheme, it is possible that the stock is not exhausted whereas without taxation, it would have been exhausted. The government faces a trade off while choosing the optimal taxation rate. Indeed, the taxation rate has a positive direct effect on the government’s payoff as it increases its share of the resource and a negative indirect effect as it decreases the probability to discover and the quantity extracted. Furthermore, the government’s payoff strictly increases with the quantity extracted and the exploration effort as it does not bear any costs. The government will be better off with a quantity extracted and an exploration effort higher than the optimal ones. However if the (PC) binds, the exploration and extraction levels that maximize the government’s payoff are the same as without
3 The country delegates the petroleum activities using a PSA

The government delegates the exploration and production activities to a firm using a PSA. The contract specifies the share of resources $\alpha \in [0,1]$ used for the costs reimbursement (the cost oil) and the share $\beta \in [0,1]$ of the remaining resources (profit oil) the firm gets after the cost oil payment. Thus, the contract defines the maximum level of costs reimbursement (exploration and production costs) and the share of profit left to the firm after the cost oil payment.

The expected utility with no uncertainty on the proven reserves:

The firm’s expected utility: $U_F = -c_e(e) + \rho(e)[CO - c_p(q) + \beta((1 - \alpha)pq + EO)]$

The government’s expected utility: $U_G = \rho(e)[pq - CO - \beta((1 - \alpha)pq + EO)]$

where $CO = \text{Min}[\alpha pq, C]$ (cost oil) and $EO = \text{Max}[\alpha pq - C, 0]$ (excess oil)

- If $\alpha pq - C \geq 0$, the payoff functions are:
  
  $U_{Fa} = -c_e(e) + \rho(e)[C - c_p(q) + \beta((1 - \alpha)pq + \alpha pq - C)]$
  
  $U_{Ga} = \rho(e)[pq - C - \beta((1 - \alpha)pq + \alpha pq - C)]$

  The exploration and production costs are entirely reimbursed.

- If $\alpha pq - C \leq 0$, the payoff functions are:

  $U_{Fa} = -c_e(e) + \rho(e)[\alpha pq - c_p(q) + \beta(1 - \alpha)pq]$

  $U_{Ga} = \rho(e)[pq - \alpha pq - \beta(1 - \alpha)pq]$

  The exploration and production costs are partly reimbursed.

If $\alpha pq - C = 0$, then the payoff functions are exactly the same: $U_{Fa} = U_{Fb}$ and $U_{Ga} = U_{Gb}$. $\alpha pq - c_p(q) - c_e(e)$ first increases with $q$ and then decreases with $q$ and is concave in $q$. We define $\hat{q}$ such as $\alpha \hat{q} c_p'\hat{q} - c_p(q)$ is maximum in $q$. $\hat{q}$ may take two values $q_1$ and $q_2$: $q_1$ is such as $\alpha pq_1 - c_p'(q_1) > 0$ and $q_2$ is such as $\alpha pq_2 - c_p'(q_2) < 0$. For a fixed exploration effort, if the quantity extracted is lower than $q_1$, then the costs

\[\text{If } \beta = 0, \text{ the firm gets a negative utility. If } \beta = 1, \text{ then the firm’s utility is the same as if it owns the resource. Thus, the exploration and extraction levels are } (e^*, q^*) \text{ and the government gets nothing.}\]
are only partly reimbursed and the firm’s utility function is $U_{Fb}$. Then if the firm produces a quantity between $q_1$ and $q_2$, the costs are entirely reimbursed and the firm’s utility function is $U_{Fa}$. Finally, if the firm produces higher quantity than $q_2$, the costs are only partly reimbursed and the firm’s utility function is $U_{Fb}$.

**Step of the game**

- The government proposes a contract $(\alpha, \beta)$.
- The firm chooses the exploration effort $e$ that costs $c_e(e)$ and a discovery occurs with probability $\rho(e)$.
- If there is no discovery the firm suffers from a loss: $c_e(e)$. If there is a discovery, the firm learns the size of the proven reserves and decides its optimal extraction level.

Using backward induction, we look for subgame perfect nash equilibria. The firm chooses its extraction strategy for a given discovery, exploration level and contract. This extraction level can be such as the total costs are above or below the cost oil. Then, given the expected production gains, the contract and the discovery’s distribution, the firm chooses its optimal exploration effort. Finally, knowing the firm’s strategy, the government chooses the contract $(\alpha, \beta)$ that maximizes its payoff.

### 3.1 The optimal extraction rule

As the firm uses backwards induction to set its optimal exploration and extraction levels, we first define, the optimal extraction rule. For a given exploration effort and contract, the firm chooses the extraction level that maximizes its utility. This extraction level should be such as the total costs are above or below the cost oil\(^4\). Depending on the sign of $\alpha pq - C$, the firm’s (and the government) utility function changes. Thus, we define two optimal production levels, one under $\alpha pq > C$ and another one under $\alpha pq < C$. The firm maximizes its utility subject to a resource constraint $(RC)$ (the production cannot exceeds the discovery) and a cost oil constraint $(COC)$.

\[
\max_q \quad U_F = -c_e(e) + \rho(e)[CO - c_p(q) + (1 - \alpha)\beta pq + \beta EO] \text{ s.t. } q \leq R \quad (RC)
\]

with $CO = \min[\alpha pq, C]$ and $EO = \max[\alpha pq - C, 0]$

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\(^4\)Actually, we will show that the firm can never choose between the two strategies
The costs are entirely reimbursed: \( \alpha pq - C \geq 0 \)
\[
\text{Max}_q \quad U_{Fa} = -c_e(e) + \rho(e)[c_e(e) + \beta(pq - c_e(e) - c_p(q))] \quad \text{s.t.}
\]
\[
\alpha pq \geq c_e(e) + c_p(q) \quad (COC_1)
\]
\[
q \leq R \quad (RC)
\]

If the stock is sufficient the firm extracts \( q^* \), if not, it exhausts the stock. Furthermore, if while producing \( q^* \) the \((COC_1)\) is not satisfied, the firm decreases the production until the \((COC_1)\) binds. If \( \alpha \) is high enough to ensure that the \((COC_1)\) is satisfied, the costs are entirely reimbursed and the extraction level is independent from the contract and is exactly the same as without taxation.

The costs are not entirely reimbursed: \( \alpha pq - C \leq 0 \)
\[
\text{Max}_{e,q} \quad U_{Fb} = -c_e(e) + \rho(e)[(\alpha + (1-\alpha)\beta)pq - c_p(q)] \quad \text{s.t.}
\]
\[
\alpha pq \leq c_e(e) + c_p(q) \quad (COC_2)
\]
\[
q \leq R \quad (RC)
\]

If the stock is sufficient, the firm extracts \( q_{psa} \) such as \( (\alpha+(1-\alpha)\beta) p - c'_p(q_{psa}) = 0 \)
\( q_{psa} \leq q^* \quad (q_{psa} = q^* \text{ for } \alpha = 1 \text{ or } \beta = 1) \) and if \( (1-\alpha)(1-\beta) = \tau \), the production is the same as if the government used a simple taxation rate.
\( q_{psa} \) increases with \( \alpha \) and \( \beta \). Furthermore, if while producing \( q_{psa} \) the \((COC_2)\) is not satisfied, the firm increases the production until the \((COC_2)\) binds.

\[
\alpha pq^* - c_p(q^*) < \alpha pq_{psa} - c_p(q_{psa}) \Leftrightarrow \alpha pq^* - c_p(q^*) - c_e(e) < \alpha pq_{psa} - c_p(q_{psa}) - c_e(e)
\]

For a given exploration effort and contract, the firm can never choose between extracting \( q^* \) and have its costs entirely reimbursed or extracting \( q_{psa} \) and have its costs partly reimbursed. For a given exploration effort and contract, only one strategy (or none) is feasible.

If the \((COC)\) binds: \( \alpha pq - c_p(q) = c_e(e) \), the firm’s utility function is exactly the same in both optimization problem.
In the first case, \((COC_1)\) binds if while producing \( q^* \), \((COC_1)\) is not satisfied \( \Leftrightarrow \alpha pq^* - c_p(q^*) - c_e(e) < 0 \). As \( \alpha p - c'_p(q^*) < 0 \), the firm has the incentive to decrease its extraction until \((COC_1)\) binds and produces \( q_2 \). In the second case, \((COC_2)\) binds if while producing \( q_{psa} \), \((COC_2)\) is not satisfied \( \Leftrightarrow \alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0 \). As \( \alpha p - c'_p(q_{psa}) < 0 \), the firm has the incentive to increase its production level until \((COC_2)\) binds and also produces \( q_2 \). If the firm produces \( q_1 \), the \((COC)\) also binds but its utility is lower than when it produces \( q_2 \).
3 types of contracts exist (see figure 1)\(^5\):
\[
\alpha pq^* - c_p(q^*) - c_e(e) > 0 \text{ and } \alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0.
\]
\[
\alpha pq_{psa} - c_p(q_{psa}) - c_e(e) < 0 \text{ and } \alpha pq^* - c_p(q^*) - c_e(e) < 0.
\]
\[
\alpha pq^* - c_p(q^*) - c_e(e) < 0 \text{ and } \alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0.
\]

As there is uncertainty on the size of the discovery, the firm and the government maximize their expected utility which is the sum of possible payoff depending on the size of the discovery. But \(q_1\) and \(q_2\) can be used to switch from one utility function to another: the costs are entirely reimbursed \((U_{Fa})\) or partly reimbursed \((U_{Fb})\).

To define the expected payoff functions we use the optimal extraction rule previously defined and the definition of \(q_1\) and \(q_2\). If the quantity extracted is below \(q_1\), then the costs are only partly reimbursed and the utility function is \(U_{Fb}\). Then if the firm produces a quantity between \(q_1\) and \(q_2\), the costs are entirely reimbursed and the utility function is \(U_{Fa}\). Finally, if the firm produces higher quantity than \(q_2\), the costs are only partly reimbursed and the utility function is \(U_{Fb}\).

For each contract, the firm and government’s expected utility functions are computed at the optimal level of extraction. From its expected utility function, the firm chooses its optimal exploration effort for a given expected production outcome and contract. The government knowing the firm’s exploration and production strategy decides the level of \((\alpha, \beta)\) that maximizes its expected utility function under the firm’s participation constraint.

Each of the three contracts may involve a different value of \((\alpha, \beta)\) and may induce different levels of exploration and production. Comparing its payoff under the three contracts, the government should be able to choose the PSA to implement and compare its payoff to the one induced by a standard taxation scheme. Furthermore, we should be able to compare the exploration and production levels to the one without taxation.

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\(^5\)We do not consider the case where \(\alpha = 0\). If \(\alpha = 0\), then the optimal share of profit oil is such as \(\beta^* = (1 - \tau^*)\), and the PSA is similar to a standard taxation scheme.
3.2 The optimal exploration rules and the optimal contracts

3.2.1 Contract 1:

In the first contract $\alpha$ and $\beta$ are such as::

\[ \alpha pq - c_p(q) > 0 \] and \[ \alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0 \]

\[ U_i = \int_{R_{q_1}}^{R_{q_2}} U_{ib}(R)f(R) \, dR + \int_{q_1}^{q^*} U_{ia}(R)f(R) \, dR + \int_{q^*} R f(R) \, dR \text{ where } i = F, G \]

As soon as the stock is large enough to produce $q^*$, $q^*$ is produced. If the stock is too small, the firm exhausts the stock whether its costs are entirely reimbursed or not.

- Firm’s optimization problem:

\[ U_F = -c_e(e) + \rho(e) \left[ \int_{R_{q_1}}^{q_1} [(\alpha + (1 - \alpha)\beta)pR - c_p(R)]f(R) \, dR + \int_{R_{q_1}}^{q^*} \beta(pR - c_p(R))f(R) \, dR \right] \]
\[
\begin{align*}
&+ \int_{q^*}^{R} \beta(pq^* - c_p(q^*))f(R) \, dR + \int_{q_1}^{R} c_e(e)(1 - \beta)f(R) \, dR \\
\frac{\partial U_F}{\partial e} &= -c'_e(e) + \rho'(e)A + \rho(e)(1 - \beta) \int_{q_1}^{R} c'_e(e)f(R) \, dR = 0 \quad (1)
\end{align*}
\]

with \( A \) is such as \( U_F = -c_e(e) + \rho(e)A \).

From (1), \( \alpha \) increases the level of exploration and \( \beta \) has an ambiguous effect on the exploration effort. If the discovery is lower than \( q_1 \), an increase in \( \beta \) increases the exploration effort. However, if the discovery is higher than \( q_1 \) (the costs are entirely reimbursed), \( \beta \) has an ambiguous effect on the exploration effort. On the one hand, it increases the incentive to discover as it increases the share left to the firm. On the other hand, to increase the profit left after the cost oil payment, the firm has the incentive to decrease its exploration effort. Furthermore as \( q^* \) is independent from \( \alpha \) and \( \beta \), the contract only influences the level of exploration. We denote \( e_1(\alpha, \beta) \), the optimal exploration level induced by the first contract and defined by (1).

- Government’s optimization problem

\[
\begin{align*}
Max_{\alpha, \beta} U_G &= \rho(e_1)(1 - \beta) \left[ \int_{q_1}^{R} (1 - \alpha)pR f(R) \, dR + \int_{q_1}^{q^*} [pR - c_p(R)] f(R) \, dR \\
&+ \int_{q^*}^{R} [pq^* - c_p(q^*)]f(R) \, dR - \int_{q_1}^{R} c_e(e_1) f(R) \, dR \right] s/c U_F \geq 0 \quad (\lambda)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial L}{\partial \alpha} &= \frac{d e_1}{d \alpha} \left[ \rho'(e_1)B - \rho(e_1) \int_{q_1}^{R} c'_e(e_1)f(R) \, dR \right] - \rho(e_1)(1 - \lambda) \int_{q_1}^{R} pR f(R) \, dR = 0 \quad (2)
\end{align*}
\]

\[
\begin{align*}
\frac{\partial L}{\partial \beta} &= \frac{d e_1}{d \beta} (1 - \beta) \left[ \rho'(e_1)B - \rho(e_1) \int_{q_1}^{R} c'_e(e_1)f(R) \, dR \right] - \rho(e_1)(1 - \lambda)B = 0 \quad (3)
\end{align*}
\]

With \( B \) is such as \( U_G = \rho(e_1)(1 - \beta)B \).

Contrary to a standard taxation scheme, the government’s payoff is no longer strictly increasing with the exploration effort and the quantity produced. Indeed, the government under a PSA has to pay for the exploration cost if the cost oil is high enough and receives a share of the revenue net of costs when the costs are not
(2) and (3) are not compatible, this problem entails a corner solution.

If $\beta = 1$ then the government gets nothing, if $\beta = 0$, then the firm’s participation constraint is not satisfied and finally, $\alpha = 0$ is not compatible with $\alpha pq^* - c_p(q^*) - c_e(e_1) > 0$. Thus the solution is $\alpha = 1$.

The government would be better off with a higher exploration effort and at equilibrium an increase in the profit oil share left to the firm increases the exploration effort. Furthermore, the exploration effort is lower than without taxation. Indeed, using (2) and (3), one can show that at the equilibrium $\frac{de_1}{d\beta^*} > 0$. If $\frac{de_1}{d\beta^*} < 0$ then $\rho(e)B$ decreases with $e_2$. Thus, the government would be better off with a lower exploration effort. As $\frac{de_1}{d\alpha^*} > 0$, the government can always decrease $\alpha$ to decrease the exploration effort. In this case, both $\rho(e)B$ and the resources left for the profit oil would be higher, thus $\frac{de_1}{d\beta^*} < 0$ cannot be an equilibrium because in this case the government always has the incentive to decrease $\alpha$.

Using (1) and $\frac{de_1}{d\beta^*} > 0$, one can easily show $e_1 < e^*$.

**Proposition 1** The first contract is such as the cost oil is set to 100%. If the net revenue is negative, the firm gets all the revenue from the extraction and partly covers its cost. If the net revenue is positive, then, it is shared between the firm and the government. The firm produces the same quantity as without taxation but lowers its exploration effort. The government only gets a revenue when a discovery occurs and if the revenue from the extraction covers the costs.

This contract cannot be implemented only if $pq^* - c_p(q^*) - c_e(e_1) < 0 \iff pq^* - c_p(q^*) - c_e(e^*) < 0$. Even without taxation, the profit would have been negative. If the price and the technology are such as without taxation $(q^*,e^*)$ is possible, then the government can always implement this first PSA.

Compared to a standard taxation scheme, the quantity is not distorted and the quantity produced is higher. However, one cannot conclude whether the exploration effort is lower or higher than under a standard taxation scheme.

One may consider a special case where $q_1 < R$ meaning that the costs are always reimbursed. If a discovery occurs and the optimally extracts, the gains are always
positive \((pq - cp(q) - ce(e) > 0)\). In this case, the contract does not affect the production level and the exploration effort increases with the share of profit oil left to the firm. As we consider the \((COC)\) slack, an increase in \(\alpha\) has no effects on the production and exploration levels. However, \(\alpha\) should be high enough for the \((COC)\) to be satisfied. Furthermore, such a PSA can only be implemented if the price of the resource, the firm efficiency, the minimum level of discovery and \(\alpha\) are high enough.

In practice, the cost oil is viewed as an instrument to favor investment but if the cost oil is high enough, it has no impact on the production and exploration levels. If the share of resources allocated to costs is high enough for the costs to be entirely reimbursed while the firm produces the optimal quantity, increasing further the share allocated to costs has no effects. This PSA, is exactly the same as a proportional tax on revenue net of costs (exploration and production costs) where the exploration costs are reimbursed and \((1 - \beta)\) the tax rate.

**Proposition 2** If the cost oil is high enough, the firm efficient and the market conditions favourable, the cost oil has no impact on the production and exploration levels. If the costs are always reimbursed, increasing further the cost oil has no effects. The quantity produced is not distorted and the exploration level is lower than without taxation and can be lower or higher than under a standard taxation scheme.

### 3.2.2 Contract 2

In the second contract, \(\alpha\) and \(\beta\) are such as:

\[
\alpha pq^* - cp(q^*) - ce(e) < 0 \quad \text{and} \quad \alpha pq_{psa} - cp(q_{psa}) - ce(e) < 0
\]

\[
U_i = \int_{q_1}^{q_2} U_{ib}(R)f(R)\,dR + \int_{q_1}^{q_2} U_{ia}(R)f(R)\,dR + \int_{q_2}^{\bar{R}} U_{ib}(q_{psa})f(R)\,dR
\]

where \(i = F, G\)

The firm cannot produce such as the price equals the marginal production cost and be entirely reimbursed. If \(R \in [R, q_{psa}]\), the firm exhausts the stock as \((\alpha + (1 - \alpha)\beta)p - c_p'(R) > 0 \quad (\Leftrightarrow \ p - c_p'(R) > 0)\). If the stock is higher than \(q_{psa}\), the best strategy is to produce \(q_{psa}\).

- Firm’s optimization problem

\(\alpha < 1\) because if \(\alpha = 1\), then \(pq_{psa} - cp(q_{psa}) - ce(e) < 0\), and the profit is never positive.
\[ U_F = -c_e(e) + \rho(e) \left[ \int_{q_1}^{q_2} [(\alpha + (1 - \alpha) \beta)pR - c_p(R)]f(R) \, dR + \int_{q_1}^{q_2} [(\alpha + (1 - \alpha) \beta)p \rho(R) - c_p(R)]f(R) \, dR \right] \]

\[ + \int_{q_2}^{q_1} [(\alpha + (1 - \alpha) \beta)pR - c_p(R)]f(R) \, dR + \int_{q_2}^{q_1} [(\alpha + (1 - \alpha) \beta)p q_{psa} - c_p(q_{psa})]f(R) \, dR \]

\[ + (1 - \beta) \int_{q_1}^{q_2} c_e(e) f(R) \, dR \]

\[ \frac{\partial U_F}{\partial e} = -c'_e(e) + \rho'(e) C + \rho(e)(1 - \beta) \int_{q_1}^{q_2} c'_e(e) f(R) \, dR = 0 \quad (4) \]

with \( C \) is such as \( U_F = -c_e(e) + \rho(e) C \)

From (4), \( \alpha \) increases the level of exploration and production. \( \beta \) still has an ambiguous effect. Indeed, an increase in \( \beta \) increases the incentive to explore as both the share of resource left to the firm and the production \( q_{psa} \) increases, but it also decreases the incentive to explore to increase the net revenue when the costs are entirely reimbursed. Contrary to the first contract, the second contract has an impact on the production level, the higher the cost oil and the share of the profit oil left to the firm are, the higher \( q_{psa} \) is. We denote \( e_2(\alpha, \beta) \), the optimal exploration level induced by the second contract and defined by (4).

- Government’s optimization problem

\[ Max_{\alpha, \beta} U_G = (1 - \beta) \rho(e_2) \left[ (1 - \alpha) \int_{q_1}^{q_2} pR f(R) \, dR + \int_{q_2}^{q_1} p q_{psa} f(R) \, dR + \int_{q_2}^{q_1} p R f(R) \, dR \right] \]

\[ + \int_{q_2}^{q_1} [pR - c_p(R) - c_e(e_2)]f(R) \, dR \]

\[ s/c \quad U_F \geq 0 \quad (\lambda) \]

\[ \frac{\partial C}{\partial \alpha} = \frac{d e_2}{d \alpha} \left[ \rho'(e_2) C - \rho(e_2)c'_e(e_2) \int_{q_1}^{q_2} f(R) \, dR \right] + \rho(e_2)(1 - \alpha) \int_{q_2}^{q_1} \pi p \frac{d q_{psa}}{d \alpha} \, d\alpha \int_{q_2}^{q_1} f(R) \, dR \]

\[ + (\lambda - 1) \rho(e_2) E = 0 \quad (5) \]

\[ \frac{\partial C}{\partial \beta} = \frac{d e_2}{d \beta}(1 - \beta) \left[ \rho'(e_2) C - \rho(e_2)c'_e(e_2) \int_{q_1}^{q_2} f(R) \, dR \right] + \rho(e_2)(1 - \beta)(1 - \alpha) \int_{q_2}^{q_1} \pi p \frac{d q_{psa}}{d \beta} \, d\beta \int_{q_2}^{q_1} f(R) \, dR \]

\[ + (\lambda - 1) \rho(e_2) D = 0 \quad (6) \]

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With $D$ is such as $U_G = (1-\beta)\rho(e_2)D$ and $E = \int_0^{q_1} pR f(R) dR + \int_{q_2}^{q_{psa}} pR f(R) dR + \int_{q_{psa}}^{\bar{R}} pq_{psa} f(R) dR$

One can show using (5) and (6) that $\frac{de_2}{d\beta^*} < 0$

At the equilibrium, an increase in $\beta^*$ decreases the exploration effort but, we cannot conclude whether, the exploration level is lower or higher than the one without taxation. Furthermore, the government would have been better off with a lower exploration effort but it cannot decrease the exploration effort without costs. The government faces a trade off between the exploration level desired, the production level and the share of the profit oil it gets. By offering a favorable contract to the firm to increase the firm’s incentive to produce, the government may induce a too high exploration effort.

Proposition 3 By setting a relatively low cost oil, the government can induce a distorted extraction (lower than $q^*$) and the exploration effort can be lower or higher than the ones with a standard taxation scheme or without taxation. In this case, the exploration effort increases with the cost oil and at the equilibrium decreases with share of the profit oil left to the firm. Furthermore, both the share of profit oil left to the firm and the cost oil increases the production level.

One may consider a special case where, $R > q_2$ meaning that the costs are never fully reimbursed. In this case, the contract is similar to a taxation scheme and $(1-\alpha)(1-\beta) = \tau^*$. The government gets exactly the same utility as if a taxation scheme was used. For example, if $\tau^* = 50\%$, several PSA can be used such as $\alpha' = 30\%$ and $\beta' = 28\%$ or $\alpha'' = 10\%$ and $\beta'' = 44\%$.

By setting a really low cost oil, the government is able to replicate the same exploration and extraction levels induced by a taxation scheme. In this case, $\beta$ should be sufficiently high to reward the firm and reach the exploration and production levels desired. This contract can always be used, and implies a really low cost oil. Under this PSA, the cost oil and the share of profit oil left to the firm are substitute instruments except that the cost oil has to be low enough for the (COC) to be satisfied. As in the standard taxation scheme, the government faces a trade off between leaving the firm with a low share of resource and having low levels of exploration and production.

Proposition 4 If the cost oil is low enough (the costs are never fully reimbursed),
the PSA induces the same exploration and extraction levels as in a taxation scheme: 
\((1 - \alpha_b)(1 - \beta_b) = \tau^*\). Both the profit and the cost oil increase the production and exploration levels. Furthermore, the share of profit oil left to the firm should be high enough to compensate for the low cost oil.

For the government, delegating the oil field exploitation through a PSA, can be at least as well as through a taxation scheme. For very low values of \(\alpha\) (contract 2), the PSA induces the same exploration and production levels as in a taxation scheme. Hence, as soon as the government chooses a PSA that induces another exploration and extraction levels, this implies that this PSA leaves the government with a higher utility than a standard taxation scheme.

**Proposition 5** If the cost oil is really low (the costs are never fully reimbursed), the PSA can induce the same exploration and extraction levels than the ones induced by a taxation scheme. Hence, an optimal PSA can do at least as well as an optimal taxation scheme.

### 3.2.3 Contract 3

In the third contract, \(\alpha pq^* - c_p(q^*) - c_e(e) < 0\) and \(\alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0\)^7.

\[
U_i = \int_{q_1}^{q_2} U_{ib}(R) f(R) dR + \int_{q_1}^{q_2} U_{ia}(R) f(R) dR + \int \left( U_{ia}(q_2) f(R) - U_{ib}(q_2) f(R) \right) dR \text{ where } i = F, G
\]

The firm cannot produce such as the price equals the marginal production cost and be entirely reimbursed. Furthermore, if the firm produces \(q_{psa}\), its costs are entirely reimbursed and it gets a share \(\beta (pq_{psa} - c_p(q_{psa}))\). Thus, the firm has the incentive to produce as much as possible until the cost oil is reached and as long as the price is higher than the marginal production cost. When \(R \in [q_2, \bar{R}]\) if the firm produces higher quantity than \(q_2\) its costs are not entirely reimbursed and thus the optimal level of production is \(q_2\).

- Firm’s optimization problem

\[
U_F = -c_e(e) + \rho(e) \left[ \int_{q_1}^{q_2} [(\alpha + (1 - \alpha)\beta)pR - c_p(R)] f(R) dR + \int_{q_1}^{q_2} \beta[pR - c_p(R)] f(R) dR \right]
\]

^7\(\alpha \in [0, 1]\), if \(\alpha = 1\), then \(pq^* - c_p(q^*) - c_e(e) < 0\) and the profit is never positive and if \(\alpha = 0\) is not compatible with \(\alpha pq_{psa} - c_p(q_{psa}) - c_e(e) > 0\)
\[
\frac{\partial U_F}{\partial \varepsilon} = -c_e'(e) + \rho'(e)F + \rho(e) \left[ \int_{q_1}^{q_2} \beta(p - c'_p(q_2)) \frac{dq_2}{de} f(R) dR \right] \\
+ \frac{1}{\alpha} \left[ \int_{q_1}^{q_2} pR f(R) dR + \int_{q_1}^{q_2} [pR - c_p(R)] f(R) dR \right] = 0 \quad (7)
\]

By setting its exploration effort, the firm has to take into account the effect on the production level. Indeed, an increase in the exploration effort decreases the production level \(q_2\). When the cost oil equals the total costs, there is a trade off between the production and the exploration levels. \(\alpha\) and \(\beta\) have an ambiguous effect on the exploration effort.

- Government’s optimization problem

\[
Max_{\alpha,\beta} U_G = (1 - \beta) \rho(e) \left[ \int_{q_1}^{q_2} (1 - \alpha)pR f(R) dR + \int_{q_1}^{q_2} [pR - c_p(R)] f(R) dR \right] \\
+ \frac{1}{\alpha} \left[ \int_{q_1}^{q_2} pq_2 - c_p(q_2) \right] f(R) dR - \int_{q_1}^{q_2} c_e(e) f(R) dR \right] \quad s/c \quad U_F \geq 0 \quad (\lambda)
\]

\[
\frac{\partial U}{\partial \alpha} = \frac{d e}{d \alpha} \left[ \rho'(e)G - \rho(e)c'_e(e) \int_{q_1}^{q_2} f(R) dR \right] \\
+ \rho(e)(1 - \beta) \left[ - \int_{q_1}^{q_2} pR f(R) dR + \int_{q_1}^{q_2} (p - c'_p(q_2)) \frac{dq_2}{d\alpha} f(R) dR \right] \\
+ \lambda \rho(e) \left[ (1 - \beta) \int_{q_1}^{q_2} pR f(R) dR + \int_{q_1}^{q_2} \beta[p - c'_p(q_2)] \frac{dq_2}{d\alpha} f(R) dR \right] = 0 \quad (8)
\]

\[
\frac{\partial U}{\partial \beta} = \frac{d e}{d \beta} \left[ \rho'(e)G - \rho(e)c'_e(e) \int_{q_1}^{q_2} f(R) dR \right]
\]
\[ + \rho(e)(1 - \beta) \left[ \int_{q^2}^{q_R} (p - c_p'(q)) \frac{dq_2}{d\beta} f(R) \, dR \right] + (\lambda - 1)\rho(e)G = 0 \quad (9) \]

With \( G \) is such as \( U_G = (1 - \beta)\rho(e)G \)

**Proposition 6** For intermediate value of the cost oil, the PSA can induce the firm to produce and explore such as the cost oil equals the total costs. Thus the firm faces a trade off between exploring and producing. Both the profit oil left to the firm and the cost oil have ambiguous effects on the production and exploration levels. However, by lowering the cost oil, the government should be able to avoid this case.

## 4 Conclusion

This paper shows that the PSA can do at least as well as a taxation scheme. Using a PSA, the government can induce different exploration and extraction levels depending on the share of resources allocated to the cost reimbursement and the one allocated to the firm. By setting a really low cost oil, the government is able to replicate the taxation scheme. By setting the cost oil to 100\%, the government is able to induce the same extraction level as if there is no taxation and a lower exploration level. Furthermore, there exists a threshold of \( \alpha \) beyond which increasing the cost oil has no effect on the exploration effort. Finally, the PSA can give the incentive to the firm to produce and explore such as the costs equal to the cost oil.

We assume that the firm and the government are risk neutral. However, it could be interesting to relax this assumption. Indeed, the government may be risk averse, especially in developing countries where the fiscal revenue from petroleum activities is high. Furthermore, the large international oil companies are able to diversify their portfolios by investing in fields with different characteristics (risk-return), and may be less risk averse than the government. In our paper, delegation to international oil companies was needed because of the government’s lack of efficiency in petroleum activities. However, if the government is risk averse, even if it is as efficient as the firm, it may want to delegate to avoid the exploration risk.

Finally, we could introduce uncertainty on price or on costs. Under a PSA, this information is crucial, as those exogenous variables are used to differentiate between the 3 contracts. In the second contract, the cost oil must be such as \( \alpha p_{q_{psa}} - c_p(q_{psa}) - c_e(e_2) < 0 \), if the price or the costs are different from their expected value and such as \( \alpha p_{q_{psa}} - c_p(q_{psa}) - c_e(e_2) > 0 \), then firm produces and
explores such as its costs equal the cost oil. As the game is sequential, when the government chooses the contract to implement, it may have less information than when the firm chooses its exploration and production levels.

References


