Reconciling Consumption Inequality with Income Inequality[*]

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Abstract

The rise in consumption inequality in response to the increase in income inequality over the last three decades in the U.S. is puzzling to standard incomplete market models with expected utility. Incomplete spanning models tend to predict too much consumption inequality while debt-constrained market models tend to result in too little consumption inequality. We show that a model with two-sided lack of commitment and chance attitudes, as emphasized by prospect theory, can explain the relationship and avoids the systematic bias of the expected-utility models. With chance attitudes, such as optimism and pessimism, the agents increase the weight of high and low outcomes compared to their objective probabilities. For realistic values of risk aversion and for chance attitudes found in experimental economics, lenders' incentive to share decreases. Without inducing a systematic bias for consumption inequality, the latter effect endogenously amplifies the increase in consumption inequality relative to the standard expected utility model and thereby improves the fit to the data.

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1 Introduction

The increase in consumption inequality in response to the increase in income inequality in the U.S. over the last three decades is hard to explain with the standard macroeconomic models (Blundell, Pistaferri, and Preston, 2008, Cutler and Katz, 1992, Krueger and Perri, 2006). Krueger and Perri (2006) found that incomplete-spanning models like Aiyagari (1994) by two-fold overstated the increase in consumption inequality, whereas models with two-sided lack of commitment or debt-constrained markets (Alvarez and Jermann, 2000) by four-fold understated the rise in consumption inequality. Rationalizing the link between income and consumption inequality in a macroeconomic model is, however, essential to optimally designing government policies, such as a progressive tax system to protect households' after-tax income against undesirable fluctuations (Krueger and Perri, 2011).

In macroeconomic research, economists have almost universally relied on the expected utility theory, as axiomatized by von Neumann and Morgenstern (1947) and Savage (1954).[1] Under this theory, stochastic outcomes are weighted with their objective probabilities. With this weighting, people's attitudes toward risk are captured solely by outcome sensitivity (curvature of utility), whereas people's sensitivity toward risk, such as their optimism or pessimism, is ignored. However, empirical studies of people's attitudes toward risk have found that chance attitudes are at least as important as the curvature of utility, and they have identified a critical regularity: people tend to be risk-seeking for low-probability gains and risk-averse for low-probability losses (Conte, Hey, and Moffatt, 2011, Tversky and Kahneman, 1992, Wu and Gonzalez, 1996).

In this paper, we ask whether a model with chance attitudes can reflect the relationship between income and consumption inequality. As the key innovative idea, we integrate chance attitudes into an environment with exogenous idiosyncratic income risk and two-sided lack of commitment (Kocherlakota, 1996). At the heart of our model is the rank-dependent utility (RDU), which was originally proposed by Quiggin (1982) and Schmeidler (1989) as an alternative to the expected utility. In this more general specification of preferences, agents not only weigh stochastic outcomes according to their objective probabilities but also employ a nonlinear weighting function to

[1]The exceptions are Barberis, Huang, Santos, and Santos (2001) and Hansen, Sargent, Turmuhambetova, and Williams (2006), who analyzed the implications of loss aversion for asset prices and for optimal policy decisions, respectively. The application of non-expected utility theories to dynamic settings is challenging as we discuss in Section 2.

determine the decision weights of outcomes. The decision weights depend crucially on the ranking of outcomes. Relatively high outcomes and relatively low outcomes that both appear with low probabilities are found to have higher weights, compared with their objective probabilities. Correspondingly, outcomes that are close to the mean have lower weights.

To reduce their consumption risk, households in our model can voluntarily participate in risk-sharing arrangements with other households, or alternatively, they can save all by themselves as an outside option. Financial market imperfections are one friction behind this assumption (Alvarez and Jermann, 2000). With this friction, households face endogenous borrowing limits that prevent them from hedging the consumption risk completely. Therefore, a more attractive outside option for the lenders is related to more severe borrowing limits and a higher consumption risk.

The equilibrium consumption risk is significantly different between the RDU model and the expected utility model. Under two-sided lack of commitment, the standard expected utility model exhibits too much risk sharing and thus results in too little consumption inequality than is observed in the data. As our main result, we find that the RDU model with relatively pessimistic agents is not prone to the systematic mistake of the expected utility model and improves the fit to the data.

Empirical studies on chance attitudes have simultaneously estimated the functional form of a weighting function that captures the chance attitudes and the coefficient of risk aversion that captures the curvature of the utility function. The coefficient of the relative risk aversion of the utility function is commonly found to be less than one and individuals tend to be relatively more pessimistic than optimistic (Wakker, 2010). For a coefficient of risk aversion less than one, if outcomes were weighed with true probabilities, then the risk aversion would be unreasonably low, and consumption inequality would be implausibly high. Relatively more pessimism than optimism stemming from the weighting function has a diminishing effect on consumption inequality. In our model, relatively pessimistic agents place additional weight on the low-income state in the future which tends to decrease consumption inequality but does not completely compensate for the overshooting in consumption inequality caused by a relative risk aversion coefficient less than one. Overall, we thus demonstrate that the willingness of lenders to share nevertheless decreases compared to the expected utility model. The latter effect is reflected in the optimal consumption allocation by an increase in consumption inequality.

We confront the RDU model with the U.S. consumption and income data over a 24-year period from 1980 to 2003, and we build on Krueger and Perri (2006) to estimate within-group income and consumption inequality using the Consumer Expenditure Interview Survey. Within-group consumption and income inequality are calculated as the residuals of regressions that control for observable characteristics of households, such as level of education or years of experience of household members. The parameters of the RDU model that describe people's risk attitudes, i.e., the parameter of the weighting function and the constant coefficient of relative risk aversion of the utility function, are not treated as free parameters but are taken directly from microeconometric estimations of actual observed behavior in experiments (Conte, Hey, and Moffatt, 2011, Wu and Gonzalez, 1996).

The RDU model with relative pessimism explains the consumption inequality pattern observed in the data substantially better and can better reflect the puzzling link between income and consumption inequality. This result is found to be robust with respect to modeling assumptions and across inequality measures that we consider.

The remainder of the paper is organized as follows. In the next section, we provide a review of prospect theory, and afterwards, we lay down the economic environment. We highlight the effect of chance attitudes in a simplified environment in Section 4. We present our calibration strategy and our quantitative results in Section 5. The final section concludes the paper.

2 Review of Cumulative Prospect Theory

The prospect theory for risk and uncertainty, as proposed by Kahneman and Tversky (1979), Tversky and Kahneman (1992), is motivated by examples that reveal that the expected utility theory, as axiomatized by von Neumann and Morgenstern (1947), has some descriptive shortcomings. Perhaps the most famous example is the Allais paradox (Allais, 1953), which illustrates that actual human behavior contradicts the independence axiom of expected utility also known as the sure-thing principle. This principle postulates that the preference relationship between lotteries is not affected by addition or subtraction of a common consequence. The sure-thing principle is obeyed by the expected utility theory, which makes the theory unsuitable for a comprehensive study of economic decisions under uncertainty and risk.[2]

[2] Prelec (1990) reported a typical finding that violates the sure-thing principle. The vast majority of individuals prefers gamble $S = (0.02, \$20{,}000)$, which yields \$20,000 with probability 0.02, over gamble $R = (0.01, \$30{,}000)$,

The cumulative prospect theory provides a theoretical description for plausible behavior of economic agents. Restricted to the domain of gains, the theory is equivalent to the rank-dependent utility theory (RDU) that was first axiomatized by Quiggin (1982) under the name of *Anticipated Utility Theory*. Under this theory, individual attitudes toward risk are captured not only by the curvature of utility, as under the expected utility theory, but also by nonlinear probability weights.

Consider a lottery $(\{x_j\},\{p_j\})$ that pays x_j with probability p_j. The rank-dependent utility of the lottery for a given probability weighting function, $w:[0,1]\to[0,1]$ with $w(0)=0$ and $w(1)=1$ is defined as follows. First, the lottery outcomes are ranked from the worst to the best, $x_1 \le x_2 \le ... \le x_n$. Second, for each outcome x_j the decision weight π_j is computed as the marginal w-contribution of the outcome probability to the rank:

$$\pi_j = w\left(\sum_{k=j}^{n} p_k\right) - w\left(\sum_{k=j+1}^{n} p_k\right),$$

where the rank of an outcome is the probability of yielding something that is strictly better than this outcome. Third, the rank-dependent utility is computed as the weighted sum of the outcomes

$$u^{RD}(\{x_j\},\{p_j\}) = \sum_{j=1}^{n} \pi_j x_j$$

The weighting function is commonly found to have an inverse-S shape (see Figure 7 in the Appendix). For the concave part of the weighting function, an increase in the rank decreases the decision weight and thus generates optimism and risk seeking. Similarly, convexity results in pessimism and risk aversion (Wakker, 2010).[3]

Non-expected utility theories like the rank-dependent utility theory have been applied to analyze dynamic decisions. A fundamental question in dynamic settings is about a way in which preferences of agents are updated with the flow of information. Based on the updated preferences the agents formulate their decisions. Non-expected utility theories are commonly axiomatized in

which yields \$30,000 with probability 0.01. The majority also prefers $R' =$ (0.01, \$30,000; 0.32, \$20,000) over $S' =$ (0.34, \$20,000). The sure-thing principle is violated because S' and R' are designed by adding a 0.32 chance to win \$20,000 to both of the original gambles, S and R.

[3] Coming back to the example by Prelec (1990), the choices of the individuals can be explained by cumulative prospect theory and by a typical inverse-S shaped weighting function that is concave below $p^\star = 0.34$ because it then follows immediately: $w(0.02) - w(0.01) > w(0.34) - w(0.33)$. For the evidence on individual choices under risk that are captured by the convex part of the weighting function, see Kahneman and Tversky (1979).

accordance with one of the two principles: dynamic consistency and consequentialism (Machina, 1989). Dynamic consistency requires that substrategies of optimal strategies are optimal (Eichberger, Grant, and Kelsey, 2007). Consequentialism demands that only outcomes that are still possible can matter for the updated preferences (Dominiak, Duersch, and Lefort, 2012). If both, dynamic consistency and consequentialism are preserved together with other standard assumptions, the preferences should have the expected-utility form.

No consensus has emerged so far in the decision-theoretical literature on the most plausible axiomatization. While Machina (1989) argues to drop consequentialism and to preserve dynamic consistency, other authors like Eichberger, Grant, and Kelsey (2007) preserve consequentialism rather than dynamic consistency. A recent behavioral experiment reveals that the majority of subjects violate dynamic consistency rather then consequentialism (Dominiak, Duersch, and Lefort, 2012).

Dynamic consistency and consequentialism can be illustrated with the following example. Consider the two-period decision tree in Figure 1. For the Kahneman-Tversky weighting function (Figure 7), the preference relationship at the initial node D_0 is $BD \succ BC \succ A$. At the subsequent decision node, D_1, however, the preference relationship will be reversed $C \succ D$. Thus, in this example the dynamic consistency is not preserved. For expected utility, the decision weights are objective probabilities of the outcomes and the decisions are dynamically consistent.

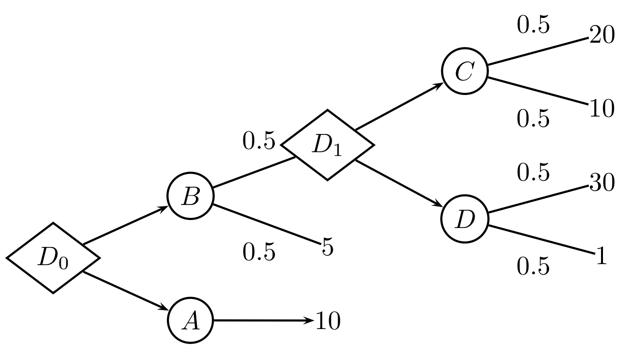

Figure 1: Decision tree

Suppose that the dynamic consistency is preserved, that is $BD \succ BC$ and $D \succ C$. As the payoff of the lower outcome at B is increased, the preference relationship eventually changes to $BC \succ BD$ and therefore $C \succ D$. This result, however, illustrates the lack of consequentialism, as the preference relationship at D_1 does not exclusively depend on the possible outcomes at D_1.

As pointed out by Machina (1989), the violation of dynamic consistency for non-expected utility roots in the principle of consequentialism that is implicitly imposed in the evaluation at the second decision node.

We rely on the most common method of evaluation the outcomes known as backward induction (Wakker, 2010). According to the method, the decision tree is analyzed backwards and risky outcomes are subsequently substituted by their certainty equivalents. For example, when evaluating the choices at D_0, first, at D_1 the outcome C (which is strictly preferable to D) is substituted with its certainty equivalent of 14.69, and this leads to the choice of A over B at D_0. By construction, backward induction preserves consequentialism, but it does not preserve dynamic consistency.

One concept related to dynamic consistency is time consistency, which is characterized by a time-invariant decision function. Similar to Zimper (2012) and consistent with backward induction, we address this type of consistency by considering sustainable allocations. The notion of sustainability requires that the incentives to reverse original plans are explicitly taken into account.

Cumulative prospect theory has been employed in a number of economic studies including health insurance (Ryan and Vaithianathan, 2003, Wakker, Thaler, and Tversky, 1997) and life insurance decisions (Bleichrodt and Eeckhoudt, 2006). In the next section, we consider the optimal insurance of income risk as another important example. We develop a model with two-sided lack of commitment and rank-dependent utility in which agents can engage in risk-sharing arrangements to hedge against their idiosyncratic income risk.

3 Economic Environment

In this section, we describe our economic environment. As in Thomas and Worrall (1988), we consider voluntary risk-sharing arrangements. Following Quiggin (1982), we develop a model based on the rank-dependent utility (RDU) as the decision-theoretical framework.

Consider an exchange economy with a continuum of *ex-ante* identical agents. Time is discrete. The income of agent i is given by a stochastic process $\{y_t^i\}_{t=0}^{\infty}$, where the set of possible income realizations in each period is finite $y_t^i \in Y_t \equiv \{y_{t1}, ..., y_{tn}\}$. Income is independent across agents and evolves across time according to a first-order Markov chain with time-varying exogenous states (Krueger and Perri, 2006, 2011). The income is risky for an agent but not uncertain, i.e. the parameters of the Markov chain are public knowledge.

As the principal novel feature, we take into account the empirically observed inclination of agents toward optimism and pessimism as emphasized by the prospect theory (Conte, Hey, and Moffatt, 2011, Tversky and Kahneman, 1992, Wu and Gonzalez, 1996). Consider a stochastic consumption allocation $c \equiv \{c_t\}_{t=0}^{\infty}$ with a finite number of realizations in each period. We denote the realization j of period t consumption as c_{tj} and the unconditional probability of the realization as p_{tj}. The rank-dependent utility of the consumption allocation is constructed as follows. First, within each period, the realizations are ordered from the lowest to the highest according to the period utility function $u : \mathbb{R}^+ \to \mathbb{R}$, which is assumed to be increasing and strictly concave. Then, the period t allocation is valued according to one-period rank-dependent utility (Wakker, 2010)

$$u^{RD}(c_t) = \sum_{j=1}^{n} \pi_{tj} u(c_{tj}),$$

where the decision weights are defined as $\pi_{tj} = w(\sum_{k=j}^{n} p_{tk}) - w(\sum_{k=j+1}^{n} p_{tk})$, and $w : [0,1] \to [0,1]$ is the probability weighting function. The weighting function is assumed to be strictly increasing and to satisfy $w(0) = 0$ and $w(1) = 1$. Finally, agents evaluate the consumption allocation according to the following time-separable discounted utility function:

$$\text{RDU}(c) = \sum_{t=0}^{\infty} \beta^t u^{RD}(c_t), \tag{1}$$

where $0 < \beta < 1$ is the time discount factor. The *ex-ante* rank-dependent utility, $\text{RDU}(c)$, also serves as the social welfare measure. Expected utility (EU) is a particular case of rank-dependent utility that emerges for the identity weighting function $w(p) = p$.

We restrict attention to bounded memory where agents' information set only contains their current income, but not their income history (Zimper, 2012). Correspondingly, the weighting function in (1) is time-invariant as in Abel (2002), Cecchetti, Lam, and Mark (2000).[4]

We employ the most common evaluation method of computing the rank-dependent utility in a dynamic setting that is known as backward induction (Post, van den Assem, Baltussen, and Thaler,

[4]Even if the parameters of income process were unknown to the agents, the lack-of-memory assumption would prevent them from learning resulting in time-invariant weighting-function. Zimper (2012) further shows that the bounded memory assumption is the key to establish a unique stationary equilibrium price function in a Lucas-fruit tree economy with a neo-additive weighting function.

2008, Wakker, 2010). Under this method, the decision tree is analyzed backwards and recursively each decision note is replaced by its certainty equivalent. This means that we can treat the infinite sum in (1) as the sum of one-period RDUs.[5]

In principle, modeling of agents' decisions under risk requires a probability space for which a (possibly non-additive) probability measure is defined over all possible consumption streams. Such construction is standard for additive probability measures of EU theory but much more complicated for non-additive probability measures of the RDU. We avoid these complications by the additive specification of an intertemporal RDU function (1).

We analyze risk-sharing possibilities under two-sided lack of commitment by introducing voluntary participation constraints. The constraints characterize the intertemporal trade-off between the current-period consumption and the value of risk sharing provided by the arrangement in the future periods. Without eliminating the intertemporal trade-off, the limited-memory assumption implies that the current risk-sharing transfers do not depend on transfers received in the past and that current consumption is measurable only with respect to current income.[6]

A risk-sharing arrangement is sustainable if each agent, after observing the current period income, at least weakly prefers to follow the arrangement to defect into autarky. The autarky option is characterized by two features. First, if an agent deviates into autarky, then that agent is excluded from risk sharing with other households. Second, after T periods of staying in autarky, the agent may reengage in the risk-sharing agreement. The notion of sustainable risk-sharing arrangements is formally introduced by the following definition.

Definition 1 *A consumption allocation $\{\{c_t^i\}_{t=0}^{\infty}\}$ is sustainable if*

(i) for each agent i in any period t, the consumption allocation is weakly preferable to the outside option

$$\mathrm{RDU}_t(\{c_{t+\tau}^i\}_{\tau=0}^{\infty}) \geq \mathrm{RDU}_t(\{c_{at,t+\tau}^i\}_{\tau=0}^{\infty}) \quad \forall i, t, \tag{2}$$

where the allocation of $\{c_{t+\tau}^i\}_{\tau=0}^{\infty}$ on the left-hand side is evaluated according to rank-dependent

[5] An alternative method is forward induction. Here, the consumption streams are evaluated as a whole rather than period-by-period as in case of backward induction. One can show that forward and backward induction coincide only for additive or neo-additive probability measures which is one of the mayor difficulties for non-expected utility theories (Wakker, 2010).

[6] For expected utility, the resulting history-independent insurance contracts are also modeled by Kimball (1988), Coate and Ravallion (1993), and Ligon, Thomas, and Worrall (2002). Note that these type of contracts are not optimal when the memory of agents is not bounded.

utility conditional on all the information available in period t

$$\mathrm{RDU}_t(\{c^i_{t+\tau}\}_{\tau=0}^{\infty}) = \sum_{\tau=0}^{\infty} \beta^\tau u_t^{RD}(c^i_{t+\tau}),$$

and the allocation of $\{c^i_{at,t+\tau}\}_{\tau=0}^{\infty} = (\{y^i_{t+\tau}\}_{\tau=0}^{T-1}, \{c^i_{t+\tau}\}_{\tau=T}^{\infty})$ *on the right-hand side is evaluated according to*

$$\mathrm{RDU}_t(\{c^i_{at,t+\tau}\}_{\tau=0}^{\infty}) = \sum_{\tau=0}^{T-1} \beta^\tau u_t^{RD}(y^i_{t+\tau}) + \sum_{\tau=T}^{\infty} \beta^\tau u_t^{RD}(c^i_{t+\tau});^{7} \text{ and}$$

(ii) the allocation is resource-feasible

$$\int c^i_t \,\mathrm{d}\, i \leq \int y^i_t \,\mathrm{d}\, i \quad \forall t. \tag{3}$$

The participation constraints (2) are the key element of the definition. In any particular period t agents know the true probabilities of further consumption and income realizations. Their decision weights for all possible realizations are based on these conditional probabilities. Thus, a consumption allocation is sustainable if the rank-dependent utility of the consumption allocation computed according to the decision weights is at least as large as the corresponding rank-dependent utility of the outside option.

Among the sustainable allocations, we are interested in the optimal allocation that is defined as follows.

Definition 2 *A socially optimal arrangement under voluntary participation is a consumption allocation that provides the highest social welfare among the set of sustainable allocations.*

In the next section, we employ a neo-additive weighting function to parameterize rank-dependent utility. The neo-additive weighting function is linear in the interior of domain $w(p) = \lambda + (1 - \lambda - \gamma)p$ for $p \in (0, 1)$ and is discontinuous at the endpoints, $w(0) = 0$ and $w(1) = 1$. The decision weights associated with the neo-additive weighting function are the following: $\pi_h = \lambda + (1 - \lambda - \gamma)p_h$ for the highest state, $\pi_l = \gamma + (1 - \lambda - \gamma)p_l$ for the lowest state, and $\pi_j = (1 - \lambda - \gamma)p_j$ for all other states,

[7] The expression reflects that an agent after T periods weakly prefers a sustainable allocation to autarky.

where $\{p_j\}$ is the true objective probability distribution. The parameter λ denotes the degree of optimism, and γ denotes the degree of pessimism. We consider $\lambda \geq 0$, $\gamma \geq 0$, and $\lambda + \gamma \leq 1$. The rank-dependent utility is related to the standard expected utility in the following way:

$$\mathrm{RDU}(c) = \lambda \max_{c \in S}\{U(c)\} + \gamma \min_{c \in S}\{U(c)\} + (1 - \lambda - \gamma)\,\mathrm{E}[U(c)], \tag{4}$$

where $U(c) = \sum_{t=0}^{\infty} \beta^t u(c_t)$ and S is the set of resource-feasible allocations. In this expression, $\mathrm{E}[U(c)]$ denotes the standard expected utility, which is based on unconditional objective probabilities.[8]

4 Analytical Results

Empirical studies on inequality find that the consumption inequality is smaller than the income inequality, and they are positively correlated (Blundell, Pistaferri, and Preston, 2008, Cutler and Katz, 1992). In an expected utility model, Krueger and Perri (2006) demonstrate that models with two-sided lack of commitment tend to underpredict the increase in consumption inequality in response to the recent rise in income inequality in the U.S. from the year 1980 to the year 2003. In this section, we demonstrate that our model has the potential to reconcile the changes in consumption and income inequality found in the data when (1) agents are biased towards pessimism and (2) incentive constraints matter.

To illustrate the main mechanism underlying the result, in this section we abstract from a number of features. First, we assume that the income process consists of three time-invariant states $y_h = \bar{y} + \sigma_y$, $y_m = \bar{y}$, and $y_l = \bar{y} - \sigma_y$, where $\sigma_y > 0$ is the standard deviation of the process. The states are equally likely and the income realizations are independent across time. Second, we suppose that once an agent deviates to autarky, that agent is precluded from risk sharing forever. Third, we employ a neo-additive weighting function.[9] A full-scale model is studied in the next section.

With the three time-independent income realizations, the problem is recursive. The social

[8] When the neo-additive weighting function is applied to models with uncertainty, $\zeta \equiv \lambda + \gamma$ represents the degree of ambiguity, which is the lack of confidence the agents have in the expected utility measure (Ellsberg, 1961).

[9] In the case of three states, any weighting function of backward-induction rank-dependent utility can identically be characterized by two neo-additive parameters: the degree of pessimism, γ, and the degree of optimism, λ, with $w(2/3) = (2 + \lambda - 2\gamma)/3$ and $w(1/3) = (1 + 2\lambda - \gamma)/3$.

welfare is the following

$$\mathrm{RDU}(c) = \frac{1}{1-\beta}\left(\lambda \max\{u(c_h), u(c_m), u(c_l)\} + \gamma \min\{u(c_h), u(c_m), u(c_l)\}\right.$$
$$\left. +(1-\lambda-\gamma)\frac{1}{3}(u(c_h)+u(c_m)+u(c_l))\right), \quad (5)$$

where c_h, c_m and c_l are the consumption of agents with current-period income y_h, y_m, and y_l, respectively. The participation constraints for agents with high and low current-period income are

$$u(c_h) + \beta\,\mathrm{RDU}(c) \geq u(y_h) + \beta\,\mathrm{RDU}(y) \quad (6)$$
$$u(c_m) + \beta\,\mathrm{RDU}(c) \geq u(y_m) + \beta\,\mathrm{RDU}(y) \quad (7)$$
$$u(c_l) + \beta\,\mathrm{RDU}(c) \geq u(y_l) + \beta\,\mathrm{RDU}(y) \quad (8)$$

where

$$\mathrm{RDU}(y) = \frac{1}{1-\beta}\left(\frac{1+2\lambda-\gamma}{3}u(y_h) + \frac{1-\lambda-\gamma}{3}u(y_m) + \frac{1-\lambda+2\gamma}{3}u(y_l)\right),$$

and the resource constraint is

$$\frac{1}{3}(c_h + c_m + c_l) = \frac{1}{3}(y_h + y_m + y_l). \quad (9)$$

An optimal consumption allocation $\{\hat{c}_h, \hat{c}_m, \hat{c}_l\}$ maximizes the social welfare (5) subject to the participation constraints (6)-(8), and the resource feasibility constraint (9).

The optimal allocation exists but may not be unique.[10] We restrict our attention to empirically plausible allocations such that agents with higher income consume more, i.e., $c_h \geq c_m \geq c_l$. Without this restriction, it is straightforward to deduce that, for example, if $\{\hat{c}_h, \hat{c}_m, \hat{c}_l\}$ with $\hat{c}_h < \hat{c}_l$ is an optimal allocation, then $\{\hat{\hat{c}}_h, \hat{c}_m, \hat{\hat{c}}_l\}$, where $\hat{\hat{c}}_h = \hat{c}_l$ and $\hat{\hat{c}}_l = \hat{c}_h$, is also optimal.

At the socially optimal allocation, the voluntary participation constraints (6)-(8) may not be binding. When the constraints are not binding, the optimal allocation is the first-best allocation. With optimism and pessimism, the first-best allocation does not necessary exhibit perfect risk sharing. When agents are pessimistic and pay particular attention to the lowest consumption realization, they prefer to smooth consumption across states. However, when agents are optimistic and

[10]The existence follows from continuity of the objective function and compactness of the feasible set.

risk-seeking, the optimal consumption allocation exhibits inequality. In the following proposition, we characterize the first-best allocation.

Proposition 1 *Consider the socially optimal arrangement. If $\lambda \leq \gamma$, the first-best allocation is perfect risk sharing. If $\lambda > \gamma$, the optimal consumption allocation exhibits inequality.*

Proof. If $c_h > c_m > c_l$ the first-best allocation is characterized by the first-order conditions

$$(1 + 2\lambda - \gamma)u'(c_h) = (1 - \lambda - \gamma)u'(c_m) = (1 - \lambda + 2\gamma)u'(c_l) \tag{10}$$

together with the resource feasibility (9). For $\lambda > \gamma$, these conditions imply that $c_h > c_m$ and $c_m < c_l$, and the latter inequality is a contradiction. It follows that $c_h > c_m = c_l$. If $\lambda \leq \gamma$, condition (10) implies that $c_h < c_l$, which is a contradiction. Hence, for $\lambda \leq \gamma$, the optimal allocation satisfies $c_h = c_m = c_l$. ■

In the expected utility model, the first-best allocation always exhibits perfect risk sharing. Comparing the first-best allocations in rank-dependent utility and expected utility models, the difference emerges only when agents are relatively optimistic, $\lambda > \gamma$. Not surprisingly, relatively optimistic agents prefer a risky consumption allocation to a certain allocation.

The first-best allocation is not suitable for explaining the changes in consumption inequality because it is independent of income inequality. Furthermore, the consumption allocation exhibits inequality only if agents are relatively optimistic. This pattern, however, is at odds with the empirical literature on risk attitudes, which finds people to be relatively pessimistic, $\gamma \geq \lambda$ (Wakker, 2010).

To explain the changes in consumption inequality found in the data, we proceed with constrained-efficient allocations. First, we show that consumption inequality can be smaller than income inequality only when the participation constraints of low-income agents are not binding at the optimal allocation. This property resembles the findings under expected utility. Consumption inequality and income inequality are measured as the standard deviations of consumption and income distribution, respectively.

Proposition 2 *Consider the optimal consumption allocation. Consumption inequality can be smaller than income inequality only if the participation constraints for low-income agents (8) are* *not binding for all values of income inequality.*

Proof. If the participation constraints for low-income agents (8) were binding, then from these constraints we would obtain that $\hat{c}_l \leq y_l$. Because the weight for the middle state is smaller than for low state, the participation constraints for that state should also be binding, which implies $\hat{c}_m \leq \bar{y}$. By resource feasibility, $\hat{c}_h \geq y_h$. Hence, the consumption inequality would be greater than the income inequality. ■

In the data, the consumption inequality is smaller than the income inequality. Thus, the only empirically relevant possibilities that we are left with are the case of binding participation constraints of high-income agents and the case of binding participation constraints of high-income and of middle-income agents. For the purpose of illustration of our theoretical results, we will focus on the case when only participation constraints of high-income agents are binding.

For this case the optimal consumption allocation is characterized by $c_h > c_m = c_l$. This result follows from then efficient risk-sharing at the lower end of the income distribution, i.e. the participation constraints of middle-income and low-income agents are slack. As an immediate implication, the consumption inequality is smaller than income inequality, $\sigma_c < \sqrt{3}/2\sigma_y$.

In the next proposition, we characterize the dependence of the optimal allocation on the degrees of optimism and pessimism.

Proposition 3 *Consider the optimal consumption allocation. The consumption inequality is decreasing in the degree of pessimism, γ, if only the participation constraints for high-income agents are binding.*

The proof can be found in Appendix A.2. Intuitively, pessimistic agents dislike a risky allocation.

In the main proposition, we will show that the degree of relative pessimism diminishes the response of consumption inequality to a change in income inequality. As an intermediate step, we state a condition in the following lemma that ensures a positive correlation between consumption inequality and income inequality.

Lemma 1 *If only the participation constraints for high-income agents (6) are binding under the socially optimal arrangement, then consumption inequality is increasing with income inequality if* *(and only if) the outside option for high-income agents is increasing with income inequality, i.e.*

$$\left(1 + \frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)u'(y_h) - \left(\frac{\beta}{1-\beta}\frac{1-\lambda+2\gamma}{3}\right)u'(y_l) > 0.$$

The proof can be found in Appendix A.3. The intuitive basis is the following. The binding participation constraints for high-income agents imply that these agents obtain the same utility from the outside option and from the socially optimal arrangement. With the increase in income inequality, the agents obtain a higher utility if and only if the value of their outside option also increases with the income inequality. The increase in utility for high-income agents decreases their willingness to pay for insurance and thus results in higher consumption inequality.

Even for low degrees of relative risk aversion such as logarithmic preferences, standard debt-constrained market models with agents maximizing their expected utility (Krueger and Perri, 2006) tend to underpredict the increase in consumption inequality with the rise of income inequality, as has been observed in the U.S. from the year 1980 to the year 2003. In these models, the increase in consumption inequality would be larger if the (constant) relative risk-aversion coefficient were much smaller. It is, however, implausible to postulate the coefficient to be below one unless the risk aversion attitude is preserved by a weighting function of rank-dependent utility. Under the binding participation constraints, a decrease in the degree of relative risk aversion makes the outside option more attractive for high-income agents, which results in higher consumption inequality. Thus, even under standard expected utility, there exist coefficients of relative risk aversion coefficients smaller than one, for which the model tends to overpredict the increase in consumption inequality. When this is the case, the model with rank-dependent utility and relatively more pessimism than optimism has a diminishing effect on consumption inequality. In the following proposition, we provide conditions for this result.

Proposition 4 *Assume the utility function exhibits constant relative risk aversion, a degree of pessimism is sufficiently large, $\gamma \geq \bar{\gamma}$ and the value of the outside option for high-income agents is increasing in income inequality. The rate of change in consumption inequality with respect to the increase in income inequality is decreasing in the degree of pessimism, γ.*

The proof can be found in Appendix A.4. The intuitive basis for this result is as follows. The increase in the variance of income has two opposite effects on consumption inequality. On the one hand, the ex-ante value of the outside option decreases. On the other hand, the high income increases, which may increase the value of the outside option for high-income agents. Under binding participation constraints, the latter effect dominates the former when agents value the current gain higher than the higher risk of a low income in the future (see Lemma 1). When agents become relatively more pessimistic, they put additional weight on the low-income state in the future, and the continuation value of living in autarky decreases. This decrease must also be reflected in the optimal consumption allocation and consumption inequality decreases.

The condition $\gamma \geq \bar{\gamma}$ ensures that an increase in income inequality leads to a less than one-for-one increase in consumption inequality. The value of $\bar{\gamma}$ is decreasing in λ, ((and this is nice for us)).

In this section, we have shown that the RDU model with relatively more pessimism than optimism qualitatively has the potential to constitute the missing link between income and consumption inequality. However, whether the RDU model can indeed reconcile the inequalities for realistic values of pessimism, optimism and relative risk aversion is the quantitative question that we analyze in the following section.

5 Numerical Results

Our model with rank-dependent utility explains substantially better than the standard expected utility model the relationship between income and consumption inequality, as observed in the United States from 1980 to 2003. In this section, we describe the data and model calibration, and characterize numerically the optimal risk-sharing arrangement.

5.1 Data

We calibrate the model to match the U.S. economy for the years 1980-2003. We use the public dataset prepared by Krueger and Perri (2006), which is based on quarterly data from the Consumer Expenditure Interview Survey (CE). The dataset consists of cross-sectional consumption and income series that have been aggregated from the original survey data across a number of income and expenditure categories.

Consumption and income inequality in our model corresponds to the within-group components of consumption and income inequality in the data. As in Katz and Autor (1999) and Krueger and Perri (2006), income inequality and consumption inequality (measured as cross-sectional variances) can be decomposed into two components: between-group inequality and within-group inequality. Between-group inequality is attributed to observable characteristics of households, such as education, experience, occupation, race, and region of residence. We consider predictable and uninsurable the part of income that can be explained by these characteristics. On the contrary, within-group inequality is attributed to the residual component that cannot be explained by publicly observable characteristics. This component is considered to be stochastic and can potentially be insured. Our analysis is devoted entirely to this component.

The key stylized finding from this dataset is that consumption inequality has increased substantially less than income inequality. This finding is consistent with the majority of contributions to this line of research (Attanasio, Battistin, and Ichimura, 2007, Blundell, Pistaferri, and Preston, 2008, Primiceri and van Rens, 2009). In a recent paper, Aguiar and Bils (2011) challenged the majority opinion by arguing that consumption inequality and income inequality have moved one to one. They claimed that there is a systematic bias in reported consumption expenditures, and they proposed an approach to identify this bias. By fixing the demand elasticities of household consumption expenditures, with respect to a range of goods-expenditure categories, to values found in the CE in 1972-73, they obtained an estimate for the consumption expenditures of households from the year 1980 to the year 2003. The gap with the actual reported consumption expenditures was then interpreted as a time-dependent measurement error that is good-specific and income-specific. However, by assuming constant demand elasticities, Aguiar and Bils (2011) did not capture substitution effects across expenditure categories to smooth total consumption expenditures, and thus they tended to overstate the increase in consumption inequality. The assumption of constant demand elasticities is controversial and appears to be a rather special case (Banks, Blundell, and Lewbel, 1997, Blundell, Chen, and Kristensen, 2007). Therefore, we opt to employ the direct consumption inequality measures reported by Krueger and Perri (2006).

5.2 Calibration

Our benchmark calibration is designed to highlight the differences between our model and the debt-constrained markets model in Krueger and Perri (2006). Therefore, we set a number of corresponding parameters to the same values. A justification for this calibration is outlined in McGrattan and Prescott (2003). We consider a period utility function that exhibits constant relative risk aversion and we set the time-discount factor to $\beta = 0.96$. One period of the model corresponds to one year.

We model the income as a stochastic Markov process with nine time-varying exogenous states.[11] The autoregressive coefficient of the income process is set to $\rho = 0.8014$ (Krueger and Perri, 2011). For each year, we set the cross-sectional variance of the income process to the variance observed in the data. We then compute time-invariant transition probabilities of the Markov chain following Tauchen (1986). Consumption and income inequality are calculated as cross-sectional variances of consumption and income distribution, respectively.

In our baseline, we follow Tversky and Kahneman (1992) in choosing the functional form of the probability weighting function and the utility function, which are the following:

$$w(p) = \frac{p^\delta}{(p^\delta + (1-p)^\delta)^{1/\delta}}, \qquad u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}. \tag{11}$$

The weighting function is monotonically increasing for $\delta > 0.28$, and it nests the expected utility as a limiting case ($\delta = 1$). As a robustness exercise, we also employ a neo-additive weighting function.

We simultaneously set the parameter of the weighting function and the constant coefficient of risk aversion of the utility function according to values estimated in laboratory studies specifically designed to capture decisions under risk. As our baseline, we employ the averages of the estimated values reported in seven studies (provided in Table 4 in the appendix), including Tversky and Kahneman (1992) and Wu and Gonzalez (1996). The values are 0.74 for the Tversky–Kahneman parameter, δ, and 0.50 for the coefficient of relative risk aversion of the utility function, σ.

To permit a fair comparison with Krueger and Perri (2006), in all simulations, we match the consumption inequality of the year 1980, which is the initial year in our sample, by calibrating the

[11] For autoregressive coefficients up to 0.9, based on Monte-Carlo studies, Tauchen (1986) found that nine states were "clearly adequate". Krueger and Perri (2011) approximated the income process with five states.

duration of exclusion from the risk-sharing transfers.

For the rank-dependent utility model and our baseline calibration, the duration for exclusion from the risk-sharing arrangement is equal to 10 years. For the expected utility model, we employ a logarithmic period utility function which results in a duration for exclusion of $T = 20$ years.

5.3 Model Simulation

In Table 1 and Figure 2, we report our principal numerical results on the capacity of the RDU model and the EU model in explaining the cross-sectional variance of consumption over time.

As displayed in Figure 2, the EU model systematically understates the consumption inequality compared with the data. Between the years 1980 and 2003, the increase in consumption inequality that the EU model predicts is more than five times smaller than the actual increase. This result corresponds to the findings of Krueger and Perri (2006). Though the lack of history-dependence that follows from the bounded memory assumption leads to contracts that exhibit less risk sharing, this property alone does not improve the fit to the data. In our RDU model, we also allow for non-linear probability weighting which changes the picture.

As the first novel result here, the RDU model is better at capturing the change in consumption inequality. Over the considered period, the model predicts an increase of 3.29 percent which is remarkably close to the actual increase of 2.93 percent found in the data. This observation is also reflected in the statistics that measure the explanatory power of both models for consumption inequality (see Table 1). The mean squared error (MSE) in the EU model is 95 percent higher than in the RDU model. Furthermore, both information criteria, i.e., the Bayesian information criterion (BIC) and the Akaike information criterion (AIC), indicate a significant increase in the fit of the RDU model. The magnitude of the increase indicates that the EU model fails to explain some substantial structural variations in the data.

As can be seen from Figure 4, both the standard expected utility model (EU) and the RDU model capture the increasing trend in the insurance ratio, $1 - \mathrm{var}_i(\ln c_t^i)/\mathrm{var}_i(\ln y_t^i)$; however, the EU model systematically overstates the degree of private insurance relative to the data in the last decade. The better performance of the RDU model has an important economic implication. Progressive tax system provides public insurance against the idiosyncratic income risk (Krueger and Perri, 2011). If the tax system were optimally designed based on the expected utility model,

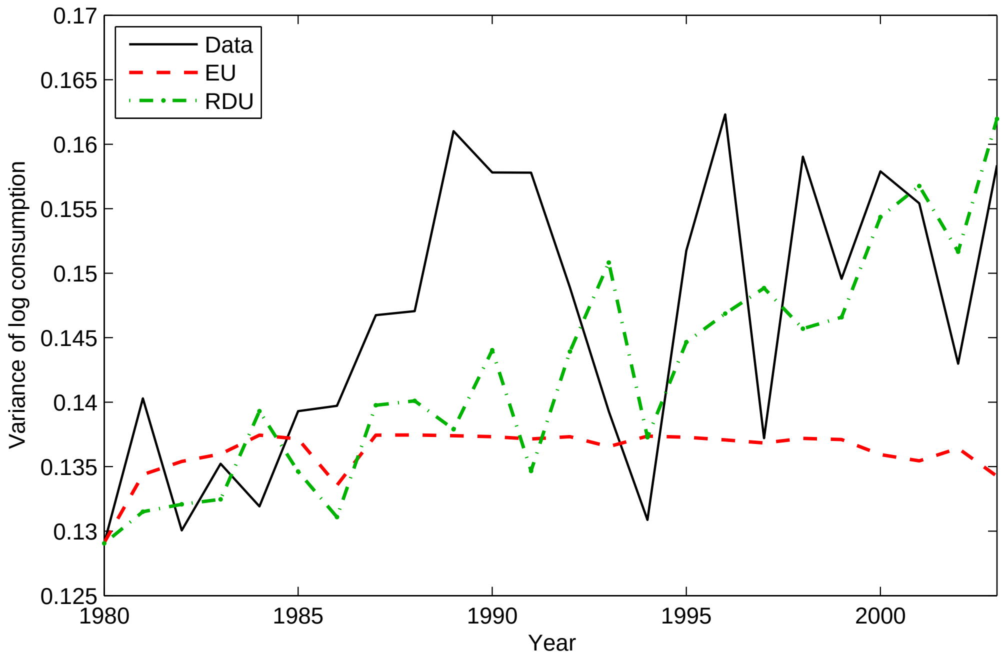

Figure 2: Consumption inequality over time in the data (solid) versus the RDU model (dashed) and the EU model (dash-dotted)

too little public insurance would be provided. According to the expected utility model, there is already a substantial amount of private insurance and introducing public insurance by a progressive tax system may thus distort private incentives leading to negative welfare effects. However, in the RDU model as it is in the reality, households are underinsured, which makes it more likely that the government intervention has positive welfare effects. Thus, the RDU model allows for more accurate assessment of welfare effects of public insurance, and this in turn increases the social welfare.

5.4 Robustness

In this subsection, we conduct several robustness exercises across different dimensions. First, we consider the Gini coefficient as an alternative measure of consumption and income inequality. Second, we use smoothed time series to analyze the implications of our decision-theoretical models for the consumption and inequality trends as in Krueger and Perri (2006). Finally, we employ a specification of the income process that comprises both, a transitory and a persistent component as estimated by Storesletten, Telmer, and Yaron (1999, 2004).

Table 1: EU model and RDU model: baseline results

	EU model	RDU model
Weighting function parameter, δ	1	0.74
Utility function parameter, σ	1	0.50
Increase in consumption inequality 1980-2003 (Data: 2.93%),	0.51%	3.29%
Mean-square error ratio, normalized	1.95	1
Akaike Information Criterion, AIC	-125.71	-139.72
Bayesian Information Criterion, BIC	-124.53	-137.36

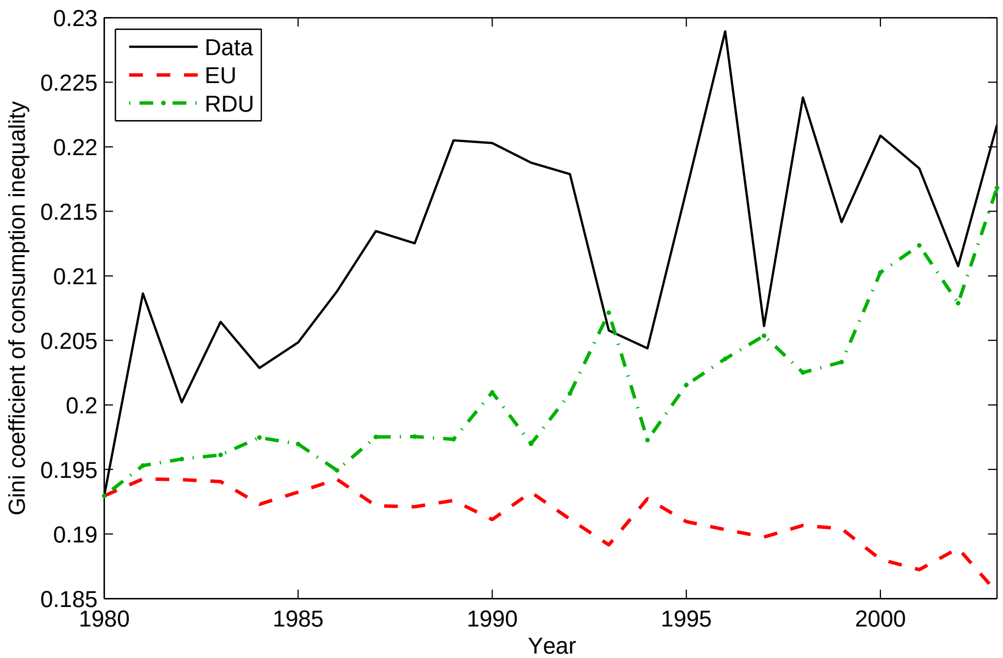

Figure 3: Gini coefficient of consumption inequality over time in the data (solid) versus the RDU model (dashed) and the EU model (dash-dotted)

Gini coefficient of consumption inequality As an alternative measure of inequality, we also consider the Gini coefficient of consumption inequality. For this exercise, we keep the parameters of our baseline calibration and adjust T in both models to match the Gini coefficient in the year 1980. In Figure 3, we plot the evolution of the Gini coefficient of consumption in the data versus the RDU model and the EU model. The primary message is similar to the analysis in which the inequality is measured by the variance of the logarithm of consumption. The RDU model captures substantially better than the EU model the changes in the Gini coefficient. In particular, the mean squared error in the RDU model is more than 250 percent lower than in the EU model (see Table 5).

Smoothed time series In this section, we focus on the long-term trend rather than capturing the year-to-year changes in the inequality measures. We follow Krueger and Perri (2006) and apply an HodrickPrescott filter to the time series of the cross-sectional variances of log income and log consumption with smoothing parameter $\lambda = 400$.[12] For the construction of the time-varying income states, as well as for the calibration strategy, we follow the same procedures as outlined above. The only difference is that we employ here the smoothed series for income inequality as an input, and we assess the performance by comparing the models' predictions with the smoothed consumption inequality time series from the data. The main results of this exercise are displayed in Figure 6 and Table 2.

Between 1980 and 2003, the EU model predicts a smaller increase in consumption inequality in comparison with the data. This result corresponds to the findings of Krueger and Perri (2006). In contrast, the RDU model predicts an increase of 2.74 percent, which comes close to the actual increase in the data that amounts to 2.15 percent. This number is also closer to the data than the 4.5 percent increase predicted by incomplete spanning (Aiyagari, 1994), as reported by Krueger and Perri (2006).

In Table 2, we report the statistics to assess the explanatory power of the RDU and the EU model. The advantage of the RDU model is even stronger when the trend in consumption inequality is the explanatory variable. The mean squared error in this case is approximately seven times smaller than in the EU model.

As can be seen in Figure 6 in the Appendix, the RDU model performs substantially better than the EU model for values of the Tversky–Kahneman parameter between 0.725 and 0.765.

Transitory and persistent income parts In this part, we employ an income specification that comprises transitory and persistent income components as in Storesletten, Telmer, and Yaron

[12] Alternatively, we also employ a value of $\lambda = 6.25$ for the smoothing parameter, which is the value proposed by Ravn and Uhlig (2002). The results on the performance of the two models are qualitatively similar.

Table 2: EU model and RDU model: smoothed time series

	EU model	RDU model
Weighting function parameter, δ	1	0.74
Utility function parameter, σ	1	0.50
Mean-square error ratio, normalized	2.53	1
Akaike information criterion, AIC	-122.61	-142.90
Bayesian information criterion, BIC	-121.43	-140.55

Table 3: Persistent and transitory component of income

	EU model	RDU model
Weighting function parameter, δ	1	0.74
Utility function parameter, σ	1	0.50
Mean-square error ratio, normalized	8.73	1
Akaike Information Criterion, AIC	-87.97	-137.97
Bayesian Information Criterion, BIC	-86.80	-135.611

(1999, 2004). More precisely, the logarithm of income is modeled as

$$\ln(y_t) = z_t + \epsilon_t, \qquad z_t = \rho z_{t-1} + \eta_t,$$

where ϵ_t, η_t are independent, serially uncorrelated and normally distributed with variances $\sigma^2_{\epsilon_t}, \sigma^2_{\eta_t}$. The persistence parameter ρ is set to 0.95 which is the point estimate found by Storesletten, Telmer, and Yaron (1999, 2004). For a given value of ρ, we identify the variances $\sigma^2_{\epsilon_t}, \sigma^2_{\eta_t}$ from the cross-sectional within-group income variance and the cross-sectional within-group auto-covariance from the CE data. The persistent part of income is captured by a stochastic Markov process with seven time-varying exogenous states. The transitory part is modeled with two time-varying exogenous states. The results on the performance of the RDU and the EU model are reported in Table 3 and confirm our findings in the baseline calibration. The mean square error for the RDU model is approximately nine times smaller and both information criteria indicate a substantially better fit of the RDU model compared with the EU model.

6 Conclusion

The rise in consumption inequality in response to the increase in income inequality over the last three decades in the U.S. is puzzling to standard incomplete market models. We have developed a model with two-sided lack of commitment and chance attitudes, as emphasized by prospect theory to analyze this relationship. Chance attitudes, such as optimism and pessimism, are captured in the theory by higher decision weights of high and low outcomes relative to objective probabilities as in the standard expected utility model.

For realistic values of risk aversion and for chance attitudes found in experimental studies, we have found that lenders' incentive to share decreases. The latter effect endogenously amplifies the increase in consumption inequality compared to the standard expected utility model and thereby improves the fit to the data. Thus, our findings provide support for the prospect theory of risk attitude.

An important question for future research is the decentralization of the optimal consumption allocation with rank-dependent utility. Under uncertainty and for a neo-additive weighting function, Zimper (2012) provided conditions for the existence and uniqueness of asset-price equilibria in a Lucas-Fruit- tree model. Our results indicate that a Kahneman-Tversky weighting function has however empirically more plausible implications than a neo-additive weighting function. Thus, the extension of Zimper's results to a more general class of non-additive probability measures is a challenging task for future research.

In this paper, we have analyzed the implications of rank-dependent utility for consumption inequality. If the optimal consumption allocation can be decentralized, it opens up another direction for future research. An interesting question here is how the consumption and saving decisions of households with non-expected utility preferences affect the distribution of wealth as in Cordoba (2008).

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A Appendix

A.1 Sustainability of the first-best allocation

Proposition A.1 *Let $\bar{\beta} < 1$ be the lowest discount factor such that for any $\beta \geq \bar{\beta}$ the socially optimal arrangement is the first-best allocation. If $\lambda \leq \gamma$, the cut-point $\bar{\beta}$ decreases in the degree of pessimism, γ.*

Proof. If $\lambda \leq \gamma$, the first-best allocation exhibits perfect risk sharing (Proposition 1) and the cut-point $\bar{\beta}$ is given by the saturated participation constraint (6). Employing the implicit function theorem, we obtain

$$\frac{\partial \bar{\beta}}{\partial \gamma} = -\frac{\bar{\beta}^2}{3} \frac{u(y_h) + u(\bar{y}) - 2u(y_l)}{u(y_h) - u(\bar{y})} < 0$$

With a decrease in the degree of pessimism, γ, the value of the outside option increases, which requires an increase in the discount factor to sustain the first-best allocation. ■

A.2 Proof of Proposition 3

Differentiating the participation constraint (6), we obtain

$$\frac{\partial \sigma_c}{\partial \gamma} = \frac{\beta}{1-\beta} \frac{1}{3C} \left(u(c_h) - u(c_l) - u(y_h) - u(\bar{y}) + 2u(y_l) \right) < 0,$$

where $c_h = \bar{y} + \sqrt{2}\sigma_c$ and $c_l = \bar{y} - \sigma_c/\sqrt{2}$, and σ_c is the standard deviation of the consumption allocation, and

$$C \equiv \left(1 + \frac{\beta}{1-\beta} \frac{1 + 2\lambda - \gamma}{3} \right) \sqrt{2} u'(c_h) - \left(\frac{\beta}{1-\beta} \frac{2 - 2\lambda + \gamma}{3} \right) \frac{1}{\sqrt{2}} u'(c_l).$$

Efficient risk sharing implies $c_h \leq y_h$ and by resource feasibility $y_l < c_l < \bar{y}$ that render the nominator of the derivative positive. As in Lemma 1, the denominator is positive at the optimal allocation.

A.3 Proof of Lemma 1

Assuming $c_h \geq c_l$, the participation constraint for high-income agents (6) is

$$\left(1+\frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)u(c_h)+\left(\frac{\beta}{1-\beta}\frac{2-2\lambda+\gamma}{3}\right)u(c_l)=$$
$$\left(1+\frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)u(y_h)+\left(\frac{\beta}{1-\beta}\frac{1-\lambda-\gamma}{3}\right)u(\bar{y})+\left(\frac{\beta}{1-\beta}\frac{1-\lambda+2\gamma}{3}\right)u(y_l) \quad \text{(A.1)}$$

Substituting for c_l from the resource-feasibility constraint (9), the left-hand side of (A.1) is a concave function of c_h. Hence, there are at most two solutions to (A.1) that are candidates for the optimal allocation. Expressing the allocations through deviations from the mean and employing the implicit function theorem, from (A.1) we obtain

$$\frac{\partial\sigma_c}{\partial\sigma_y}=\frac{\left(1+\frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)\frac{\sqrt{3}}{\sqrt{2}}u'(y_h)-\left(\frac{\beta}{1-\beta}\frac{1-\lambda+2\gamma}{3}\right)\frac{\sqrt{3}}{\sqrt{2}}u'(y_l)}{\left(1+\frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)\sqrt{2}u'(c_h)-\left(\frac{\beta}{1-\beta}\frac{2-2\lambda+\gamma}{3}\right)\frac{1}{\sqrt{2}}u'(c_l)}, \quad \text{(A.2)}$$

where σ_c and σ_y are the standard deviations of consumption and income distribution, respectively. If there are two solutions, the derivative of the left-hand side of (A.1) is positive at one solution and negative at the other. At the first best, $(1+2\lambda-\gamma)u'(c_h)=(2-2\lambda+\gamma)/2u'(c_l)$, such that the derivative of the left hand side of (A.1) with respect to c_h is positive. Because the objective function of the social planner is strictly concave, the optimal allocation corresponds to a solution that is closer to the first best, which is not sustainable, and the derivative of the left-hand side at the constrained-efficient allocation is positive.

A.4 Proof of Proposition 4

Using the participation constraint (6) to differentiate σ_c with respect to σ_y and γ gives

$$\frac{\partial\left(\partial\sigma_c/\partial\sigma_y\right)}{\partial\gamma}=\frac{1}{C^2}\left\{\frac{\beta}{1-\beta}\frac{-1}{3}\left(\frac{\sqrt{3}}{\sqrt{2}}u'(y_h)+\frac{2\sqrt{3}}{\sqrt{2}}u'(y_l)\right)C+\frac{\beta}{1-\beta}\frac{1}{3}\left(\sqrt{2}u'(c_h)+\frac{1}{\sqrt{2}}u'(c_l)\right)A\right.$$
$$\left.-\left(\left(1+\frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\sqrt{2}\right)u''(c_h)+\left(\frac{\beta}{1-\beta}\frac{2-2\lambda+\gamma}{3}\right)\frac{1}{\sqrt{2}}u''(c_l)\right)\frac{\partial c_h}{\partial\gamma}A\right\},$$

where

$$A = \left(1 + \frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)\frac{\sqrt{3}}{\sqrt{2}}u'(y_h) - \left(\frac{\beta}{1-\beta}\frac{1-\lambda+2\gamma}{3}\right)\frac{\sqrt{3}}{\sqrt{2}}u'(y_l), \text{ and}$$

$$C = \left(1 + \frac{\beta}{1-\beta}\frac{1+2\lambda-\gamma}{3}\right)\sqrt{2}u'(c_h) - \left(\frac{\beta}{1-\beta}\frac{2-2\lambda+\gamma}{3}\right)\frac{1}{\sqrt{2}}u'(c_l).$$

From Lemma 1, we obtain $A > 0$ and $C > 0$. Efficient risk sharing implies that $c_h < y_h$; combined together with $\gamma \geq (1-\lambda)(2-\sqrt{3})/(2\sqrt{3}-1)$, we get $C > A$. Constant relative risk aversion results in convex marginal utilities and ensures that

$$\frac{\sqrt{3}}{\sqrt{2}}u'(y_h) + \frac{2\sqrt{3}}{\sqrt{2}}u'(y_l) > \frac{\sqrt{3}}{\sqrt{2}}u'(c_h) + \frac{2\sqrt{3}}{\sqrt{2}}u'(c_l) > \sqrt{2}u'(c_h) + \frac{1}{\sqrt{2}}u'(c_l)$$

and thus $\partial^2\sigma_c/(\partial\sigma_y\partial\gamma) < 0$.

A.5 Additional tables and figures

Table 4: Estimates of the rank-dependent utility parameters

Study	Weighting function, δ	Utility function, σ
Battalio, Kagel, and Jiranyakul (1990)	0.72	–
Camerer (1989)	0.82	0.78
Chew and Waller (1986)	0.74	–
Conte, Hey, and Moffatt (2011)	0.74 – 1.06	0.29 – 0.82
Prelec (1990)	0.69	–
Tversky and Kahneman (1992)	0.61	0.12
Wu and Gonzalez (1996)	0.61 – 0.81	0.38 – 0.62
Average	0.74	0.50

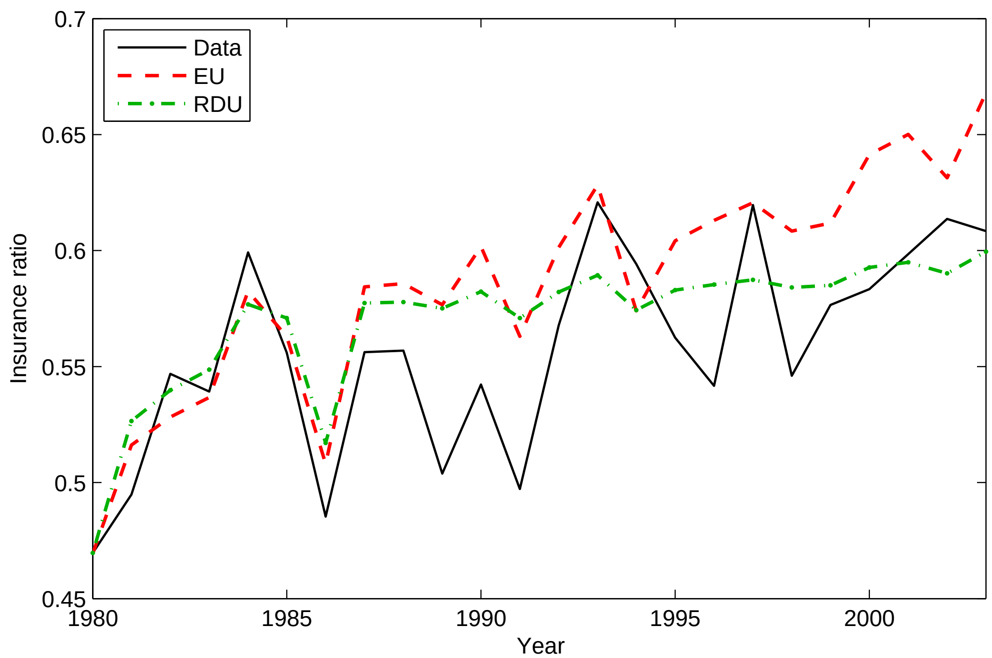

Figure 4: Insurance ratio $1 - \mathrm{var}_i(\ln c_t^i)/\,\mathrm{var}_i(\ln y_t^i)$ over time in the data (solid) versus the RDU model (dashed) and the EU model (dash-dotted)

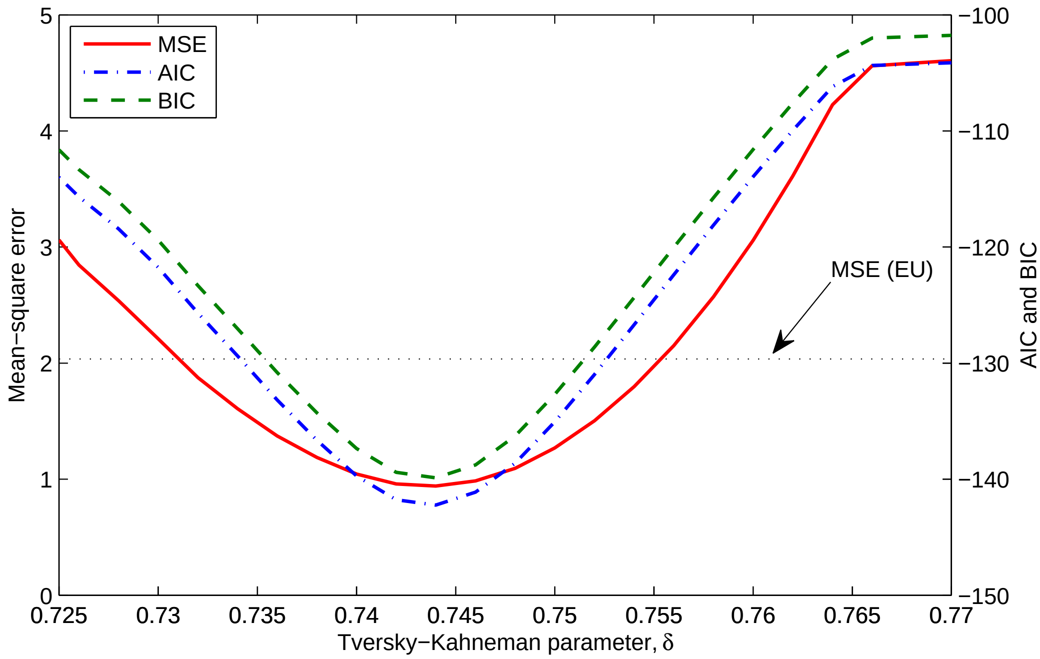

Figure 5: Mean squared error (MSE), Akaike information criterion (AIC), and Bayesian information criterion (BIC) in the RDU model fordifferent values of the Tversky–Kahneman parameter

Table 5: EU model and RDU model: Gini coefficient

	EU model	RDU model
Weighting function parameter, δ	1	0.74
Utility function parameter, σ	1	0.50
Mean-square error ratio, normalized	2.56	1

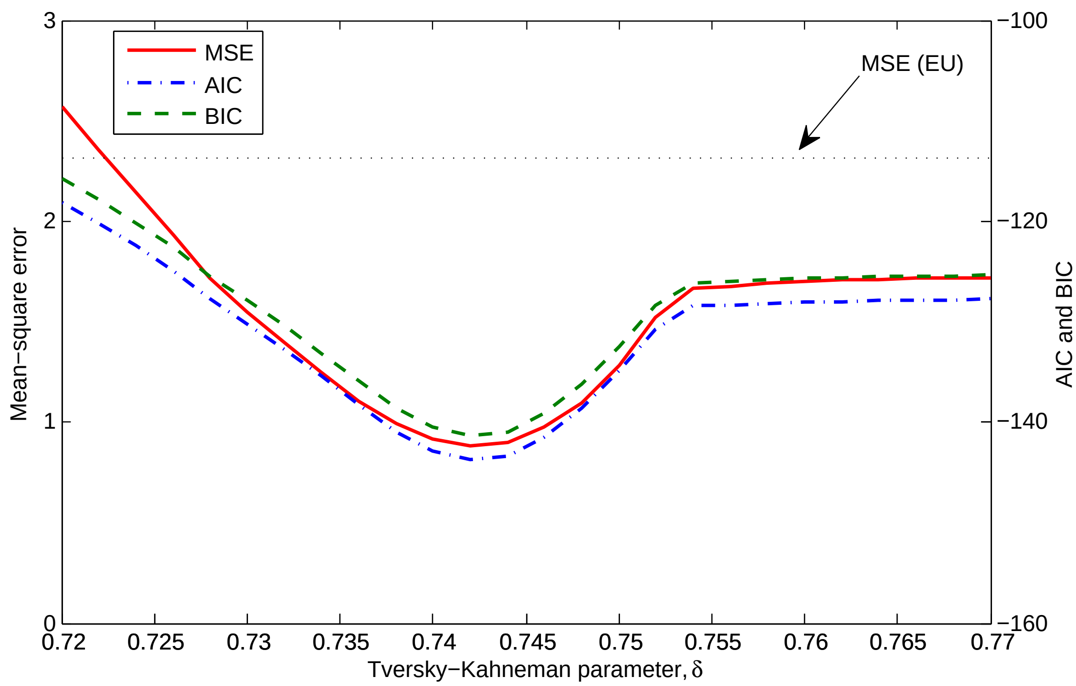

Figure 6: Smoothed time series. Mean squared error (MSE), Akaike information criterion (AIC), and Bayesian information criterion (BIC) in the RDU model for different values of the Tversky–Kahneman parameter

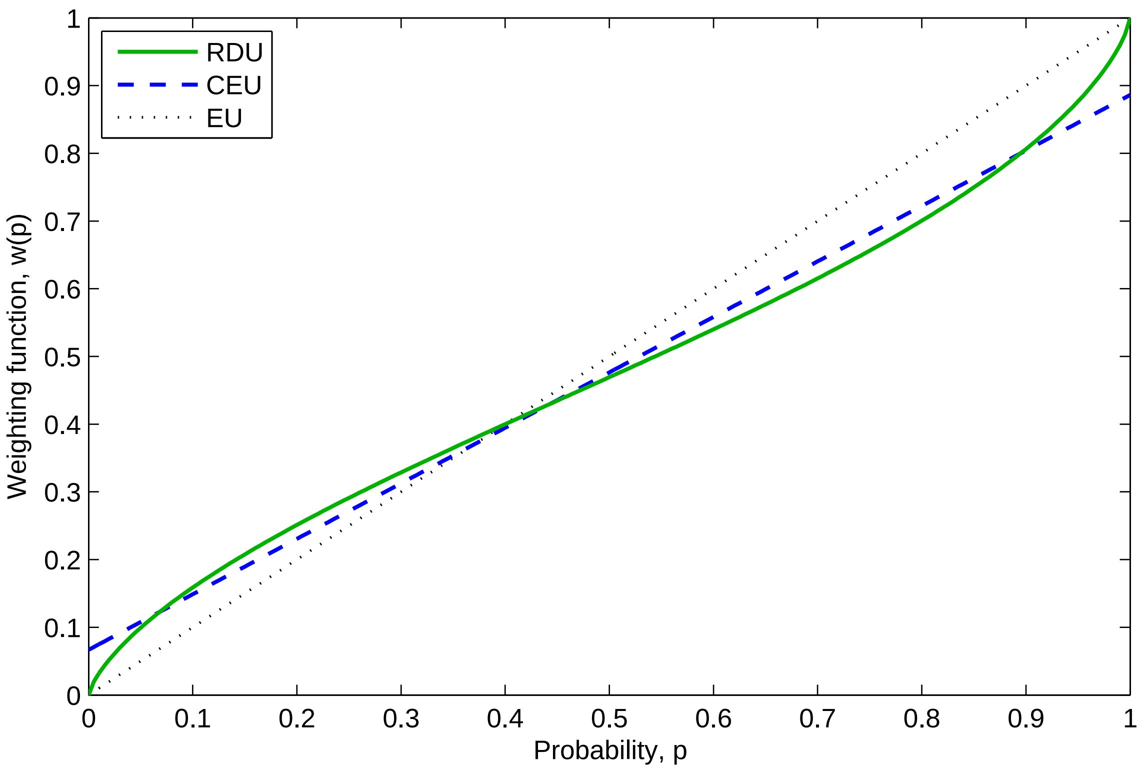

Figure 7: Tversky–Kahneman weighting function $w(p) = p^{\delta}/(p^{\delta} + (1-p)^{\delta})^{-1/\delta}$ for $\delta = 0.74$ (RDU; solid) versus neo-additive weighting function (NEO; dashed) and the expected utility (EU; dotted)