Long Term Care Insurance and Family Norms

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Abstract

Long term care (LTC) is mainly provided by the family and subsidiarily by the market and the government. To understand the role of these three institutions it is important to understand the motives and the working of family solidarity. In this paper we focus on the case when LTC is provided by children to their dependent parents out of some norm that has been inculcated to them during their childhood by some exemplary behavior of their parents towards their own parents. In the first part, we look at the interaction between the family and the market in providing for LTC. The key parameters are the probability of dependence, the probability of having a norm-abiding child and the loading factor. In the second part, we introduce the government which has a double mission: correct for a prevailing externality and redistribute resources across heterogeneous households.

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1 Introduction

It is widely agreed upon that our societies face a serious problem with long term care (LTC). Defined as a mix of medical and support services for those with disabilities and chronic care needs, LTC can be delivered at home, in an adult day care center or through another type of community program, in an assisted living facility, or in a nursing home. The source of the problem is twofold, demographic and societal. On the one hand, one witnesses a rapid increase of people aged 80+. Their relative importance in EU27 will go from 4.41% in 2008 to 12.13% in 2060. The highest figures concern Italy: 5.50 and 14.91. (Source: Eurostat, EUROPOP2008 convergence scenario). The issue of dependency arises precisely in that age bracket. On the other hand, with the drastic change in family values, the increasing number of childless households and the mobility of children, the number of dependent elderly who cannot count on the assistance of anyone is increasing. Those two parallel evolutions explain why there is a mounting demand on the government and the market to provide alternatives to the family.

LTC is the nexus of intense and complex interactions among three institutions: the state, the market and naturally the family.[1] An important empirical work particularly in the US has been devoted to the crowding out effect that social assistance can have on either the market or the family. Answer to these questions depends closely on the nature of family solidarity. Is it is based on pure altruism from children to dependent parents?[2] Does it rely on some sort of market or strategic exchange between the parents and the children as presented by Kottlikof and Spitvak (1981) or Bernheim et al. (1985)?[3] Or does it depend on some sort of social norm that comes from prevailing culture or from parental education? Those three sources of family solidarity are likely to coexist. They have to be well understood to grasp the role of the market in LTC and to design optimal policies.

[1] Brown and Finkelstein (2004a,b), Brown et al. (2006), Finkelstein and McGarry, (2004, 2005).

[2] Pestieau and Sato. (2006, 2008), Jousten et al. (2005).

[3] There exist a number of papers studying exchanges within the family. See, e.g., Stern and Engers (2002). For a survey, see Norton (2000).

We consider a framework where children can assist parents but this entails a cost. This cost is incurred *ex ante*, before the degree of disability of the parents has been revealed. One can think of irreversible occupational or residential choices. For instance, children might buy a house close to their parents, or pursue certain studies or carrier paths allowing them to assist parents if needed. All these investment decisions are taken before the degree of disability has been known. If parents turn out to be independent in their old age, the investment in family help made by the children is unproductive.

In this paper we focus on the idea that children's assistance to dependent parents is motivated by a family norm that is inculcated to them by the example of giving behavior that their parents give them during their childhood as modeled by Cox and Stark (1993, 2005) under the concept of demonstration effect. Accordingly parents make transfers to their own parents when children are present to observe such transfers. This in return conditions their own behavior when their parents age. The conjecture that the parents' behavior is aimed at inculcating desirable behavior in their children generates testable hypotheses about transfers that have been investigated by these authors using household survey microdata. We use this idea for the issue of LTC, but with an important difference, as the need for LTC is uncertain: it occurs only in case of loss of autonomy. As a consequence, the demonstration behavior turns out to be a sunk cost if the individual does not need LTC in his old days.

As it will appear presently, an individual has three ways of covering for LTC needs. First, he can hope to get some aid from his child. The amount of this aid will depend on the demonstration effect but also on the chance of having a traditional child. In other words, investing in demonstration is twice risky: it is only operative if the individual turns dependent and if his child happens to be traditional. The second way is through the private insurance market. The plus of insurance is that it is targeted on the state of dependence; its minuses are the prevalence of loading costs and the fact that it does not provide the same quality of LTC service as the family. The third way is traditional saving, that is saving for retirement. If the loading costs are prohibitive, the individual can choose to self-insure instead of buying some LTC insurance.

The paper is both positive and normative. In a first stage we want to understand the interplay between the market and the family. We there show that the level of family solidarity depends positively of the probability of dependency, the probability of having a traditional child, that is a child who abides to the family norm, and to the level of the insurance loading cost. Conversely, the insurance market size will be higher if the probability of having a traditional child is high and if the loading factor is low.

We also show that the market outcome is not optimal even in the case of identical individuals because in their choice of investment individuals only partially internalize the benefit of this investment for their elderly parents. This creates an externality that can be corrected by a Pigouvian tax on labor. In addition to this, if the private insurance is not actuarially fair, individuals might overinvest in family help. In such a case, public LTC insurance can be a useful instrument for the social planner. We show that the optimal public insurance has to trade-off the insurance motive and the correction for the family norm externality. For instance, if family help is discouraged by the introduction of a public LTC insurance, the social planner might provide less than full insurance in order to enhance the family norm. Introducing heterogeneous individuals with uneven incomes brings another role for the government: it is to correct for the above externality but also to transfer resources from high to low income households. We characterize the optimal tax schedule when the social planner can use a linear income tax, and a flat-benefit public LTC insurance.

The rest of the paper is organized as follows. In Section 2 we describe the model of family norm and the equilibrium allocation. In Section 3 we analyze the optimal allocation when all individuals in the population have the same productivity. We also discuss the role of a linear income tax and of a public health insurance. In Section 4 we consider the case when individuals differ in productivity. In Section 5 we conclude.

2 A model of family norm

We consider an overlapping generations model in which people live two periods. The first period corresponds to young age: each individual has one child, allocates time between family help and work, and allocates income across consumption, savings and long-term care insurance. The second period corresponds to old age: the individual consumes his savings. Furthermore, with probability π, the individual is dependent. In this case, he receives family help and the LTC insurance compensation.

To analyze the transmission of the family norm, we assume that parents can shape the preferences of their children through demonstration. This modeling strategy was first proposed by Stark (1995), who also found empirical evidence of the existence of such a demonstration effect (Cox and Stark, 1993, 2005).

An individual active in time t belongs to generation t. At the beginning of period t, before the dependency status of the parent has realized, the individual sets the family norm γ_t. This variable can be interpreted as an irreversible investment in the family that will be operative only if the parent turns out to be disabled. For instance, children might choose jobs and sectors that do not require too much traveling, or to move far away. They might also choose an education leading to careers which are compatible with family help. All these decisions limit the career prospects of children. Under this interpretation, γ_t is a parameter reducing individual productivity and wage, w. With probability π the parent is dependent and the investment in γ_t is productive. With probability $(1-\pi)$, this investment does not increase the utility of the parent. However, it still works as a demonstration device for children.

With probability π, a parent is dependent and receives from his child a transfer $\mu(\gamma_{t+1})$, where $\mu(.)$ is a strictly increasing and concave function representing how valuable is children's help for dependent parents. This translates the idea that the nature of the transfer is not just monetary.

Let us define ρ as the probability that a child conforms to the previous behavior of his parent by setting the same rate of intrafamily transfer, γ_t.

We will call this child a traditionalist. With probability $(1-\rho)$ the child is modern, and is not influenced by tradition. He chooses the investment that maximizes his own expected utility. For this type, the quantity γ_{t+1} is set optimally and does not depend on γ_t. The utility function of an individual depends on whether the child turns out to be traditionalist or not. To simplify, we will assume that the productivity of each individual is equal to w. We limit the analysis to a small open econonomy where the productivity of labor and the interest rate r are assumed to be constant.

Summarizing, the timing is as follows

1. At the beginning of period t, each individual has one child and γ_t is set depending on the type of the individual.

2. The individual supplies inelastically one unit of labor for a unitary wage $(1-\gamma_t)w$.

3. The disability of the parent is revealed. In case of disability, parent receives $\mu(\gamma_t)$. The individual income is allocated between current consumption c_t, savings s_t and a premium $P(I_t)$ for long term care (LTC) insurance I_t.

4. In period $t+1$, the individual is old. He consumes the gross return of his savings, $(1+r)s_t$, where $r \geq 0$ is the interest rate. If disabled, he receives from his child $\mu(\gamma_{t+1})$ and an insurance benefit equal to I_t.

The insurance premium is

$$P(I_t) = \frac{\lambda \pi I_t}{(1+r)},$$

where $\lambda \geq 1$ is the insurance company's loading factor.

Individuals in each generation can be of two types, traditionalists (denoted by T) and moderns (denoted in the following by M).

Some comments are in order concerning the meaning of γ. In our model γ is an investment that reduces the productivity of the young individual and in turn will enable him to help his parent in case of dependency. The young adult is not interested by the benefit $\mu(\gamma)$ that his dependent parent will enjoy but by the demonstration effect that such a behavior may have on his own child. This investment in time made *ex ante* could be viewed as the opportunity cost of living close to one's parent or choosing an occupation that makes one more available in case the parent becomes dependent. Quite clearly such an investment is lost if the parent stays healthy and autonomous. As said above, we also assume that the function $\mu(.)$ is strictly increasing and concave, and that $\mu'(0) = \infty$. Compared to other types of aid (public or private), aid from children is viewed as highly valuable, yet with decreasing returns (hence the concavity of $\mu(\gamma)$). Another interpretation could be that $w\gamma$ would correspond to some insurance premium that would provide an income to the aiding child in case his parent becomes disabled. In that case the premium is $w\gamma$ and the compensation $w\gamma/\pi$ allows the child to provide an aid of length $w\gamma/\pi$.[4]

2.1 Traditionalist individuals' behavior

A traditionalist young adult adopts the family norm chosen by his own parent, namely $\gamma_t^T = \gamma_{t-1}$. His expected utility function takes the form

$$u(c_t^T)+\beta\left[(1-\pi)u(d_t^T)+\pi\left[\rho H(m_t^T)+(1-\rho)H(m_t^T-\mu(\gamma_t^T)+\mu(\gamma_{t+1}^M))\right]\right], \tag{1}$$

where $c_t^T = (1-\gamma_t^T)w - P(I^T) - s_t^T$, $d_t^T = s_t^T(1+r)$ and $m_t^T = s_t^T(1+r) + I_t^T + \mu(\gamma_t^T)$. With probability $(1-\rho)$ the child is modern and the individual receives a transfer γ_{t+1}^M from his child in case of dependency. We assume

[4] An alternative specification could have been that the individual provides an aid of length γ just in case of dependency of his parent with the expectation that in case of dependency of himself he would get γ. This specification happened to be more complex analytically. Furthermore, such a modeling strategy would not be compatible with the demonstration effect: only children whose grandparents were dependent would be exposed to a family norm.

that $H(x) \leq u(x)\ \forall x$, that is to say that, given the consumption level, individuals are always worse off if dependent.

The traditionalist individual will choose s_t^T and I_t^T in order to maximize (1). The first order conditions are

$$u'(c_t^T) = (1+r)\beta\left[(1-\pi)u'(d_t^T) + \pi\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right]\right],$$

and

$$u'(c_t^T) = \frac{\beta}{\lambda}(1+r)\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right].$$

Combining those two equation one gets the following condition:

$$\left(\frac{1}{\lambda} - \pi\right)\left[\rho H'(m_t^T) + (1-\rho)H'(m_t^T - \mu(\gamma_t^T) + \mu(\gamma_{t+1}^M))\right] = (1-\pi)u'(d_t^T).$$

The individual fully insures himself if and only if $\lambda = 1$. If $\lambda > 1$, insurance is not full and the marginal utility of the individual when disabled is greater than the marginal utility in case of good health. In particular, if $\lambda \geq 1/\pi$, the individual buys no insurance and relies on self insurance. In the following we assume interior solutions by considering a loading factor $\lambda < 1/\pi$.

2.2 Modern individuals' behavior

A modern young adult (denoted by M) chooses γ_t^M, I_t^M and s_t^M in order to maximize

$$u(c_t^M) + \beta\left[(1-\pi)u(d_t^M) + \pi\left[\rho H(m_t^M) + (1-\rho)H(m_t^M - \mu(\gamma_t^M) + \mu(\gamma_{t+1}^M))\right]\right],$$

where $c_t^M = w(1-\gamma_t^M) - P(I_t^M) - s_t^M$, $d_t^M = s_t^M(1+r)$ and $m_t^M = s_t^M(1+r) + I_t^M + \mu(\gamma_t^M)$.

The first order condition with respect to γ_t^M is

$$wu'(c_t^M) = \beta\rho\pi\mu'(\gamma_t^M)H'(m_t^M). \tag{2}$$

The left hand side of this condition represents the opportunity cost of the family norm. The right hand side is the marginal benefit deriving from the presence of traditionalist children that will reproduce the family norm. If the child is modern (with probability $1-\rho$), the family norm chosen at t will have no influence on γ_{t+1}. However, given the exogenous parameters, the young individual at t anticipates that a modern child will face exactly the same problem and will optimally choose $\gamma_{t+1}^M = \gamma_t^M$.

The first order condition with respect to savings and LTC insurance are respectively

$$u'(c_t^M) = (1+r)\beta\left[(1-\pi)u'(d_t^M) + \pi H'(m_t^M)\right], \tag{3}$$

and

$$u'(c_t^M) = \frac{\beta}{\lambda}(1+r)H'(m_t^M). \tag{4}$$

The interpretation is the same as for the traditionalist type.

Assuming interior solutions, combining (2) and (4) yields the following expression

$$\mu'(\gamma_t^M) = w\frac{1+r}{\lambda\pi\rho}, \tag{5}$$

which defines implicitly the equilibrium family norm $\gamma_t^M(w,\, r,\, \lambda,\, \pi,\, \rho)$. This norm is constant over time, so that modern individuals in different generations will all choose the same level of family norm, γ^M.[5] Since $\mu(.)$ is strictly concave, γ^M decreases if the interest rate or the productivity increase. On the one hand, if the interest rate increases, this makes LTC insurance and savings a better way to transfer consumption across periods compared to the family norm. On the other hand, an increase in productivity corresponds to an increase in the opportunity cost of the time devoted to the family.

The family norm increases with the probability of having a traditionalist child and the probability of being disabled. In fact, when choosing γ^M, modern individuals consider that this help will be productive with probability $\rho\pi$. However, we know that the help set by the children has an impact on the

[5] This is due to the fact that in our small open economy, w and r are constant over time.

utility of the parents with probability π. This discrepancy creates what we call an externality here below. Note also that, if there are no traditionalist individuals ($\rho = 0$), then the family norm is equal to zero.

The family norm γ^M is also increasing with the loading factor. If the loading factor is very high, it becomes more interesting for the individual to substitute the family norm, that acts as an informal insurance, for LTC insurance. However note that, even if $\lambda = 1$, that is if LTC insurance is actuarially fair, the assumption $\mu'(0) = \infty$ implies that the family norm is always positive. Intuitively, under our assumptions, the first unit of children's help has very high returns, exceeding the one of the insurance. For small values of γ, assistance from children is always more valuable than assistance from strangers.

2.3 Steady state

In the steady state, $\gamma_t = \gamma_{t+1} = \gamma^{SS}$ $\forall t$ and for each dynasty.

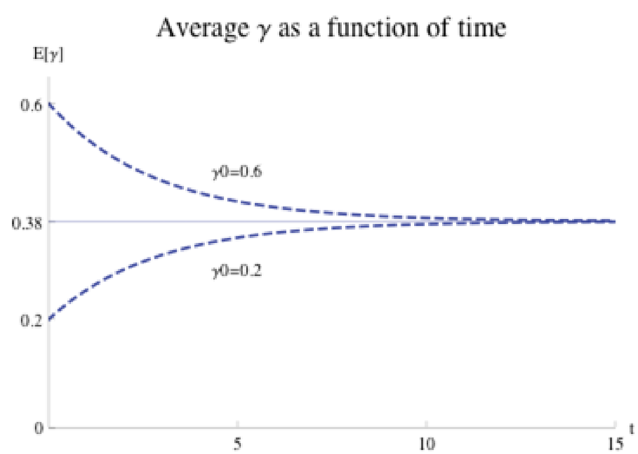
Assume that the initial norm is γ_0. In each dynasty, as soon as one individual is modern, $\gamma = \gamma^M(w, r, \lambda, \pi, \rho)$ for all subsequent generations. Traditionalist individuals will just reproduce γ^M, while modern ones face exactly the same incentives as their first modern ancestor. After t number of periods, the probability that at least one individual has been modern in a given dynasty is $1 - \rho^t$. Thus, $1 - \rho^t$ dynasties set $\gamma_t = \gamma_{t-1} = \gamma^M$. As t tends to infinity, $(1 - \rho^t)$ tends to one and the economy reaches the steady state, with $\gamma^{SS} = \gamma^M$.

In the following, we suppose that t is large enough so that the proportion of dynasties that do not set the family norm equal to γ^{SS} is negligible. We will denote the steady state family norm in absence of government intervention γ^{LF}, with $\gamma^{LF} = \gamma^{SS} = \gamma^M$. Interestingly, γ^{SS} could very well be higher than γ_0. The moderns abandon the tradition and might find it optimal to generate a higher family norm.

This point is illustrated in the Figure 1, which reports the average γ_t as a function of time, under the assumptions $\mu(\gamma) = ln(\gamma)$, $w = 1$, $r = 0.1$,$\pi = 0.4$, $\lambda = 1.5$, and $\rho = 0.7$. More precisely, at each point in time, the average family norm is equal to γ_0 for all dynasties that did not experienced a modern generation yet, and equal to γ^M if at least a modern individual belonged to the dinasty. Thus, $E(\gamma_t) = \rho^t \gamma_0 + (1 - \rho^t)\gamma^M$. Under our assumptions, $\gamma^M \simeq 0.38$. One can see that γ_t converges to its steady state value, γ^M. Furthermore, if the initial family norm is high ($\gamma_0 = 0.6$), the family norm converges to the stady state from above. If the initial norm is small ($\gamma_0 = 0.2$), convergence takes place from below. Modern children might lead to an increase in the level of family help.

It is worth to discuss two extreme cases. First, all individuals in the society might be modern ($\rho = 0$). Then, as discussed above, the family norm at the steady state would be equal to zero. Second, all individuals might be traditionalist. In this case, there would be no dynamics and γ_0 would be the steady state value of the family norm.

Figure 1



3 Identical productivities

In this section we consider a population where all dynasties have the same productivities. We first characterize the first best allocation. We then study how the first best could be centralized by a linear income tax and a public LTC insurance. We also analyze the second best allocation when not all these instruments are available.

3.1 First best

In the steady state, whichever the type of the children, the consumption in case of dependency is equal to m. The first best allocation is characterized by the solution of the problem of an utilitarian social planner maximizing the utility of the representative generation in the steady state under the economy resource constraint. In doing so, the social planner takes into account the fact that γ is only operative with probability π.[6] The first best problem is:

$$\begin{aligned}&\max_{c,d,m,\gamma} u(c)+\beta[(1-\pi)u(d)+\pi H(m)]\\ &s.t.\ \ c(1+r)+(1-\pi)d+\pi m \geq w(1-\gamma)(1+r)+\pi\mu(\gamma).\end{aligned}$$

The first order conditions with respect to the consumption levels yield the following condition

$$u'(c^{FB})=\beta(1+r)u'(d^{FB})=\beta(1+r)H'(m^{FB})=(1+r)\psi,$$

where ψ is the multiplier associated to the resource constraint. In the first best, there is perfect consumption smoothing across time and states.

The first order condition with respect to γ is

$$\begin{aligned}&w(1+r)=\pi\mu'(\gamma^{FB})\\ \iff\ &\mu'(\gamma^{FB})=w\frac{(1+r)}{\pi}.\end{aligned}$$

Comparing this expression with (5) we can compare γ^{FB} with γ^{LF}. First remark that, in the first best, γ does not depend on ρ. The social planner internalizes the fact that the help of modern children positively affects their parents, so that the social benefit of γ equals $\pi\mu(\gamma)$. At the laissez faire individuals only take into account the benefit of the family investment due to the imitation behavior of traditionalis children. They thus internalize only

[6] Alternatively, one could analyze the case when the social planner is able to impose a mutualization of family help. In such a case, γ is never wasted, since individuals with healthy parents are forced to help dependent elderly not belonging to their family. This specification would be more relevant for traditional societies with extended family. Our model applies to nuclear families.

a share of the social benefit, $\rho\pi\mu(\gamma)$. It is important to note that this result holds only if ρ is stricly greater than zero and smaller than one. If $\rho = 0$, the in the steady state the family norm is equal to zero. If $\rho = 1$, then in the steady state $\gamma^{LF} = \gamma_0$, the initial family norm. In this case $\gamma^{FB} \lesseqgtr \gamma^{LF}$, depending on the initial value of the family norm.

Second, γ^{FB} does not depend on λ, since we assume here that the government can transfer consumption freely across periods and states of the world. Overall the relationship between γ^{FB} and γ^{LF} is ambiguous. Because of the positive externally on parents, γ^{LF} tends to be too small if insurance is actuarial fair. However, if $\lambda > 1$, LTC insurance becomes less attractive and is substituted for by the family norm. More precisely $\gamma^{FB} \geq \gamma^{LF}$ if and only if $\lambda\rho \leq 1$.

Remember that ρ represents the proportion of individuals engaging in traditional behavior. It can be considered as a proxy for the traditionalism of a society. Consequently, our comparison between first best and equilibrium family norms has an easy interpretation. Societies where the loading factor and/or traditionalism are low will display family norms that are lower than the first best level. On the contrary, traditional societies that do not have access to an efficient insurance market will display an excessive degree of family help.

3.1.1 Decentralization of the first best

The first best can be decentralized by a linear income tax and a demogrant if the loading factor is equal to one ($\lambda = 1$). In this case, the only distortion in the laissez faire allocation comes from underprovision of γ. A linear tax θ on individuals' income and a lump sum transfer L decentralize the first best. The optimal θ decentralizing the first best family norm is a Pigouvian tax. This tax induces the individuals to internalize the impact of the full social benefit of the family norm:

$$\begin{aligned}
&\mu'(\gamma^*) = (1-\theta^*)w\frac{(1+r)}{\rho\pi} = w\frac{(1+r)}{\pi} = \mu'(\gamma^{FB}) \\
&\iff \theta^* = 1-\rho.
\end{aligned}$$

The optimal lump sum transfer is $L^* = \theta^* w(1-\gamma^{FB})$, so that the disposable income is not affected by the government intervention. As we noted before, this result only holds if $0 < \rho < 1$. If all individuals were modern, then agents would have no private benefit from investing in family help. There would be no use for a payroll tax. If, conversely all individuals were traditionalist, then the family norm would be equal to γ_0, and it would be impossible to obtain the optimal level of family norm. Modern individuals are necessary to generate some dynamics of the family norm, and are necessary to make policy effective. Intuitively, any policy is powerless if individuals are unable abandon a tradition.

If the loading factor is higher than one, such a system does not decentralize the first best. However, the first best can be achieved if a flat-benefit public LTC insurance is introduced. In this case, individuals pay a share θ of their income, receive a lump sum transfer when young and receive a transfer B in case of dependency when old. The public LTC insurance is assumed to be actuarially fair. The resource constraint of the social planner is thus

$$L \leq \theta w(1-\gamma) - \frac{\pi B}{(1+r)}.$$

Given the tax schedule, the representative individual chooses $\gamma^*(\theta, L, B)$, $s^*(\theta, L, B)$ and $I^*(\theta, L, B)$ such that the individual first order conditions are satisfied:

$$\begin{aligned}
&(1-\theta)wu'(c^*) = \beta\rho\pi\mu'(\gamma^*)H'(m^*) \\
&u'(c^*) = (1+r)\beta\left[(1-\pi)u'(d^*) + \pi H'(m^*)\right] \\
&\lambda u'(c^*) \geq \beta(1+r)H'(m^*),
\end{aligned}$$

where $c^* = (1-\gamma^*)(1-\theta)w + L - P(I^*) - s$, $d^* = s^*(1+r)$ and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + B$. Note that the condition on I^* allows for corner solutions, which may take place if the public LTC insurance crowds out the private one. The problem of the social planner is

$$\max_{\theta,L,B} u(c^*(\theta,L,B)) + \beta\left[(1-\pi)u(d^*(\theta,L,B)) + \pi H(m^*(\theta,L,B))\right]$$
$$s.t.\;\; L \le \theta w(1-\gamma) - \frac{\pi B}{(1+r)}.$$

Since the resource constraint is always saturated at the optimum, we can rewrite the problem as

$$\max_{\theta,B} u(c^*(\theta,B)) + \beta\left[(1-\pi)u(d^*(\theta,B)) + \pi H(m^*(\theta,B))\right]$$

where $c^*(\theta,B) = (1-\gamma^*)w - \pi B/(1+r) - \lambda\pi I^*/(1+r) - s^*$, while d^* and m^* are defined as above.

The first order conditions with respect to θ and B are

$$\begin{aligned}
&\left[\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*) - w\theta u'(c^*)\right]\frac{\partial\gamma^*}{\partial\theta} \\
&+ \left[\beta\rho\pi H'(m^*)\mu'(\gamma^*) - w(1-\theta)u'(c^*)\right]\frac{\partial\gamma^*}{\partial\theta} \\
&+ \left[(1+r)\beta\left((1-\pi)u'(d^*) + \pi H'(m^*)\right) - u'(c)\right]\frac{\partial s^*}{\partial\theta} \\
&+ \left[\beta\pi H'(m^*) - \frac{\lambda\pi}{(1+r)}u'(c^*)\right]\frac{\partial I^*}{\partial\theta} = 0,
\end{aligned}$$

and

$$\begin{aligned}
&\beta\pi H'(m^*) - \frac{\pi}{(1+r)}u'(c^*) \\
&+ \left[\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*) - w\theta u'(c^*)\right]\frac{\partial\gamma^*}{\partial B} \\
&+ \left[\beta\rho\pi H'(m^*)\mu'(\gamma^*) - w(1-\theta)u'(c^*)\right]\frac{\partial\gamma^*}{\partial B} \\
&+ \left[(1+r)\beta\left((1-\pi)u'(d^*) + \pi H'(m^*)\right) - u'(c)\right]\frac{\partial s^*}{\partial B} \\
&+ \left[\beta\pi H'(m^*) - \frac{\lambda\pi}{(1+r)}u'(c^*)\right]\frac{\partial I^*}{\partial B} = 0.
\end{aligned}$$

Using the envelope theorem and observing that either $\beta\pi H'(m) = \lambda\pi u'(c)/(1+$

r), or $I^* = 0$ (implying $\partial I^*/\partial\theta = \partial I^*/\partial B = 0$), we can rewrite the conditions above as

$$\left[\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*) - w\theta u'(c^*)\right]\frac{\partial\gamma^*}{\partial\theta} = 0, \tag{6}$$

and

$$\beta\pi H'(m^*) - \frac{\pi}{(1+r)}u'(c^*) + \left[\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*) - w\theta u'(c^*)\right]\frac{\partial\gamma^*}{\partial B} = 0. \tag{7}$$

Substituting (6) in (7), we get

$$\beta(1-\rho)\pi H'(m^*) - w\theta u'(c^*) = 0,$$

and

$$\beta H'(m^*) - \frac{1}{(1+r)}u'(c^*) = 0.$$

Since the individual first order condition with respect to γ^* is $(1-\theta)wu'(c^*) = \beta\rho\pi\mu'(\gamma)H'(m^*)$, we can rewrite the first condition as

$$\frac{(1-\rho)}{\rho} = \frac{\theta^*}{(1-\theta^*)} \iff \theta^* = 1-\rho.$$

Again, the payroll tax corrects for the family norm externality.

The second condition simply states that the public LTC smooths consumption across states. Public insurance is non distortionary since there tax includes a lump-sum transfer (see Cremer and Pestieau, 2011). Note that, if $\lambda = 1$, then public and private insurance are perfect substitutes.

3.1.2 Second best: public LTC insurance

We now study the optimal public LTC insurance scheme when the number of instruments of the social planner is limited. We assume that the public insurance is funded either through a lump-sum tax on the young or a proportional income tax.

First, suppose that the only instruments available to the social planner is a transfer B to dependent individuals financed by a lump-sum tax L on the young. Given this tax the individual chooses $\gamma^*(L,B)$, $s^*(L,B)$ and $I^*(L,B)$ such that the individual first order conditions are satisfied.

The problem of the social planner is now:

$$\max_{L,B} u(c^*(L,B)) + \beta\left[(1-\pi)u(d^*(L,B)) + \pi H(m^*(L,B))\right]$$
$$s.t. \quad -L \geq \frac{\pi B}{(1+r)}.$$

Since the resource constraint is always binding, the problem can be rewritten as

$$\max_{B} u(c^*(L,B)) + \beta\left[(1-\pi)u(d^*(L,B)) + \pi H(m^*(L,B))\right],$$

where $c^* = (1-\gamma^*)w - \pi B/(1+r) - \lambda\pi I^*/(1+r) - s^*$, and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + B$, and d^* is defined as above. The first order condition with respect to B, after using the envelope theorem, reduces to

$$\begin{aligned} &\left[-u'(c^*) + \beta(1+r)H'(m^*)\right] \\ &+\beta(1-\rho)\pi H'(m^*)\mu'(\gamma^*)\frac{d\gamma^*}{dB} = 0, \end{aligned} \tag{8}$$

Condition (8) is easy to interpret. The first term in brackets represents the insurance concern of the social planner. If this term is equal to zero, insurance is full. The second term represents the family norm externality. It is reasonable to assume that the family norm is decreasing in B, since the public LTC insurance transfers resources from the young to the dependent.[7] Consequently, the second term of (8) is always negative. For the condition to hold, one needs $[-u'(c^*) + \beta(1+r)H'(m^*)] > 0$, that is to say less than full insurance. Suppose that the social planner chooses the benefit B^{FI} (where FI stands for full insurance) equalizing the marginal utilities in all states of the word. Then, from (5), we get

$$\mu'(\gamma^{FI}) = w\frac{1+r}{\pi\rho} > \mu'(\gamma^{FB}),$$

[7] This is not necessarily true, since the public LTC insurance also discourage savings, and savings and family norm are substitutes.

implying a family norm smaller than in the first best. Consequently, to enhance the family norm, the social planner optimally chooses $B^* < B^{FI}$ and insurance is less than full. There is thus a trade-off between insuring disability and correcting for the family norm externality.

Suppose now that the transfer B to dependent individuals is financed by a payroll tax θ. Given this tax the individual chooses $\gamma^*(\theta, B)$, $s^*(\theta, B)$ and $I^*(\theta, B)$ such that the individual first order conditions are satisfied.

The problem of the social planner is now:

$$\begin{aligned} &\max_{\theta,B} u(c^*(\theta,B)) + \beta\left[(1-\pi)u(d^*(\theta,B)) + \pi H(m^*(\theta,B))\right] \\ &s.t.\;\; w\theta(1-\gamma) \geq \frac{\pi B}{(1+r)}. \end{aligned}$$

Since the resource constraint is always binding, the problem can be rewritten as

$$\max_{\theta,B} u(c^*(\theta,B)) + \beta\left[(1-\pi)u(d^*(\theta,B)) + \pi H(m^*(\theta,B))\right],$$

where $c^* = (1-\gamma^*)(1-\theta)w - \lambda\pi I^*/(1+r) - s^*$, and $m^* = s^*(1+r) + I^* + \mu(\gamma^*) + w\theta(1-\gamma)(1+r)/\pi$, and d^* is defined as above.

The first order condition with respect to θ, after using the envelope theorem, reduces to

$$\begin{aligned} &w(1-\gamma^*)\left[-u'(c^*) + \beta(1+r)H'(m^*)\right] \\ &+\beta\pi H'(m^*)\left[(1-\rho)\mu'(\gamma^*) - \theta w\frac{(1+r)}{\pi}\right]\frac{d\gamma^*}{d\theta} = 0. \end{aligned} \tag{9}$$

The first term represents the insurance motive of the social planner. This term is positive whenever there is less than full insurance. The second term represents the family norm externality. Its sign is ambigous since $d\gamma/d\theta$ might be either positive or negative. On the one hand, the tax reduces the opportunity cost of investing in family help. On the other hand the tax reduces the disposable income of young individuals and reduces the marginal benefit of family help received in the old age. Overall, it is not clear which effect dominates. Two cases might arise. If the public LTC insurance does not crowd out the private one, then equation (4) holds, and the family norm is implicitely given by condition (5). In this case, $d\gamma/d\theta > 0$. Conversely, if the public LTC insurance crowds out the private one, γ is given by (2), and $d\gamma/d\theta \gtreqless 0$.

To get some intuitition, let us define θ^{FI} as the tax decentralizing full insurance. Given this tax, the steady state family norm is such that

$$\mu'(\gamma^{FI}) = w(1-\theta^{FI})\frac{(1+r)}{\rho\pi}.$$

Thus, the left hand side of first order condition (9) evaluated in θ^{FI}, is equal to $\beta\pi H'(m^*)\left[(1-\rho)\mu'(\gamma^*) - \theta w(1+r)/\pi\right](d\gamma^*/d\theta)$, which is equal to zero if and only if $\theta^{FI} = (1-\rho)$. This is not in general true.

First consider the case where $\theta^{FI} > (1-\rho)$. In this case, the family norm γ^{FI} is too high with respect to the first best, and $\left[(1-\rho)\mu'(\gamma^{FI}) - \theta^{FI}w(1+r)/\pi\right]$ is negative. If $d\gamma/d\theta > 0$, then the sign of the left hand side of (9) is negative, so that the optimal θ is smaller than θ^{FI}. Consequently, the social planner gives up some insurance in order to reduce the family norm. Conversely, if $d\gamma/d\theta < 0$, the left hand side of the equation is positive, so that the optimal θ is greater than θ^{FI}. In this case the social planner insures individuals more than fully, in order to keep down the family norm.

If $\theta^{FI} < (1-\rho)$, we can use a similar reasoning. The equilibrium family norm is too small. If $d\gamma/d\theta > 0$, the social planner overinsures individuals in order to sustain the family norm. If $d\gamma/d\theta < 0$, the social planner provides less than full coverage.

Finally, remark that $\theta^* > 0$. Suppose this was not true. The public LTC insurance would consist of a transfer from the dependent to the young, and there would be no crowding out of private insurance. Then, insurance would be less than full and the first term of (9) would be positive. Since equation (5) holds, $d\gamma/d\theta > 0$. Then the left hand side of the above expression would be positive, which contradicts $\theta^* < 0$ being optimal.

4 Heterogeneous productivities

We now turn to the case of dynasties characterized by different productivities. We assume that there is a finite number n of productivity types in the population. A particular productivity type is denoted by w_i, with $i = 1, 2, ..., n$, and $w_1 < w_2 < ... < w_n$. Each productivity level w_i occurs with probability p_i. Moreover, $\sum_{i=1}^{n} p_i = 1$. Individuals in the same dynasty have the same productivities.

In the steady state, an individual of type i has a family norm $\gamma_i = \gamma(w_i, .)$. Furthermore, equation (5) implies that $\partial\gamma/\partial w \leq 0$. More productive individuals have a lower family norm, since their opportunity cost of devoting time to the family is higher. In the appendix we show that savings increase with w, while it is not possible to sign the derivative of private insurance purchases with respect to w.

4.1 First best

The problem of an utilitarian social planner to maximize the sum of individual utilities in the steady steady state:

$$\max_{c_i, d_i, m_i, \gamma_i} \quad E_w \left\{ u(c) + \beta[(1-\pi)u(d) + \pi H(m)] \right\}$$
$$s.t. \qquad E_w \left\{ c(1+r) + (1-\pi)d + \pi m - \gamma w(1+r) + \pi\mu(\gamma) \right\} \geq E_w[w],$$

where $E_w[u(w)] = \sum_{i=1}^{n} p_i u(w_i)$. The first order conditions with respect to the consumption levels yield the following conditions:

$$u'(c_i^{FB}) = \beta(1+r)u'(d_i^{FB}) = \beta(1+r)H'(m_i^{FB}) = (1+r)\psi \qquad \forall i, \ (10)$$

where ψ is the multiplier associated with the resource constraint. These conditions imply that $c_i^{FB} = c^{FB}$, $d_i^{FB} = d^{FB}$and $m_i^{FB} = m^{FB}$. In the first best, the allocation is characterized by perfect consumption smoothing across productivities.

The first order condition with respect to the family norm can be written as

$$w_i u'(c^{FB}) = \beta\pi\mu'(\gamma_i^{FB})H'(m^{FB}).$$

Combining this condition with (10) one gets

$$\mu'(\gamma_i^{FB}) = w_i \frac{(1+r)}{\pi},$$

implying that more productive individuals should set a smaller family norm, since it is more efficient for them to devote time to labor activities. In the first best thus, the optimal family norm decreases with the individual productivity, while consumption levels are uniform across types.

4.2 Second best: linear income tax

Consider now a situation where the social planner can only use a linear tax. She collects a fraction θ of individuals' income an redistribute the tax revenue through a lump sum transfer L. Given this tax schedule, each individual i optimally chooses $\gamma_i^*(\theta, L)$, $s_i^*(\theta, L)$ and $I_i^*(\theta, L)$.

The problem of the social planner is

$$\begin{aligned}
&\max_{\theta,T} E_w \left\{u(c^*(\theta, L)) + \beta[(1-\pi)u(d^*(\theta, L)) + \pi H(m^*(\theta, L))]\right\} \\
&s.t.\ \ L \leq E_w \left[w(1-\gamma^*)\theta\right].
\end{aligned}$$

Since the budget constraint is binding at the optimal allocation one can replace L with $E_w\left[w(1-\gamma)\theta\right]$ in the problem and maximize with respect to θ only. The first order condition with respect to θ, is equal to

$$\begin{aligned}
&E_w \left\{-w(1-\gamma^*)u'(c^*) + E_w\left[w(1-\gamma^*)\right]u'(c^*)\right\} \\
&+E_w \left\{\beta(1-\rho)\pi\mu'(\gamma)H'(m^*)\frac{d\gamma^*}{d\theta} - E_w\left[w\theta\frac{d\gamma}{d\theta}\right]u'(c)\right\} = 0.
\end{aligned}$$

After manipulations the first order condition with respect to θ can be written as

$$\theta^* = \frac{-Cov_w\left[u',\,(1-\gamma^*)w\right] + \beta(1-\rho)E_w\left[\mu' H' \frac{d\gamma^*}{d\theta}\right]}{E_w\left[wu'\right] E_w\left[\frac{d\gamma^*}{d\theta}\right]} > 0. \tag{11}$$

In setting the tax the social planner takes into account not only the usual trade-off between redistribution (first term of the numerator) and efficiency (denominator), but also the family norm externality (second term of the numerator). Since the level of γ is suboptimal in the laissez faire, the social planner will set a greater tax than the one she would choose in the absence of family norms. The tax reduces labor wages and consequently enhances the time devoted to family help. Note also that the tax cannot correct for the lack of full insurance due to the loading factor. If public insurance is not available, then the social planner cannot improve on the individual insurance choices I^*.

4.3 Second best: payroll tax and public LTC insurance.

We now characterize the optimal public LTC insurance when individuals are heterogeneous. We consider a setting where individuals pay a share θ of their income, and receive a transfer B in case of dependency in the old age. The public LTC insurance is assumed to be actuarially fair. The resource constraint of the social planner is thus

$$E_w\left[\theta w(1-\gamma)\right] \geq \frac{\pi B}{(1+r)}.$$

Given the tax schedule, each individual i optimally chooses $\gamma_i^*(\theta, B)$, $s_i^*(\theta, B)$ and $I_i^*(\theta, B)$. Note that the first order condition with respect to I,(4), allows for corner solutions, which may take place if the public LTC insurance entirely crowds our the private one. In particular, starting at $B = 0$, I_i^* decreases in B, and $I_i^* = 0$ if

$$\lambda u'(c_i^*) > \beta(1+r)H'(m_i^*).$$

If $I^* > 0$, then γ is implicitely defined by (5) and does not depend on the transfer B. If $I^* = 0$, however, the level of the family norm might be affected by such a transfer. The problem of the social planner is

$$\max_{\theta,L,\tau} E_w \left\{ u(c^*) + \beta[(1-\pi)u(d^*) + \pi H(m^*)] \right\}$$
$$s.t. \;\; E_w \left[\theta w (1-\gamma^*) \right] \geq \frac{\pi B}{(1+r)}.$$

The Lagrange expression for the planning problem is

$$\mathcal{L} = E_w \left\{ u(c^*) + \beta[(1-\pi)u(d^*) + \pi H(m^*)] \right\} - \psi \left\{ -E_w \left[\theta w (1-\gamma^*) \right] + \frac{\pi B}{(1+r)} \right\},$$

where $\psi \geq 0$ is the Lagrangian multiplier associated with the revenue constraint. The first order conditions with respect to θ and B yield

$$\frac{\partial \mathcal{L}}{\partial \theta} = E_w \left[-w(1-\gamma^*)u'(c^*) + \beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial \gamma^*}{\partial \theta} \right]$$
$$-\psi E_w \left[w\theta \frac{\partial \gamma^*}{\partial \theta} - w(1-\gamma^*) \right] = 0, \tag{12}$$

and

$$\frac{\partial \mathcal{L}}{\partial B} = E_w \left[\beta \pi H'(m^*) + \beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial \gamma^*}{\partial B} \right]$$
$$-\psi E_w \left[\frac{\pi}{(1+r)} + w\theta \frac{\partial \gamma^*}{\partial B} \right] = 0. \tag{13}$$

Using the above first-order conditions, we can write

$$\frac{\partial \tilde{\mathcal{L}}}{\partial \theta} \equiv \frac{\partial \mathcal{L}}{\partial \theta} + \frac{\partial \mathcal{L}}{\partial \theta}\frac{\partial \theta}{\partial B} = 0, \tag{14}$$

where $\partial B/\partial \theta = (1+r)E_w \left[w(1-\gamma^*) \right] / \pi$ is obtained from the resource constraint of the government. We define

$$\frac{\partial \gamma^C}{\partial \theta} \equiv \frac{\partial \gamma^*}{\partial \theta} + \frac{\partial \gamma^*}{\partial B}\frac{\partial B}{\partial \theta}.$$

The sign of $\partial \gamma^c / \partial \theta$ is ambiguous whenever the public insurance crowds out the private one. Combining (12) and (13), we can rewrite (14) as

$$\frac{\partial\tilde{\mathcal{L}}}{\partial\theta} = E_w\left[-w(1-\gamma^*)u'(c^*) + \beta\pi H'(m^*)\beta(1-\rho)\pi\mu'(\gamma^*)H'(m^*)\frac{\partial\gamma^C}{\partial\theta}\right]$$

$$+E_w\left[\beta(1-r)H'(m^*)\right]E_w\left[w(1-\gamma^*)\right] - \psi E_w\left[w\theta\frac{\partial\gamma^C}{\partial\theta}\right] = 0.$$

After manipulations we obtain the following conditon for the optimal θ.

$$\theta^* = \frac{-Cov_w\left[u',\,(1-\gamma^*)w\right] + E_w\left[\beta(1-r)H' - u'\right] + \beta(1-\rho)E_w\left[\mu' H'\frac{\partial\gamma^C}{\partial\theta}\right]}{\psi E_w\left[w\frac{\partial\gamma^C}{\partial\theta}\right]}. \tag{15}$$

When setting the tax the social planner takes into account not only the the usual trade-off between redistribution (first term of the numerator) and efficiency (denominator), but also the insurance motive and the family norm externality (second and third term of the numerator). With respect to the formula in (11), the optimal tax here also depends on an insurance term: the social planner can affect the level of insurance through B.

It can be easily shown that $\theta^* > 0$. Assume this was not true. In this case, the government would make a transfer from disabled individuals to the young (and more intensively to the high-income young). In this case, no crowding out of the private LTC insurance takes place and (5) holds, so that $\partial\gamma^c/\partial\theta > 0$ for each individuals. Furthermore, insurance would not be full and $[\beta(1-r)H' - u'] > 0$. Thus the right hand side of (15) is positive, which is a contradiction with $\theta^* < 0$.

Note that, in the absence of a demogrant, the public insurance plays a redistributive role. If a demogrant was available, B would have no redistributive role. In addition to that, if $\lambda = 1$, a demogrant would ensure redistribution and the public LTC insurance would be a redundant instrument.

5 Conclusion

The purpose of this paper was to analyze a particular example of caring initiative taken by children to the benefit of their aged and disabled parents. The motivation of children is not altruism but the hope that their caring behavior will influence their own children in doing the same if later they also need help. The caring initiative we have in mind is an investment or a decision that is made before the occurrence of disability. This can be a particular residential location, an occupational choice or some type of training that can be highly useful in case parents become disabled. If parents remain healthy those investments are of little value and can be treated as sunk costs. Besides the uncertainty over disability there is a second uncertainty that concerns the tradition-abiding behavior of children. Indeed we can realistically expect that a fraction of children will not follow the example of their parents in making such caring decision. In addition to this *ex ante* investment, individuals can provide for their old days, disabled or not, by saving and by buying a private LTC insurance.

Given this setting, we first look at the case where all individuals are alike ex ante. We show that the laissez faire solution is not optimal because in making their decisions individuals neglect the future actions of the non-traditional children. This calls for a Pigovian instrument. If the private LTC insurance is not actuarially fair, we show that a linear tax and a public LTC insurance decentralize the first best. If the social planner can only rely on a public LTC insurance (funded through a lump-sum transfer or a proportional linear tax), then it is not possible to decentralize the first best. There is a trade-off between providing adequate insurance coverage and giving incentives for family help.

Then, we turn to the case where individuals differ in earnings. The role of the government in this case is to correct for the above externality but also to redistribute resources. We consider a number of instruments: a linear income tax, and a flat-benefit LTC social benefit. We obtain the second best values of these instruments. Not surprisingly, the optimality of social insurance depends on the loading costs. The payroll tax plays a double role:

it finances public LTC expenditures but it is also a subsidy on the caring investment.

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Appendix: comparative statics with respect to w.

Equation (5) permits us to recover $\partial\gamma/\partial w = ((1+r)/\lambda\rho\pi)/\mu''(\gamma) < 0$. Total derivation of (3) and (4) yields

$$\frac{\partial I}{\partial w}U_{sI} + \frac{\partial s}{\partial w}U_{ss} = 0$$

$$\frac{\partial I}{\partial w}U_{II} + \frac{\partial s}{\partial w}U_{Is} = 0$$

In order to solve this system, define

$$\begin{aligned} A &\equiv \begin{bmatrix} U_{II} & U_{Is} \\ U_{sI} & U_{ss} \end{bmatrix} \\ &= \begin{bmatrix} \beta\pi H''(m) + (\lambda\pi)^2 u''(c)/(1+r)^2 & \beta\pi(1+r)H''(m) + \lambda\pi u''(c)/(1+r) \\ \beta\pi(1+r)H''(m) + \lambda\pi u''(c)/(1+r) & (1+r)^2\beta\left[(1-\pi)u''(d) + \pi H''(m)\right] + u''(c) \end{bmatrix}, \end{aligned}$$

and

$$B \equiv \begin{bmatrix} -\frac{\partial U_I}{\partial w} \\ -\frac{\partial U_s}{\partial w} \end{bmatrix} = \begin{bmatrix} (1-\gamma)\frac{\pi\lambda}{(1+r)}u''(c) - w\frac{\partial\gamma}{\partial w}\frac{\pi\lambda}{(1+r)}u''(c) - \beta\pi H''(m)\mu'(\gamma)\frac{\partial\gamma}{\partial w} \\ (1-\gamma)u''(c) - w\frac{\partial\gamma}{\partial w}u''(c) - \beta\pi(1+r)H''(m)\mu'(\gamma)\frac{\partial\gamma}{\partial w} \end{bmatrix}.$$

Using this notation, we can write

$$\frac{\partial I}{\partial w} = \frac{det\begin{bmatrix} -\frac{\partial U_I}{\partial w} & U_{Is} \\ -\frac{\partial U_s}{\partial w} & U_{ss} \end{bmatrix}}{det\,[A]}$$

and

$$\frac{\partial s}{\partial w} = \frac{det\begin{bmatrix} U_{II} & -\frac{\partial I}{\partial w} \\ U_{sI} & -\frac{\partial s}{\partial w} \end{bmatrix}}{det\,[A]}.$$

Straightforward calculations yield (under the assumption that $\lambda\pi < 1$)

$$det\,[A] \;=\; \beta^2\pi(1-\pi)(1+r)^2 H''(m)u''(d) + \beta\pi H''(m)u''(c)$$

$$+ \quad \beta\lambda^2\pi^2(1-\pi)u''(c)u''(d) + \beta\lambda\pi^2(2+\lambda\pi)u''(c)H''(m) > 0,$$

$$\begin{aligned} det \begin{bmatrix} -\frac{\partial U_I}{\partial w} & U_{Is} \\ -\frac{\partial U_s}{\partial w} & U_{ss} \end{bmatrix} &= \beta\lambda\pi(1+r)(1-\pi)(1-\gamma)u''(c)u''(d) - \beta\lambda\pi(1-\pi)(1+r)w\frac{\partial\gamma}{\partial w}u''(c)u''(d) \\ &- \beta^2\pi(1-\pi)(1+r)^2\mu'(\gamma)\frac{\partial\gamma}{\partial w}H''(m)u''(d) \\ &+ \beta\pi u''(c)H''(m)\,(\lambda\pi-1)\left((1+r)(1-\gamma)-(1+r)w\frac{\partial\gamma}{\partial w}+\mu'(\gamma)\frac{\partial\gamma}{\partial w}\right) \lesseqgtr 0. \end{aligned}$$

and

$$det \begin{bmatrix} U_{II} & -\frac{\partial I}{\partial w} \\ U_{sI} & -\frac{\partial s}{\partial w} \end{bmatrix} > 0$$

.

Thus, savings increase in the productivity parameter, while the sign of $\partial I/\partial w$ is ambigous.