Bidimensional Screening with intrinsically motivated workers

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PRELIMINARY AND INCOMPLETE DRAFT

Abstract

We study a principal-agent model with bidimensional screening. A principal delegates a task to a worker. The worker has private information on her productivity level and on her intrinsic motivation for the task to be performed. We solve the discrete case with two productivity levels and two degrees of motivation and completely characterize the optimal contract.

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1 Introduction

It is argued that efficient selection of workers is more effective, from the principal's point of view, than optimally designing incentives once the worker has been hired. In different words, it is possible for firms to partially solve agency problems by hiring agents with specific preferences (see Brehm and Gates 1997, Prendergast 2007, 2008). This seems particularly important in a labor market where intrinsic motivation of potential workers plays a crucial role.

We study a principal-agent relationship in which the agents differ in both ability and intrinsic motivation for the task to be performed, with each characteristic being independently and discretely distributed. Since workers' characteristics can not be observed by the firm, they can not be contracted upon. Instead, we assume that the firm can observe and verify the effort levels provided by the different types of workers. Thus, the principal offer to agents contracts consisting in different combinations of wage rate and effort provision. Our goal is then to characterize the set of contracts that are compatible with worker's

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self-selection in such a bidimensional asymmetric information framework and, in particular, to analyze which types of workers are hired and which are the optimal incentive schemes that the firm offers them.

The contribution of our work is twofold: from an economic point of view, it adds to the recent and rapidly growing literature on the selection of workers with intrinsic motivation; from a technical point of view, it explicitly solves the principal-agent problem in a labor market where workers are characterized by two different dimensions of private information.

From the economic point of view, our analysis focuses on the design of optimal incentive schemes for workers endowed with an intrinsic motivation for the task to be accomplished. The issue of intrinsic motivation has received considerable attention in recent years, although attention has been primarily devoted to the problem of moral hazard.

This question is tackled precisely by Besley and Gathak (2005) who show that, with motivated workers, an organization has less need to rely on high-powered incentives (i.e. bonus payments) to elicit the desired effort level. They assume that motivation only arises in a mission-oriented sector that produces collective goods (such as education and health services) and that workers in such not-for-profit sector enjoy a non-pecuniary benefit related to the provision of effort since they feel personally involved in production. It is shown that, in the mission-oriented sector, assortative matching emerges (workers and principals share the same mission) , which raises organizational productivity and lowers incentive pay with respect to the for-profit sector where workers only care about extrinsic rewards.

Many other papers like Murdock (2002) and Ghatak and Mueller (2011), focus attention on moral hazard, while we devote attention to the screening problem.

Heyes (2005) and Delfgaauw and Dur (2007) are the first papers that tackle the issue of the selection of workers, who are privately informed about their vocation, and study how intrinsic motivation affects a firm's optimal wage schemes. In particular, they show that as a worker's motivation increases the minimum wage that a worker is willing to accept decreases. Therefore, as the wage increases, the average quality of the pool of potential applicants for the job deteriorates, since workers with lower motivation are willing to accept the job. Delfgaauw and Dur (2007), in particular, point out that the optimal wage scheme entails a trade-off between the probability of filling a vacancy and the expected motivation of job applicants. Delfgaauw and Dur (2010) consider a richer framework where workers are heterogeneous with respect to both their intrinsic motivation to work at a firm and their ability or productivity. Their analysis focuses on the issue of managerial self selection into public vs private sectors in the case of full information on the workers' characteristics: the key result is that, if the demand for public sector output is not too high, then the return to managerial ability is always lower in the public sector as compared to the private sector. Therefore, attracting a more able managerial workforce to the public sector by increasing remuneration up to the private sector levels is not cost-efficient. Finally, Barigozzi and Raggi (2012) and Barigozzi and Turati (2012) consider the same framework with incomplete information about the two workers' characteristics. Since neither motivation nor productivity are observable by the employer, the wage rate must be independent of the workers' types and thus a fixed wage must be offered to all potential applicants. The authors examine what happens to both average productivity and average motivation of active workers as the wage increases and they show that nonmonotonicities arise. In particular, they find that average productivity can be decreasing whereas average vocation can be increasing in the wage rate for some subinterval of the possible remuneration levels. As a consequence, in the presence of multidimensional asymmetric information, an increase in the remuneration might determine a simultaneous decrease in both expected vocation and expected productivity of applicants.

Our paper is most closely related to Handy and Katz (1998) and Delfgaauw and Dur (2008). The first authors set up a theoretical model to explain the stylized fact that in the not-for-profit sector there appears to be lower managerial and professional wages than in the for-profit sector.[1] They argue that non-profits attract motivated managers by offering them compensation packages involving lower money wages and a larger component of institution specific fringe benefits compared with those offered by the for-profit sectors in the economy. But, rather than considering an agency setup, they impose an exogenously given ranking of effort levels and reservation wages for the different types of managers, and it is this ranking which drives the main results.

Delfgaauw and Dur (2008) aims at explaining the observed difference in attitude of civil servants. Indeed, civil servant are either viewed as completely lazy, because of the lack of incentives in public organizations or as highly motivated to provide a service to the community. The authors characterize the optimal incentive schemes offered by a cost minimizing public agency when workers differ in laziness and public service motivation. They show that when effort is observable and contractible and when the production required by the agency is sufficiently high so that at least two types of agents must be hired, the agency attracts dedicated workers as well as the economy's laziest workers, by offering contracts that are both distorted.[2] We depart from the previous model in two main ways: first our principal's objective function is profit maximization rather than cost minimization, second our screening problem is unrestricted. Indeed, Delfgaauw and Dur (2008) simplify their analysis by imposing that the principal is interested in hiring only two types of agents rather the whole set of types, and they also assume that there are only three, rather than four, types of workers (the devoted and lazy type is excluded).

[1] They also try to support the evidence that managers and professionals working for non-profits differ from those in for-profits in characteristics that are suggestive of devotion to the organizations and its cause, while they do not significantly differ in attributes like problem solving, ability and intelligence. They consider managers who can be either devoted or indifferent and have high or low ability and analyze self-selection mechanisms aimed at enhancing commitment on the part of managers of nonprofit institutions, when organizations are not able to screen for devotion but can have a costless although not completely accurate test for ability.

[2] Indeed, dedicated workers are asked to exert higher effort than in the private, perfectly competitive sector whereas lazy workers' effort is distorted downwards in order to make their contract unattractive for dedicated workers.

We find that it is always optimal for the principal to offer contracts leading to full separation and full participation of types. When full separation becomes impossible to realize, then the principal resorts to bunching and offers the same contract to different types of employees. Optimal contracts are never based on the exclusion of either the less able or the less motivated types of workers. Hence, in our setting screening along two different dimension of private information is not too costly, neither in terms of informational rents that the principal has to leave to the most motivated and/or able type, nor in terms of downward distortions of effort levels that less motivated and/or able types are required to provide.

The literature on the analysis of optimal screening of agents with unknown characteristics has flourished in the last two decades of the twentieth century. Nonetheless, this problem have been examined under the assumption of unidimensional asymmetric information. The interesting and possibly more realistic cases where agents have several unobservable characteristics and where principals use several instruments have not been studied as extensively as the unidimensional case because of their technical difficulties. In particular, it has been impossible to extend to the multidimensional environment most of the qualitative results and regularity conditions that make the unidimensional case easy tractable. This is possibly one of the reasons of the diminishing interest on multidimensional screening in the theoretical literature.

In our model, we consider a bidimensional adverse selection problem with a discrete type space where each characteristics comes from a binary distribution. Types distributions are assumed to be independent but no regularity requirements (i.e. symmetry) are imposed. Most importantly, we are in a situation in which the number of dimensions of private information is larger than the number of instruments available to the principal (namely the contractible effort level).

When the dimensionality of actions is the same as the dimensionality of private information and the type space is discrete, Armstrong and Rochet (1999) provide a complete characterization with economic application to multiproduct nonlinear pricing or multiproduct monopoly regulation. When, instead, the dimensionality of actions is smaller than the dimensionality of private information and the type space is continuous, Laffont et al. (1987) explicitly solve a model of bidimensional optimal nonlinear pricing by a regulated monopoly when consumers' utility is linear quadratic (has our workers') and when consumers' types are uniformly distributed on a unit square. Moreover, for the continuous case, Armstrong (1996), Rochet and Chonè (1998) and Basov (2001, 2005) present several useful techniques to solve the problem of multidimensional screening. Despite the poor economic intuitions, these papers provide existence proofs and characterization results showing that, when there are several dimensions of private information and when this information is continuously distributed, exclusion is generic and full separation of types is impossible. In other words, it is generally optimal for the principal not to serve the lower part of the agent's distribution and to offer the same contract to different types of consumers.

Our analysis owes much to Armstrong (1999), who considers optimal regulation of a monopoly that is privately informed about both its cost and demand functions. He solves a discrete model distinguishing between two main classes of problems. If cost uncertainty is relatively more important that demand uncertainty, then optimal prices are always weakly above marginal costs. Conversely, if demand uncertainty is more significant than cost uncertainty, then pooling and submarginal cost pricing could be optimal. The aim of Armstrong (1999)'s analysis is to show that it is the combination of cost and demand uncertainty that brings about the intriguing result of submarginal cost pricing.[3] To focus on this aspect, he explicitly ignores the problem of exclusion by restricting parameter values in such a way that it is never optimal for the regulator to shut down some types of firm. Interestingly, in our model, we do not need to impose analogous requirements: the problem is sufficiently well-behaved so that full participation always dominates exclusion and separation of types always dominates pooling.

The rest of the paper is organized as follows. In Section 2, we set up the model, describe the first-best (Section 2.1.1) and two benchmark cases in which there is asymmetric information on one dimension only, be it ability (Section 2.1.2) or intrinsic motivation (Section 2.1.3). In Section 3, we consider the interaction between the two sources of incomplete information. We distinguish between the two polar cases in which: (*i*) motivation uncertainty dominates over productivity uncertainty (Section 3.1) or (*ii*) productivity uncertainty dominates over motivation uncertainty (Section 3.2). First we characterize the optimal contracts with full separation and full participation and then we introduce bunching and/or exclusion.

2 The model

We consider a principal-agent model with bidimensional adverse selection. Both the principal and the agent are risk neutral. The principal (he) is willing to hire an agent (she) to perform a given task.

The production function is such that the only input is labor supplied by the agent. We call e the *observable and measurable* task level that the agent is asked to provide.[4] The production function displays constant returns to effort in such a way that

$$q(e) = e.$$

The principal's payoff function can be written as

$$\pi = e - w,$$

where the price of output is assumed to be exogenous and normalized to 1, and w is the salary paid to the hired worker. Obviously, the principal's profit depends on the type of the agent as will be clear in

[3] Indeed, with either cost uncertainty or demand uncertainty alone, prices are always set weakly above marginal costs.

[4] In particular, the variable e can be interpreted as a job-specific requirement like the amount of hours of labor the agent is asked to devote to production or the speed at which a production line is run in a factory.

the sequel.

Suppose that agents differ in two characteristics, productivity and intrinsic motivation. As for productivity, we interpret a highly productive (or highly efficient) potential worker as an agent incurring in a low cost of providing a given effort level. Workers can have two possible levels of productivity $\theta_i \in \{\theta_L, \theta_H\}$: they can be highly productive, i.e. they can have a low cost of effort θ_L, with probability ν, or they can be less productive and have a high cost of effort θ_H, with probability $1-\nu$, where $\theta_H > \theta_L > 0$. As for intrinsic motivation, we consider it as the "enjoyment of one's personal contribution to outcome, be it out of impure (warm-glow) or pure altruism"[5]. Again, we assume that motivation can take two possible values $\gamma_j \in \{\gamma_L, \gamma_H\}$ and, in particular, we focus attention on situations in which agents can be either intrinsically motivated, with motivation parameter taking value $\gamma_H = \gamma$, with probability μ, or not motivated, with the motivation parameter taking value $\gamma_L = 0$, with probability $1-\mu$, where $\gamma > 0$. We assume for simplicity that motivation and productivity have independent distributions. So there are four types of agents denoted as $ij = \{LH, LL, HH, HL\}$ where the first index indicates productivity and the second motivation.

Both productivity and motivation affect the agents' utility, but do not have any impact on the agents' *reservation utility*, which is normalized to zero for all possible types.

Workers' utility is quasi-linear in income and takes the form

$$u_{ij} = w_{ij} - \frac{1}{2}\theta_i e^2 + \gamma_j e.$$

The marginal rate of substitution between effort and wage is given by

$$MRS_{e,w} = -\frac{\partial u_{ij}/\partial e}{\partial u_{ij}/\partial w} = \theta_i e - \gamma_j,$$

which is positive for $e > \frac{\gamma_j}{\theta_i}$ (it is always positive for non-motivated agents such that $\gamma_j = 0$). Thus, when the effort required by the principal is sufficiently high, motivated workers' indifference curves have the standard positive slope in the space (e, w) and effort is a "bad". Alternatively, the agents' utility is decreasing in effort if $e < \frac{\gamma_j}{\theta_i}$.

Also notice that productivity θ_i enters utility with a convex term, whereas motivation γ_j enters utility with a linear term. Providing effort represents a net cost when

$$-\frac{1}{2}\theta_i e^2 + \gamma_j e < 0.$$

[5] We refer to Delfgaauw and Dur (2010, page 658). The same interpretation of intrinsic motivation can be found in Besley and Ghatak (2005) and Delfgaauw and Dur (2007, 2008 and 2010, in the last paper only as for Section 5). A slightly different view of intrinsic motivation (which suits our model as well) is given by Delfgaauw and Dur (2007, page 607), who argue that intrinsic motivation might arise because "the firm has some unique trait that is valued differently by different workers, giving the firm monopsony power. (...) Monopsony power arises naturally when intrinsic motivation is firm-specific. When it is related to an occupation rather than to working at a particular firm, monopsony power arises only if there is no other firm (in the neighborhood) offering similar jobs". In turn, the link between workers' motivation and market power justifies our hypothesis concerning profit maximization and wage setting on the part of the principal.

Obviously, the above condition is satisfied for any effort level $e > 0$ if workers are not motivated and $\gamma_j = 0$; if instead $\gamma_j > 0$, then it is satisfied for effort levels such that $e > \frac{2\gamma_j}{\theta_i}$. Thus, only if the effort required by the principal is sufficiently high do motivated workers experience a disutility loss from effort provision and do workers need a positive wage to be willing to exert such effort. Conversely, if the effort required is sufficiently low, motivated workers could perform their task also when receiving a negative reward (in other words they would be ready to volunteer to be hired by the firm).

Finally, notice that agents' utility function is well-behaved in the sense that it satisfies the (double) single-crossing property.

Remark 1 *The single-crossing property is satisfied both with respect to the productivity parameter and with respect to motivation. In fact $MRS_{e,w}$ is increasing in θ and decreasing in γ.*

By considering the impact of productivity and motivation together on the workers' effort and on the firm's output, we can say that the more efficient type is worker LH (with high productivity and motivation) whereas the less efficient type is worker HL (with low productivity and without motivation). Workers of types LL and HH are in between. Thus, we expect that the contracts offered by the principal will specify effort levels such that $e_{LH} \geq \max\{e_{LL}, e_{HH}\} \geq \min\{e_{LL}, e_{HH}\} \geq e_{HL}$. The ranking between workers LL and HH will be considered in the sequel.

In what follows, we assume that the principal offers the agent a menu of contracts of the form $\{e, w(e)\}$. Applying the Revelation Principle we will focus on four contracts such that a worker of type ij exerts effort e_{ij} and receives a wage $w(e_{ij}) = w_{ij}$.

2.1 Benchmark cases

2.1.1 Full information

In the first-best, efficiency and motivation are both observable. For $i = L, H$ and $j = L, H$, the principal solves

$$\max_{(e_{ij}, w_{ij})} \pi = e_{ij} - w_{ij} \tag{FB}$$
$$\text{s.t. } u_{ij} \geq 0$$

which is maximized for a level of effort equal to

$$e_{ij}^{FB} = \frac{1+\gamma_j}{\theta_i} \tag{1}$$

and where the wage levels are set such that each worker receives her zero reservation utility

$$w_{ij}^{FB} = \frac{(1+\gamma_j)(1-\gamma_j)}{2\theta_i}.$$

Note that all wages are non-negative if $\gamma_j \leq 1$.

It is immediate to check that $e_{LH}^{FB} > e_{HH}^{FB} > e_{HL}^{FB}$ and $e_{LH}^{FB} > e_{LL}^{FB} > e_{HL}^{FB}$. Also note that, for intermediate types, one has

$$e_{LL}^{FB} \leq e_{HH}^{FB} \text{ if and only if } \gamma \geq \frac{\theta_H - \theta_L}{\theta_L} \equiv \gamma^{FB} \tag{2}$$

and

$$e_{LL}^{FB} \geq e_{HH}^{FB} \text{ if and only if } \gamma \leq \gamma^{FB}, \tag{3}$$

Given that $\gamma \leq 1$, a necessary condition for (2) is that $\gamma^{FB} < 1$ or else $\theta_H < 2\theta_L$, while a sufficient condition for (3) is that $\gamma^{FB} \geq 1$ or $\theta_H \geq 2\theta_L$. Alternatively, $e_{HH}^{FB} \geq e_{LL}^{FB}$ if and only if $e_{HH}^{FB} \leq \frac{\gamma}{\theta_H - \theta_L}$ and $e_{LL}^{FB} \leq \frac{\gamma}{\theta_H - \theta_L}$ both hold. On the contrary, $e_{LL}^{FB} \geq e_{HH}^{FB}$ if and only if both $e_{LL}^{FB} \geq \frac{\gamma}{\theta_H - \theta_L}$ and $e_{HH}^{FB} \geq \frac{\gamma}{\theta_H - \theta_L}$ are satisfied.[6] Therefore (2) is equivalent to

$$e_{LL}^{FB} \leq e_{HH}^{FB} \leq \frac{\gamma}{\theta_H - \theta_L} \tag{4}$$

while (3) is equivalent to

$$e_{LL}^{FB} \geq e_{HH}^{FB} \geq \frac{\gamma}{\theta_H - \theta_L}. \tag{5}$$

Remark 2 *Ordering of effort levels in first-best.*

- *If $\gamma \leq \gamma^{FB}$ and $e_{LL}^{FB} \geq e_{HH}^{FB}$, then the ordering of optimal effort levels is $e_{LH}^{FB} > e_{LL}^{FB} \geq e_{HH}^{FB} > e_{HL}^{FB}$.*
- *If $\gamma \geq \gamma^{FB} \Leftrightarrow e_{HH}^{FB} \geq e_{LL}^{FB}$ and $\theta_H < 2\theta_L$, then the ordering of optimal effort levels is $e_{LH}^{FB} > e_{HH}^{FB} \geq e_{LL}^{FB} > e_{HL}^{FB}$.*

Thus, when motivation is sufficiently high then worker HH (with low productivity and motivation) is more efficient than worker LL (with high productivity and without motivation); conversely, when motivation is low enough, then worker HH is less efficient than worker LL.

At the first-best, the requirement $\gamma \leq 1$ implies that motivated agents face a disutility cost from effort and receive a non-negative wage. Intuitively, given Program (FB) and expression (1) we can interpret $1 + \gamma$ as the total marginal productivity of the effort. When $\gamma \leq 1$, the contribution of the worker's intrinsic motivation on the marginal productivity of effort does not dominate the standard one.

Finally, at the first-best, the principal's payoff is given by

$$\pi^{FB} = \frac{\left(1 + \gamma_j\right)^2}{2\theta_i}$$

which is obviously higher the higher the productivity of the agent and the higher her motivation.

[6] Take $e_{HH} \leq e_{LL}$: this is equivalent to $\frac{1+\gamma}{\theta_H} \leq \frac{1}{\theta_L}$ or else to $\theta_L + \theta_L\gamma \leq \theta_H$. It follows that $\theta_L\gamma \leq \theta_H - \theta_L$ or else that $\frac{\gamma}{(\theta_H - \theta_L)} \leq \frac{1}{\theta_L} \equiv e_{LL}^{FB}$. Similarly, taking $\theta_L\gamma \leq \theta_H - \theta_L$ and adding on both sides of the inequality $\gamma(\theta_H - \theta_L)$ yields $\theta_H\gamma \leq (1+\gamma)(\theta_H - \theta_L)$ whereby $\frac{\gamma}{(\theta_H - \theta_L)} \leq \frac{1+\gamma}{\theta_H} \equiv e_{HH}^{FB}$. The same reasoning can be applied to the opposite case in which $e_{HH} \geq e_{LL}$.

2.1.2 Adverse selection on productivity levels

Suppose that workers' motivation γ_j is observable to the principal but productivity θ_i is not, we call this case Benchmark 1, or B1. For $j = L, H$ the principal solves

$$\max_{(e_{Hj},w_{Hj});(e_{Lj},w_{Lj})} E(\pi) = \nu(e_{Lj} - w_{Lj}) + (1-\nu)(e_{Hj} - w_{Hj})$$

s.t.

$$w_{Hj} - \frac{1}{2}\theta_H e_{Hj}^2 + \gamma_j e_{Hj} \geq w_{Lj} - \frac{1}{2}\theta_H e_{Lj}^2 + \gamma_j e_{Lj}, \tag{IC_{θ_H}}$$

$$w_{Lj} - \frac{1}{2}\theta_L e_{Lj}^2 + \gamma_j e_{Lj} \geq w_{Hj} - \frac{1}{2}\theta_L e_{Hj}^2 + \gamma_j e_{Hj}, \tag{IC_{θ_L}}$$

$$w_{Hj} - \frac{1}{2}\theta_H e_{Hj}^2 + \gamma_j e_{Hj} \geq 0, \tag{PC_{θ_H}}$$

$$w_{Lj} - \frac{1}{2}\theta_L e_{Lj}^2 + \gamma_j e_{Lj} \geq 0 \tag{PC_{θ_L}}$$

According to standard solution techniques, assume that the only binding constraints are the participation constraint of the least productive agent PC_{θ_H} and the incentive compatibility constraint of the more productive one IC_{θ_L}. Thus, for $j = L, H$, wage schedules satisfy

$$w_{Hj} = \frac{1}{2}\theta_H e_{Hj}^2 - \gamma_j e_{Hj}$$

$$w_{Lj} = \frac{1}{2}\theta_L e_{Lj}^2 - \gamma_j e_{Lj} + \underbrace{\frac{1}{2}(\theta_H - \theta_L) e_{Hj}^2}_{\text{Info rent worker } Lj} \tag{6}$$

Here the informational rent of agent Lj is increasing in $\theta_H - \theta_L$ and in e_{Hj}. This implies that the effort of agent e_{Hj} is distorted downward in the optimal second-best contract. Substituting the previous wage schedules into the principal's problem and solving for the effort levels we find

$$e_{Lj}^{B1} = \frac{1+\gamma_j}{\theta_L} = e_{Lj}^{FB}$$

and

$$e_{Hj}^{B1} = \frac{(1+\gamma_j)(1-\nu)}{(\theta_H - \nu\theta_L)},$$

where the result of "no distortion at the top" holds. Moreover, $e_{Hj}^{B1} < e_{Hj}^{FB}$, implying that we observe the usual downward distortion in the effort exerted by the less efficient worker (here the low-productivity one). It is easy to verify that the monotonicity condition $e_{Lj}^{B1} > e_{Hj}^{B1}$ holds and that the omitted constraints PC_{θ_H} and IC_{θ_L} both hold.

Finally, it is straightforward to show that full participation of low-productivity workers is always optimal and that, as long as $e_{Hj} > 0$, it is never in the principal's interest to exclude type Hj workers, that is the types characterized by low productivity.[7]

[7] In fact, the principal's benefit from keeping workers Hj is the expected profit from those workers $(1-\nu)\left(e_{Hj} - w_{Hj}\right)$,

2.1.3 Adverse selection on motivation

Suppose now that workers' productivity θ_i is observable to the principal but motivation γ_j is not, we call this case Benchmark 2, or B2. For $i = L, H$ the principal solves

$$\max_{(e_{iH},w_{iH});(e_{iL},w_{iL})} E(\pi) = \mu(e_{iH} - w_{iH}) + (1-\mu)(e_{iL} - w_{iL})$$

s.t.

$$w_{iH} - \frac{1}{2}\theta_i e_{iH}^2 + \gamma e_{iH} \geq w_{iL} - \frac{1}{2}\theta_i e_{iL}^2 + \gamma e_{iL}, \qquad (IC_{\gamma_H})$$

$$w_{iL} - \frac{1}{2}\theta_i e_{iL}^2 \geq w_{iH} - \frac{1}{2}\theta_i e_{iH}^2, \qquad (IC_{\gamma_L})$$

$$w_{iH} - \frac{1}{2}\theta_i e_{iH}^2 + \gamma e_{iH} \geq 0, \qquad (PC_{\gamma_H})$$

$$w_{iL} - \frac{1}{2}\theta_i e_{iL}^2 \geq 0 \qquad (PC_{\gamma_L})$$

Again, we consider that the only binding constraints are the participation constraint of the non-motivated agent PC_{γ_L} and the incentive compatibility constraint of the motivated one IC_{γ_H}.[8] Thus, for $i = L, H$, wage schedules satisfy

$$w_{iL} = \frac{1}{2}\theta_i e_{iL}^2$$

$$w_{iH} = \frac{1}{2}\theta_i e_{iH}^2 - \gamma e_{iH} + \underbrace{\gamma e_{iL}}_{\text{Info rent worker } iH} \qquad (7)$$

The informational rent of agent iH is increasing in e_{iL}. This explains why the effort of agent iL is distorted downward in the optimal second-best contract. Substituting the previous wage schedules into the principal's problem and solving for effort levels we find

$$e_{iH}^{B2} = \frac{1+\gamma}{\theta_i} = e_{iH}^{FB}$$

and

$$e_{iL}^{B2} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\theta_i}$$

where the result of "no-distortion at the top" holds. Moreover, $e_{iL}^{B2} < e_{iL}^{FB}$ is true, implying that we observe the expected downward distortion in the effort exerted by the less efficient worker (here the non-motivated one), and $e_{iL}^{B2} > 0$ for

$$\gamma < \frac{1-\mu}{\mu} \equiv \gamma^{B2}$$

whereas the cost from letting them participate is the informational rent appearing in expression (6) multiplied by the proportion of workers receiving the rent, that is $\frac{1}{2}\nu(\theta_H - \theta_L)e_{Hj}^2$. By substituting expression (6) for the wage in $(1-\nu)(e_{Hj} - w_{Hj})$, it can be easily checked that the principal always offer a contract to low-productivity workers, independently of their motivation.

[8] Remind that motivated agents have interest in mimicking non-motivated ones whenever the effort required to them is larger than $\frac{2\gamma}{\theta_i}$.

where $\gamma < \gamma^{B2}$ always holds if $\mu < \frac{1}{2}$. In words, when γ is sufficiently high, the informational rent that the principal must pay to the motivated types is so costly that he prefers to exclude non-motivated workers. However, the necessary (and sufficient) condition for full participation is always satisfied if the proportion μ of motivated workers is sufficiently low.[9]

Finally, we can easily verify that the omitted constraints PC_{γ_H} and IC_{γ_L} are both satisfied.

3 Screening on productivity and motivation

Suppose now that both the workers' productivity θ_i and motivation γ_j are the agents' private information, we call this situation the second-best. The principal offers the worker a choice of four effort/wage combinations. For $i = L, H$ and $j = L, H$, the principal's program is

$$\begin{aligned} \max_{(e_{ij},w_{ij})} E(\pi) = \quad & \nu\mu(e_{LH} - w_{LH}) + \nu(1-\mu)(e_{LL} - w_{LL}) + \\ & (1-\nu)\mu(e_{HH} - w_{HH}) + (1-\nu)(1-\mu)(e_{HL} - w_{HL}) \end{aligned} \tag{8}$$

subject to four participation constraints PC_{ij} and twelve incentive compatibility constraints $IC_{ijvsi'j'}$. For type LH the constraints are

$$w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq 0 \tag{PC_{LH}}$$

and

$$w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 + \gamma e_{LL} \tag{IC_{LHvsLL}}$$

$$w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq w_{HH} - \frac{1}{2}\theta_L e_{HH}^2 + \gamma e_{HH} \tag{IC_{LHvsHH}}$$

$$w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}\theta_L e_{HL}^2 + \gamma e_{HL}. \tag{IC_{LHvsHL}}$$

For type LL:

$$w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq 0 \tag{PC_{LL}}$$

and

$$w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 \tag{IC_{LLvsLH}}$$

$$w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq w_{HH} - \frac{1}{2}\theta_L e_{HH}^2 \tag{IC_{LLvsHH}}$$

$$w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq w_{HL} - \frac{1}{2}\theta_L e_{HL}^2. \tag{IC_{LLvsHL}}$$

[9] Let us now consider the necessary and sufficient conditions for full participation. The principal's benefit from keeping workers iL is the expected profit from those workers $(1-\mu)(e_{iL} - w_{iL})$, whereas the cost from letting them participate is the informational rent appearing in expression (7) multiplied by the fraction of motivated workers, that is $\mu\gamma e_{iL}$. By substituting expression (7) for the wage in $(1-\mu)(e_{iL} - w_{iL})$, it can be easily checked that the principal prefers full participation, independently of productivity levels, if the necessary condition for participation of worker HL, $e_{iL}^{B2} > 0$, is met.

For type HH:

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq 0 \tag{PC_{HH}}$$

and

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{LH} - \frac{1}{2}\theta_H e_{LH}^2 + \gamma e_{LH} \tag{IC_{HHvsLH}}$$

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{LL} - \frac{1}{2}\theta_H e_{LL}^2 + \gamma e_{LL} \tag{IC_{HHvsLL}}$$

$$w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL}. \tag{IC_{HHvsHL}}$$

Finally, for type HL one has

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq 0 \tag{PC_{HL}}$$

and

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{LH} - \frac{1}{2}\theta_H e_{LH}^2 \tag{IC_{HLvsLH}}$$

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{LL} - \frac{1}{2}\theta_H e_{LL}^2 \tag{IC_{HLvsLL}}$$

$$w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq w_{HH} - \frac{1}{2}\theta_H e_{HH}^2. \tag{IC_{HLvsHH}}$$

Adding incentive compatibility constraints two by two yields a partial ranking of effort levels. In particular, summing IC_{LLvsHL} with IC_{HLvsLL} and IC_{HHvsLH} with IC_{LHvsHH} one has $e_{Lj} \geq e_{Hj}$ $\forall j = L, H$, meaning that, given motivation, effort required must be higher the higher productive efficiency. In the same way, adding IC_{HHvsHL} with IC_{HLvsHH} and IC_{LHvsLL} with IC_{LLvsLH} yields $e_{iH} \geq e_{iL}$ $\forall i = L, H$. Namely, for a given productivity level, effort is higher the higher the motivation.

Hence the following monotonicity condition holds

$$e_{LH} \geq \max\{e_{LL}; e_{HH}\} \geq \min\{e_{LL}; e_{HH}\} \geq e_{HL}. \tag{9}$$

The monotonicity condition (9) allows us to eliminate global downward incentive constraints and focus only on local ones. Indeed, adding IC_{LHvsHH} and IC_{HHvsHL} one obtains

$$w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL} + \frac{1}{2}(\theta_H - \theta_L) e_{HH}^2.$$

But, when $e_{HH} \geq e_{HL}$, the right-hand side of the above inequality is greater than $w_{HL} - \frac{1}{2}\theta_L e_{HL}^2 + \gamma e_{HL}$, which in turn implies that the global downward incentive constraint IC_{LHvsHL} is satisfied when the two local incentives constraints IC_{LHvsHH} and IC_{HHvsHL} are.[10]

What about intermediate types HH and LL? Adding IC_{LLvsHH} and IC_{HHvsLL} one has

$$\frac{1}{2}(\theta_H - \theta_L)(e_{LL} - e_{HH})(e_{LL} + e_{HH}) - \gamma(e_{LL} - e_{HH}) \geq 0,$$

[10] The same conclusion holds taking the two local incentives IC_{LHvsLL} and IC_{LLvsHL}.

which is satisfied for either

$$e_{HH} > e_{LL} \text{ and } e_{LL} + e_{HH} \leq \frac{2\gamma}{\theta_H - \theta_L}, \tag{10}$$

or

$$e_{LL} > e_{HH} \text{ and } e_{LL} + e_{HH} \geq \frac{2\gamma}{\theta_H - \theta_L}. \tag{11}$$

Intuitively, if $e_{HH} > e_{LL}$ holds at the second-best, then motivation has a larger impact on effort and output provision than productivity. On the contrary, if $e_{LL} > e_{HH}$ holds at the second-best, then productivity has a larger impact on effort and output provision than motivation. In what follows, we will show that condition (10) implies (4) and that condition (11) implies (5). Thus, when effort levels are aligned in a given way at the first best, then we expect the same ordering of effort levels to arise at the second best.

Lemma 1 *Second-best effort levels are always aligned with first-best ones.*

Using the same arguments as before, one can get rid of other global constraints. Suppose that condition (10) is verified: then, it is easy to show that the sum of the local constraints IC_{LHvsHH} and IC_{HHvsLL} implies that the global constraint IC_{LHvsLL} is satisfied as well. In addition, IC_{HHvsLL} and IC_{LLvsHL} imply IC_{HHvsHL}. By the same token, suppose that condition (11) holds: then, one can prove that constraints IC_{LHvsLL} and IC_{LLvsHH} imply constraint IC_{LHvsHH} and also that IC_{LLvsHH} and IC_{HHvsHL} can be used to eliminate IC_{LLvsHL}.

Moreover, taking participation constraints into account one can write that

$$\underbrace{w_{HH} - \frac{1}{2}\theta_H e_{HH}^2 + \gamma e_{HH} \geq w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 + \gamma e_{HL}}_{IC_{HHvsHL}} > \underbrace{w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq 0}_{PC_{HL}}$$

so the participation constraint PC_{HH} is automatically satisfied when PC_{HL} holds. Also

$$\underbrace{w_{LH} - \frac{1}{2}\theta_L e_{LH}^2 + \gamma e_{LH} \geq w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 + \gamma e_{LL}}_{IC_{LHvsLL}} > \underbrace{w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq 0}_{PC_{LL}},$$

thus the participation constraint PC_{LH} is automatically satisfied when PC_{LL} is. Finally, one can write:

$$\underbrace{w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 \geq w_{HL} - \frac{1}{2}\theta_L e_{HL}^2}_{IC_{LLvsHL}} > \underbrace{w_{HL} - \frac{1}{2}\theta_H e_{HL}^2 \geq 0}_{PC_{HL}}$$

which implies that once incentive constraint IC_{LLvsHL} and participation constraint PC_{HL} are satisfied, then also participation constraint PC_{LL} is. So, when all worker types are expected to be hired by the principal, it is only necessary to consider the participation constraint of the worst type HL.

We will solve a relaxed program in which only downward incentive constraints will bind. There are two different cases to be investigated. In the following proposition we provide an interpretation of the two cases by considering binding incentive constraints for types that are at the extreme (LH and HL) and at the intermediate position (LL and HH) of the monotonicity condition (9).

Proposition 1 ***Case 1***. *When motivation has a higher impact on effort provision than productivity, then condition (10) holds and a separating equilibrium with $e_{HH}^{SB} > e_{LL}^{SB}$ is attained. The binding downward incentive constraints for extreme types are those of high productivity types mimicking low-productivity ones, that is IC_{LHvsHH} and IC_{LLvsHL}. The relevant intermediate downward incentive constraint is IC_{HHvsLL}.*

If motivation has a higher impact on effort and output provision than productivity, then we solve a bidimensional screening problem which embeds and generalizes the two sub-problems with adverse selection on the workers' productivity only (Benchmark B1 in Subsection 2.1.2). These two sub-problems are now considered simultaneously and linked by the intermediate incentive constraint IC_{HHvsLL}. Figure 1 describes the binding incentives constraints in this case.

Insert Figure 1 about here

Proposition 2 ***Case 2***. *When productivity has a higher impact on effort provision than motivation, then condition (11) holds and a separating equilibrium with $e_{LL}^{SB} > e_{HH}^{SB}$ is attained. The binding downward incentive constraints for extreme types are those of motivated types mimicking non-motivated ones, that is IC_{LHvsLL} and IC_{HHvsHL}. As for the relevant intermediate downward incentive constraint, two sub-cases must be considered: (i)* ***Case 2a.*** *When the binding intermediate incentive constraint is IC_{LLvsHH}. (ii)* ***Case 2b.*** *When the binding intermediate incentive constraint is IC_{LLvsHL}.*

If productivity has a higher impact on effort and output provision than motivation, then we have to solve a bidimensional screening problem that consists of the two sub-programs related to adverse selection on workers' motivation (as in Benchmark B2 in Subsection 2.1.3) together with an intermediate incentive constraint that ca be either IC_{LLvsHH} (see Figure 2a) or IC_{LLvsHL} (as in Figure 2b). This is peculiar to our setting where the principal anticipates that the effort required from type HH can be so low that a negative or a very low wage is offered to her. In Case 2a, γ is low so that the effort required from HH is high enough, in turn this implies that type LL is willing to mimic type HH. This subcase occurs when condition $e_{HH}^{SB}+ e_{HL}^{SB} \geq \frac{2\gamma}{\theta_H - \theta_L}$ holds. The less intuitive subcase 2b occurs when the effort required from type HH is such that her salary is not attracting for type LL. As a consequence the latter worker obtains a higher utility when mimicking type HL. Subcase 2b occurs when γ is sufficiently high or $e_{HH}^{SB}+ e_{HL}^{SB} < \frac{2\gamma}{\theta_H - \theta_L}$.

Insert Figure 2a and 2b about here

But it might also be unfeasible to separate intermediate types HH and LL: then we will also study pooling equilibria with $e_{LL}^{SB} = e_{HH}^{SB} = e_p^{SB}$. As in Case 2 above, we must distinguish two sub-cases, the first one where the extreme binding incentive constraint is IC_{HHvsHL}, which is relevant when $e_p^{SB} + e_{HL}^{SB} \geq$

$\frac{2\gamma}{\theta_H-\theta_L}$ (see Figure 3a) and the second one where the extreme binding incentive constraint is IC_{LLvsHL}, occurring when $e_p^{SB}+e_{HL}^{SB}\leq\frac{2\gamma}{\theta_H-\theta_L}$ (see Figure 3b).

Insert Figure 3a and 3b about here

To draw the previous figures we considered that $\theta_H<2\theta_L$ for Case 1 and $\theta_H>2\theta_L$ for Case 2. Remind that $\theta_H<2\theta_L$ is a necessary condition for $e_{LL}^{FB}<e_{HH}^{FB}$ whereas $\theta_H>2\theta_L$ is a sufficient condition for $e_{LL}^{FB}>e_{HH}^{FB}$.

Interestingly we will show that, when motivation and productivity have a similar impact on effort provision, then separation of types LL and HH becomes impossible and Case 2a converges to the pooling sub-case with binding IC_{HHvsHL}, whereas Case 1 and Case 2b both converge to the pooling sub-case with binding IC_{LLvsHL}.

Figure 4 below illustrates the relative positions of the separating Case 1, Case 2a and Case 2b and the pooling cases.

Insert Figure 4 about here

3.1 Case 1: motivation prevails

A separating equilibrium with $e_{HH}^{SB}>e_{LL}^{SB}$ occurs if and only if Condition (10) holds, that is if $e_{LL}^{SB}+e_{HH}^{SB}\leq\frac{2\gamma}{(\theta_H-\theta_L)}$. Then, the constraints that are expected to bind at the optimum are IC_{HHvsLL}, PC_{HL}, IC_{LHvsHH} and IC_{LLvsHL}.

Note that, when IC_{HHvsLL}, PC_{HL}, IC_{LHvsHH} and IC_{LLvsHL} hold, the global incentive constraints IC_{LHvsLL} and IC_{HHvsHL} are automatically satisfied. Imposing that the previous constraints be binding and solving them for the wage levels, one obtains

$$w_{HL}=\frac{1}{2}\theta_He_{HL}^2, \tag{12}$$

$$w_{LL}=\frac{1}{2}\theta_Le_{LL}^2+\underbrace{\frac{1}{2}\left(\theta_H-\theta_L\right)e_{HL}^2}_{\text{Info rent worker }LL}, \tag{13}$$

$$w_{HH}=\frac{1}{2}\theta_He_{HH}^2-\gamma e_{HH}\underbrace{-\frac{1}{2}\left(\theta_H-\theta_L\right)e_{LL}^2+\gamma e_{LL}+\frac{1}{2}\left(\theta_H-\theta_L\right)e_{HL}^2}_{\text{Info rent worker }HH} \tag{14}$$

and finally

$$\begin{aligned} w_{LH} \;=\; & \frac{1}{2}\theta_Le_{LH}^2-\gamma e_{LH} \\ & \underbrace{+\frac{1}{2}\left(\theta_H-\theta_L\right)e_{HH}^2-\frac{1}{2}\left(\theta_H-\theta_L\right)e_{LL}^2+\gamma e_{LL}+\frac{1}{2}\left(\theta_H-\theta_L\right)e_{HL}^2}_{\text{Info rent worker }LH} \end{aligned} \tag{15}$$

All types except HL receive an informational rent. Since the informational rent is always increasing in the effort exerted by the types that can be mimicked, as usual, we will observe a downward distortion with respect to the first best for all the effort levels except the one of worker LH.

From expression (14), the rent of type HH is given by two terms: the last one is the rent received from type LL (by mimicking HL) and the first one is $-\frac{1}{2}(\theta_H - \theta_L) e_{LL}^2 + \gamma e_{LL}$. This latter represents the part of the rent specific to type HH mimicking LL which is positive if and only if $e_{LL} < \frac{2\gamma}{(\theta_H - \theta_L)}$ holds and increasing in e_{LL} if and only if $e_{LL} < \frac{\gamma}{(\theta_H - \theta_L)}$ is verified. It is immediate to see that both requirements are satisfied when Condition (10) holds.[11] This part of the informational rent of type HH specific to type HH mimicking LL is the unique one that contains both a term depending on $(\theta_H - \theta_L)$ (as the informational rent in Benchmark B1) and a term depending on γ (as the rent in Benchmark B2).[12] Conversely, the rent of type LL and the part of type LH rent which is specific to type LH mimicking type HH (the two rents coming from the incentive compatibility constraints of the extreme types), only depend on $(\theta_H - \theta_L)$ and not on γ. This happens since we are studying a program that replicates the two subcases in Benchmark B1 and link them by the means of the constraint IC_{HHvsLL} (see also Figure 1).

Substituting the wage schedules (12), (13), (14) and (15) into the principal's objective function and maximizing with respect to effort levels yields

$$e_{LH}^{SB1} = \frac{1+\gamma}{\theta_L} = e_{LH}^{FB}, \tag{16}$$

$$e_{HH}^{SB1} = \frac{(1-\nu)(1+\gamma)}{(\theta_H - \nu\theta_L)} = e_{HH}^{B1} \tag{17}$$

$$e_{LL}^{SB1} = \frac{\nu(1-\mu) - \mu\gamma}{\nu(1-\mu)\theta_L - \mu(\theta_H - \theta_L)}, \tag{18}$$

and

$$e_{HL}^{SB1} = \frac{(1-\nu)(1-\mu)}{\theta_H - (1-(1-\nu)(1-\mu))\theta_L}. \tag{19}$$

Interestingly, e_{HH}^{SB1} is equivalent to the effort level we obtained for type HH in the case of adverse selection on the workers' productivity levels only. This again confirms that we are studying a program which extends to bidimensional screening the two subprograms analyzed in Benchmark B1.

All effort levels, except the one of the most efficient type of agent LH, are strictly less than the corresponding first-best levels. Hence we have the familiar result of "no distortion at the top" and a downward distortion in effort levels for all other agent's types. Intuitively and given that we are studying here a problem with bidimensional adverse selection, the effort required from the workers LL and HL are characterized by a larger downward distortion than in program B1.

[11] Indeed, from $e_{HH} > e_{LL}$ and $e_{HH} + e_{LL} \leq \frac{2\gamma}{\theta_H - \theta_L}$ it follows that $e_{LL} < e_{HH} > \frac{\gamma}{\theta_H - \theta_L}$.

[12] The same observation holds for type LL in Case 2a as described in Subsection 3.2.1.

The following Proposition refers to existence and characterization of a fully separating equilibrium with full participation in Case 1.

Proposition 3 ***Full participation and full separation in Case 1.*** *A solution to the principal's program which entails full participation and full separation of types, which satisfies the monotonicity condition $e_{LH} > e_{HH} > e_{LL} > e_{HL} > 0$ and which is such that effort levels are given by expressions from (16) to (19) exists if and only if $\theta_H < \theta_L \min\{\rho_1, \rho_2\}$ and $\underline{\gamma}^{SB1} < \gamma < \overline{\gamma}^{SB1}$ with*

$$\begin{aligned} \underline{\gamma}^{SB1} &\equiv \frac{(\mu(1-\nu)+\nu(1-\mu))(\theta_H-\theta_L)}{(\nu\mu(\theta_H-\theta_L)+(1-\nu)(1-(1-\nu)(1-\mu))\theta_L)} \\ \overline{\gamma}^{SB1} &\equiv \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta_H-\theta_L)}{\mu(\theta_H-(1-(1-\nu)(1-\mu))\theta_L)} \\ \rho_1 &\equiv \frac{(1-(1-\nu)(1-\mu))}{\mu} \\ \rho_2 &\equiv \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)} \end{aligned}.$$

Proof. See the Appendix A.1. ■

Interestingly, both $\gamma^{FB} < \underline{\gamma}^{SB1}$ and $\min\{\rho_1, \rho_2\} < 2$ hold, so that the alignment of second-best efforts levels with the ranking obtained in first-best under Condition (4) necessarily holds. Moreover, notice that the lower bound of the existence range $\underline{\gamma}^{SB1}$ comes from the condition assuring that $e^{SB1}_{HH} > e^{SB1}_{LL}$ holds, whereas the upper bound $\overline{\gamma}^{SB1}$ corresponds to $e^{SB1}_{LH} < e^{SB1}_{LL}$ being satisfied.

It is possible to show that the solution characterized by full participation and full separation of types always yields the highest profits to the principal, who will then always implement it when possible.

Lemma 2 *In Case 1, the principal's profits are such that the equilibrium with full participation and full separation dominates the equilibria with full participation and pooling (be it pooling of types LL and HL or types HH and LL) which dominate the equilibrium with exclusion of worker HL, which dominates the equilibrium with pooling of types LL and HL and exclusion of type HL, which in turn dominates the equilibrium with exclusion of both worker HL and LL.*

Proof. The proof of the previous and the following results are available in a separated file that can be requested to the authors. ■

When the equilibrium with full participation and full separation of types is not viable, the principal will have to resort to different optimal contracts involving pooling of types. In particular, if $\gamma \leq \underline{\gamma}^{SB1}$, the principal is forced to offer the same contract to both types HH and LL, whereas if $\gamma \geq \overline{\gamma}^{SB1}$, we expect a pooling equilibrium where types HL and LL receive the same contract. Let us consider this instances in what follows.

Lemma 3 (i) ***Full participation and Pooling between types HH and LL in Case 1.*** *A solution to the principal's program which entails full participation, pooling between types HH and LL and IC_{LLvsHL}*

binding, which satisfies the monotonicity condition $e_{LH} > e_{HH} = e_{LL} > e_{HL} > 0$ *and where effort levels are given by expressions (16), (19) and*

$$e_{LL}^{SB1} = e_{HH}^{SB1} \equiv e_{\underline{p}}^{SB1} \frac{(\nu (1-\mu) + \mu (1-\nu)) - \mu\nu\gamma}{(\nu (1-\mu) + \mu (1-\nu)) \theta_L}$$

is chosen if and only if $\theta_H < \theta_L \rho_1$ *and* $\gamma^{FB} \leq \gamma \leq \underline{\gamma}^{SB1}$.

(*ii*) ***Full participation and Pooling between types LL and HL in Case 1.*** *A solution to the principal's program which entails full participation, pooling between types LL and HL, which satisfies the monotonicity condition* $e_{LH} > e_{HH} > e_{LL} = e_{HL} > 0$ *and which is such that effort levels are given by expressions (16), (17) and*

$$e_{LL}^{SB1} = e_{HL}^{SB1} \equiv e_{\overline{p}}^{SB1} = \frac{(1-\mu) - \mu\gamma}{(1-\mu) \theta_H}$$

is chosen if and only if $\theta_H < \theta_L \min\{\rho_1, \rho_2\}$ *and* $\overline{\gamma}^{SB1} \leq \gamma \leq \min\{\gamma^{B2}, 1\}$ *with*

$$\gamma^{B2} = \frac{1-\mu}{\mu}.$$

Notice that γ^{B2} is the threshold value such that $e_{iL}^{B2} > 0$ in benchmark B2. Moreover, $\gamma^{B2} \geq 1$ if and only if $\mu \leq \frac{1}{2}$, therefore the principal always proposes a pooling contract to types LL and HL when motivation is sufficiently high (for $\gamma \geq \overline{\gamma}^{SB1}$) and the probability of being motivated is sufficiently low (for $\mu \leq \frac{1}{2}$). Conversely, when $\mu > \frac{1}{2}$ and $\gamma^{B2} < 1$, then for $\gamma \geq \gamma^{B2}$ the principal will exclude type HL and fully separate the remaining types.

Corollary 1 ***Exclusion of type HL in Case 1.*** *A solution to the principal's program which entails separation and exclusion of type HL, which satisfies the monotonicity condition* $e_{LH} > e_{HH} > e_{LL} > e_{HL} = 0$ *and which is such that effort levels are given by expressions from (16) to (18) is chosen if and only if* $\mu > \frac{1}{2}$, $\theta_H < \theta_L \rho_1$ *and* $\gamma^{B2} < \gamma \leq 1$.

As a final remark on Case 1 note that, since workers' utility is linear in the wage rate, the effort levels described by expressions from (16) to (19) remain the optimal ones both with pooling and with exclusion of types. This is due to the fact that, solving programs with pooling between some types or programs with exclusion of some workers does affect the optimal rents and thus the optimal wages, but it does not modify the optimal efforts required from workers that are hired with separating contracts. Put differently, indifference curves of workers that are hired with separating contracts move up and down on the contract plane (e, w), but the optimal effort level does not change.

The different separating and pooling equilibria occurring in Case 1 are described in Figure 5 as a function of motivation.

Insert Figure 5 about here

3.2 Case 2: productivity prevails

Full separation with $e_{LL}^{SB} > e_{HH}^{SB}$ occurs if and only if Condition (11) holds, that is if $e_{LL}^{SB} + e_{HH}^{SB} \geq \frac{2\gamma}{\theta_H - \theta_L}$. The relevant downward incentive constraints that one assumes to be binding are IC_{LHvsLL}, IC_{HHvsHL}, IC_{LLvsHH} or eventually IC_{LLvsHL} (whichever one binds first), together with participation constraint PC_{HL}.

Solving for wages one has

$$w_{HL} = \frac{1}{2}\theta_H e_{HL}^2, \tag{20}$$

$$w_{HH} = \frac{1}{2}\theta_H e_{HH}^2 - \gamma e_{HH} + \underbrace{\gamma e_{HL}}_{\text{Info rent worker } HH}, \tag{21}$$

and

$$w_{LL} - \frac{1}{2}\theta_L e_{LL}^2 = \max\left\{w_{HH} - \frac{1}{2}\theta_L e_{HH}^2; w_{HL} - \frac{1}{2}\theta_L e_{HL}^2\right\}. \tag{22}$$

If $\max\left\{w_{HH} - \frac{1}{2}\theta_L e_{HH}^2;\ w_{HL} - \frac{1}{2}\theta_L e_{HL}^2\ \right\} = w_{HH} - \frac{1}{2}\theta_L e_{HH}^2$, which occurs for $e_{HH}^{SB} + e_{HL}^{SB} \geq \frac{2\gamma}{\theta_H - \theta_L}$, then it means that the constraint IC_{LLvsHH} is binding (we are in the standard case of binding downward incentive constraints depicted in Figure 2a, which we call Case 2a); conversely, if $\max\left\{w_{HH} - \frac{1}{2}\theta_L e_{HH}^2\ ;\ w_{HL} - \frac{1}{2}\theta_L e_{HL}^2\ \right\}$ $= w_{HL} - \frac{1}{2}\theta_L e_{HL}^2$, then it must be that $e_{HH}^{SB} + e_{HL}^{SB} \leq \frac{2\gamma}{\theta_H - \theta_L}$ and that the constraint IC_{LLvsHL} is binding instead (we are in a case where one of the downward incentive constraints is not binding, as shown in Figure 2b, which will be called Case 2b). Then, there are two possible subcases to be considered according to which one of the two previous inequalities is satisfied.

3.2.1 Case 2a with IC_{LLvsHH} binding

Suppose that $e_{HH}^{SB} + e_{HL}^{SB} \geq \frac{2\gamma}{\theta_H - \theta_L}$, in which case IC_{LLvsHH} is binding. This represents the most intuitive case where the downward incentive constraint between the intermediate types LL and HH is binding. From (22), one obtains

$$w_{LL} = \frac{1}{2}\theta_L e_{LL}^2 + \underbrace{\frac{1}{2}\left(\theta_H - \theta_L\right) e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LL} \tag{23}$$

and

$$w_{LH} = \frac{1}{2}\theta_L e_{LH}^2 - \gamma e_{LH} + \underbrace{\gamma e_{LL} + \frac{1}{2}\left(\theta_H - \theta_L\right) e_{HH}^2 - \gamma e_{HH} + \gamma e_{HL}}_{\text{Info rent worker } LH}. \tag{24}$$

Again, all types except HL receive an informational rent. In order to have the usual downward distortion with respect to the first-best for all the effort levels except worker LH, the informational rents need to be increasing in the effort exerted by the types that can be mimicked. From expression (23), the rent of type LL is given by two terms: the last one is the rent received from type HH (by mimicking HL) and the first one is $\frac{1}{2}\left(\theta_H - \theta_L\right) e_{HH}^2 - \gamma e_{HH}$. This latter term represents the part of the rent specific to type LL when mimicking HH and it is increasing in e_{HH} if and only if $e_{HH}^{SB2} > \frac{\gamma}{\theta_H - \theta_L}$ and positive if and only if $e_{HH}^{SB2} > \frac{2\gamma}{\theta_H - \theta_L}$. Only when e_{HH}^{SB2} is sufficiently high type LL benefits from mimicking type HH. Contrary to Case1 before, such conditions are not automatically satisfied and must be checked ex-post. [13]This is peculiar to our setting where the principal takes into account that the effort required from type HH can be so low that a negative or a very low wage is offered to her.

As we observed for worker HH in Case 1 before, the portion of type LL informational rent which is specific to type LL mimicking HH is the only one that contains both a term depending on $(\theta_H - \theta_L)$ (as the informational rent in Subsection 2.1.2) and a term depending on γ (as the rent in Subsection 2.1.3). Conversely, the rent of type HH and the portion of the type LH one which is specific to type LH mimicking HH (the two rents coming from the incentive compatibility constraints of the extreme types), only depend on γ and not on $(\theta_H - \theta_L)$. This happens since we are studying a program that essentially replicates and links together the two subcases in Benchmark B2 (see also Figure 2a).

Substituting the wage functions into the objective function and deriving w.r.t. effort levels we obtain

$$e_{LH}^{SB2a} = \frac{1+\gamma}{\theta_L} \tag{25}$$

$$e_{LL}^{SB2a} = \frac{(1-\mu) - \mu\gamma}{(1-\mu)\,\theta_L}, \tag{26}$$

$$e_{HH}^{SB2a} = \frac{(1-\nu)\,\mu + (1-(1-\nu)\,(1-\mu))\,\gamma}{(1-\nu)\,\mu\theta_H + \nu\,(\theta_H - \theta_L)} \tag{27}$$

and

$$e_{HL}^{SB2a} = \frac{(1-\nu)\,(1-\mu) - (1-(1-\nu)\,(1-\mu))\,\gamma}{(1-\nu)\,(1-\mu)\,\theta_H}. \tag{28}$$

Note, again, that all effort levels, but the one of the most efficient type of agent LH, are strictly less than the corresponding first best levels. Hence the result of "no distortion at the top" and downward distortion in effort levels for all other agent's types is still obtained. Moreover, e_{LL}^{SB2a} is equivalent to the effort level we obtained for type LL in the case of adverse selection on the workers' motivation only (benchmark B2). And, as before, the effort required from the less efficient workers (here types HH and HL) are characterized by a larger downward distortion than in program B2.

We are then able to state the following Proposition.

Proposition 4 ***Full participation and full separation in Case 2a.*** *A solution to the principal's program which entails full participation, full separation of types and IC_{LLvsHH} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} > 0$ and which is such that effort levels are given by expressions from (25) to (28) exists if and only if $\underline{\gamma}^{SB2a} < \gamma < \overline{\gamma}^{SB2a}$ with*

$$\begin{aligned} \underline{\gamma}^{SB2a} &\equiv \tfrac{\nu(1-\nu)(1-\mu)(\theta_H - \theta_L)}{(1-(1-\nu)(1-\mu))(\theta_H - \nu\theta_L)} \\ \overline{\gamma}^{SB2a} &= \min\left\{\gamma_1^{SB2a}, \gamma_2^{SB2a}, \gamma_3^{SB2a}\right\} \end{aligned}$$

[13]It can be checked that $e_{HH}^{SB2} > \frac{2\gamma}{\theta_H - \theta_L}$ if and only if $\gamma < \frac{\mu(1-\nu)(\theta_H - \theta_L)}{\nu(\theta_H - \theta_L) + \mu(1-\nu)(\theta_H + \theta_L)}$.

and

$$\begin{array}{ll} \gamma_1^{SB2a} \equiv & \frac{(1-\nu)(1-\mu)}{(1-(1-\nu)(1-\mu))} \\ \gamma_2^{SB2a} \equiv & \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta_H-\theta_L)}{\mu(1-(1-\nu)(1-\mu))(\theta_H-\theta_L)+\theta_L(\nu(1-\mu)+\mu(1-\nu))} \\ \gamma_3^{SB2a} \equiv & \frac{(1-\nu)(1-\mu)(\theta_H-\theta_L)(2(1-\nu)\mu\theta_H+\nu(\theta_H-\theta_L))}{\mu(1-\mu)(1-\nu)^2\theta_H(\theta_H+\theta_L)+(\nu+\mu^2(1-\nu))(1-\nu)\theta_H(\theta_H-\theta_L)+(1-(1-\nu)(1-\mu))\nu(\theta_H-\theta_L)^2} \end{array} .$$

Proof. The proof of the previous and the following results are available in a separated file that can be requested to the authors. ■

Observe that the lower bound $\underline{\gamma}^{SB2a}$ corresponds to condition $e_{HH}^{SB2a} > e_{HL}^{SB2a}$, while $\gamma < \gamma_1^{SB2a}$ is equivalent to $e_{HL}^{SB2a} > 0$, $\gamma < \gamma_2^{SB2a}$ is equivalent to $e_{LL}^{SB2a} > e_{HH}^{SB2a}$ and finally $\gamma \leq \gamma_3^{SB2a}$ ensures that the requirement $e_{HH}^{SB} + e_{HL}^{SB} \geq \frac{2\gamma}{\theta_H - \theta_L}$ holds.

Paralleling Case 1, one can show that it is always in the interest of the principal to implement the solution with full participation and full separation.

Lemma 4 *In Case 2a, the principal's profits are such that the equilibrium with full participation and full separation dominates the equilibria with full participation and pooling (be it pooling of types HH and HL or types HH and LL) which dominate the equilibrium with exclusion of worker HL, which dominates the equilibrium with pooling of types HH and LL and exclusion of type HL, which in turn dominates the equilibrium with exclusion of both worker HL and HH.*

Consider now the instances in which the equilibrium with full participation and full separation of types is not viable. In particular, if $\gamma \leq \underline{\gamma}^{SB2a}$, then we expect a pooling equilibrium where types HH and HL receive the same contract. Conversely, if $\gamma \geq \overline{\gamma}^{SB2a}$, we expect a pooling equilibrium where types HH and LL receive the same contract. But note that when $\overline{\gamma}^{SB2a} = \gamma_1^{SB2a}$ and $\gamma_1^{SB2a} < \gamma < \min\left\{\gamma_2^{SB2a}, \gamma_3^{SB2a}\right\}$, the sufficient condition for the exclusion of worker HL is satisfied. This leads us to consider an alternative solution in which both pooling of types HH and LL and exclusion of type HL are implemented.

Lemma 5 (i) ***Full participation and pooling between types HH and HL in Case 2a.*** *A solution to the principal's program which entails full participation, pooling between types HH and HL and IC_{LLvsHH} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} = e_{HL} > 0$ and which is such that effort levels are given by expressions (25), (26) and*

$$e_{HH}^{SB2a} = e_{HL}^{SB2a} \equiv e_{\underline{p}}^{SB2a} = \frac{1-\nu}{\theta_H - \nu\theta_L}$$

is chosen if and only if $\gamma \leq \underline{\gamma}^{SB2a}$.

(ii) ***Full participation and pooling between types HH and LL in Case 2a.*** *A solution to the principal's program which entails full participation, pooling between types HH and LL and IC_{HHvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} = e_{HH} > e_{HL} > 0$ and which is such that effort levels are given by expressions (25), (28) and*

$$e_{HH}^{SB2a} = e_{LL}^{SB2a} \equiv e_{\overline{p}}^{SB2a} = \frac{(\nu(1-\mu) + \mu(1-\nu))(1+\gamma)}{\nu\mu(\theta_H - \theta_L) + (\nu(1-\mu) + (1-\nu)\mu)\theta_H}$$

is chosen only if $\gamma \geq \overline{\gamma}^{SB2a}$ *and* $\overline{\gamma}^{SB2a} \neq \gamma_1^{SB2a}$.

Note that when $\overline{\gamma}^{SB2a} = \gamma_1^{SB2a}$ and $\gamma_1^{SB2a} < \gamma < \min\left\{\gamma_2^{SB2a}, \gamma_3^{SB2a}\right\}$, the sufficient condition for the exclusion of worker HL is satisfied. This leads us to consider alternative solutions such that either pooling of types HH and LL and exclusion of type HL or separation but exclusion of type HL or else exclusion of both types HL and HH are implemented.[14]

3.2.2 Case 2b with IC_{LLvsHL} binding

Suppose now that the incentive constraint IC_{LLvsHL} is binding and that $e_{HH}^{SB2b} + e_{HL}^{SB2b} \leq \frac{2\gamma}{\theta_H - \theta_L}$ holds (see Figure 2b). This represents the less intuitive subcase where the effort required from type HH is so low that her salary is not attracting for type LL. As a consequence the latter obtains a higher utility when mimicking type HL. From (22), the relevant wage levels are now

$$w_{LL} = \frac{1}{2}\theta_L e_{LL}^2 + \underbrace{\frac{1}{2}\left(\theta_H - \theta_L\right) e_{HL}^2}_{\text{Info rent worker } LL} \tag{29}$$

and

$$w_{LH} = \frac{1}{2}\theta_L e_{LH}^2 - \gamma e_{LH} + \underbrace{\gamma e_{LL} + \frac{1}{2}\left(\theta_H - \theta_L\right) e_{HL}^2}_{\text{Info rent worker } LH} \tag{30}$$

together with w_{HH} and w_{HL} as defined above by expressions (20) and (22).

As before, all types except HL receive an informational rent. However, here we do not observe a downward distortion with respect to the first-best effort levels neither for type LH nor for HH. Indeed, since the incentive constraint IC_{LLvsHH} is not binding, it is useless to distort effort level e_{HH} downward in order to reduce the informational rent left of worker LL. Thus, the wage of worker LL is exactly the same we observe in Case 1 (see expression 13) even if the ranking between equilibrium efforts here is different.

From expression (29), and contrary to the previous Case 2a, the rent of type LL is only given by one term which depends on the effort exerted by worker HL (no rent depending on HH appears). More precisely, both the informational rents of types HH and LL depend on the effort of worker HL; however in w_{LL} the rent is $\frac{1}{2}\left(\theta_H - \theta_L\right) e_{HL}^2$ (as the one in expression (6) in Subsection 2.1.2), whereas in w_{HH} the rent is γe_{HL} (as the one in expression (7) in Subsection 2.1.3).

Substituting the wage functions into the objective function and deriving w.r.t. effort levels we obtain

$$e_{LH}^{SB2b} = \frac{1+\gamma}{\theta_L} = e_{LH}^{FB}, \tag{31}$$

$$e_{LL}^{SB2b} = \frac{(1-\mu) - \gamma\mu}{(1-\mu)\,\theta_L}, \tag{32}$$

[14] In the region $\gamma \geq \overline{\gamma}^{SB2a}$, we do not fully characterize the optimum because several different cases might arise, and the analysis becomes cumbersome without being very insightful.

$$e_{HH}^{SB2b} = \frac{1+\gamma}{\theta_H} = e_{HH}^{FB} \tag{33}$$

and

$$e_{HL}^{SB2b} = \frac{(1-\nu)\left((1-\mu)-\gamma\mu\right)}{\nu\left(\theta_H-\theta_L\right)+\theta_H\left(1-\mu\right)\left(1-\nu\right)}. \tag{34}$$

Note that e_{LL}^{SB2b} has the same expression as e_{LL}^{SB2a} and as e_{LL}^{B2} in the benchmark case with adverse selection on motivation. As already mentioned, both e_{LH}^{SB2b} and e_{HH}^{SB2b} are equal to their first-best levels, while both $e_{LL}^{FB} > e_{LL}^{SB2b}$ and $e_{HL}^{FB} > e_{HL}^{SB2b}$ hold.

Proposition 5 ***Full participation and full separation in Case 2b.*** *A solution to the principal's program which entails full participation, full separation of types and IC_{LLvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} > 0$ and which is such that effort levels are given by expressions from (31) to (34) exists if and only if $\mu < \frac{1}{2}$, $\theta_H > \rho_3\theta_L$ and $\underline{\gamma}^{SB2b} \leq \gamma < \overline{\gamma}^{SB2b}$, with*

$$\begin{array}{rl} \underline{\gamma}^{SB2b} \equiv & \frac{(\theta_H-\theta_L)(2\theta_H(1-\mu)(1-\nu)+\nu(\theta_H-\theta_L))}{((\nu(\theta_H-\theta_L)+\theta_H(1-\mu)(1-\nu))(\theta_H+\theta_L)+\theta_H(\theta_H-\theta_L)(1-\nu)\mu)} \\ \overline{\gamma}^{SB2b} \equiv & \frac{(1-\mu)(\theta_H-\theta_L)}{\theta_L+\mu(\theta_H-\theta_L)} \\ \rho_3 \equiv & \frac{(1-\mu(1+\nu))}{(1-2\mu)(1-\mu(1-\nu))} \end{array}.$$

Proof. The proof of the previous and the following results are available in a separated file that can be requested to the authors. ■

As in the aforementioned Cases 1 and 2a, the principal's profits can be ranked in such a way that full participation is always preferred to exclusion and separation is always preferred to pooling.

Lemma 6 *In Case 2b, the principal's profits are such that the equilibrium with full participation and full separation dominates the equilibrium with full participation and pooling of types HH and LL, which dominates the equilibrium with exclusion of worker HL, which dominates the equilibrium with pooling of types HH and LL and exclusion of type HL, which in turn dominates the equilibrium with exclusion of both worker HL and HH.*

What happens when full participation and full separation are not viable? Below $\underline{\gamma}^{SB2b}$ one expects the principal to exclude the least efficient types, namely HL and possibly HH too, while above $\overline{\gamma}^{SB2b}$, one expects to have a pooling equilibrium where types LL and HH are given the same contract and, possibly, the least efficient type HL is excluded.

Lemma 7 (i) ***Full separation and exclusion of type*** HL ***in Case 2b.*** *A solution to the principal's program which entails full separation but exclusion of type HL and IC_{LLvsHL} binding, which satisfies the monotonicity condition $e_{LH} > e_{LL} > e_{HH} > e_{HL} = 0$ and which is such that effort levels are given by expressions from (31) to (33) is chosen when $\mu < \frac{1}{2}$ and $\underline{\underline{\gamma}}^{SB2b} \leq \gamma < \underline{\gamma}^{SB2b}$, where*

$$\underline{\underline{\gamma}}^{SB2b} \equiv \frac{(\theta_H-\theta_L)}{(\theta_H+\theta_L)}.$$

The equilibrium characterized by exclusion of both types HL and HH, which is such that effort levels are given by expressions (31) to (32) is chosen when $\mu < \frac{1}{2}$ *and* $\gamma < \underline{\underline{\gamma}}^{SB2b}$.

(*ii*) ***Full participation and Pooling between HH and LL in Case 2b.*** *An equilibrium with full participation and pooling between types LL and HH and* IC_{LLvsHL} *binding, with effort levels described by expressions (31), (**??**) and*

$$e_{LL}^{SB2b} = e_{HH}^{SB2b} \equiv e_{\overline{p}}^{SB2b} = \frac{(\nu(1-\mu) + \mu(1-\nu)) - \gamma\mu\nu}{(\nu(1-\mu) + \mu(1-\nu))\,\theta_L} = e_{\underline{p}}^{SB1} \tag{35}$$

is chosen when $\overline{\overline{\gamma}}^{SB2b} \leq \gamma \leq \gamma^{FB}$, *where*

$$\overline{\overline{\gamma}}^{SB2b} \equiv \frac{(\nu(1-\mu) + \mu(1-\nu))(\theta_H - \theta_L)((\theta_H - \theta_L) + 2(1-\nu)(1-\mu)\theta_L)}{(\theta_H - (1-(1-\nu)(1-\mu))\theta_L)(2(\nu(1-\mu) + \mu(1-\nu))\theta_L + \mu\nu(\theta_H - \theta_L))} > \overline{\gamma}^{SB2b}.$$

Observe that Lemma (7.*ii*) describes precisely the same pooling equilibrium obtained in Case 1 for low motivation levels.

Finally bridging the gap between $\overline{\gamma}^{SB2b}$ and $\overline{\overline{\gamma}}^{SB2b}$ the following happens.

Lemma 8 ***Pooling between HH and LL and exclusion of HL in Case 2b.*** *An equilibrium with pooling between types LL and HH, exclusion of type HL and* PC_{LL} *binding, with effort levels described by expressions (31) and (35) is chosen when* $\max\left\{\overline{\gamma}^{SB2b}, \overline{\gamma_0}^{SB2b}\right\} \leq \gamma < \overline{\overline{\gamma}}^{SB2b}$, *where*

$$\overline{\gamma_0}^{SB2b} \equiv \frac{(\theta_H - \theta_L)(\nu(1-\mu) + \mu(1-\nu))}{(\nu\mu(\theta_H - \theta_L) + 2(\nu(1-\mu) + \mu(1-\nu))\theta_L)}.$$

If $\max\left\{\overline{\gamma}^{SB2b}, \overline{\gamma_0}^{SB2b}\right\} = \overline{\gamma_0}^{SB2b}$, *then the solution which excludes both types HH and HL is implemented when* $\overline{\gamma}^{SB2b} < \gamma < \overline{\gamma_0}^{SB2b}$.

A Appendix

A.1 Proof of Proposition

All effort levels are strictly positive, except for e_{LL}^{SB1}. Notice that, in order for e_{LL}^{SB1} to be a maximum of $E(\pi)$, it is necessary to impose that both the numerator and the denominator of its expression (**??**) be positive[15], that is it must be

$$\gamma < \frac{\nu(1-\mu)}{\mu} = \gamma_0$$

where $\gamma_0 > 1$ for $\mu > \frac{\nu}{1+\nu}$ (thus $\mu > \frac{\nu}{1+\nu}$ implies that $\gamma < \gamma_0$ is always verified) and $\frac{(\theta_H - \theta_L)}{\theta_L} < \frac{\nu(1-\mu)}{\mu}$ or

$$\theta_H < \theta_L \frac{(1-(1-\nu)(1-\mu))}{\mu} = \rho_1 \theta_L$$

[15] This can be easily seen by collecting e_{LL} in the principal's objective function once the wage schedules have been substituted and observing the sign of the coefficient of e_{LL}^2.

with $\rho_1 > 1$. In short, it must be that $\frac{\nu(1-\mu)}{\mu} > \max\left\{\gamma; \frac{(\theta_H-\theta_L)}{\theta_L}\right\}$.

As far as the monotonicity conditions are concerned, $e_{HH}^{SB1} > e_{LL}^{SB1}$ if and only if

$$\gamma > \frac{(\mu(1-\nu)+\nu(1-\mu))(\theta_H-\theta_L)}{(\nu\mu(\theta_H-\theta_L)+(1-\nu)(1-(1-\nu)(1-\mu))\theta_L)} = \underline{\gamma}^{SB1},$$

where $\underline{\gamma}^{SB1} < 1$, is always the case for $(3\mu\nu-\nu-\mu) \geq 0$, that is for $\nu > \frac{1}{3}$ and $\mu \geq \frac{\nu}{(3\nu-1)}$, whereas for $(3\mu\nu-\nu-\mu) < 0$ it is true when

$$\theta_H < \theta_L \frac{((\mu+\nu-3\mu\nu)+(1-\nu)(1-(1-\nu)(1-\mu)))}{(\mu+\nu-3\mu\nu)} = \theta_L\rho_2$$

with $\rho_2 > \rho_1$ if and only if $\mu > \frac{\nu}{(1+\nu)}$ (which is always the case when $\mu \geq \frac{1}{2}$). Hence, it must be that $\theta_H < \min\{\rho_1, \rho_2\}\theta_L$; conversely, when $\rho_2 \leq \rho_1$ and $\rho_2\theta_L < \theta_H < \rho_1\theta_L$ then a solution with $e_{HH}^{SB1} > e_{LL}^{SB1}$ does not exist.

Moreover, $e_{LH}^{SB1} < e_{LL}^{SB1}$ for

$$\gamma < \frac{(1-\mu)(1-(1-\nu)(1-\mu))(\theta_H-\theta_L)}{\mu((\theta_H-\theta_L)+\theta_L(1-\mu)(1-\nu))} = \overline{\gamma}^{SB1},$$

with condition $\gamma < \overline{\gamma}^{SB1}$ being relevant only when $\mu \geq \frac{1}{2}$, whereas when $\mu < \frac{1}{2}$ it can be discarded because it is always satisfied. Finally note that the chain of inequalities $\underline{\gamma}^{SB1} < \overline{\gamma}^{SB1} < \gamma_0$ holds provided that the denominator of e_{LL}^{SB1} be positive (which is our starting requirement).

We need to verify that condition $e_{LL}^{SB1} + e_{HH}^{SB1} \leq \frac{2\gamma}{\theta_H-\theta_L}$ is satisfied, knowing that $e_{LL}^{SB1} \leq \frac{\gamma}{\theta_H-\theta_L}$ is. Now, suppose that $\max\left\{\gamma; \frac{\theta_H-\theta_L}{\theta_L}\right\} = \gamma$ or $\gamma > \frac{\theta_H-\theta_L}{\theta_L}$. But the latter inequality is equivalent to condition 2, which in turn is equivalent to $e_{LL}^{FB} < e_{HH}^{FB} < \frac{\gamma}{\theta_H-\theta_L}$. This implies that first best and second best intermediate effort levels have the same ordering and that $e_{LL}^{SB1} < e_{HH}^{SB1} < \frac{\gamma}{\theta_H-\theta_L}$ holds, whereby condition $e_{LL}^{SB1} + e_{HH}^{SB1} < \frac{2\gamma}{\theta_H-\theta_L}$ always holds. Conversely, suppose that $\max\left\{\gamma; \frac{\theta_H-\theta_L}{\theta_L}\right\} = \frac{\theta_H-\theta_L}{\theta_L}$ or $\gamma < \frac{\theta_H-\theta_L}{\theta_L}$. And the latter condition is equivalent to 3 which can also be restated as $e_{LL}^{FB} > e_{HH}^{FB} > \frac{\gamma}{\theta_H-\theta_L}$. This implies that at the first best one has the reverse (with respect to the second best) ordering of intermediate effort levels so the condition $e_{LL}^{SB1} + e_{HH}^{SB1} \leq \frac{2\gamma}{\theta_H-\theta_L}$ need not be satisfied. A sufficient condition would be $e_{HH}^{SB1} \leq \frac{\gamma}{\theta_H-\theta_L}$ which holds if and only if

$$\gamma \geq \frac{(1-\nu)(\theta_H-\theta_L)}{(1-\nu)\theta_L+\nu(\theta_H-\theta_L)} = \gamma_3$$

where $\gamma_3 < \underline{\gamma}^{SB1}$ always holds and $\gamma_3 < 1$ if and only if $-(\theta_H - 2\theta_L - 2\nu\theta_H + 3\nu\theta_L) > 0$. Therefore the sufficient condition $e_{HH}^{SB1} \leq \frac{\gamma}{\theta_H-\theta_L}$ is verified and condition $e_{LL}^{SB1} + e_{HH}^{SB1} \leq \frac{2\gamma}{\theta_H-\theta_L}$ is always satisfied if $\underline{\gamma}^{SB1} \leq \gamma \leq \overline{\gamma}^{SB1}$ holds.

We are also interested in the necessary and sufficient condition for full participation, requiring that the principal's payoff is larger when she hires all workers' types than when she hires only some of them. By following the same reasoning applied in Section 2.1.2 as for Remark **??**, we must compare the costs and benefits from participation of the less efficient worker, that is type HL. In particular, the principal's benefit from keeping worker HL is the expected profit $(1-\mu)(1-\nu)(e_{HL}-w_{HL})$, whereas the cost from letting HL participate is the informational rent paid to the other three workers' types $[1-(1-\mu)(1-\nu)]\frac{1}{2}(\theta_H-\theta_L)e_{HL}^2$. By substituting expression (12) for the wage w_{HL} and using expression (19), it can be easily checked that the principal always prefers full participation to exclusion of type HL.

Note that, to derive conditions for existence and to characterize the equilibrium with exclusion of type HL we proceed as in the case with full participation but we obviously drop worker HL from the principal's maximization program and we omit the constraint $e_{LH}^{SB1} < e_{LL}^{SB1}$. Since the higher bound of the existence range for an equilibrium with full participation $\overline{\gamma}^{SB1}$ comes precisely from the condition $e_{LH}^{SB1} < e_{LL}^{SB1}$, we obtain that the range for existence of a separating equilibrium with exclusion of HL is larger on the right side w.r.t. the range $\left[\underline{\gamma}^{SB1}, \overline{\gamma}^{SB1}\right]$. Moreover, since the workers' utility functions are quasi-linear, we find that with exclusion the optimal efforts of the remaining types are given by the same expressions from (16) to (18). Obviously the optimal wages of the remaining types will be *lower* than expressions from (15) to (13), since the portions of the three informational rents that depend on e_{LH}^{SB1} disappear.

The same reasoning can be used to find out conditions for existence and characterization of an equilibrium with full participation but where the principal offers the same contract to workers LH and LL. In this case the pooling effort $e_{LL}^{SB1} = e_{LH}^{SB1}$ and the pooling wage $w_{LL}^{SB1} = w_{LH}^{SB1}$ are in between expressions (18) and (16), and (13) and (15) respectively. Again, conditions such that an equilibrium with pooling of workers LH and LL exists are less stringent than the ones we obtained in Proposition 3 because the condition $e_{LH}^{SB1} < e_{LL}^{SB1}$ is no longer relevant.

Similar conclusions can be drawn from the program with exclusion of both workers HL and LL and from the program with exclusion of worker HL and a pooling contract offered to workers HH and LL.

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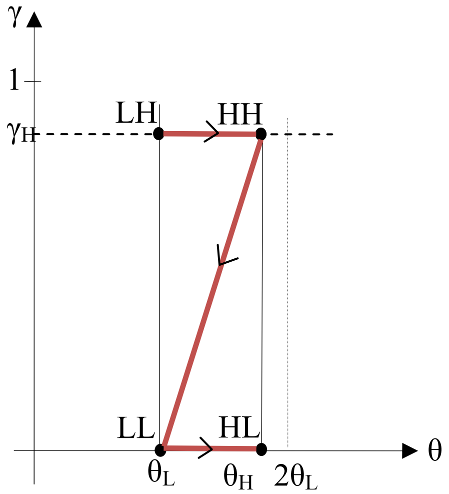

Figure 1: $e_{HH} > e_{LL}$.

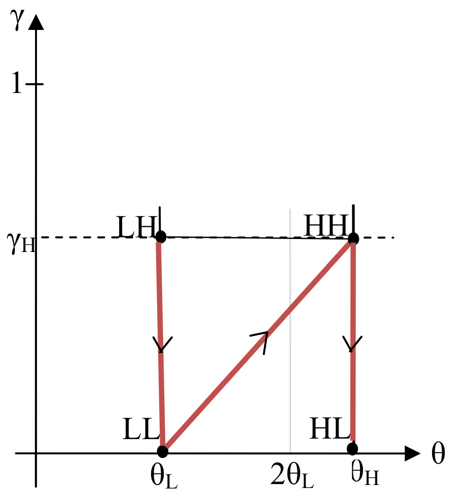

Figure 2a: $e_{LL} > e_{HH}$ and $e_{HH} + e_{HL} > 2\gamma/(\theta_H - \theta_L)$.

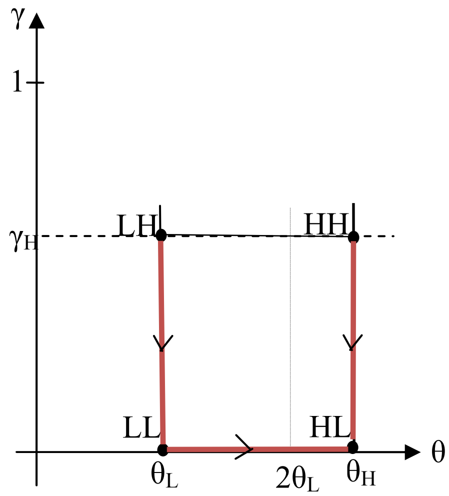

Figure 2b: $e_{LL} > e_{HH}$ and $e_{HH} + e_{HL} < 2\gamma/(\theta_H - \theta_L)$.

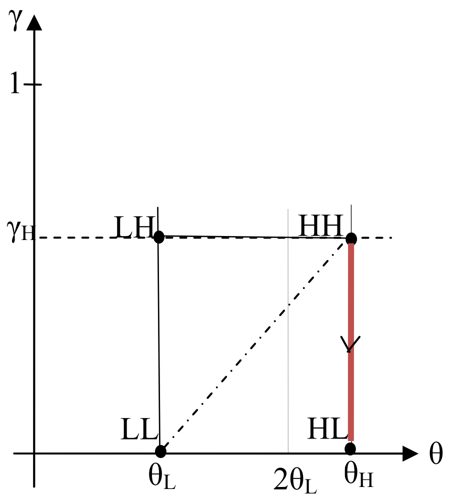

Figure 3a: $e_{LL} = e_{HH} = e_p$ and $e_p + e_{HL} > 2\gamma/(\theta_H - \theta_L)$.

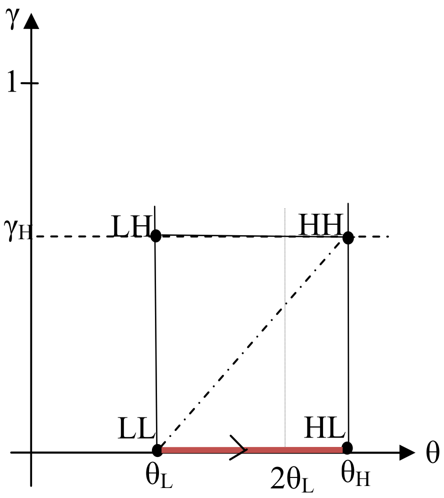

Figure 3b: $e_{LL} = e_{HH} = e_p$ and $e_p + e_{HL} < 2\gamma/(\theta_H - \theta_L)$.

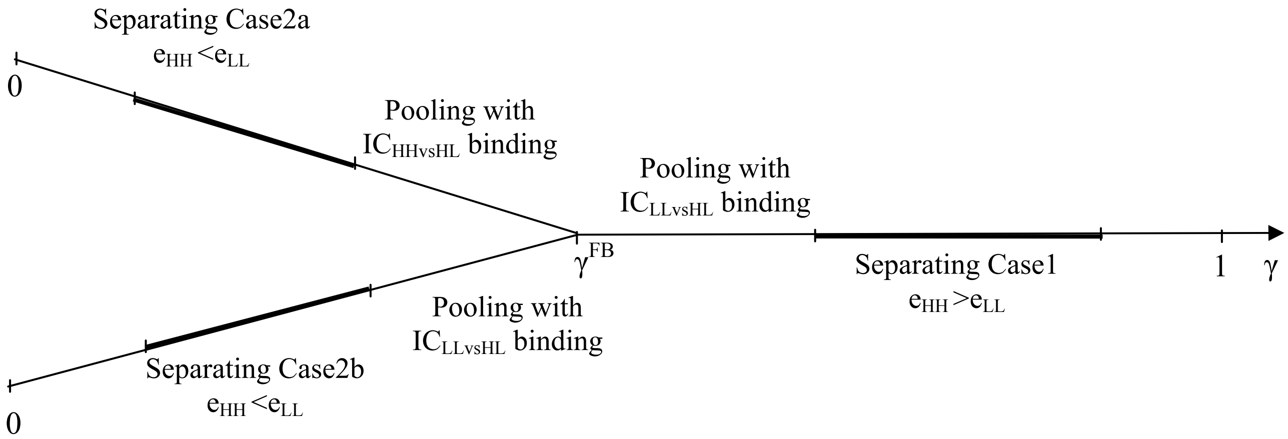

Figure 4: existence of the different separating and pooling equilibria as a function of motivation.

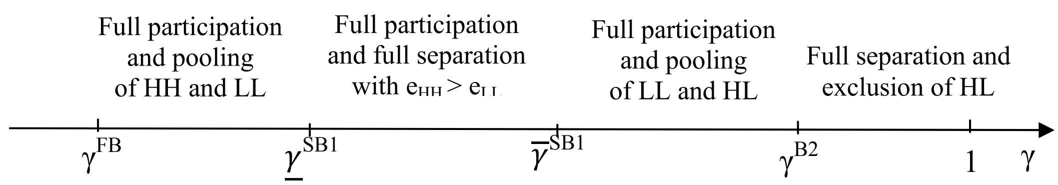

Figure 5. Case 1 with $e_{HH} \geq e_{LL}$: separating and pooling equilibria as a function of motivation. When pooling between HH and LL occurs, then the constraint IC_{LLvsHL} is binding.