Entry of New Drugs, Optimal Insurance Coverage and Reference Pricing Regulation

David Bardey*, Bruno Jullien† and Jean-Marie Lozachmeur‡

March 15, 2012

Abstract

Internal reference pricing for pharmaceuticals is now a widely used regulation tool for the pricing and reimbursement of pharmaceuticals. This type of regulation is usually used as a complement of copayment rates to decrease pharmaceutical expenditures. In this paper we determine, from a social planner's point of view, the optimal policy mix between reference pricing and copayment rate when drugs are horizontally differentiated. Our results show that in the short run *i.e.* for a fixed number of drugs, the copayment system is chosen independently of the reference price system and the reference pricing should be maximal. However, in the long run, there may be room for less than maximal reference pricing. This occurs when the equilibrium number of drugs in the therapeutic class is

*University of Rosario (Bogota) and Toulouse School of Economics.
†Toulouse School of Economics (IDEI and GREMAQ-CNRS).
‡Toulouse School of Economics (IDEI and GREMAQ-CNRS).

too small and patients would be better off with more differentiated drugs.

Jel codes: I18, L11, L15, L51.

1 Introduction

The pharmaceutical expenditures have dramatically increased in most developed countries during the two last decades. From 1995 to 2005, the real annual growth rate in pharmaceutical spending has been 4.6% on average in OECD's countries. Pharmaceutical expenditures represent on average 1.5% of the GDP in OCDE's countries in 2005. This trend is mainly due to an explosion of expenditure in *R&D* and its risen cost (Pammolli and Riccaboni, 2004). Over the last years, several countries - such as Germany, Italy or Australia - have tried to reduce pharmaceutical expenditures by adopting a reference pricing regulation. The common feature of reference pricing regulations is that drugs are divided into different therapeutic classes according to their active agents and/or their indications. While modalities vary,[1] internal reference pricing (as opposed to external reference pricing) consists in determining a reference price as a weighted sum of drugs' prices adopted in the same therapeutic class. If the price of a drug is higher than this reference price, patients pay the full difference between the price of the drug and the reference price. As a consequence, this regulatory scheme is often perceived as a complement to copayment rates in order to encourage patients in consuming low price drugs. In a static view, where no innovation or drugs entries are considered, reference pricing mechanically generates a fall in drug reimbursement and prices. In the long run, where innovation by pharmaceutical firms is driven by the prospect of future profits,

[1] See Lopez-Casanovas and Puig-Junoy (2000), Danzon (2001) and Danzon and Ketcham (2004) for more details on reference pricing and its applications.

the impact of such a regulation is far from being obvious.

The goal of this paper is to determine, from a social planner's point of view, the optimal policy mix between reference pricing and copayment rates. We build a model where there are several pharmaceutical firms selling an horizontal differentiated drug that belong to the same therapeutic class. In other words, all the patients value differently the drugs because of their different secondary effects. In line with the Germany's modality, we consider a reference pricing system that consists in a combination of extreme value prices in the market. We call maximal reference pricing a system where the reference price is the minimum price of the class. On the contrary, minimal pricing reference describes a system in which the reference price is equal to the maximum price adopted in the class (this corresponds to the case where there is no reference pricing at all.). On the top of this price regulation mechanism, risk averse consumers benefit from an insurance plan consisting in a premium and a linear copayment rate.[2] Our results show that in the short run, the copayment rate is chosen independently of the reference price regulation and the reference pricing should be maximal. Indeed, as long as the number of available drugs in the therapeutic class is fixed, the social planner is only willing to lower drugs prices since he cannot improve the drugs diversity. However, in the long run, there may be room for less than maximal reference pricing. This occurs when the number of products in the therapeutic class is too small. The desirable level of reference pricing is the result of a trade-off between an adverse effect - a new drug decreases adverse

[2] According to the fact that we consider unitary consumption, quantities do not matter.

or secondary effects - and a fixed cost effect generated by drugs' entries. We particularly emphasize the role of risk aversion by means of two illustrating examples: the higher is the level of risk aversion, the more likely it is desirable to have less than maximal reference pricing

Our paper is related to and mainly borrows from two stands of literature. The first one deals with the relationship between drug price regulation and the resulting equilibrium, static or dynamic, in the drugs markets. The second one investigates the interplay between insurance coverage and innovation in the pharmaceutical sector.

In line in the first stand, Brekke *et al.* (2007) develop a general set-up containing horizontal and vertical differentiations. They consider a two reference pricing modalities, namely generic and therapeutic reference pricing, both associated to a copayment rate. Their aim is to compare the equilibrium allocations with and without reference pricing. They reveal that the therapeutic reference pricing modality leads to the lowest prices. From this result, they show that the therapeutic reference pricing makes entry of new drugs less likely.

In a model that distinguish between pioneer and followers drugs but that does not consider generic, Bardey *et al.* (2010) evaluate the long run impact of reference pricing on pharmaceutical innovation, delays of introduction, patients' health and expenditures. These authors show that reference pricing regulation allows to lower prices and therefore delays pioneer drugs and me-toos entries. Nevertheless, as me-toos are more delayed, it may favor costly pioneer drugs and may increase health expenditures in the long run. This result is also men-

tioned in Pammolli and Riccaboni (2004) who explore the interplay between technological advances and cost containment policy. They conclude that reference pricing regulation reduce the rents coming from horizontal differentiation and favor arrivals major of breakthroughs. Kyle (2005) explores the empirical puzzle of the pharmaceuticals entry patterns. She finds that laboratories which invest in *R&D* in countries that use price control reach fewer markets and with longer delays. She concludes that price controls have a significant and strong impact on entries into markets.

In line with the second stand, *i.e.* without considering regulation on drugs prices, Lakdawalla and Sood (2005) explain that health insurance coverage resembles a two-part pricing contract. Indeed, *ex-ante* policy holders pay an up-front fee in exchange for a fixed unit price paid *ex-post*. Even when moral hazard behaviors are present, they reveal that a complete and competitive health insurance for innovative drugs can lead to an optimal innovation amount. Garber *et al.* (2006) extends Lakdawalla and Sood's analysis when the monopoly innovator can only charge linear prices. Their results show that when copayment rate ensures optimal utilization, laboratories may have excessive incentives to innovate and it can be optimal to limit drugs prices.

The aim of this present paper is to combine the previous stands mentioned in a same framework. More precisely, we resume the Miraldo (2007)'s issue to understand the interplay between copayment and reference pricing. We investigate how interacts the optimal health insurance coverage and the optimal drug price regulation in a long run perspective, following Lakdawalla and Sood (2005) and

Garber *et al.* (2006). The next section presents the set-up. Section 3 is devoted to the drugs price competition model in a context of horizontal differentiation when reference pricing regulation and copayment coexist. Section 4 characterizes the optimal regulation in the short and the long run. Section 5 is devoted to comparative static analysis. Lastly, section 6 concludes and discusses some possible extensions.

2 The set-up

Consider a collection of policyholders that can be ill with a probability π and healthy with probability $1-\pi$. We normalize the size of the population to 1. Policyholders are indexed by j uniformly distributed over $J \equiv [0,1]$. In case of illness, patients choose a drug among a continuum of N treatments, denoted by a subscript $i \in I \equiv (1,N)$. When choosing a drug i in case of illness, a patient's net income is $w_s^i = w - \rho - p_i$ where w is an exogenous income, ρ denotes the premium paid to the insurance scheme and p_i is the net price paid -*i.e.* the out-of-pocket amount- for drug i. Denoting by a the copayment rate (and thus $1-a$ is the reimbursement rate) and P_i the retail price of drug i, one has $p_i = aP_i$. When healthy, the net income is $w_h = w - \rho$. In the state of illness, policyholders are horizontally differentiated: when consuming drug i, a policy holder j is affected by an adverse or secondary effect which depends on a individual stochastic variable $\tilde{x}_j^i \in X$, observed by the policy holder before consumption takes place. The distribution of $X^N \equiv \left(\tilde{x}_j^1, \tilde{x}_j^2, ..., \tilde{x}_j^N\right)$ may depend

on j and N, but as we shall see in a symmetric way.

The policy holder j's expected utility when consuming drug i is:

$$U = \pi u\left(w_s^i - L\left(\tilde{x}_j^i\right)\right) + (1-\pi)u(w_h),$$

where the function $L\left(\tilde{x}_j^i\right)$ represents the monetary equivalent loss incurred by a patient j consuming drug i.

At a symmetric equilibrium where $p_i = p$ for every $i \in (1, N)$, consumers minimize the loss so that the aggregate expected utility function writes:

$$EU = \pi E\left[u\left(w_s - \min_{i \in I} L\left(\tilde{x}_j^i\right)\right)\right] + (1-\pi)u(w_h). \tag{1}$$

where E is the expectation operator over $J \otimes X^N$. It is worth stressing that our model can be interpreted in two ways. Either individuals know their type j before the risk occurrence and the aggregate utility represents a utilitarian objective. Either individuals do not know their type before the risk occurrence and the aggregate utility represents their expected utility when individuals are risk neutral towards their type.

Throughout the paper, we use an exponential VNM utility function of the CARA form:

$$u(\omega) = -\exp(-\sigma\omega),\ k = h, s,$$

where σ represents the absolute risk aversion parameter and ω represents the

net wealth. Equation (1) thus yields:

$$EU = -\pi \exp\left(-\sigma\left(w_s - \min_{i\in I} L\left(\tilde{x}_j^i\right)\right)\right) - (1-\pi)\exp(-\sigma w_h),$$

so that, for any symmetric equilibrium prices -*i.e.* $p_i = p$- the aggregate utility is:

$$EU = -\pi\phi\left(N,\sigma\right)\exp\left(-\sigma w_s\right) - (1-\pi)\exp(-\sigma w_h), \tag{2}$$

where $\phi\left(N,\sigma\right) = E\exp\left(\sigma \min_{i\in I} L\left(\tilde{x}_j^i\right)\right)$. For analytical tractability, we state the following assumption on the distribution of secondary effects.[3]

Assumption E $L\left(\tilde{x}_j^i\right) = \theta\tilde{x}_j^i$ are identically and independently distributed across policyholders and drugs over $\mathbb{R}+$ and follow an exponantial distribution with distribution function $F\left(x\right) = 1 - \exp\left(-\tau x\right)$.

COMMENTS ON THE ROLE OF τ?

When this assumption is fulfilled, the probability that a drug $k \in I$ consumed by patient j has the minimum adverse effect is

$$\Pr\left(\min_{i\in I} L\left(\tilde{x}_j^i\right) = x_j^k\right) = \left(1 - F\left(x_j^k\right)\right)^{N-1}.$$

Integrating over all the possible realizations of the random variable x_j^k, one thus

[3] In section 4.2, we provide necessary conditions on the distribution function that generalize our main result.

has

$$\begin{aligned}\phi(N,\sigma) &= N\int_0^{\infty} f(x)\exp(\sigma\theta x)\left(1-F(x)\right)^{N-1}dx \\ &= N\int_0^{\infty}\tau\exp\left(\left(\sigma\theta-N\tau\right)x\right).\end{aligned}$$

Provided that $\tau N > \theta\sigma$, in such a case, one finally obtains

$$\phi(N,\sigma) = \frac{\tau N}{\tau N-\sigma\theta}. \tag{3}$$

Ceteris paribus, note that higher is the number of drugs avalaible in the therapeutic class, and higher are the patients' welfare due to a decrease of the secondary effects. On the contrary, the negative consequences of secondary effects on patients' welfare both increase in the differentiation and in the risk aversion parameters.

3 Price competition in the pharmaceutical markets

We assume in this section that an equilibrium exists and that the market is fully covered. Roughly speaking, all policyholders choose a drug in the therapeutic class when ill[4]. The case in which patients may be excluded from the drugs' market is relegated section XX. Thus, the demand for one drug is the mass of

[4] This assumption is quite realistic in countries chacterized by high levels of coverage, *i.e.* low levels of out-of-pockets.

individuals who prefers this drug to any other. We then focus on symmetric equilibria where firms set the same prices. Consider any drug $i \in (1, N)$ and suppose that the net out-of-pocket paid by patients for all other drugs is p. Our assumptions on the utility functions (in particular, the fact that the utility of ill patients are symmetric and quasi-linear in prices) then implies that the demand D_i when the out-of-pocket paid for consuming drug i is p_i, can be expressed as follows at any symmetric equilibrium:

$$D_i = D\left(p_i - p, N\right),$$

where D is a function independent of i satisfying $D\left(0, N\right) = 1/N$ and $\partial D/\partial p_i < 0$ for any p_i, p and N. [5]

Let us now consider an internal reference pricing regulation. The reference price is given by a weighted sum of the minimal and the maximal gross price of the drugs in the market. Formally, at any symmetric equilibrium, the reference price is given by:

$$R = r \inf\left\{P_i, P\right\} + (1 - r) \max\left\{P_i, P\right\}, \tag{4}$$

where $r \in [0, 1]$ is the weight put on the minimal price. P_i is the gross price of drug i while P is the gross price of the other drugs present in the market. When $r = 1$, there is "maximal" reference pricing. This is the most stringent policy

[5] More formally $D\left(p_i - p, N\right)$ is the probability that the "total loss" $L\left(\tilde{x}_j^i\right) + p_i$ be less that $\min_{l \neq i} L\left(\tilde{x}_j^l\right) + p$.

from the perspective of pharmaceutical firms. On the contrary, when $r = 0$, there is no reference pricing.[6] In other words, all the treatments are reimbursed at a rate $1 - a$.

By definition, the out-of-pocket paid by patients is defined by:

$$p_i = P_i - (1 - a) \min [P_i, R]$$

so that,

$$p_i - p = P_i - P - (1 - a) (\min [P_i, R] - \min [P, R])$$

Using the definition of R in equation (4) this yields:

$$(p_i - p) = [a + (1 - a)r] (P_i - P),$$

so that the total demand for drug i is implicitly given by:

$$D_i = D (\alpha (P_i - P), N),$$

where $\alpha = a + (1 - a)r \in [a, 1]$. α measures the distortion introduced by the insurance system on the market for drugs. At given prices, decreasing α amounts to increase the amount of the bill reimbursed by health insurance. We shall refer to α as the "pass-through rate" resulting from the joint regulation of insurance coverage and reference pricing. This rate determines the fraction of the price P

[6] For instance, in Germany r is set to 2/3.

which is actually paid by individuals.

Laboratories compete in prices. Therefore we have:

$$P_i \in \arg\max P_i D\left(\alpha\left(P_i - P\right), N\right).$$

As the symmetric equilibrium yields $D_i = 1/N$, we obtain:

Proposition 1 *The drug price in a symmetric equilibrium is* $P = \varphi(N)/\alpha$ *where* $\varphi(N) = -D(0,N)/\left(\partial D(0,N)/\partial p_i\right)$ *is the inverse semi-elasticity of the residual demand and* α *is the "pass through rate".*

It is worth noticing that the equilibrium obtained is identical to the one in a standard general model of horizontal differentiation in which the differentiation parameter would be $\varphi(N)/\alpha$. Here, the pass-through rate increases in the weight put on the minimal price and in the copayment rate. In other words, decreasing the insurance coverage or relaxing the strength of reference price regulation lead to a lower price competition intensity. Our model can also be viewed as an *expost* moral hazard analysis, where instead of coming from a traditional quantity effect, *i.e.* patients consume more treatments when they are benefit from a higher level of coverage, the patients' overconsumption is due to a price effect. In other words, the reference princing regulation as the level of coverage both affect the drug price, changing the patients' level of drug consumption.

Lemma 2 *Under assumption 2, at a symmetric equilibrium, the drugs' price*

and the laboratories' profit are respectively:

$$P = \frac{\theta}{\tau (N-1) \alpha}$$

and,

$$\Pi (N) = \frac{\pi \varphi (N)}{\alpha N} = \frac{\pi \theta}{\alpha \tau N (N-1)}. \tag{5}$$

Proof. With assumption 2, the probability that a drug k is chosen by a patient j is given by

$$\begin{aligned} \Pr\left(-p_k - \theta x_j^k \leq -p_l - \theta \tilde{x}_j^l\right) &= \Pr\left(\tilde{x}_j^l \geq \frac{p_k - p_l}{\theta} + x_j^k\right) \text{ for every } l \neq k \in I \\ &= \left(1 - F\left(x_j^k\right)\right)^{N-1}. \end{aligned}$$

Integrating over all possible realizations of x_j^k, the expected demand of drug k is thus:

$$\begin{aligned} D_k &= \int_0^\infty f(x) \left(1 - F\left(\frac{p_k - p_l}{\theta} + x\right)\right)^{N-1} dx \\ &= \frac{1}{N} \exp\left(-\frac{\tau (N-1)}{\theta} (p_k - p_l)\right). \end{aligned}$$

It yields $\varphi (N) = \theta / (\tau (N-1))$. Q.E.D. ■

The closed form solution obtained under assumption 2 is useful to point out in a more obvious way the different effects at work. *Ceteris paribus*, the differentiation parameter θ increases the drugs' price and the profit obtained by laboratories at a symmetric equilibrium. However, the overall differentiation

$\varphi(N)$ decreases in the number of drugs available in the therapeutic class.

4 Optimal public policy

In a first section, we determine the optimal policy in a static framework, considering the market structure as given *i.e.* the number of drugs is fixed. Next, we characterize the long run equilibrium in which the number of drugs present in the market is endogenous.

4.1 Exogenous market structure

The social objective is to maximize the policyholder's expected utility as defined in (2). Note that the optimum can be decentralized in two institutional *scenarii*. The first one corresponds to a regulator who designs simultaneously the reference price and the copayment rate. The second one is a two stage game where the regulator first fixes the reference price and in a second stage, the insurer (either public or private) chooses the insurance policy. This last stage may correspond to the result of a perfect competition on the health insurance market with a positive loading factor or to the social insurance policy subject to a budget constraint in presence of distortive taxes.[7]

The resource constraint imposes $\rho = \pi(1-a)(1+\lambda)P$ where λ is to be interpreted as a loading factor if the insurer is private or as a shadow cost of

[7] Of course the second scenario is equivalent to the first one as long as insurers offer health insurance contract before patients know their type.

public fund if the insurer is public. The objective of the regulator is thus

$$\begin{aligned}
&\max_{a,z} EU = (1-\pi)u(w_h) + \pi\phi\left(N,\sigma\right)u\left(w_s\right) \\
&\text{s.t } \rho = \pi(1+\lambda)(1-a)\varphi\left(N\right)/\alpha, \\
&a \in (0,1)\,,\ \alpha \in (a,1)\,.
\end{aligned}$$

The first order conditions with respect to a and α respectively yield:

$$\begin{aligned}
\frac{\partial EU}{\partial a} &= \left[(1-\pi)\,u'\left(w_h\right) + \pi\phi\left(N,\sigma\right)u'(w_s)\right]\pi(1+\lambda)\varphi\left(N\right)/\alpha - \pi\phi\left(N,\sigma\right)\varphi\left(N\right)/\alpha u'\left(w_s\right) \leq (\mathbf{6}) \\
\frac{\partial EU}{\partial \alpha} &= \frac{\pi\phi\left(N,\sigma\right)\varphi\left(N\right)u'(w_s)}{\alpha^2}\left[\left(\frac{(1-\pi)\,u'(w_h) + \pi\phi\left(N,\sigma\right)u'(w_s)}{\phi\left(N,\sigma\right)u'(w_s)}\right)(1+\lambda)\,(1-a) + a\right] \geq 0
\end{aligned}$$

Lemma 3 *There exist a threshold value of $\tilde{\lambda}$ such that for $\lambda < \tilde{\lambda}$, $a^* = 0$.*

Proof. The first order condition in a can be rewritten

$$\begin{aligned}
\frac{\partial EU}{\partial a} &= \left[(1-\pi)\,u'\left(w_h\right) + \pi\phi\left(N,\sigma\right)u'(w_s)\right](1+\lambda) - \phi\left(N,\sigma\right)u'\left(w_s\right) \\
&= (1-\pi)\left[u'\left(w_h\right) - \phi\left(N,\sigma\right)u'(w_s)\right] + \lambda\left[(1-\pi)\,u'\left(w_h\right) + \pi\phi\left(N,\sigma\right)u'(w_s)\right].
\end{aligned}$$

For $\lambda = 0$, we have

$$\frac{\partial EU}{\partial a} = (1-\pi)\left[u'\left(w_h\right) - \phi\left(N,\sigma\right)u'(w_s)\right].$$

Assuming an interior solution would yield

$$\begin{aligned} u'(w_h) &= \phi(N,\sigma)\, u'(w_s) \\ &\Longleftrightarrow \\ \frac{u'(w_h)}{u'(w_s)} &= \phi(N,\sigma) \\ &\Longleftrightarrow \\ \exp(-\sigma a p) &= \phi(N,\sigma) \\ &\Longleftrightarrow \\ -\sigma a p &= \ln \phi(N,\sigma). \end{aligned}$$

As $\phi(N,\sigma) > 1$, it yields a contradiction. For $\lambda \to \infty$, $\partial EU/\partial a \to \infty$. The intermediate value theorem implies the result of Proposition X. ■

This result means that for $\lambda < \tilde{\lambda}$, the increase of the premium is not sufficient to require a positive copayment rate. It is due to the fact that the benefit coming from the risk smoothing dominates the premium increase when adverse effects are taken into account. On the contrary, for higher values of the distortions, i.e. $\lambda > \tilde{\lambda}$, this increase of the premium generates stronger consequences on the premium and a strictly positive copayment becomes necesarry to outweigh the risk smoothing and the premium effects. In such a case, the first order condition in α becomes

$$\frac{\partial EU}{\partial \alpha} = \pi\varphi(N)\left(\frac{(1-\pi)\, u'(w_h) + \pi\phi(N,\sigma)\, u'(w_s)}{\alpha^2}\right)(1+\lambda) > 0$$

We will call such situation Regime A that consists in two corner solutions and summarized in the following proposition.

Proposition 4 *Regime A: The optimal policy in Regime A is characterized by* $a = 0$ *and* $r = 1$.

Let us now assuming that $\lambda > \tilde{\lambda}$ and consequently that the first order condition with respect to a yields an interior solution. Rearranging (6) give:

$$EU' = \frac{\phi(N,\sigma)\, u'(w_s)}{1+\lambda}, \tag{7}$$

where $EU' = (1-\pi)\, u'(w_h) + \pi\phi(N,\sigma)\, u'(w_s)$ is the policyholders' expected marginal utility of income. Using (7) , this implies that $\partial EU/\partial\alpha = \pi\phi(N,\sigma)\, u'(w_s)\theta(N)/\alpha^2 < 0$. In other words, the optimal α is equal to its maximal value i.e. $r = 1$. The optimal policy in such a context is named Regime B and is characterized in the following proposition:

Proposition 5 *For* N *fixed and* $\lambda > \tilde{\lambda}$*, the optimal regulation in Regime B is characterized by:*

(i) The optimal copayment rate a *is given by* (7).

(ii) The reference pricing weight is $r = 1$.

The optimal copayment rate, determined by condition (7), is the result of a trade-off between risk pooling (ex-ante efficiency) and an increase in the size of the risk (ex-post efficiency)[8]. An increase in a reduces the (desirable) risk

[8] See Geoffard (2006) for a complete analysis dealing with this trade-off.

pooling for risk averse policyholders but also diminishes the size of the risk by reducing the price paid by patients. Consequently, for a given market structure i.e. N fixed, the weight r does not generate any distortions whereas the copayment rate is determined according to a standard trade-off to mitigate ex post moral inefficiencies. Then, in a static framework, it is always optimal to set the strongest reference pricing i.e. $r = 1$ so as to decrease the price and lower the risky loss magnitude.[9] It should be noted that profits are not taken into account in the welfare criterion. We make this assumption in order to make a parallel between the short and the long run analysis because in the latter, profits are equal to zero. If profits were taken into account in the welfare objective, there may be some room for less than maximal reference pricing if the social weight put on the level of profits is high enough relatively to the shadow cost of public fund λ.

4.2 Endogenous market structure

In this section, we endogeneize the market structure in the therapeutic class and we focus on the free-entry equilibrium that occurs in the long run. Laboratories enter into the market as long as the profits earned in the drug market outweigh a sunk cost $K(N)$ generated by the R&D activities. For the entry equilibrium to be well defined we assume that:

Assumption FE $NK(N)/\varphi(N)$ is increasing with N.

[9] Usually in *ex-post* moral hazard analysis dealing with health insurance contracts, the increase in the size of the risk comes from a quantity effect. Here, the mechanism at work is similar but is due to a price effect.

This assumption ensures that a strictly positive number of laboratories enter the market. Equalizing the sunk cost to equilibrium profits given by equation (5), the number of firms for a free-entry equilibrium is implicitly given by (up to the integer):

$$K(N) = \frac{\pi\varphi(N)}{\alpha N}.$$

Therefore, the optimal policy is now the solution of the following program:

$$\begin{aligned}
&\max_{a,\alpha} EU = (1-\pi)u(w_h) + \pi\phi(N,\sigma)u(w_s) \\
&\text{s.t } K(N) = \pi\varphi(N)/\alpha N \\
&\alpha \in [a,1].
\end{aligned}$$

Using the free-entry equilibrium condition, the problem amounts to solve:

$$\begin{aligned}
&\max_{a,N} EU = (1-\pi)u\left(w - (1+\lambda)(1-a)NK(N)\right) \\
&+\pi\phi(N,\sigma)u\left(w - (1+\lambda)(1-a)NK(N) - NK(N)\frac{a}{\pi}\right), \\
\text{s.to} \quad &(\mu_1)\ \ NK(N) - \pi\varphi(N) \geq 0, \\
&(\mu_2)\ \ aNK(N) - \pi\varphi(N) \leq 0,
\end{aligned}$$

where μ_1 and μ_2 are the Lagrangian multipliers respectively associated to the lower and upper bound constraints on z. In the following, we first illustrate the interior optimum and then provide a necessary and sufficient condition for an interior solution to exist.

4.2.1 Optimum

Assume first that $\mu_1 = \mu_2 = 0$. The first-order conditions with respect to a and N give respectively:

$$EU'(.) = \frac{\phi(N,\sigma)\, u'(w_s)}{1+\lambda}, \tag{8}$$

$$\pi \frac{\partial \phi(N,\sigma)}{\partial N} u(w_s) = \left[NK'(N) + K(N)\right](1+\lambda)\, EU'(.) \tag{9}$$

Equation (8) describes the same trade-off between risk pooling and size of the risk in the preceding section, but obvioulsy taken at a different value of N. The second equation captures the trade-off between the social benefits and costs generated by the entry of a new drug, respectively described by the LHS and the RHS of equation (**??**). The social marginal cost is simply the welfare effect of an additional sunk cost, weighted by the social cost of public fund. It affects all policy holders, in both states of the world. We call it fixed cost effect. The marginal benefit $\pi \partial \phi(N,\sigma)/\partial N$ captures the marginal reduction of adverse effects generated by a new drug entry. We name it adverse effect. In the following, we analyze whether the optimal number of drugs is interior or not, i.e. wether there is less than maximal reference pricing or not.

Denote $\underline{N}$ the number of drugs given implicitely by the free entry condition $\underline{N} K(\underline{N}) = \pi\theta(\underline{N})$ i.e. where $r = 1$. It is worth stressing that this number is independant of the level of absolute risk aversion σ. Substituting (9) in (8), a necessary condition for an interior solution for r is

$$\frac{\partial EU}{\partial N} = \pi \frac{\partial \phi(\underline{N}, \sigma)}{\partial N} u(w_s) - \left[\underline{N} K'\left(\underline{N}\right) + K\left(\underline{N}\right)\right] \phi\left(\underline{N}, \sigma\right) u'(w_s) > 0 \qquad (10)$$

We can now state the following proposition:[10]

Proposition 6 *Suppose that* $K(N) = kN^{\epsilon}$ *and assumption2 is fulfilled. There is less than maximal reference pricing if* $\sigma \in \left(\bar{\sigma}, \frac{\tau}{\theta}\underline{N}\right)$ *where*

$$\bar{\sigma} = \frac{\tau}{\theta}\left(\frac{1 + \epsilon \underline{N}}{1 + \epsilon}\right),$$

where $\underline{N} > 1$ *is implicitley given by*

$$\underline{N}^{\epsilon+1}\left(\underline{N} - 1\right) = \frac{\pi\theta}{k\tau}.$$

Proof. See appendix. ■

Now, let us analyze how the optimal level of copayment and the optimal number of drugs available in the therapeutic class vary with the risk aversion parameter.

Proposition 7 *When* $K(N) = k$*, the risk aversion parameter:*

i) increases the optimal number of drugs,

ii) increases the copayment rate if and only if $a \leq 1/\phi(\underline{N}, \sigma)$*,*

iii) decreases the reference pricing weight r*.*

[10] Detailed computations can be found in appendix.

Proof. See appendix. ■

Proposition 8 *When $K(N) = k$, the sunk cost:*

i) decreases the optimal number of drugs,

ii) increases the optimal copayment if and only if $a \leq 1/\phi(\underline{N}, \sigma)$,

iii) increases the reference pricing if $a \geq 1/\phi(\underline{N}, \sigma)$.

5 Extensions

5.1 Adverse effects distribution

Proposition 9 *Assume that $\tilde{x}_j^i$ are identically and independently distributed across policy holders and drugs, with atomless cumulative distribution function F on a support $[0, \bar{x}]$ (where $\bar{x}$ can be infinite) and density f. Assume $E(\exp(\sigma\theta x))$ exists for σ small. Then there exists σ_N (possibly infinite) such that $\phi(N, \sigma_N)$ is defined for $\sigma \in [0, \sigma_N[$. If $1 - F(x)$ is log-log concave, then $-\frac{\pi}{\sigma}\frac{\partial\phi(N,\sigma)}{\partial N}/\phi(N, \sigma)$ is increasing.*

6 Conclusion

The model provided allows to shed light on various mechanisms and underline several results dealing with the reference pricing issue. First, on the short run, it is always optimal for the Government to implement the strongest reference pricing regulation in order to lower health care expenditure. On the long run, the regulator takes into account the drug entry process. It mainly depends on

the amount of the sunk cost (and its slope) and the inefficiency generated by the adverse effect. In terms of policy making, our model points out the importance to have good estimations of the social value of drugs diversity (which strongly depends positively on the level of policy holders' risk aversion) and of the sunk cost associated to drug entries in order to set optimally the reference pricing strength.

This model could be extended in several ways. It could be interesting to introduce some vertical differentiation between drugs. In this case, following Brekke *et al.* (2007), we could compare more explicitly than we do the two modalities of reference pricing *i.e.* generic versus therapeutic. Another extension would be to open the black box of the entry process by modeling completely the innovation process (Aghion and Tirole, 1994).

In this paper, we determine the optimal public regulation in short and long run equilibrium analysis. Nevertheless, as in Bardey *et al.* (2010), it would be useful to complete this analysis by introducing delays in order to take into account the impact of health insurance and drug price regulation on drugs introduction delay. Indeed, these delays of introduction may have huge impact in terms of welfare. It is in our research agenda.

References

[1] Aghion P and Tirole J, 1994, "Opening the Black Box of Innovation", *European, Economic Review*,

[2] Anderson S., de Palma A. and Thisse J. (1992), Discrete Choice Theory of Product Differentiation, MIT Press

[3] Bardey D, Bommier A and B Jullien, "Retail price regulation and innovation: Reference pricing in the pharmaceutical industry", *Journal of Health Economics*, Elsevier, vol. 29(2), 303-316.

[4] Blomqvist A.(1997), "Optimal non-linear health insurance", *Journal of Health Economic*s, 16, 303-321.

[5] Brekke K., Konigbauer I. and Straume O. (2007), "Reference Pricing of Pharmaceuticals", *Journal of Health Economic*s, 26, 613-642.

[6] Danzon P. M. (2001). "Reference Pricing: Theory and Evidence," in G. Lopez-Casasnovas and B. Jonsson (eds.), *Reference Pricing and Pharmaceutical Policy*, New York: Springer.

[7] Danzon P. and Ketchman J. (2004), "Reference Pricing of Pharmaceuticals for Medicare: Evidence from Germany, the Netherlands and New Zeland", *Frontiers in Health Policy Research 7:2,* bepress.

[8] Danzon P. and Liu H. (1996), "Reference Pricing and Physician Drug Budgets: The German Experience in Controlling Pharmaceutical Expenditures", working paper, University of Pennsylvania.

[9] Danzon P. and Pauly M. (2002), "Health Insurance and the Growth in Pharmaceutical Expenditures", *Journal of Law and Economics*, vol XLV, 587-613.

[10] DiMasi J.A, 2000, "New drug innovation and pharmaceutical industry structure: trends in the output of pharmaceutical firms", *Drug Information Journal* 34, 1169-1194.

[11] Dimasi J.A., Hansen R.W. and Grabowski H.G. (2003), "The price of innovation: new estimates of drug development costs", *Journal of Health Economics*, 22: 151-185.

[12] Dimasi J.A., Hansen R.W., Grabowski H.G. and Lasagna L. (1991), "Cost of Innovation in the Pharmaceutical Industry", *Journal of Health Economics*, 10(2):, 107-142.

[13] DiMasi JA, Paquette C (2004). "The economics of follow-on drug research and development", *Pharmacoeconomics*. 22 Supl 2:1-14.

[14] Geoffard P-Y, 2006, "Incentive and Selection Effects in Health Insurance" chap. 10 in Elgar Companion to Health Economics, Andrew Jones, ed, Elgar.

[15] Hurwitz M. and Caves R. (1988) "Persuasion or Information? Promotion and the Shares of Brand-Name and Generic Pharmaceuticals", *Journal of Law and Economics*, 31, 299-320.

[16] Laffont J.-J. and Tirole J. (1993), A Theory of Incentives in Procurement and Regulation, MIT Press.

[17] Lopez-Casanovas G. and Puig-Junoy J. (2000), "Review of the literature on reference pricing", *Health Policy*, 54, 87-123.

[18] Pammolli F. and M. Riccaboni (2004), "Market Structure And Drug Innovation", *Health Affairs*, 23(1), 48-50

[19] Pauly M. (1968), "The Economics of Moral Hazard: Comment", *American Economic Review*, 58, 531-537.

[20] Zeckhauser R. (1970) "Medical Insurance: A Case Study of the Trade-off Between Risk Spreading and Appropriate Incentives", *Journal of Economic Theory*, 2, 10-26.

7 Appendix

7.1 Proof of Proposition

One has

$$\pi \frac{\partial \phi(\underline{N}, \sigma)}{\partial N} \frac{u(w_s)}{u'(w_s)} = \frac{\pi\theta}{N(\tau N - \sigma\theta)}$$

so that (10) is fulfilled if

$$\frac{\pi\theta}{N(\tau N - \sigma\theta)} > \underline{N}K'(\underline{N}) + K(\underline{N})$$

or if

$$\begin{aligned} \sigma &> \bar{\sigma} = \frac{\tau}{\theta}\underline{N} - \frac{\pi}{\underline{N}(NK'(\underline{N}) + K(\underline{N}))} \\ &= \frac{\tau}{\theta}\underline{N} - \frac{\pi}{(1+\epsilon)k\underline{N}^{\epsilon+1}} \end{aligned}$$

where we used $K(N) = kN^{\epsilon}$. The free entry condition yields

$$k\underline{N}^{\epsilon+1} = \frac{\pi\theta}{\tau(\underline{N} - 1)}$$

so that

$$\bar{\sigma} = \frac{\tau}{\theta}\left(\frac{1 + \epsilon\underline{N}}{1+\epsilon}\right)$$

7.2 Proof of proposition9

Note first that if $1-F(x)$ is log-log concave then

$$\frac{d^2\left[\log\left(1-F(x)\right)\right]}{dx^2}=\frac{d}{dx}\left[\frac{\frac{f(x)}{1-F(x)}}{-\log\left(1-F(x)\right)}\right]<0. \tag{11}$$

for any $x\in[0,\bar{x}]$.

Given that $\tilde{x}_j^i$ are iid, one has:

$$\phi(N,\sigma)=\int_0^{\bar{x}}\exp(\sigma\theta x)\,N\left(1-F(x)\right)^{N-1}f(x)\,dx. \tag{12}$$

Since $\phi(N,\sigma)$ increases with σ, it follows immediately that $\phi(N,\sigma)<NE(\exp(\sigma\theta x))$ so that $\phi(N,\sigma)$ exists for σ small. Notice also that $\lim_{\sigma\to\sigma_N}\phi(N,\sigma)=+\infty$. Integrating (12) by part yields (WE NEED THAT THE INTEGRAL EXISTS IF INFINITE SUPPORT)

$$\phi(N,\sigma)=-\left\{\left(1-F(x)\right)^N\exp\sigma\theta x\right\}_0^{\bar{x}}+\sigma\theta\int_0^{\bar{x}}\left(1-F(x)\right)^N\exp(\sigma\theta x)\,dx.$$

(IF THE INTEGRAL EXISTS THEN $\left(1-F(x)\right)^N\exp\sigma\theta x$ TENDS TO ZERO AT THE UPPER BOUNDARY) so that

$$\phi(N,\sigma)=1+\sigma\theta\int_0^{\bar{x}}\left(1-F(x)\right)^N\exp[\sigma\theta x]\,dx.$$

Suppose that

$$\sigma\theta\int_0^{\bar{x}}\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N\exp\left[\sigma\theta x\right]dx$$

exists (TO BE PROVED), then

$$-\frac{\pi}{\sigma}\frac{\partial\log\left(\phi\left(N,\sigma\right)\right)}{\partial N}=\pi\theta\frac{\int_0^{\bar{x}}-\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N\exp\left[\sigma\theta x\right]dx}{\int_0^{\bar{x}}\exp\left(\sigma\theta x\right)N\left(1-F\left(x\right)\right)^{N-1}f\left(x\right)dx}.$$

This increases with σ if

$$\frac{\int_0^{\bar{x}}-\theta x\exp\left[\sigma\theta x\right]\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N dx}{\int_0^{\bar{x}}-\exp\left[\sigma\theta x\right]\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N dx}>\frac{\int_0^{\bar{x}}\theta x\exp\left(\sigma\theta x\right)\left(1-F\left(x\right)\right)^{N-1}f\left(x\right)dx}{\int_0^{\bar{x}}\exp\left(\sigma\theta x\right)\left(1-F\left(x\right)\right)^{N-1}f\left(x\right)dx}$$

or if

$$\int_0^{\bar{x}}\theta xh\left(x\right)dx>\int_0^{\bar{x}}\theta xl\left(x\right)dx \tag{13}$$

where $h\left(x\right)$ and $l\left(x\right)$ are two densities defined by:

$$\begin{aligned}h\left(x\right) &= \frac{-\exp\left(\theta x\right)\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N}{\int-\exp\left(\sigma\theta x\right)\log\left(1-F\left(x\right)\right)\left(1-F\left(x\right)\right)^N dx},\\ l\left(x\right) &= \frac{\exp\left(\sigma\theta x\right)\left(1-F\left(x\right)\right)^{N-1}f\left(x\right)dx}{\int\exp\left(\sigma\theta x\right)\left(1-F\left(x\right)\right)^{N-1}f\left(x\right)dx}.\end{aligned}$$

Since θx is increasing, a sufficient condition for (13) to hold is that the distribution with density $h\left(x\right)$ first order stochastically dominates (FOSD) the distribution with density $l\left(x\right)$ *i.e.*

$$\int_0^z h(x)\,dx < \int_0^z l(x)\,dx$$

for any $z \in [0, \bar{x}]$. Using the Rothchild-Stiglitz single crossing property, this is true if

$$\frac{d}{dz}\left.\frac{l(z)}{h(z)}\right|_{l=h} < 0 \text{ for every } z \in [0, \bar{x}].$$

Since

$$\frac{l(z)}{h(z)} = \frac{\frac{f(z)}{1-F(z)}}{-\log(1-F(z))} * \frac{\int_0^{\bar{x}} -\exp(\sigma\theta x)\log(1-F(x))(1-F(x))^N\,dx}{\int_0^{\bar{x}} \exp(\sigma\theta x)(1-F(x))^{N-1} f(x)\,dx},$$

straitforward differentiation with respect to z yields

$$\text{sign}\left(\frac{d}{dz}\left.\frac{l(z)}{h(z)}\right|_{l=h}\right) = \text{sign}\left[\frac{d}{dz}\left(\frac{\frac{f(z)}{1-F(z)}}{-\log(1-F(z))}\right)\right]$$

which is negative if $(1-F(x))$ is log-log concave as shown by (11).

8 Proof of Proposition 7

$$EU'(.) = \frac{\phi(N,\sigma)\,u'(w_s)}{1+\lambda}, \tag{14}$$

$$\pi\frac{\partial\phi(N,\sigma)}{\partial N}u(w_s) = [NK'(N) + K(N)](1+\lambda)\,EU'(.) \tag{15}$$

The two first order conditions write $\nabla = (\nabla_a(a,N), \nabla_N(a,N))$:

$$\begin{aligned}
\nabla_a(a,N) &= (1+\lambda)\left[\begin{array}{c}(1-\pi)\,u'\left(w-\pi(1-a)\frac{NK(N)}{\pi}(1+\lambda)\right)\\ +\pi\phi(N,\sigma)\,u'\left(w-\pi(1-a)\frac{NK(N)}{\pi}(1+\lambda)-a\frac{NK(N)}{\pi}\right)\end{array}\right]\\
&\quad -\phi(N,\sigma)\,u'\left(w-\pi(1-a)\frac{NK(N)}{\pi}(1+\lambda)-a\frac{NK(N)}{\pi}\right),\\
\nabla_N(a,N) &= \pi\frac{\partial\phi(N,\sigma)}{\partial N}u\left(w-\pi(1-a)\frac{NK(N)}{\pi}(1+\lambda)-\frac{aNK(N)}{\pi}\right)-(NK(N))'(1+\lambda)\,EU'(.)
\end{aligned}$$

or,

$$\begin{aligned}
\nabla_a &= (1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau N}\exp\left(\sigma a\frac{kN}{\pi}\right)+\pi(1+\lambda)-1=0\\
\nabla_N &= \pi\theta-kN(\tau N-\sigma\theta)=0
\end{aligned}$$

Differentiations with respect to a, N and σ give

$$\frac{da}{d\sigma}=\frac{\begin{pmatrix}-\nabla_{a\sigma} & \nabla_{aN}\\ -\nabla_{N\sigma} & \nabla_{NN}\end{pmatrix}}{|\det|}$$

and,

$$\frac{dN}{d\sigma}=\frac{\begin{pmatrix}\nabla_{aa} & -\partial\nabla_{a\sigma}\\ \nabla_{Na} & -\nabla_{N\sigma}\end{pmatrix}}{|\det|}$$

with

$$det = \begin{vmatrix} \nabla_{aa} & \nabla_{aN} \\ \nabla_{Na} & \nabla_{NN} \end{vmatrix}.$$

Moreover, we have

$$\begin{aligned}
\nabla_{aa} &= (1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\sigma\frac{k}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right), \\
\nabla_{aN} &= (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\sigma\theta}{\tau N^2}(1+a)\right], \\
\nabla_{Na} &= 0, \\
\nabla_{NN} &= -2k\tau N + k\sigma\theta = -k(2\tau N-\sigma\theta) < 0, \\
\nabla_{a\sigma} &= (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[-\frac{\theta}{\tau N}(1-a)\right], \\
\nabla_{N\sigma} &= \theta kN, \\
\nabla_{ak} &= (1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau N}\frac{\sigma aN}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right), \\
\nabla_{Nk} &= -N(\tau N-\sigma\theta).
\end{aligned}$$

First, let us calculate det. It gives

$$\begin{aligned}
det &= \begin{vmatrix} \nabla_{aa} & \nabla_{aN} \\ \nabla_{Na} & \nabla_{NN} \end{vmatrix} \\
&= -(1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\sigma\frac{k^2}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right)[(2\tau N-\sigma\theta)] < 0.
\end{aligned}$$

Therefore, we have

$$\frac{dN}{d\sigma} = \frac{\begin{pmatrix} (1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\sigma\frac{k}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right) & (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\theta}{\tau N}(1-a)\right] \\ 0 & -\theta kN \end{pmatrix}}{|\det|}$$

This yields:

$$\frac{dN}{d\sigma} = \frac{-\theta kN(1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\sigma\frac{k}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right)}{\det} > 0$$

Moreover,

$$\frac{da}{d\sigma} = \frac{\begin{pmatrix} (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\theta}{\tau N}(1-a)\right] & (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\sigma\theta}{\tau N^2}(1+a)\right] \\ -\theta kN & -k(2\tau N-\sigma\theta) \end{pmatrix}}{|\det|}$$

This yields

$$\begin{aligned} \frac{da}{d\sigma} &= \frac{1}{|\det|}(1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[-\frac{\theta}{\tau N}(1-a)k(2\tau N-\sigma\theta)+\theta kN\left[\frac{\sigma\theta}{\tau N^2}(1+a)\right]\right], \\ &= \frac{1}{|\det|}\left[(1+\lambda)(1-\pi)\frac{k\theta}{\tau N}\exp\left(\sigma a\frac{kN}{\pi}\right)[-(1-a)(2\tau N-\sigma\theta)+\sigma\theta(1+a)]\right], \\ &\quad \frac{-2}{|\det|}\left[(1+\lambda)(1-\pi)\frac{k\theta}{\tau N}\exp\left(\sigma a\frac{kN}{\pi}\right)[\tau N(1-a)-\sigma\theta]\right]. \end{aligned}$$

It yields

$$\begin{aligned}\frac{da}{d\sigma} &\geq 0 \Leftrightarrow \tau N(1-a) - \sigma\theta \geq 0 \\ &\Leftrightarrow a \leq \frac{\tau N - \sigma\theta}{\tau N} \\ &\Leftrightarrow a \leq \frac{1}{\phi(N,\sigma)}.\end{aligned}$$

The free entry condition says

$$(a + (1-a)r)\tau k N(N-1) - \pi\theta = 0$$

so that

$$\tau k N(N-1)(1-r)\frac{da}{d\sigma} + \tau k N(N-1)(1-a)\frac{dr}{d\sigma} + \alpha\tau k(2N-1)\frac{dN}{d\sigma} = 0$$

or

$$\begin{aligned}\frac{dr}{d\sigma} &= -\frac{1}{(1-a)}\left[(1-r)\frac{da}{d\sigma} + \alpha\frac{(2N-1)}{N(N-1)}\frac{dN}{d\sigma}\right] \\ &= \frac{1}{(1-a)}\left[\begin{array}{c} 2(1+\lambda)(1-\pi)(1-r)\frac{k\theta}{\tau N}\exp\left(\sigma a\frac{kN}{\pi}\right)[\tau N(1-a) - \sigma\theta] \\ +\alpha\frac{(2N-1)}{N(N-1)}(1+\lambda)(1-\pi)\theta k N\frac{(\tau N - \sigma\theta)}{\tau}\sigma\frac{k}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right)\end{array}\right] \\ &= \frac{(1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)}{(1-a)(N-1)}\frac{k\theta}{\tau N}\left[\begin{array}{c} 2(1-r)[\tau N(1-a) - \sigma\theta](N-1) \\ +\alpha(2N-1)N(\tau N - \sigma\theta)\sigma\frac{k}{\pi}\end{array}\right]\end{aligned}$$

Using (), this yields

$$\begin{aligned}
\frac{dr}{d\sigma} &= \frac{(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[2(1-r)\left[\tau N(1-a)-\sigma\theta\right](N-1)+\alpha(2N-1)\sigma\theta\right] \\
&= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right](N-1)+\alpha\left(N-\frac{1}{2}\right)\sigma\theta\right] \\
&= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right](N-1)+\alpha(N-1)\sigma\theta+\frac{1}{2}\alpha\sigma\theta\right] \\
&= \frac{(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(N-1)\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right]+\alpha\sigma\theta\right]+\frac{1}{2}\alpha\sigma\theta\right] \\
&= \frac{(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(N-1)\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right]+\alpha\sigma\theta\right]+\frac{1}{2}\alpha\sigma\theta\right].
\end{aligned}$$

As $\pi\theta = kN(\tau N - \sigma\theta)$, we have

$$\begin{aligned}
\frac{dr}{d\sigma} &= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right](N-1)+(a+(1-a)r)\left(N-\frac{1}{2}\right)\sigma\theta\right. \\
&= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(N-1)\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right]+(a+(1-a)r)\sigma\theta\right]+\frac{1}{2}\sigma\theta\alpha\right] \\
&= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(N-1)\left[(1-r)\left[\tau N(1-a)-\sigma\theta\right]+(a+(1-a)r)\sigma\theta\right]+\frac{1}{2}\sigma\theta\alpha\right] \\
&= \frac{2(1+\lambda)(1-\pi)k\theta\exp\left(\sigma a\frac{kN}{\pi}\right)}{\tau N|\det|(1-a)(N-1)}\left[(N-1)\left[(\tau N-\sigma\theta)\left[(1-a)(1-r)\right]+\sigma\theta r\right]+\frac{1}{2}\sigma\theta\alpha\right]
\end{aligned}$$

so that

$$\frac{dr}{d\sigma} < 0.$$

Differentiations with respect to a, N and k give

$$\begin{aligned}
\frac{da}{dk} &= \frac{\begin{pmatrix} -\nabla_{ak} & \nabla_{aN} \\ -\nabla_{Nk} & \nabla_{NN} \end{pmatrix}}{|\det|} \\
&= \frac{\begin{pmatrix} -(1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\frac{\sigma a}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right) & (1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\sigma\theta}{\tau N^2}(1+a)\right] \\ N(\tau N-\sigma\theta) & -k(2\tau N-\sigma\theta) \end{pmatrix}}{|\det|} \\
&= \frac{1}{|\det|}\begin{bmatrix} (1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)}{\tau}\frac{\sigma a}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right)[k(2\tau N-\sigma\theta)] \\ -N(\tau N-\sigma\theta)(1+\lambda)(1-\pi)\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{\sigma\theta}{\tau N^2}(1+a)\right] \end{bmatrix} \\
&= \frac{1}{|\det|}\left((1+\lambda)(1-\pi)\frac{(\tau N-\sigma\theta)\sigma}{\tau}\exp\left(\sigma a\frac{kN}{\pi}\right)\left[\frac{a}{\pi}[k(2\tau N-\sigma\theta)]-\left[\frac{\theta}{N}(1+a)\right]\right]\right)
\end{aligned}$$

Therefore,

$$sign\left(\frac{da}{dk}\right) = -sign\left(\frac{a}{\pi}[k(2\tau N-\sigma\theta)]-\left[\frac{\theta}{N}(1+a)\right]\right).$$

Then,

$$\begin{aligned}
\frac{da}{dk} &\geq 0 \\
&\Longleftrightarrow \\
\left[\frac{\theta}{N}(1+a)\right] &\geq \frac{a}{\pi}[k(2\tau N-\sigma\theta)] \\
&\Longleftrightarrow \\
[\theta\pi(1+a)] &\geq aN[k(2\tau N-\sigma\theta)]
\end{aligned}$$

The first order condition in N implies

$$\pi\theta = kN\left(\tau N - \sigma\theta\right)$$

$$\begin{array}{rcl} \dfrac{da}{dk} & \geq & 0 \\ & \Longleftrightarrow & \\ \left(\tau N - \sigma\theta\right)\left(1+a\right) & \geq & a\left[2\tau N - \sigma\theta\right] \\ & \Longleftrightarrow & \\ a & \leq & \dfrac{1}{\phi\left(N,\sigma\right)}. \end{array}$$

The impact of k on N is given by

$$\begin{array}{rcl} \dfrac{dN}{dk} & = & \dfrac{\begin{pmatrix} \nabla_{aa} & -\nabla_{ak} \\ \nabla_{Na} & -\nabla_{Nk} \end{pmatrix}}{|\det|} \\ & & \dfrac{\begin{pmatrix} \left(1+\lambda\right)\left(1-\pi\right)\frac{\left(\tau N-\sigma\theta\right)}{\tau}\sigma\frac{k}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right) & -\left(1+\lambda\right)\left(1-\pi\right)\frac{\left(\tau N-\sigma\theta\right)}{\tau}\frac{\sigma a}{\pi}\exp\left(\sigma a\frac{kN}{\pi}\right) \\ 0 & N\left(\tau N-\sigma\theta\right) \end{pmatrix}}{|\det|} \\ & = & \dfrac{1}{|\det|}\left[\left(1+\lambda\right)\left(1-\pi\right)\dfrac{n\left(\tau N-\sigma\theta\right)^{2}}{\tau}\sigma\dfrac{k}{\pi}\exp\left(\sigma a\dfrac{kN}{\pi}\right)\right] < 0. \end{array}$$

Finally, the impact of k on r

$$\begin{aligned}
\frac{dr}{dk} &= \frac{-1}{\tau kN\left(N-1\right)\left(1-a\right)}\left[\tau kN\left(N-1\right)\left(1-r\right)\frac{da}{dk}+\left(a+\left(1-a\right)r\right)\tau k\left(2N-1\right)\frac{dN}{dk}+\left(a+\left(1-a\right)r\right)\right. \\
&= \frac{-\left(1+\lambda\right)\left(1-\pi\right)\left(\tau N-\sigma\theta\right)\sigma\exp\left(\sigma a\frac{kN}{\pi}\right)}{\pi\tau N\left(N-1\right)\left(1-a\right)\left|\det\right|}\left[\begin{array}{r} N\left(N-1\right)\left(1-r\right)\left(\left(\left[ka\left(2\tau N-\sigma\theta\right)-\left[\frac{\theta\pi}{N}\left(1+a\right)\right]\right]\right)\right. \\ +\left(a+\left(1-a\right)r\right)\left(2N-1\right)\left(\left[N\left(\tau N-\sigma\theta\right)k\right]\right) \\ -\left(a+\left(1-a\right)r\right)N\left(N-1\right)\left(k\left[\left(2\tau N-\sigma\theta\right)\right]\right)\end{array}\right. \\
&= \frac{-\left(1+\lambda\right)\left(1-\pi\right)\left(\tau N-\sigma\theta\right)\sigma\exp\left(\sigma a\frac{kN}{\pi}\right)}{\pi\tau N\left(N-1\right)\left(1-a\right)\left|\det\right|}\left[\begin{array}{r} N\left(N-1\right)\left(1-r\right)\left(\left(\left[ka\left(2\tau N-\sigma\theta\right)-\left(a+\left(1-a\right)r\right)\right.\right.\right. \\ +\left(a+\left(1-a\right)r\right)\left(2N-1\right)\left(\left[N\left(\tau N-\right.\right.\right. \\ -\left(a+\left(1-a\right)r\right)N\left(N-1\right)\left(k\left[\left(2\tau N-\right.\right.\right.\end{array}\right. \\
&= \frac{-\left(1+\lambda\right)\left(1-\pi\right)k\left(\tau N-\sigma\theta\right)\sigma\exp\left(\sigma a\frac{kN}{\pi}\right)}{\pi\tau\left(N-1\right)\left(1-a\right)\left|\det\right|}\left[\begin{array}{r} \left(N-1\right)\left(1-r\right)\left[a\left(2\tau N-\sigma\theta\right)-\left(a+\left(1-a\right)r\right)\tau\left(\right.\right. \\ +\left(a+\left(1-a\right)r\right)\left(2N-1\right)\left(\tau N-\sigma\theta\right) \\ -\left(a+\left(1-a\right)r\right)\left(N-1\right)\left(2\tau N-\sigma\theta\right)\end{array}\right. \\
&= \frac{-\left(1+\lambda\right)\left(1-\pi\right)k\left(\tau N-\sigma\theta\right)\sigma\exp\left(\sigma a\frac{kN}{\pi}\right)}{\pi\tau\left(N-1\right)\left(1-a\right)\left|\det\right|}\left[\begin{array}{r} \left(N-1\right)\left(1-r\right)\left(a\left(2\tau N-\sigma\theta\right)-\left(a+\left(1-a\right)r\right)\tau\left(\right.\right. \\ +\left(a+\left(1-a\right)r\right)N\left(\tau-\sigma\theta\right)\end{array}\right. \\
&= \frac{-\left(1+\lambda\right)\left(1-\pi\right)k\left(\tau N-\sigma\theta\right)\sigma\exp\left(\sigma a\frac{kN}{\pi}\right)}{\pi\tau\left(N-1\right)\left(1-a\right)\left|\det\right|}\left[\begin{array}{c} \left(N-1\right)\left(1-r\right)\left(\left(\sigma\theta-\tau N+\tau Na\right)\right) \\ +\left(a+\left(1-a\right)r\right)N\left(\tau-\sigma\theta\right)\end{array}\right]
\end{aligned}$$

A sufficient condition to have $dr/dk \geq 0$ is

$$\begin{aligned}
\sigma\theta-\tau N+\tau Na &\geq 0 \\
&\Longleftrightarrow \\
a &\geq \frac{1}{\phi\left(N,\sigma\right)}
\end{aligned}$$