Social-Welfare-Improving and/or Poverty-Reducing Directions of Commodity Tax Reforms in France

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Abstract

Using the marginal tax reforms framework, we investigate the existence of social-welfare-improving and/or poverty-reducing directions of indirect tax reforms in France. We first estimate elasticities through an econometric model of demand using a detailed households consumption survey. We then calibrate our theoretical model using the parameters of the French tax system and offer empirical estimations of the marginal cost of public funds. We then search for directions of tax reform by comparing consumption dominance curves at different orders reflecting the ethical considerations of the government.

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1 Introduction

In most countries, governments make extensive use of indirect taxation through value-added taxes. This is especially true in European countries. For example, in France, the value-added tax system is responsible for about half of total tax revenue and thus constitutes the most important source of government's revenue. Given this heavy reliance on value-added taxation, measuring its efficiency and equity effects is an important issue for the tax policy. In the present paper, we consider marginal tax reforms. A first feature of this framework is that the initial or observed tax system is not assumed to be optimal. A second feature is that 'small' increases in tax rates are considered. In this context, both efficiency and equity effects of tax reforms can be identified through simple analytical developments and evaluated using only a limited amount of statistical information. Note also small tax changes are often preferred to large ones by policy makers (see e.g. Feldstein, 1975; Slesnick, 1998).

A key component in evaluations of tax reforms is the marginal cost of public funds (MCF). The MCF measure the efficiency cost incurred by society in raising additional public funds. It is a summary measure of additional distortions in the allocation of resources that result from the collection of one additional euro of public funds. Since no assumption is made about the optimality of the tax system, there is no unique number representing the MCF for the tax system as a whole. Instead, there is one MCF for each tax rate. Thus, the MCF associated with one particular tax rate, say tax rate k, reflects the importance of the efficiency cost resulting from the collection of public funds through an increase in tax rate k. By comparing the value of the MCF associated with different tax rates, one can directly identify directions of efficient tax reforms. In particular, efficiency enhancing revenue-neutral tax reforms can be obtained by increasing tax rates with a relatively low MCF and decreasing tax rates with a relatively high MCF, while maintaining government's revenue constant.

However, enhancing the efficiency of the tax system is not sufficient to enhance social welfare. One can tackle this difficulty by searching only for Pareto-improving tax reforms. Since Pareto-improving tax reforms must make no loser, it is easy to understand that, in practice, this approach yields no results (see Ahmad and Stern, 1984). Therefore, in empirical evaluations of tax reforms, there is a need for a social welfare function to capture equity issues. Following Ahmad and Stern (1984), social preferences can be characterized by a functional form such as a power function with a constant inequality aversion parameter. An alternative approach that we will follow is based on a less restrictive specification of social preferences. It has been developed by Yitzhaki (1990), Yitzhaki and Thirsk (1990), Yitzhaki and Slemrod (1991), Mayshar and Yitzhaki (1995, 1996), Yitzhaki and Lewis (1996), Makdissi and Wodon (2002), Duclos *et al.* (2005), among others. The attractive feature of Yitzhaki *et* *al.*'s approach is that it allows for the identification of social welfare improving tax reforms without imposing other restriction than that the social welfare function satisfies some ethical principle.

In section 2, we present the theoretical framework and derive a computable analytical formula for the MCF. We then establish a formal, but implementable, sufficient condition for a tax reforms to constitute a socially-improving tax reform for all social welfare functions satisfying a clearly specified set of ethical principle. In section 3, we describe the data material and the econometric model of demand that we have used to estimate the demand elasticity. We then calibrate our theoretical model using the parameters of the French indirect tax system and offer empirical estimations of the marginal cost of public funds. We then search for directions of tax reform by comparing consumption dominance curves at different orders reflecting the ethical principle the social welfare function satisfies. Finally, section 4 concludes.

2 Theoretical framework

2.1 Economic model

We consider a simple static indirect tax model for a competitive closed economy *à la* Diamond and Mirrlees (1971). There are H households, L consumption goods, a private production sector, and the government. Each household $h = 1, ..., H$ is endowed with a single fixed factor, $y^h > 0$, taken to be the *numéraire* (i.e. with its price normalized to unity) and interpreted as its exogenous wealth. Thus $\mathbf{y} = \left(y^1, ..., y^H\right)$ represents the distribution of wealth among households. With no loss of generality we consider that the distribution of wealth is ordered from the poorer household to the wealthier household. Furthermore, $y^H = y^{\max}$ The private production sector is assumed perfectly competitive. It uses aggregate supply of factor, $Y = \sum_{h=1}^{H} y^h$, as the only input to produce the L consumption goods through a linear technology. In this context, profit maximization implies that producer prices $\mathbf{p} = (p_1, ..., p_L) >> 0$ are constant, and that pure profits are zero in competitive equilibrium.

Moreover, we assume that households have identical, rational, continuous and strictly convex preferences over the set of all nonnegative bundles of consumption goods $\mathbb{R}_+^L$. Households' preferences can therefore be represented by a strictly quasi-concave utility function $u : \mathbb{R}_+^L \to \mathbb{R}$. Taking as given the vector of consumer prices, $\mathbf{q} = (q_1, \ldots, q_L) >> 0$, each household h chooses a consumption bundle $\mathbf{x}^h = \left(x_1^h, ..., x_L^h\right)$ that maximizes its utility $u\left(\mathbf{x}^h\right)$ under the budget constraint $y^h = \sum_{\ell=1}^{L} q_\ell . x_\ell^h$. From the solution to the household's maximization problem, we get the indirect utility function

$$v^h = v\left(\mathbf{q}, y^h\right) = u\left(\mathbf{x}\left(\mathbf{q}, y^h\right)\right), \tag{1}$$

where $\mathbf{x}\left(\mathbf{q}, y^h\right) = \left(x_1\left(\mathbf{q}, y^h\right), \ldots, x_L\left(\mathbf{q}, y^h\right)\right)$ is Marshallian demand. Thus $\mathbf{v} = \left(v^1, ..., v^H\right)$ represents the distribution of welfare among households. Since the indirect utility function is strictly increasing in wealth, $\mathbf{v}$ is ordered in the same way than $\mathbf{y}$ (from the smallest to the largest).

The government is assumed to raise public funds through taxes on consumption goods. The tax system is characterized by a vector of (non-negative) tax rates, $\mathbf{t} = (t_1, \ldots, t_L) \geq 0$, where $t_\ell = \frac{q_\ell - p_\ell}{p_\ell}$. The government's total tax revenue is given by

$$R = \sum\nolimits_{\ell=1}^{L} t_\ell . p_\ell . X_\ell\left(\mathbf{q}, \mathbf{y}\right), \tag{2}$$

where $X_\ell\left(\mathbf{q}, \mathbf{y}\right) = \sum_{h=1}^{H} x_\ell\left(\mathbf{q}, y^h\right)$ is the aggregate Marshallian demand of commodity ℓ. We also use the following notation: $R_\ell^h = t_\ell . p_\ell . x_\ell\left(\mathbf{q}, y^h\right)$, $R^h = \sum_{\ell=1}^{L} R_\ell^h$ and $R_\ell = \sum_{h=1}^{H} R_\ell^h$.

It will be proved useful to consider the money-metric indirect utility function, or equivalent income, constructed by means of the expenditure function $e\left(\mathbf{q}, u\right)$, which gives the minimal expenditure a household needs to achieve the utility level u at prices $\mathbf{q}$.[1] Choosing an arbitrary vector of strictly positive prices $\overline{\mathbf{q}} >> 0$, household h's equivalent income is given by

$$y_E^h = y_E\left(\overline{\mathbf{q}}, \mathbf{q}, y^h\right) = e\left(\overline{\mathbf{q}}, v\left(\mathbf{q}, y^h\right)\right). \tag{3}$$

Note that it is a strictly increasing function of the indirect utility level whatever the chosen reference vector $\overline{\mathbf{q}}$. Hence, it is ordinally equivalent to the indirect utility level and is itself an indirect utility function for households preferences.

2.2 The marginal cost of public funds

Consider the impact of a marginal commodity tax reform which consists in a small increase $dt_k > 0$ in the k'th tax rate. The reform will have both an impact on government revenue and households welfare. Hence, the MCF (associated to the k'th tax rate) is defined as the ratio of the change households' equivalent income (giving a monetary evaluation of the welfare loss) to the change in government revenue due to the increase in t_k:

$$MCF_k = \frac{\sum_{h=1}^{H} \frac{\partial y_E^h}{\partial t_k}}{\frac{\partial R}{\partial t_k}}. \tag{4}$$

We can also derive a household-specific measure of the marginal cost of public funds: $MCF_k^h = \frac{dy_e^h}{dR^h}$, with $MCF_k = \sum_{h=1}^{H} \alpha^h . MCF_k^h$, where $\alpha^h = \frac{dR^h}{dR}$ is the percentage of additional public funds raised from household h.

[1] As is well-known, the expenditure function can be deduced from the indirect utility function using the identity $v(\mathbf{q}, v\left(\mathbf{q}, u\right)) = u$.

The change in total government revenue is given by[2]

$$\frac{\partial R}{\partial t_k} = \frac{1}{t_k}\left[R_k + \frac{t_k}{1+t_k}\sum\nolimits_{\ell=1}^{L} R_\ell.\epsilon_{\ell k}\right] \tag{5}$$

where

$$\epsilon_{\ell k} = \frac{\partial X_\ell(\mathbf{q},\mathbf{y})}{\partial q_k}\frac{q_k}{X_\ell(\mathbf{q},\mathbf{y})} \tag{6}$$

is the (uncompensated) elasticity of the aggregate Marshallian demand of good ℓ with respect to the consumer price of good k. For an household, it is written $\epsilon_{\ell k}^h = \frac{\partial x_\ell(\mathbf{q},y^h)}{\partial q_k}\frac{q_k}{x_\ell(\mathbf{q},y^h)}$, and we have $\epsilon_{\ell k} = \sum_{h=1}^{H}\frac{x_\ell(\mathbf{q},y^h)}{X_\ell(\mathbf{q},\mathbf{y})}\epsilon_{\ell k}^h$. Observe that dR only depends on exogenous parameters of the model (Marshallian demand elasticity, tax bases and tax rates).

By chain rule, the change in household h's equivalent income is given by:

$$\frac{\partial y_E^h}{\partial t_k} = \frac{\partial e\left(\overline{\mathbf{q}}, v\left(\mathbf{q},y^h\right)\right)}{\partial u}\frac{\partial v\left(\mathbf{q},y^h\right)}{\partial t_k}. \tag{7}$$

Whenever the reference price vector is the current consumer price vector, i.e. $\overline{\mathbf{q}} = \mathbf{q}$, using the identity $e\left(q, v\left(\mathbf{q},y^h\right)\right) = y^h$, we have $\partial e\left(q, v\left(\mathbf{q},y^h\right)\right)/\partial u = \left[\partial v\left(\mathbf{q},y^h\right)/\partial y^h\right]^{-1}$. Moreover, from the household's utility maximization problem, we have $\frac{\partial v(\mathbf{q},y^h)}{\partial t_k} = \frac{\partial v(\mathbf{q},y^h)}{\partial y^h}p_k.x_k^h\left(\mathbf{q},y^h\right)$. Hence, the change in equivalent income simplifies to

$$\frac{\partial y_E^h}{\partial t_k} = p_k.x_k\left(\mathbf{q},y^h\right). \tag{8}$$

Observe that dy_E^h can directly be computed from observed data (tax bases and tax rates).

Up to this point, we have all the elements to derive an analytical formula for the MCF which only depends on the exogenous parameters of the model (Marshallian demand elasticity, tax bases and tax rates). Substituting (8) and (5) in (4) yields:[3]

$$MCF_k = \frac{1}{1+\frac{t_k}{1+t_k}\sum_{\ell=1}^{L}\frac{R_\ell}{R_k}\epsilon_{\ell k}}. \tag{9}$$

This formula can be used to provide empirical estimations of the MCF for each tax rate k.

3 Social-Welfare-Improving and/or Poverty-Reducing Reforms

Marginal tax reforms for poverty alleviation for couples of goods have been analyzed in many articles – see for instance Yitzhaki and Slemrod (1991), Makdissi and Wodon (2002), Duclos,

[2]Note also that $\frac{\partial R^h}{\partial t_k} = \frac{1}{t_k}\left[R_k^h + \frac{t_k}{1+t_k}\sum_{\ell=1}^{L}R_\ell^h.\epsilon_{\ell k}^h\right]$ and thus $\alpha^h = \frac{1+\frac{t_k}{1+t_k}\sum_{\ell=1}^{L}\frac{R_\ell^h}{R_k^h}\epsilon_{\ell k}^h}{1+\frac{t_k}{1+t_k}\sum_{\ell=1}^{L}\frac{R_\ell}{R_k}.\epsilon_{\ell k}}\frac{R_k^h}{R_k}$.

[3]We also have the household-specific marginal cost of public funds: $MCF_k^h = \frac{1}{1+\frac{t_k}{1+t_k}\sum_{\ell=1}^{L}\frac{R_\ell^h}{R_k^h}\epsilon_{\ell k}^h}$.

and Makdissi and Wodon (2008), among other. The problem of finding poverty-reducing tax reforms is almost equivalent to finding welfare-improving tax reforms within a marginal analysis. The duality between welfare and poverty ordering has been first brought out by Foster and Shorrocks (1988) and further extended to a trial relationship between welfare, inequality and poverty by Duclos and Makdissi (2004). Those dual (trial) relationships have been also explored in the context of indirect marginal tax reforms.

3.1 Revenue-neutral tax reforms

To avoid any speculation about the impact of additionnal public expenditure, we restrict our attention to the class of tax reforms in which the governments revenue remains constant, namely revenue-neutral tax reforms satisfying

$$dR = \sum_{k=1}^{L} \frac{\partial R}{\partial t_k} dt_k = \sum_{k=1}^{L} \frac{R_k}{MCF_k} \frac{dt_k}{t_k} = 0. \tag{10}$$

Thus the general marginal tax reform $\mathbf{dt} = (dt_1, ..., dt_L)$ is revenue-neutral if and only if, for any $k = 1, ..., L$ with $dt_k \neq 0$,

$$\frac{dt_k}{t_k} = - \sum_{\ell=1\ (\ell \neq k)}^{L} \gamma_{k\ell} \frac{R_\ell}{R_k} \frac{dt_\ell}{t_\ell}, \tag{11}$$

where

$$\gamma_{k\ell} = \frac{MCF_k}{MCF_\ell} \tag{12}$$

is the ratio of the MCF from the k'th tax rate to the MCF from the ℓ'th tax rate. Observe that the ratio $\gamma_{k\ell}$ may be interpreted as an indicator of relative efficiency. If $\gamma_{k\ell} < 1$, the k'th tax instrument is more (resp. less, as) efficient than the ℓ'th tax instrument.

In the case of a reform of a paire of tax rates, say the k'th tax rate and the ℓ'th tax rate, the condition simplifies to

$$\frac{dt_k}{t_k} = -\gamma_{k\ell} \frac{R_\ell}{R_k} \frac{dt_\ell}{t_\ell}. \tag{13}$$

In our empirical investigation, we will first compute the marginal cost of funds for each tax rate. This will allow us to identify efficient tax reforms. Among these reforms, we will identify those which are fair according to clearly specified ethical principles. The approach is described below.

3.2 Social welfare and poverty functions

We assume that the public policy is evaluated with reference to a generalized utilitarian social welfare function:

$$W = \int_0^{y^{\max}} W\left(v^h\right) dF(y), \tag{14}$$

where $F(y)$ is the c.d.f. computed on the entire support $[0, y^{\max}]$. The social welfare function may also be written in its reduced form:

$$W = \int_0^{y^{\max}} w\left(y^h\right) dF(y), \tag{15}$$

where $w\left(y^h\right) = W\left(v\left(\mathbf{q}, y^h\right)\right)$. In order to investigate the duality between social welfare and poverty orderings, we consider a truncated social welfare, that is a:

$$W_z = \int_0^{y^{\widetilde{h}}} w\left(y^h\right) dF(y), \tag{16}$$

where $\widetilde{h}$ is the index of the household with wealth $y^{\widetilde{h}}$ equal to the poverty line $z \in [0, y^{\max}]$. This specification allows us to examine W_z as a social welfare function for households' with wealth below the poverty line only. If there is no troncature, i.e. $z = y^{\max}$, then $W_z \equiv W$.

On the other hand, the poverty function is assumed to be a non-negative and additive function defined over equivalent incomes:

$$P_z = \int_0^{y^{\widetilde{h}}} p\left(y_E^h\right) dF(y). \tag{17}$$

It is noteworthy that W_z and P_z are dual to each other, in the sense that the preorder $\succeq_{W_z}$– allowing for ranking two social situations – coincide with the ranking provided by the poverty preorder $\succeq_{P_z}$. For instance, if the income vectors $\mathbf{y}$ yields unambiguously less poverty than the vector $\mathbf{x}$, such that $z = y^{\widetilde{h}}$, then $\mathbf{x} \succeq_{P_z} \mathbf{y} \iff \mathbf{x} \succeq_{W_z} \mathbf{y}$. Notice that our approach is robust over a large range of equivalent scales.[4]

In the sequel, we will introduce changes in social welfare and/or social poverty that results from a general marginal indirect tax reform. For that purpose, we have to incorporate more structure to our two types of preorders, in the same manner than Foster and Shorrocks (1988). We first begin with the assumption that welfare/poverty functions belong to the following set $\mathscr{C}^s$ involving *additive and symmetric* real-valued functions being continuous and $s-$times differentiable almost everywhere over $[0, y^{\max}]$. As a consequence, the preorders may be endowed with a wide spectrum of value judgments reflecting the preference of the government towards redistribution. For that purpose, we introduce the two following sets:

$$\mathscr{U}^s\ [\mathscr{P}^s]\ := \left\{ f \in \mathscr{C}^s \;\middle|\; (-1)^s f^{(s)}(x) := (-1)^s \frac{\partial f^{(s-1)}(x)}{\partial x} \leq\ [\geq]\ 0,\ \forall x \in \mathbb{R} \right\}. \tag{18}$$

[4]Let $m(k)$ be an equivalence scale adjusted for differences in demographic characteristics (e.g. handicap, gender, ethnicity, age, etc). We then have, $\widetilde{y}(q, y, k) = \frac{y_E(q,y)}{m(k)}$. The corresponding additive poverty index is defined as:

$$P_z \equiv P(F, z) = \int_0^a p\left(\widetilde{y}(q, y, k), z\right) dF(y, k),$$

where $F(y, k)$ is the joint cumulative distribution function of y and k. Therefore, any functional forms of $m(k)$ are compatible with additive poverty indices.

The normative implications resulting from $W_z \in \mathscr{U}^s$ and $P_z \in \mathscr{P}^s$ is the following. When $s = 1$, we get the well-known Pareto principle, which postulates that an exogenous *manna* transferred to any given household increases social welfare and reduces overall poverty. The fact that W_z and P_z are included in $\mathscr{C}^s$ (for which functions are symmetric) entails that W_z and P_z respect a little more demanding requirement compared with the Pareto principle, i.e., Pen's parade. Symmetry is an anonymity principle postulating that the social planner is indifferent to the label h associated with y^h. Thus, if the distribution $\mathbf{x}$ is obtained from distribution $\mathbf{y}$ by transferring \$1 to an extreme poor household provides more increase of welfare [decrease of poverty] than transferring the same amount to a less poor household: $W_z \in \mathscr{U}^1$ and $P_z \in \mathscr{P}^1$ and $\mathbf{x} \succeq^1_{P_z} \mathbf{y} \Longleftrightarrow \mathbf{x} \succeq^1_{W_z} \mathbf{y}$.

The two first derivatives postulate that the individual social welfare [poverty] function is increasing [decreasing] and concave [convex]. This interpretation is equivalent to saying that the overall poverty index P_z satisfies the progressive transfer principle. This axiom, the so-called Pigou (1912)-Dalton (1920) principle ($s = 2$), postulates that P_z declines [W_z increases] whenever a (positive income) transfer occurs from higher-income household to a lower-income one (their rank within the income distribution remains the same). If $\mathbf{x}$ denotes the income distribution obtained from $\mathbf{y}$ after this kind of transfer, then $P_z \in \mathscr{P}^2$, $W_z \in \mathscr{U}^2$ and $\mathbf{x} \succeq^2_{P_z} \mathbf{y} \Longleftrightarrow \mathbf{x} \succeq^2_{W_z} \mathbf{y}$. The order $s = 3$, is known to be Kolm's diminishing transfer principle: the decrease of P_z [increase of W_z] is more valuable if the Pigou-Dalton transfer occurs between poor people rather than less poor households. If $\mathbf{x}$ denotes the income distribution obtained from $\mathbf{y}$ after kolm's diminishing transfer, then $P_z \in \mathscr{P}^3$, $W_z \in \mathscr{U}^3$ and $\mathbf{x} \succeq^3_{P_z} \mathbf{y} \Longleftrightarrow \mathbf{x} \succeq^3_{W_z} \mathbf{y}$.

Fishburn and Willig (1984) introduced the generalized transfer principle, which provides social preorders close to Rawls' egalitarian rule insofar as s increases. In this case, if $\mathbf{x}$ is the income distribution resulting from a generalized transfer principle of order s obtained from $\mathbf{y}$, then $P_z \in \mathscr{P}^s$, $W_z \in \mathscr{U}^s$ and $\mathbf{x} \succeq^s_{P_z} \mathbf{y} \Longleftrightarrow \mathbf{x} \succeq^s_{W_z} \mathbf{y}$.

In the sequel, we will make use of Consumption Dominance curves introduced by Makdissi and Wodon (2002), $CD-$curves from now onwards, in order to test for indirect tax reforms for many commodities. Those tax reforms will be analyzed throughout a dominance layout in which a close interrelation will be bring out between $CD-$curves and the preorders $\succeq^s_{P_z}$ and $\succeq^s_{W_z}$.

3.3 Bivariate indirect marginal tax reforms

We can now derive the impact of a general revenue-neutral tax reform on social welfare and poverty. The change in social welfare is a weighted-sum of households' change in equivalent income:[5]

$$dW_z = \int_0^{y^{\tilde{h}}} w'(y) \cdot dy_E \cdot dF(y), \tag{19}$$

where, for any given household h, $w'\left(y^h\right) = W'\left(v\left(\mathbf{q}, y^h\right)\right) \frac{\partial v\left(\mathbf{q}, y^h\right)}{\partial y^h}$ is household h's social marginal utility of income and $dy_E^h = \sum_{\ell=1}^{L} \frac{\partial y_E^h}{\partial t_\ell} dt_\ell = \sum_{\ell=1}^{L} p_\ell . x_\ell\left(\mathbf{q}, y^h\right) dt_\ell$ is the change in household h's equivalent income. In the same way, the change in poverty is given by

$$dP_z = \int_0^{y^{\tilde{h}}} p'(y_E) \cdot dy_E \cdot dF(y), \tag{20}$$

where, for any given household h, $p'\left(y_E^h\right)$ is the decrease (resp. increase) in poverty following an increase (decrease) in houshold h's equivalent income or, equivalently, household h's variation of utility.

Makdissi and Wodon (2002) introduced $CD-$curves for any order of restricted stochastic dominance in the context of poverty-reducing indirect tax reforms (dominance restricted by the truncature z). The point of the CD-curve of order 1 for household h's consumption of good ℓ, represents the ratio between his consumption of good ℓ and the aggregate consumption of good ℓ:

$$CD_\ell^1(y^h) = \frac{x_\ell\left(\mathbf{q}, y^h\right)}{X_\ell\left(\mathbf{q}, \mathbf{y}\right)} = \frac{\frac{\partial y_E^h}{\partial t_\ell}}{\sum_{h=1}^{H} \frac{\partial y_E^h}{\partial t_\ell}}. \tag{21}$$

The second-order CD-curve for good ℓ is

$$CD_\ell^2(y) = \int_0^y CD_\ell^1(u) du, \tag{22}$$

which represents the share of total consumption of good ℓ consumed by the households with income being less than the one of household h. Observe that $CD_\ell^2(y^{\max}) = 1$. The s-order CD-curve of good ℓ is given by:

$$CD_\ell^s(y) = \int_0^y CD_\ell^{s-1}(u) du. \tag{23}$$

In the context of revenue-neutral tax reforms of couples of tax rates we have the following central result, derived by Makdissi and Wodon (2002).

Theorem 3.1 *Consider a revenue-neutral tax reform which consists in an increase in the k'th tax rate together with a decrease in the ℓ'th tax rate, such that* $\frac{dt_k}{t_k} = -\gamma_{k\ell} \frac{R_\ell}{R_k} \frac{dt_\ell}{t_\ell} > 0$.

[5] Totally differentiating W_z with respect to tax rates we get: $dW_z = \int_0^{y^{\tilde{h}}} W'\left(v\left(\mathbf{q}, y^h\right)\right) dv\left(\mathbf{q}, y^h\right) d(Fy) = \int_0^{y^{\tilde{h}}} W'\left(v\left(\mathbf{q}, y^h\right)\right) \left[\sum_{\ell=1}^{L} \frac{\partial v\left(\mathbf{q}, y^h\right)}{\partial t_\ell} dt_\ell\right] dF(y)$. Observe also that, from the household's utility maximization problem, we have $\frac{\partial v\left(\mathbf{q}, y^h\right)}{\partial t_\ell} = \frac{\partial v\left(\mathbf{q}, y^h\right)}{\partial y^h} p_\ell . x_\ell^h\left(\mathbf{q}, y^h\right)$.

The revenue-neutral tax reform is s-order social-welfare-improving [poverty-reducing] for all social welfare functions $W_z \in \mathscr{U}^s$ $[P_z \in \mathscr{P}^s]$ and $s \in \{1,2,3,\ldots\}$ if, and only if,

$$CD^s_\ell(y) - \gamma_{k\ell} CD^s_k(y) \geqslant 0, \quad \forall y \leqslant z \in [0, y^{\max}].$$

Proof.
Makdissi and Wodon (2002). ■

This result is appealing since it enables any couple of goods to tested in order to obtain a social improvement and/or a decrease of poverty. A more general approach would be to handle all possible goods and to test whether it is possible to find a multivariate indirect tax reform.

3.4 Multivariate indirect marginal tax reforms

The difficulty to study a multiple good economy rather than a tow-good one relies on optimization problems, which have been studied by Ahmad and Stern (1984) and Mayshar and Yitzhaki (1995) in the case of social welfare functions. In the sequel, we will apprehend the same question using poverty indices, that is, poverty reducing tax reforms in an economy with L commodities, $\ell = 1, \ldots, L$.

Duclos, Makdissi and Wodon (2008) suggested the idea of revenue-neutral multivariate tax reform based on $CD-$curves without making neither explicit demonstrations nor applications. Let us recall the implication of budget neutrality defined previously:

$$\gamma_{ij} = \sum_{j \neq i}^{L} \frac{1 + \frac{1}{X_j}\sum_{k=1}^{L}\frac{\partial X_k}{\partial t_k}}{1 + \frac{1}{X_i}\sum_{k=1}^{L}\frac{\partial X_k}{\partial t_k}} \text{ , for all } i \neq j \in \{1, \ldots, L\} \text{ ,}$$

such that $dR = 0$ and:

$$dt_i = -\sum_{j \neq i}^{L} \frac{X_j}{X_i} \gamma_{ij} dt_j < 0 \ . \tag{24}$$

Compared with the standard approach initiated by Makdissi and Wodon (2002), we have a combination of decrease and increase in the tax rates, dtj $\forall j \neq i \in \{1, \ldots, L\}$. We then get the following:

Theorem 3.2 *Let* $\mathbf{y}$ *and* $\mathbf{x}$ *denote the income distributions that prevail before the reform and after it, respectively. Let* $dt_i = -\sum_{j \neq i}^{M} \frac{X_j}{X_i}\gamma_{ij} dt_j < 0$ *for all* $i \neq j \in \{1, \ldots, L\}$ *be a multivariate revenue-neutral marginal tax reform such that* $z \leqslant z^{\max}$ *and* $z^{\max} \in [0, y^{\max}]$. *Then, the two following propositions hold.*

(i) $\forall P_z \in \mathscr{P}^s,\ s \in \mathbb{N}^* : \left[\mathbf{x} \succeq^s_{P_z} \mathbf{y}\right] \Longleftrightarrow \left[\sum_{j \neq i}^{L}\left(CD^s_j(y) - \gamma_{ij} CD^s_i(y)\right) \leqslant 0\right]$.

(ii) $\forall SW_z \in \mathscr{U}^s,\ s \in \mathbb{N}^* : \left[\mathbf{x} \succeq^s_{W_z} \mathbf{y}\right] \Longleftrightarrow \left[\sum_{j \neq i}^{L}\left(CD^s_j(y) - \gamma_{ij} CD^s_i(y)\right) \geqslant 0\right]$.

Proof.
See Appendix C. ■

4 Empirical analysis and results

In this part, we simulate the effect on welfare of some tax reforms in France. After presenting the data and some arrangements, we estimate a demand system in order to compute marginal cost and public funds.

4.1 Data

We use the 2005/2006 Survey of Family Expenditure ("Budget Famille" provided by INSEE). This data base reports a detailed set of consumption (200 items) and demographic variables (1000 items) over 10,240 household in France – an overview is proposed in Appendix A (A1). As depicted in the Appendix (A2), we consider seven main groups of commodities. We exclude items that don't correspond to classic consumption like gifts or savings. Expenditure (defined to be the budget) is noted m, it is the sum of consumption expenditures – different from y which is the household's income.

Because the survey is conducted monthly, we are able to merge the data base with monthly-price indexes, also provided by the INSEE in a first time. However, merging each group with this price index provides an insufficient variability in prices, since we have vectors of 10,240 elements with only 12 different values. This is why the price of each group is modeled as a Stone index weighted by the consumption within group for each household. In other words, the price variability between households, for a group, is generated by their difference in consumption inside the group.

Recall that the reported values (we call $\tilde{x}$) in the survey include all indirect taxs (VAT, excises, insurance tax). Trannoy and Ruiz (2008), write the problem as $\tilde{x}_i qx = px_i \Pi_j (1+\tau_j)$ where τ_j is the $j-$th indirect tax type (VAT, excises,insurance tax) and provided these values. With these elements, we compute the gouvernement revenue collected for each good and the implicite tax right.

4.2 Elasticities and marginal cost of public funds

We present here a brief presentation about the estimation strategy.[6] Before discussing the demand system one may tackle the censoring data problem before the so-called "problem of zeros" – which arises when handling micro data surveys and the particular error structure

[6] A detailed debate about the methodology is discussed in a forthcoming paper by Khoudmi and Mulkay (2012). All estimations are performed using STATA and are available upon request.

encountered in the multi-equation case (k equations each one indexed i). Our problem can be formally described as:

$$d_{ih}^* = z'\alpha + \nu_{ih}$$
$$w_{ih}^* = x'\beta + \epsilon_{ih} \ ,$$

with

$$d_{ih} = 1 \text{ if } w_{ih} > 0$$
$$d_{ih} = 1 \text{ otherwise}$$
$$w_{ih} = d_{ih} w_{ih}^* \ ,$$

where d_{ih}^* and w_{ih}^* are respectively the resulting latent selection and dependent variables, w_{ih} and d_{ih} are their observed counterparts. Assume the concatenated error vector $[\nu', \epsilon']' = [\nu_1 \ldots \nu_i \ldots \nu_k, \epsilon_1 \ldots \epsilon_i \ldots \epsilon_k]'$ is distributed as a $2k-$variate normal distribution:

$$\begin{pmatrix} \nu \\ \epsilon \end{pmatrix} \sim Nid \left[0; \begin{pmatrix} \Sigma_{\nu\nu} & \Sigma'_{\nu\epsilon} \\ \Sigma_{\nu\epsilon} & \Sigma_{\epsilon\epsilon} \end{pmatrix} \right]$$

The estimation method have to include this correlation structure. One direct way is to estimate simultaneously the selection and the result systems as Yen (2005) with a Full Information Maximum Likelihood (FIML), which is numerically intensive, specially in the case of the result system we choose (see bellow). The other way is to consider a two step estimation *à la* Heckman (1979) like Tauchmann (2010) by including a correction term – we call multivariate inverse Mills ratio η_j – in the results equations such as:[7].

$$w_{ih} = d_h x'\beta + d_h \sum_{j=1}^{m} \delta_{ij} * \eta_j \ , \tag{25}$$

where δ_{ij} is parameter associated to the multivariate Mills ratio computed for the selection equation j introduced in the equation i. Note that the estimations in the case of a SUR is conditionnated on the individual pattern $d_h = [d_{1h}, \ldots, d_{kh}, \ldots, d_{Kh}]'$.

[7] Recall the detailed formula from Tauchman (2010)

$$w_{ih} = d_h x'\beta + d_h \sum_{j=1}^{m} \delta_{ij} \psi_{jh} \phi(z'_{jh}\alpha_j) \frac{\Phi^{m-1}(A_{jh}, R_{jh})}{\Phi^m(.)}$$

ϕ_{ih} is the selection variable recoded such as $2d_{ih} - 1$. $\Phi^k - 1$ stand for a c.d.f for a $(k-1)-$dimensional normal distribution (all computation was performed numerically). A the numerator stand for a $k-1$ vector of elements $\psi_{lh}(z'_{lh}\alpha - \sigma_{lh}^{\nu\nu} z'_{lh}\alpha)/(1-(\sigma^{\nu\nu})^2)^1/2$ where $l = 1, \ldots, k$ with $l \neq j$. R is defined by $\Psi_{jh}\Xi_{jh}\Psi_{jh}$ With Ξ a partial correlation matrix $Cor(\nu_h|_{jh})$ and Ψ a diagonal matrix with the elements ψ. Finally, $\Phi^k(.)$ stand for the joint probability of the observed pattern $Pr(d_h)$

Note that α and $\Sigma_{\nu\nu}$ have to be estimated simultaneously.[8] After reducing the selection problem to three equations,[9] this multivariate selection system as a multivariate Probit estimation was performed numerically using Cappellari and Jenkins (2006) framework involving Geweke-Hajivassiliou-Keane sampling method – see Hajivassiliou (1993) or Train (2003) – through simulated maximum likelihood maximization.

Let's turn now to the result system which is a demand system specified as the well documented Banks *and al.* QAIDS (Quadratic Almost Ideal Demand System). In order to estimate this system, note that (i) to reduce biais due to measurement errors and to allow for endogeneity on $\ln m$, one may instrument this last by $\ln x$ (ii) households characteristics are introduced by a demographic translation as discussed by Polak and Wales (1982). According to these elements, the prices index design and Tauchman (2010) formula, the original system can be written in a slightly transformed form:

$$w_{ij} = d_h \left(\rho^{\alpha}_{ih} + \sum_j \rho^{\gamma}_{ij} \mathrm{ln} q_{jh} + \rho^{\beta}_i \mathrm{ln} \left[\frac{m}{a(q)} \right] + \frac{\rho^{\lambda_i}}{b(q)} \left\{ \mathrm{ln} \left[\frac{m}{a(q)} \right] \right\}^2 + \sum_j \eta_j \right) \tag{26}$$

With $\mathrm{ln}a(q)$ as a translog form

$$\mathrm{ln}a(q) = \rho^{\alpha}_0 + \sum_i \rho^{\alpha}_{ih} \mathrm{ln} q_{ih} + \sum_i \sum_j \rho^{\gamma}_{ij} \mathrm{ln} q_{ih} \mathrm{ln} q_{jh} \tag{27}$$

And the $b(q)$ as Cobb Douglass price agregator

$$b(q) = \prod_i q_i^{\rho^{\beta}_i} \tag{28}$$

Remind the usual preference constraints for a well behaved utility function :

$$\sum_i \rho^{\alpha}_i = 0 \quad \sum_i \rho^{\gamma}_{ij} = 0 \tag{Adding up}$$

$$\sum_j \rho^{\gamma}_{ij} = 0 \tag{Homogeniety}$$

$$\gamma_{ij} = \rho^{\gamma}_{ji} \tag{Symmetry}$$

With $\rho\alpha_{ih}$ term incorporating demographic effects througt variables χ_{jh} and parameters α_{id} constrainted such as :

$$\rho^{\alpha}_{ih} = \rho^{\alpha}_i + \sum_j \rho^{\alpha}_{ij} \chi_{jh} \tag{29}$$

[8] The two step methodology have been performed in a multivariate case by Heins and Wessels (1990) estimating an univariate probit equation by equation. Unfortunately, this is true only if $\Sigma_{\nu\nu}$ is diagonal, but preliminary analysis with tetrachoric correlations and our final results invalidate this assumption.

[9] Since only Alcohol and tobacco, Clothing and Transport present a high zero consumption ratio. It is intuitive that the other goods will be almost perfectly explained.

$$\sum_i \rho_{id}^{\alpha} = 0 \qquad \text{(DEM)}$$

In the same manner than Poi (2002, 2008), we estimate the parameters for six equation over seven to avoid singularity and the parameter for the dropped equation are imposed using constraints. To allow for correlations over the obtained equations $\Sigma_{\epsilon\epsilon}$, we use a SUR non linear regression thought iterated feasible generalized least squares[10]. After estimation[11], elasticities are computed for all households and we present the mean weighted by the shares.

	Expenditure	Food	AlcToba.	Cloth.	Housing	Transport	Enter.	Other
Food	0.766	-1.687	-0.047	-0.126	0.937	-0.895	-0.363	1.415
AlcToba.	0.250	0.575	-1.544	0.996	-1.705	-0.491	-1.021	2.939
Cloth.	1.448	-0.443	0.625	-1.964	1.128	0.597	0.384	-1.775
Housing	0.572	0.717	-0.604	0.622	-1.801	0.006	-0.107	0.595
Transport	0.868	-1.180	0.066	0.116	1.264	-0.820	-0.508	0.193
Enter.	1.600	-0.226	0.305	-0.152	0.009	0.513	-0.513	-1.534
Other	1.065	0.955	0.275	-0.352	-0.978	-0.037	0.244	-1.173

Applying formula (9) lead us to compute the marginal cost of public funds and their ratio γ_{ij} and dt_i/dt_j given by the Proposition 1.1. (i on lines and j on columns).

	Direct	Food	AlcToba.	Cloth.	Housing	Transport	Enter.	Other
Food	1.292	1.000	0.657	1.248	1.264	1.081	1.013	1.252
AlcToba.	1.965	1.521	1.000	1.898	1.923	1.644	1.541	1.904
Cloth.	1.035	0.801	0.527	1.000	1.013	0.866	0.812	1.003
Housing	1.022	0.791	0.520	0.987	1.000	0.855	0.802	0.990
Transport	1.195	0.925	0.608	1.154	1.169	1.000	0.937	1.158
Enter.	1.275	0.987	0.649	1.231	1.248	1.067	1.000	1.235
Other	1.032	0.799	0.525	0.997	1.010	0.864	0.810	1.000

	Food	AlcToba.	Cloth.	Housing	Transport	Enter.	Other
Food	1.000	0.056	0.548	1.881	0.874	1.267	1.112
AlcToba.	17.716	1.000	9.713	33.316	15.487	22.441	19.704
Cloth.	1.824	0.103	1.000	3.430	1.594	2.310	2.029
Housing	0.532	0.030	0.292	1.000	0.465	0.674	0.591
Transport	1.144	0.065	0.627	2.151	1.000	1.449	1.272
Enter.	0.789	0.045	0.433	1.485	0.690	1.000	0.878
Other	0.899	0.051	0.493	1.691	0.786	1.139	1.000

[10] Because of the high sensitivity in the final estimation, we also exclude households representing 1% top and down consumption in each good

[11] Note From The Authors : Obtained results are somewhere encouraging (signs and magnitudes) but have to be improved (actually, more specifications are under work) for the final version)

4.3 Confidence Intervals and $p-$values

We first notice that the households' consumption x_i^h of good i has been rescaled in order to take into account the sampling weight w^h of each household h: $\widetilde{x}_i^h = x_i^h w^h$. The same thing holds for the aggregate consumption $\widetilde{X}_i = \sum_h w^h x_i^h$. Let us rewrite our $CD-$curves in function of the c.d.f.:

$$CD_\ell^1(y^{\max}) = \begin{cases} \frac{\widetilde{x}_i(y)}{\widetilde{X}_i} f(y^{\max}) & \text{if } s=1 \quad \text{for all} \quad \ell \in \{1,\ldots,L\} \\ \\ \int_0^{y^{\max}} CD_\ell^{s-1}(y)dy & \text{if } s \geqslant 2 \quad \text{for all} \quad \ell \in \{1,\ldots,L\} \ . \end{cases} \tag{30}$$

Following Fishburn (1976), it is possible to integrate by parts successively the c.d.f., such that for all $s \in \{2,3,4\ldots\}$:

$$CD_\ell^s(y^{\max}) = \frac{1}{(s-2)!}\int_0^{y^{\max}} \left(\frac{\widetilde{x}_i(y)}{\widetilde{X}_i}\right)(y^{\max}-y)^{s-2}\, dF(y)\ , \ \text{for all} \ \ \ell \in \{1,2,\ldots,L\}\ .$$

For a population of H households, an estimator may be of the following form:

$$\widehat{CD}_\ell^s(y^{\max}) = \frac{1}{H(s-2)!}\sum_{h=1}^{H}\left(\frac{\widetilde{x}_i(y)}{\widetilde{X}_i}\right)(y^{\max}-y)^{s-2}.$$

Let $\hat{\theta}$ be the first-order sample moment of the random variable $\theta := (y^{\max}-Y)^{s-2}$, that is, $\hat{\theta} = \frac{1}{H}\sum_{n=1}^{H}\theta^n$. It follows from the central-limit theorem, and $H \to \infty$, that $\sqrt{H}(\hat{\theta}-\theta) \sim \mathcal{N}(0, H\sigma^2)$, where $\sigma^2 := var(\hat{\theta})$. It is then clear that $\widehat{CD}_\ell^s(y^{\max}) =: \phi(\hat{\theta})$, where ϕ is a function which is assumed to be differentiable. Accordingly, from Rao's delta method:

$$\sqrt{H}(\phi(\hat{\theta})-\phi(\theta)) \sim \mathcal{N}(0, H\Sigma), \ \text{where the variance} \ \ H\Sigma = \frac{1}{H}\frac{\partial\phi(\hat{\theta})}{\partial\hat{\theta}}\sigma^2\frac{\partial\phi(\hat{\theta})}{\partial\hat{\theta}}.$$

After simple computations, we get an (unbiased) estimator for the variance of $\hat{\theta}$ expressed as:

$$\hat{\sigma}^2 = \frac{1}{H-1}\left(\kappa - \hat{\theta}^2\right)\ , \quad \text{with} \ \ \kappa := \frac{1}{H}\sum_{h=1}^{H}(\theta^h)^2\ .$$

We deduce that the variance of the last point $y^{\max}$ of the $CD-$curve is:

$$\hat{\Sigma} = \frac{1}{(s-2)!^2}\frac{\kappa-\hat{\theta}^2}{H-1}\ .$$

Making the same reasoning, we deduce the variance of all points of the $CD-$curve, that is: $var\left(\widehat{CD}_\ell^1\left(y^{H-1}\right)\right)$, $var\left(\widehat{CD}_\ell^1\left(y^{H-2}\right)\right)$, and so on. Sure, for the first 30 points of the $CD-$curve, as normality is not ensured, we cannot carry out accurate confidence intervals. A linear interpolation may be used in that case.

As an example, let us plot the confidence interval of the $CD-$curve of order 3 of the good 1 (food) against the $CD-$curve of order 3 of good 5 (transport).

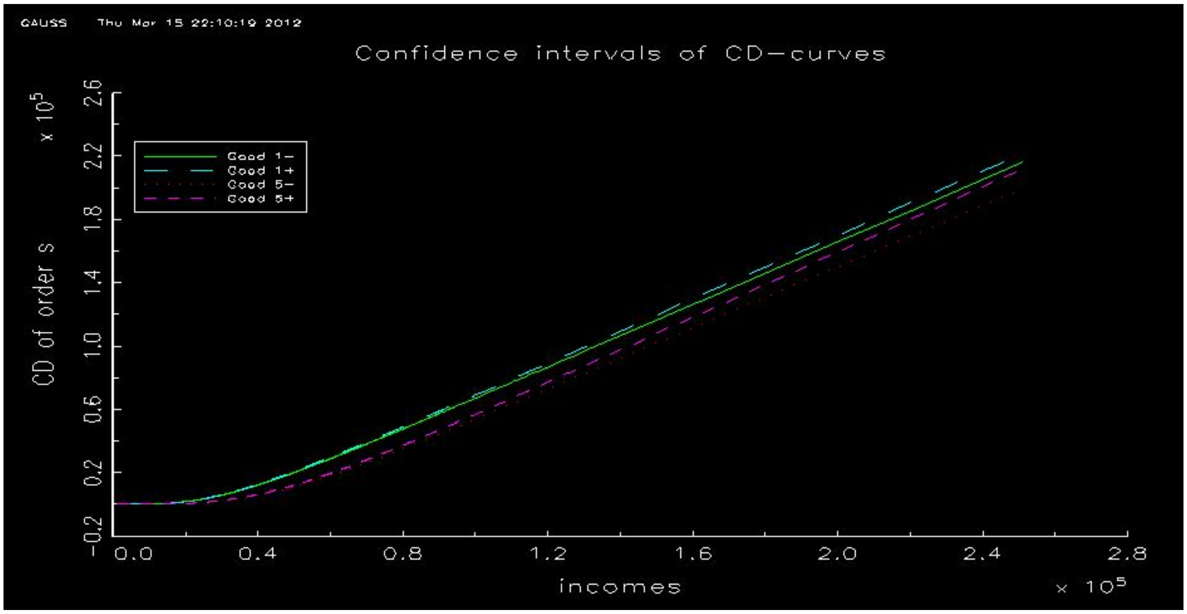


Fig. 1 Confidence intervals of $CD-$curves of order 3.

The two curves at the top represent the confidence interval of food, the two other ones that of transport. As can be seen, the thickness of the two intervals do not cross before the poverty line. We may interpret this as follows: it seems possible to increase the tax on transport and use the fiscal benefit to subside food. To reach these types of conclusion one needs to implement robust stochastic dominance tests.

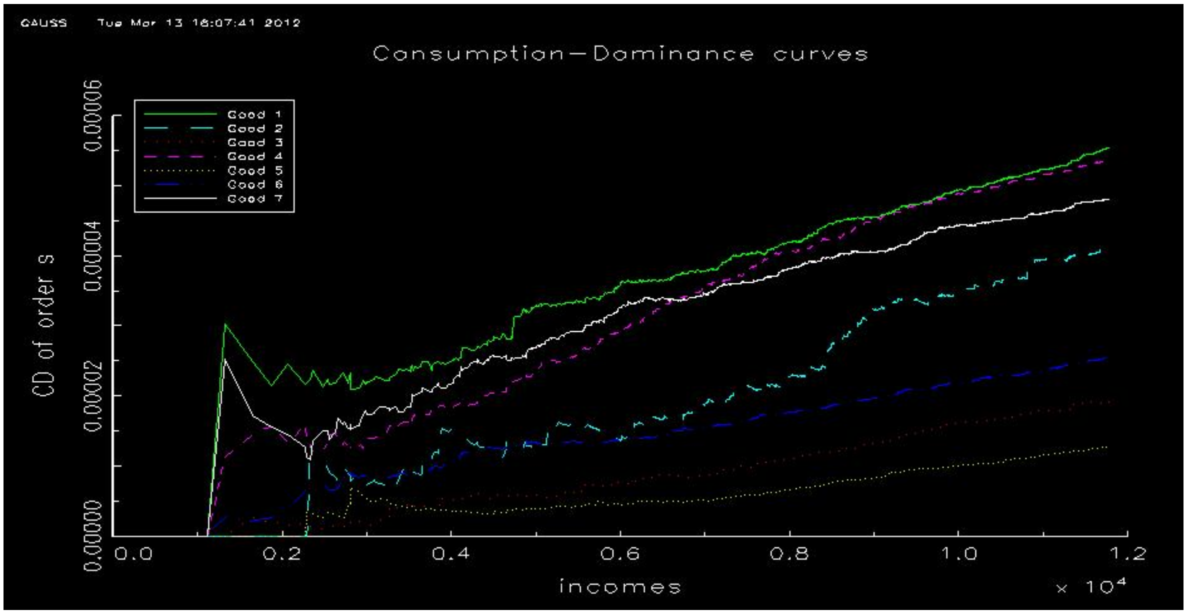
To statistically test for a reform at the any order, it is possible to compute a $p-$value for a dominance test according to Davidson and Duclos (2000) or Barret and Donald (2003) framework. Davidson and Duclos (2000) asymptotic procedure have the advantage of making use of the covariances between the estimates made at each points and include the definition of a set of points. Barret and Donald developed a Kolmogorov-Smirnov type test with a numerical procedure which evaluate the entire support but ignores the covariance structure.

This why we choose, in the sequel, to consider the Davidson and Duclos (2000) test poverty dominance purpose, where the discretization is more suitable when restricting the domain and to consider Barret and Donald (2003) bootstrap procedure for welfare dominance, where the evaluation requires the whole support $[0, y^{\max}]$.

4.4 Restricted stochastic dominance tests: poverty analysis

Davidson and Duclos (2000) test of restricted dominance is performed with z being fixed to half the income median ($\frac{1}{2}F^{-1}(0.5)$). In this respect, the $CD-$curves are plotted up to z (the first 30 points have been withdrawn to avoid bad computations):

Fig. 2 $CD-$curves of order 2.



As can be seen in the previous graph, commodity 1 is above the other ones. Subsequently, from Theorem 1, an increase of welfare (or a decrease of poverty) may be achieved in taxing the other goods ans subsidizing the first one. For instance taxing commodity 5 or 3 seems to be the better strategy. In order to check these assumptions, one has to take into account the variance of the sample, i.e., to perform statical tests. The following Table reports the normal c.d.f. of the $T-$statistics, which measures the difference between the two curves at z, with the independence assumption between the two samples. In words, if the $P-$value is above 95% then we accept that the $CD-$curve of good i (line i) dominates that of good j (column j) multiplied by the ratio of marginal costs of funds γ_{ij}, i.e., $CD_i^s(y) \geqslant \gamma_{ij} CD_j^s(y)$:

Table 1. Second-order dominance test 5% level*

	$j \rightarrow$ 1	2	3	4	5	6	7
$i=1$		1	1	0	1	1	0
$i=2$			0	0	0.9998	0	0
$i=3$				0	1	0.9999	0
$i=4$					1	1	0.5866
$i=5$						0	0
$i=6$						0	0

*Source: EBF $(2005-2006)$; $z=\frac{1}{2}F^{-1}(0.5)$

We perform the same test at the order 3 (the poverty line remains half the income median):

Table 2. Third-order dominance test 5% level*

	$j \rightarrow$ 1	2	3	4	5	6	7
$i = 1$		1	1	0	1	1	0
$i = 2$			0	0	1	0	0
$i = 3$				0	1	0.9997	0
$i = 4$					1	1	0.1684
$i = 5$						0	0
$i = 6$						0	0

*Source: EBF $(2005 - 2006)$; $z = \frac{1}{2}F^{-1}(0.5)$

At the order $s = 4$:

Table 3. Fourth-order dominance test 5% level*

	$j \rightarrow$ 1	2	3	4	5	6	7
$i = 1$		1	1	0.0105	1	1	0.001
$i = 2$			0	0	0.9997	0	0
$i = 3$				0	1	0.9349	0
$i = 4$					1	1	0.0589
$i = 5$						0	0
$i = 6$						0	0

*Source: EBF $(2005 - 2006)$; $z = \frac{1}{2}F^{-1}(0.5)$

In order to check whether the $CD-$curves cannot provide a non-ambiguous ranking between goods 1 and 4 or 1 and 7, the following graph depicts those relationships.

Fig. 3 $CD-$curves of order 4.

4.5 Non-restricted stochastic dominance tests: welfare analysis

We know turn to the welfare improving tax reforms. As shown by Foster and Shorrocks (1987) and underlined in Theorem 1, welfare and poverty are dual to each other if, and only if, welfare is truncated at z in the same manner as the poverty index. Relaxing the troncature imposed at point z is implies that we test for welfare-improving marginal reforms on the entire support $[0, y^{\max}]$. For this purpose, we make use Barret and Donald's (2003) bootstrap test of stochastic dominance (with 1,000 replications).

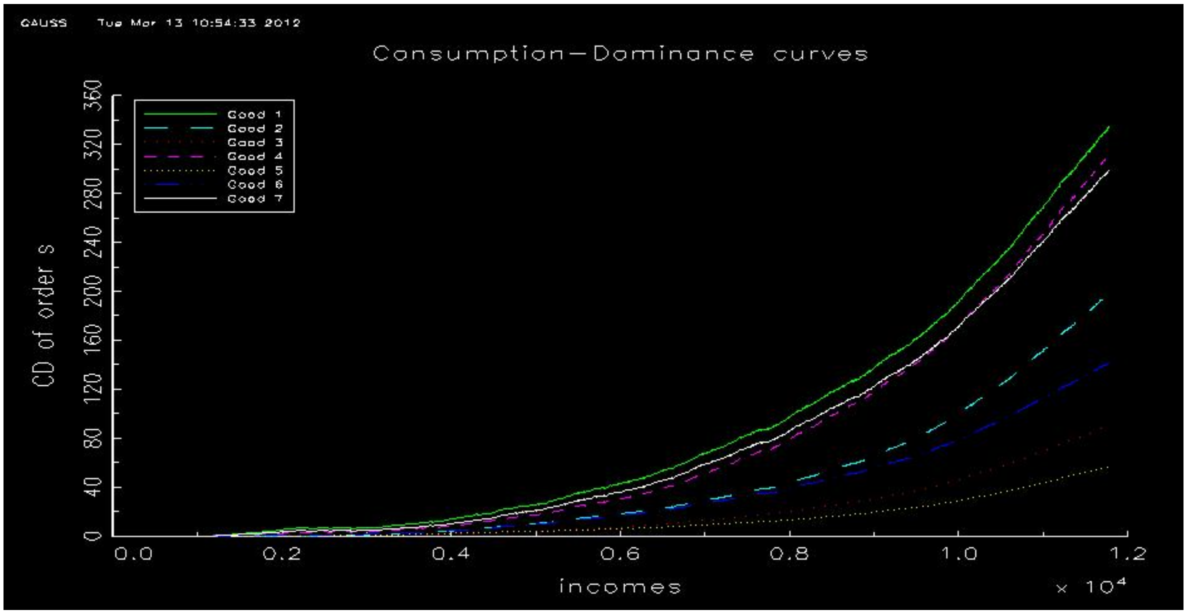


Table 4. Second-order dominance test 5% level*

	$j \rightarrow$ 1	2	3	4	5	6	7
$i = 1$		0	0	0	0	0	0
$i = 2$			0.001	0	0.875	0	0.415
$i = 3$				0	0	0	0
$i = 4$					0	0	1
$i = 5$						0	0
$i = 6$						0	1

*Source: EBF (2005 – 2006) ; B=1,000

Table 5. Third-order dominance test 5% level*

	$j \rightarrow$ 1	2	3	4	5	6	7
$i = 1$		0	0	0	0	0	0
$i = 2$			0	0	0	0	0
$i = 3$				0	0	0	0
$i = 4$					0	0	1
$i = 5$						0	0
$i = 6$						0	1

*Source: EBF (2005 – 2006) ; B=1,000

Table 6. Fourth-order dominance test 5% level*

	$j\rightarrow$ 1	2	3	4	5	6	7
$i=1$		0	0	0	0	0	0
$i=2$			0	0	0	0	0
$i=3$				0	0	0	0
$i=4$					0	0	1
$i=5$						0	0
$i=6$						0	1

*Source: EBF $(2005-2006)$; B=1,000

4.6 Multivariate reforms

The previous subsections clearly indicate that good (food) dominates the other goods, i.e., welfare-improving or poverty-reducing tax reforms may be more easy to reach by subsidizing the group of food commodities. In consequence, we show that it is possible to tax transport or clothings in order to decease the tax on food to alleviate overall poverty. The question of taxing simultaneous commodities remain open. Following Theorem 3.2, it is possible to check for multivariate tax reforms with a linear combination of $CD-$curves, those curves being weighted by the ratio of marginal costs of funds. This combination must be nonpositive: $\sum_{j\neq i}^{L}\left(CD_j^s(y)-\gamma_{ij}CD_i^s(y)\right)\leqslant 0$.

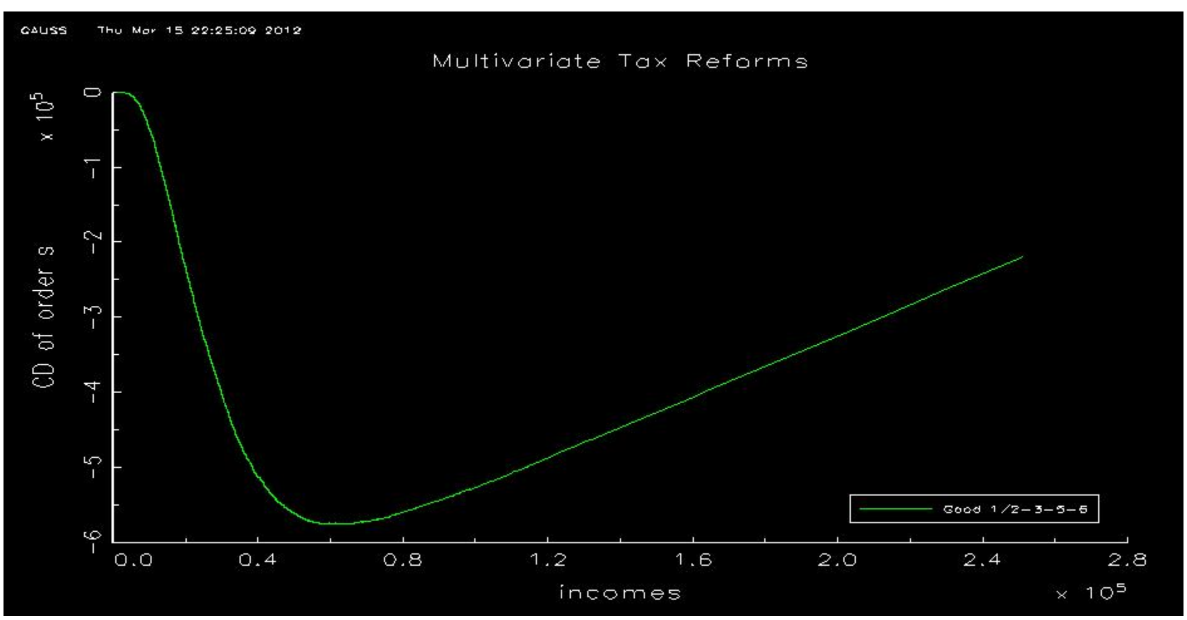


Fig. 4 Multivariate tax reform.

As can be seen in Figure 4, if we increase the tax on {alcohol-tobacco, clothing, transport, entertainment} and if we use the proceed to subsidize food, then the multivariate tax reform is welfare-improving *and* poverty-reducing: the linear combination of the $CD-$curves lies nowhere above zero.

Appendix A

A1 BDF Individual Characteristics

Variable	Label	Mean	Std Dev	Min	Max
m	Total expenditure	27730	18143	1116	251173
revtot	Total income	32394	24901	-1113	688617
Se	= 1 if household head (hhh) is male	0.649	0.477	0	1
Ag	Age of household head	50.257	16.867	16	99
Chi	Number of childrens	0.804	1.089	0	8
Ad	Number of adults	1.687	0.542	1	6
N1	=1 if french nationality	0.909	0.287	0	1
N2	=1 if foreign nationality	0.091	0.287	0	1
C1	=1 if agricultor or independant	0.056	0.230	0	1
C2	=1 if white collar	0.299	0.458	0	1
C3	=1 if worker or employed	0.311	0.463	0	1
C4	=1 if other (student. reteired)	0.334	0.472	0	1
Z1	=1 if Paris region	0.357	0.479	0	1
Z2	=1 if other region	0.643	0.479	0	1
St1	=1 if city more 100 000 inhabitant	0.446	0.497	0	1
St2	=1 if city less 100 000 inhabitant	0.301	0.459	0	1
St3	=1 if rural	0.253	0.435	0	1
D1	=1 if hhh high school or higher diploma	0.253	0.435	0	1
D2	=1 if hhh not High school	0.747	0.435	0	1
H1	=1 if individual house	0.608	0.488	0	1
H2	=1 if collective house	0.350	0.477	0	1
H3	=1 if other house type	0.042	0.201	0	1
S1	=1 if interview in winter	0.289	0.453	0	1
S2	=1 if interview in spring	0.245	0.430	0	1
S3	=1 if interview in autumn	0.244	0.430	0	1
S4	=1 if interview in summer	0.221	0.415	0	1
T	Time trend	6.244	3.534	1	12

A2 BDF Consumption. Government Revenue and Taxation

.	Food	Alc and Tob	Cloth.	Housing	Transport	Enter	Other
Consumption for the overall sample (10240 obs)							
$\bar{x}_i$	4 185	728	2 207	6 493	4 442	5 990	3 685
x_i^{median}	3 728	245	1 247	5 171	1 611	4 314	2 801
σ_{x_i}	2 692	1 349	3 120	5 315	7 298	6 530	4 146
x_i^{min}	0	0	0	0	0	0	0
x_i^{max}	24 761	38 043	79 558	70 792	96 647	161 488	219 041
$\bar{w}_i$	0.1726	0.0287	0.0711	0.2625	0.1242	0.1998	0.1412
w_i^{median}	0.1564	0.0101	0.0521	0.2288	0.0207	0.1821	0.1209
σ_{x_i}	0.0997	0.0466	0.0724	0.1578	0.1440	0.1180	0.0920
w_i^{min}	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
w_i^{max}	0.7534	0.6212	0.6379	0.9319	0.8452	0.9054	0.8721
Consumption for the conditionnated and cleaned from outliers sample (4029 obs)							
$\bar{x}_i$	4 744	1 009	2 444	6 118	2 532	6 021	3 828
x_i^{median}	4 529	650	1 751	5 107	1 824	5 042	3 212
σ_{x_i}	2 285	1 011	2 277	4 000	2 251	4 211	2 363
x_i^{min}	629	60	78	1 040	78	595	516
x_i^{max}	11 412	5 083	12 423	20 597	13 961	23 446	14 542
$\bar{w}_i$	0.2050	0.0448	0.0950	0.2568	0.1112	0.2375	0.1610
w_i^{median}	0.1954	0.0291	0.0783	0.2310	0.0825	0.2246	0.1475
σ_{x_i}	0.0882	0.0446	0.0713	0.1352	0.1010	0.1124	0.0769
w_i^{min}	0.0200	0.0013	0.0024	0.0368	0.0023	0.0227	0.0178
w_i^{max}	0.5754	0.4831	0.4803	0.7685	1.1662	0.7083	0.5678
Computed Taxation and Revenue for the overal sample with weights provided by INSEE							
t_i	0.0594	1.1479	0.1960	0.0980	0.3250	0.1458	0.1049
R_i (10^9 Euros)	5.651	9.376	8.194	13.868	25.001	17.338	8.868
X_i (10^9 Euros)	100.717	17.544	49.997	155.326	101.930	136.280	93.397

Appendix B

B1 Multivariate Probit

	AlcToba	Cloth	Transprt
Chi	-0.019 **	0.416	0.180
Ad	0.404	1.167	0.932
Ag	0.043	0.028	0.034
Se	0.099	-0.359	0.023 **
Chi2	-0.010 *	-0.064	-0.039
Ad2	-0.043	-0.170	-0.142
Ag2	0.000	0.000	-0.001
St1	0.070	0.014 **	0.030 **
St2	0.011 *	-0.022 **	-0.025 **
D1	0.029 *	-0.018 **	-0.020 **
A1	-0.015 *	-0.011 **	0.020 **
H1	-0.042 *	-0.064 **	-0.001 **
H2	-0.100 *	-0.019 **	-0.026 **
S1	0.005 *	-0.001 **	0.009 **
S2	0.047 *	0.048 **	0.030 **
S3	0.110	0.018 **	0.029 **
T	-0.005 *	0.002 **	-0.013
constant	-0.991	-0.272 **	-0.230 **

** stand for not significant at 10%

* stand for significant at 10%

nothing stand for significant at 5%

The estimated selection correlation matrix $\hat{\Sigma}_{\nu\nu}$ (with all elements significative)

.	ν_{AlTo}	ν_{Cloth}	ν_{Trsrt}
ν_{AlTo}	1		
ν_{Cloth}	0.084	1	
ν_{Trsprt}	0.105	0.201	1

B2 QUAIDS Model Results

	Food	AlcTobac	Cloth	Housing	Trsprt	Enter	Other
ρ_i^{α}	-0.155	-0.070	0.290	-0.114	0.085	1.057	-0.093
ρ_i^{β}	-0.092	-0.084	0.088	-0.232	-0.050	0.336	0.034
$\rho_{1,i}^{\gamma}$	-0.122	0.038	-0.058	0.220	-0.116	-0.109	0.146
$\rho_{2,i}^{\gamma}$	0.038	0.012	0.017	-0.043	0.023	-0.079	0.032
$\rho_{3,i}^{\gamma}$	-0.058	0.017	-0.064	0.080	-0.001	0.073	-0.047
$\rho_{4,i}^{\gamma}$	0.220	-0.043	0.080	-0.120	0.150	-0.120	-0.167
$\rho_{5,i}^{\gamma}$	-0.116	0.023**	-0.001	0.150	0.039	-0.075**	-0.021
$\rho_{6,i}^{\gamma}$	-0.109	-0.079	0.073	-0.120	-0.075	0.256	0.054
$\rho_{7,i}^{\gamma}$	0.146	0.032	-0.047	-0.167	-0.021	0.054	0.295
ρ_i^{λ}	-0.008	-0.009	0.007	-0.021	-0.006	0.032	0.004
$\rho_{Ag,i}^{\alpha}$	0.002	-0.016	0.006	-0.010	-0.025	-0.015	0.057
$\rho_{En,i}^{\alpha}$	0.004	0.001	0.000	-0.009	-0.011	-0.006	0.020
$\rho_{Ad,i}^{\alpha}$	-0.021**	-0.008	-0.004	0.004	-0.004**	0.026	0.007
$\rho_{Se,i}^{\alpha}$	-0.014	-0.056	0.053	0.005	-0.066	-0.051	0.130
$\rho_{D1,i}^{\alpha}$	0.010	0.017	-0.063	0.001	0.114**	0.100	-0.179
$\rho_{N1,i}^{\alpha}$	-0.107	0.000**	-0.002	0.194	-0.166	-0.101	0.181
η_{AlcTob}	0.003	0.000	0.000	-0.003	0.000	0.000	.
η_{Cloth}	0.010	-0.001	0.002	-0.011	-0.003	0.003	.
η_{Trsprt}	0.024	-0.003	0.001**	-0.013	-0.006	-0.026	.

** stand for not significant at 10%

* stand for significant at 10%

nothing stand for significant at 5%

Appendix C

(Sufficiency):

(i) The overall poverty variation is, assuming that $dt_\ell \equiv dq_\ell$:

$$dP_z = \int_0^a \left[\sum_{\ell=1}^{L} \frac{\partial p\left(y_E(\mathbf{q}, y), z\right)}{\partial y_E} \cdot \frac{\partial y_E(\mathbf{q}, y)}{\partial t_\ell} dt_\ell \right] dF(y) \ .$$

Denoting $\frac{\partial p(y_E(\mathbf{q},y),z)}{\partial y_E} =: p^{(1)}$, we get from Roy's identity:

$$dP_z = \int_0^a (-1)^1 p^{(1)} x_i dt_i dF(y) + \sum_{j \neq i}^{L} \int_0^a (-1)^1 p^{(1)} x_j dt_j dF(y).$$

From the multivariate differential efficiency cost of funds (24), we obtain:

$$dP_z = \int_0^a (-1)^1 p^{(1)} x_i \left(- \sum_{j \neq i}^{L} \frac{X_j}{X_i} \gamma_{ij} dt_j \right) dF(y) + \sum_{j \neq i}^{L} \int_0^a (-1)^1 p^{(1)} x_j dt_j dF(y).$$

Rearranging terms provides:

$$dP_z = \sum_{j\neq i}^{L} \int_0^a (-1)^1 p^{(1)} \frac{x_j}{X_j} X_j dt_j dF(y) - \sum_{j\neq i}^{L} \int_0^a (-1)^1 p^{(1)} \frac{x_i}{X_i} X_j \gamma_{ij} dt_j dF(y).$$

From the definition of the $CD-$curves, we get that:

$$dP_z = \sum_{j\neq i}^{L} \int_0^a (-1)^1 p^{(1)} \left[CD_j^1(y) - \gamma_{ij} CD_i^1(y)\right] X_j dt_j dy.$$

Integrating by parts s times yields inductively:

$$dP_z = \sum_{j\neq i}^{L} \int_0^a (-1)^s p^{(s)} \left[CD_j^s(y) - \gamma_{ij} CD_i^s(y)\right] X_j dt_j dy.$$

Remark that $X_j > 0$, $dt_j > 0$ and $(-1)^s p^{(s)} \geqslant 0$, then it is sufficient for $dP_z \leqslant 0$ to get

$$-\sum_{j\neq i}^{L} \left[\gamma_{ij} CD_i^s(y) - CD_j^s(y)\right] X_j dt_j \leqslant 0 \ , \ \forall y \leqslant z^{\max} \ , \ \forall j \neq i \in \{1, \ldots, L\} \ .$$

Let $\xi_j := -X_j dt_j$ such that $\{\xi\}_{j\neq i=1}^{L}$ is a sequence of real numbers and let $\eta_j := \left[\gamma_{ij} CD_i^s(y) - CD_j^s(y)\right]$. From Abel's Lemma, if $\xi_L \leqslant \xi_{L-1} \leqslant \cdots \leqslant \xi_1 \leqslant 0$, then it is sufficient that $\sum_{j\neq i}^{\ell} \eta_j \geqslant 0$ for all $\ell \in \{1, \ldots, L\} \setminus \{i\}$ to get $\sum_{j\neq i}^{\ell} \eta_j \xi_j \leqslant 0$. Therefore, the sufficient conditions to get $dP_z \leqslant 0$ are:

$$\sum_{j\neq i}^{L} \left[CD_j^s(y) - \gamma_{ij} CD_i^s(y)\right] \leqslant 0 \ , \ \forall y \leqslant z^{\max} \ , \ \forall j \neq i \in \{1, \ldots, L\} \ . \tag{31}$$

(ii) *Mutatis mutandis* as in (i).

(Necessity): Let us take a set of functions $p\left(y_E(\mathbf{q}, y), z\right)$, for which the $(s-1)$-th derivative is:

$$p^{(s-1)}\left(y_E(\mathbf{q}, y), z\right) = \begin{cases} (-1)^{s-1} \epsilon & \text{if } y \leqslant \overline{y} \\ (-1)^{s-1} (\overline{y} + \epsilon - y) & \text{if } \overline{y} < y \leqslant \overline{y} + \epsilon \ , \\ 0 & y > \overline{y} + \epsilon. \end{cases}$$

Poverty indices whose functions $p\left(y_E(\mathbf{q}, y), z\right)$ have the above form for $p^{(s-1)}\left(y_E(\mathbf{q}, y), z\right)$ belong to the class Π^s. This yields:

$$p^{(s)}\left(y_E(\mathbf{q}, y), z\right) = \begin{cases} 0 & \text{if } y \leqslant \overline{y} \\ (-1)^{s} & \text{if } \overline{y} < y \leqslant \overline{y} + \epsilon \ , \\ 0 & y > \overline{y} + \epsilon. \end{cases} \tag{32}$$

Imagine that the last sum in (31) is strictly positive on an interval $[\overline{y}, \overline{y} + \epsilon]$ for some ℓ, for $\overline{y} < z^{\max}$, and for ϵ that can be arbitrarily close to 0. For $p\left(y_E(\mathbf{q}, y), z\right)$ indices with s-order derivatives defined as in (32), the marginal tax reform induces an increase of poverty. Hence, the sum in (31) cannot be strictly positive for some ℓ, $y \in [\overline{y}, \overline{y} + \epsilon]$ when $\overline{y} < z^{\max}$. This proves necessity.

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