Domestic Abuse: Instrumental Violence and Economic Incentives

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Abstract

A large fraction of domestically abused women report that their partners interfere with their participation in education and employment. As of yet, mainstream economics has not dealt in any systematic way with this phenomenon and its implications for welfare policy. This paper puts forward a theoretical framework that rationalizes why men may use violence “instrumentally” to prevent their partners from entering employment or from increasing hours of work. The model predicts a non-monotonic relationship between the gender wage gap and domestic violence. We explore the implication of this result in the context of various welfare policies. There are unlikely to be any magic bullets or one-size-fit-all solutions when it comes to reducing the incidence of domestic violence. Instead, specific measures and incentives may have to be targeted at different types of households.

Keywords: Instrumental partner-violence, Non-cooperative family decision-making, welfare policy.

JEL Classification: J12, J22, D19.

1. Introduction

Domestic violence is an everyday occurrence throughout the world. There is a clear gender pattern in that women more often suffer abuse, and abuse that is of a severe physical type. Data from around the world suggest that 20 - 60 percent of women experience domestic violence at some stage in their lives (World Health Organization, 1996). The costs of domestic violence are substantial and can be divided into four categories (Buvinic et al, 1999). First, direct costs include the value of goods and services used in treating or preventing domestic violence.¹ Second, non-monetary costs arise from increased mortality through suicide, abuse of alcohol and drugs, and from depressive disorders. Third, economic multiplier effects result from decreased female labor participation, increased absenteeism, and reduced productivity at work. Lastly, social multiplier effects follow from the impact of domestic violence on children and the erosion of social capital.

¹The paper benefited from comments from seminar participants at the Universities of Munich, Konstanz and St Gallen, and at the CESifo Area Conference on Employment and Social Protection 2011.

1Estimates for the United States suggest that the direct service-related costs of domestic violence range between $5 and $10 billion annually (Laurence and Spalter-Roth, 1996).

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A growing literature has considered domestic violence from an economic theory perspective. The main theories that have been put forward or used in the recent economic literature can be broadly placed into three categories: “exchange theory”, “signaling theory”, and “cue-triggered theory”. Exchange theory (Tauchen et al., 1991; Farmer and Thieffenthaler, 1997; Aizer, 2010) posits that some males have preferences for inflicting pain or injury onto their female partners, and emphasizes intra-household bargains whereby husbands effectively bribe their wives into accepting some level of violence by offering side payments in return. Thus, acts of violence may become part of a Pareto-improving trade between spouses. The key prediction of exchange theories is that increasing a woman’s relative wage increases her bargaining power and monotonically decreases the level of violence by improving her outside option. In signaling theory (Bloch and Rao, 2002), a male’s “satisfaction” with his marriage is private information. While satisfied husbands would never engage in violence, dissatisfied husbands have less aversion towards using violence and may do so in order to signal their dissatisfaction, thereby extracting transfers from the wife’s family. In the behavioral “cue-triggered theory” (Card and Dahl, 2011), males may fundamentally have a preference against being abusive but may become violent as a result of “losing control” in response to some negative cues, the exposure to which they control in order to maximize their own ex ante utility.

In this paper, we put forward an alternative theory of domestic violence. Our aim is to show how economic incentives for domestic violence may emerge endogenously from the internal organization of the family. The theory we present is predicated on the idea that violent households are characterized by spousal disagreements regarding their respective economic roles, modeled here as having its root cause in the time allocated to the provision of family-specific public goods. To this effect, we present a model which depicts family behavior as a noncooperative game. However, we also assume that partners have caring preferences, which implies that, for couples with “near complete caring”, equilibrium behavior is nearly “completely cooperative”. Hence the caring parameter effectively parameterizes the degree of noncooperation. In the model, each partner derives utility from own private consumption and from a household public good which is produced using time-inputs by both partners. In the noncooperative equilibrium, spouses do not provide the efficient level of the family public good, and each partner would like the other to work less in the labor market and to contribute more time to household production activities. We allow for transfers between the partners. Transfers serve two purposes. First, a partner may use a transfer to support the other’s consumption. Second, a transfer by one partner may also serve the purpose of gaining some control over the other’s time allocation.

The only gender asymmetry in the model is that men are assumed—e.g., due to superior physical strength—to have the option of exercising violence. Violence in our model is an instrumental activity that is directly targeted at women’s labor market opportunities. This modeling approach is motivated by the observation, outlined below, that abusive males routinely target their partner’s labor market activities. In particular, we will identify instrumental incentives for violence with the husband’s equilibrium utility being locally decreasing in the wife’s earnings capacity. We assume

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2“Exchange theory” originates from sociology and is, in the context of domestic violence, associated with the work of Gelles (1974). The central premise of exchange theory is that humans are dependent on one another for outcomes that they value. Actors are self-interested, and social relations are formed and maintained because actors provide reciprocal benefits.

3In particular, equilibrium violence occurs in these models if and only if the man values the first unit of violence more than the woman dislikes it (see e.g. p. 1 in online Appendix to Aizer, 2010)
that, by exercising violence, the husband can damage his partner’s earnings capacity through tactics involving targeted physical and emotional abuse as well as direct interference with work efforts.

We obtain three main sets of results. First, we derive and analyze the set of parameter values under which domestic violence emerges as instrumental equilibrium behavior. A key result is that the husband’s incentives for abuse are effectively inversely U-shaped in the wife’s relative wage. On the one hand, when the wife has a very low wage relative to the husband, he supports her financially through a monetary transfer and she will voluntarily specialize completely in household production. Since the wife’s post-transfer time allocation choice coincides with her partner’s preferred outcome, the relationship stays violence-free. On the other hand, when the wife has a very high wage relative to the husband, her labor market income is too important for the household for him to sabotage her earnings capacity. Instead, incentives for abuse obtain when her relative wage is at an intermediate level. In this case, it is individually rational for the wife to enter employment, but the husband’s preferred choice is for her to stay specialized at household production. The husband attempts to shift the woman’s employment choice towards his preferred specialization outcome through his monetary transfer. However, from the perspective of the husband, he would be better off if her wage was lower, thus giving him an incentive to resort to violence. Hence the model effectively predicts that abuse is associated with the wife’s labor supply being contentious and economic roles in the family “hanging in the balance”. Second, we demonstrate that violence will not occur among couples’ whose behavior is characterized by complete cooperation. In the current framework, equilibrium violence requires some degree of noncooperation between partners and is ultimately triggered by the misalignment of spousal preferences regarding the intra-household allocation of time. Even though this result should not come as a surprise, it underscores the difference between our approach and economic exchange theories of domestic where violence may obtain under complete cooperation between spouses. Third, our framework serves to highlight some policy dilemmas as it allows us to think about the consequences of welfare reform. We show that various welfare policies (e.g., unconditional family cash benefits) aimed universally at all households shift the incidence of domestic violence without necessarily reducing it. In order to reduce violence, different types of households have to be targeted with different types of instruments.

Central to our model is the notion of violence by males being “instrumental” and directly related to women’s economic activity. The notion that abusive males target their partners’ work (or schooling) efforts is well-documented in the literature (Raphael 1995, 1996). Sabotage tactics used by abusive males noted in the literature include the inflicting of visible facial injuries before job interviews, destruction of homework assignments, keeping women up all night with arguments and fights before key tests or job interviews, turning off alarm clocks, destroying or hiding clothes, cutting off the victim’s hair to cause embarrassment, threatening to kidnap the children from child care centers, failing to show up as promised for child care or transportation, job-related stalking and on-the-job harassment (Tjadden and Thoennes, 1998; Zachary, 2000; Swanberg and Macke, 2006).

Interference with work effort by abusive males appears to be commonplace. Tolman and Rosen (2001) in a study of 753 female welfare recipients in Michigan document that 48 percent of

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4 The term “instrumental violence” was introduced by Gelles and Strass (1979) to refer to “...the use of pain or injury as a punishment to induce another person to carry out some act” (p. 557). In contrast “expressive violence” refers to “...the use of physical force to cause pain or injury as an end in itself” (p. 557), i.e. preference based, possibly in the context of loss of control.

5 Indeed, the so-called Work/School Abuse Scale (W/SAS) survey instrument was constructed to measure such interference with employment and education by abusive males (Riger et al., 2000).
those who had experienced severe violence in the preceding 12 months also reported some form of
direct work interference. Similarly, in a study of 1,082 applicants for public assistance in Colorado,
Pearson et al. (1999) found that 44 percent of domestic violence victims reported that their abusive
partners had prevented them from working. It is well understood that exposure to domestic violence
has major implications for victimized employees. The short-term consequences include increased
absenteeism and reduced productivity (Swanberg and Macke, 2006). Hence, hourly earnings may
be affected. Domestic violence also negatively and significantly affects the victim’s capacity to
maintain work in the long-term (Browne et al., 1999). As a consequence, potential employers may
become sceptical of a victimized woman’s inconsistent work history, making it difficult for her to
find new employment.

Our approach to modeling domestic violence entails characterizing family decision-making as
non-cooperative.6 We advocate this approach even through the dominant premise in the theory of
the family is that households are able to reach efficient outcomes (Becker 1991; Manser and Brown,
1980; McElroy and Horney, 1981; Chiappori, 1992; Chiappori et al., 2002). Indeed, in our setting,
households will reach (near) efficient outcomes if their caring is (nearly) complete. Complete or
near complete caring may well characterize a large proportion of existing couples, thus implying
that assuming efficiency is a reasonable first approximation of typical household behaviour in other
settings. It does not, however, imply that assuming efficiency is a useful approach to modeling the
behavior among couples engaging in domestic violence.7 The key point that we would stress here
is an obvious one, namely that the idea that violent behavior is efficient and welfare enhancing is
at odds with the universal view that domestic violence is a harmful activity that society should
try to prevent.

The paper proceeds as follows. In Section 2 we set out our model of domestic violence, discuss
our assumptions, and analyze the equilibrium of the model. We then in Section 3 investigate policy.
The main focus here will be to show that welfare policies shift the incidence of domestic violence in
predictable ways. Section 4 provides a simple Cobb-Douglas example which illustrates our results.
Section 5 concludes with a discussion of the empirical relevance of our results and some pointers
for future research.

2. The Model

2.1. The Formal Setup

Consider an economy consisting of households, where, in each household, there is a husband
(h) and a wife (s). Each partner (i = h, s) obtains utility from private consumption, $c_i$, and a
home-produced household public good, $Q$. For simplicity we assume separable preferences with a
common utility function over the household public good. Formally, let the preferences of partner
$i$ be represented by

$$u_i(c_i, Q) = v_i(c_i) + Z(Q),$$  

(1)

where $v_i$ is twice continuously differentiable, strictly increasing, strictly concave and $\lim_{c_i \to \infty} v_i'(c_i) = 0$ and $\lim_{c_i \to 0} v_i'(c_i) = \infty$. Each spouse has a unit of active time endowment, to be allocated be-

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6Various kinds of non-cooperative models have been put forward in the literature, e.g. by Bergstrom, 1989; Lundberg and Pollak, 1993; Konrad and Lommerund, 1995; Chen and Woolley, 2001; and Anderberg, 2007.

7Models of (expressive) domestic violence which imply efficient outcomes, either by directly assuming cooperative bargaining or via the Coase theorem, include those presented by Tauchen et al. (1991); Farmer and Thiefoenthaler (1997) and, recently, Aizer (2010).
between market work, $\ell_i$, and home production, $q_i \equiv 1 - \ell_i$. We denote the household production function by $Q = Q(q_h, q_s)$. The properties of $Z(\cdot)$ and $Q(\cdot, \cdot)$ are discussed below.

We assume “caring preferences”: each partner puts a weight of $\mu \in (0, 1/2)$ on the private utility of the spouse and a (larger) weight $(1 - \mu)$ on own private utility. The total preferences of partner $i$ are thus

$$U_i(c_i, c_{-i}, Q) = (1 - \mu)u_i(c_i, Q) + \mu u_{-i}(c_{-i}, Q).$$

(2)

Note that the limit as $\mu$ approaches one-half corresponds to a situation of full cooperation. In this case, the partners pursue the same objectives and hence will operate on the Pareto frontier. Conversely, the limit as $\mu$ approaches zero corresponds to a situation of pure noncooperation. Our model, therefore, also allows consideration of how the incidence of domestic violence varies with the degree of (non)cooperation within a partnership.

When working in the labor market, partner $i$ can earn a wage $w_i \in W_i \equiv [w_h, w_s], i = h, s$. We will refer to the wage profile $(w_h, w_s)$ as a couple’s “type” and assume that couple-types are distributed according to some continuous distribution $G$ on the support $W_h \times W_s$ (with positive density on the entire support).

The model also allows for unearned income $y_i$. However, for the baseline scenario, we assume $y_i = 0$. Positive unearned income will be considered in the context of welfare policy in Section 3. Finally, partner $i$ can make a transfer $t_i \geq 0$ to the spouse, and we let $t \equiv t_h - t_s$ denote the net transfer from the husband to the wife.

By substituting for $u_i$ and $Q$ in (2), individual $i$’s total preferences can be written as

$$U_i(c_i, c_{-i}, h, s) = (1 - \mu)u_i(c_i) + \mu u_{-i}(c_{-i}) + z(1 - \ell_h, 1 - \ell_s),$$

(3)

where $z(q_h, q_s) \equiv Z(Q(q_h, q_s))$ is the composition of the utility from $Q$ with the household production function. We assume that the partners’ time inputs into household production are “independent”:

**Assumption 1 (Household Production Technology).** The composite function $z(q_h, q_s)$ is additively separable,

$$z(q_h, q_s) = z_h(q_h) + z_s(q_s),$$

with each $z_i(\cdot)$ being twice continuously differentiable, strictly increasing and strictly concave and with $\lim_{q_i \to 0} z_i'(q_i) = \infty (i = h, s)$.

We adopt the following timing of events. First, the husband chooses whether or not to exercise domestic violence. Second, the spouses simultaneously and non-cooperatively choose transfers. Finally, the spouses simultaneously and non-cooperatively decide on how to allocate their unit time endowment between market work and the production of the household public good. The analysis that follows deals with the choices of the two family members in reverse order.

We assume complete information. In particular, the values of all the parameters are common knowledge between the two partners. The only fundamental gender asymmetry in our model is thus with respect to the husband’s ability to use violence. Violence has an investment flavor in that it has a lasting adverse effect on the victim’s earnings potential. Indeed, a positive level of domestic violence works to reduce the market wage of the wife. Since domestic violence interferes with the victim’s potential wage in the labor market, it may be thought of as purposeful behavior by the husband to achieving some control over the time allocation of the wife.
2.2. The Time Allocation

Taking unearned incomes, transfers and the spouse’s time allocation as given, partner \(i\) solves

\[
\max_{\ell_i \in [0, 1]} \{ U_i(c_i, c_{-i}, \ell_i, \ell_{-i}) | c_i = w_i\ell_i + Y_i \},
\]

where

\[
Y_i = y_i - t_i + t_{-i}.
\]

The first-order condition for an interior optimum reads

\[
(1 - \mu) v_i'(w_i\ell_i + Y_i)w_i \leq z_i'(1 - \ell_i).
\]

Note that (7), which will hold with equality when \(\ell_i > 0\) and with inequality when \(\ell_i = 0\), only involves the “own” wage and unearned income. Thus, each partner has a strictly dominant time allocation strategy, which we denote \(\ell_i(w_i, Y_i)\). By Assumption 1, we can be sure that neither partner will fully specialize in market work.

In general, the effect of \(i\)'s own wage on \(\ell_i\) is ambiguous due to conflicting income and substitution effects. We will assume, however, that the substitution effect dominates so that individual \(i\)'s labor supply is non-decreasing in the own wage. This assumption captures the idea that the husband can reduce the wife’s formal labour supply by interfering with her earnings capacity.

**Assumption 2 (Effect of Own Wage on Labor Supply).** The substitution effect dominates the income effect so that individual \(i\)'s labor supply is non-decreasing in her own wage:

\[
\frac{\partial \ell_i(w_i, Y_i)}{\partial w_i} \geq 0 \quad \text{for} \quad i = h, s.
\]

We denote the wage elasticity of labour supply by

\[
\varepsilon_i(w_i, Y_i) = \frac{\partial \ell_i(w_i, Y_i)}{\partial w_i} \frac{w_i}{\ell_i(w_i, Y_i)} \quad \text{for} \quad i = h, s,
\]

which, by Assumption 2, is also positive. We define the individual’s earnings function as

\[
m_i(w_i, Y_i) = w_i\ell_i(w_i, Y_i) \quad \text{for} \quad i = h, s.
\]

While the individual’s earnings will be decreasing in \(Y_i\), it must be that

\[
\frac{\partial m_i(w_i, Y_i)}{\partial Y_i} \in (-1, 0).
\]

This follows immediately from the fact that, with separable preferences, the individual’s consumption must be increasing in \(Y_i\). The fact that partner \(i\)'s earnings are decreasing in \(Y_i\) reflect that his/her labor supply is decreasing in \(Y_i\). Looking ahead towards the transfer decisions, this implies that each partner \(i\) can induce the other to reduce his/her labor supply by increasing the transfer \(t_i\).

It is also straightforward to demonstrate that partner \(i\)'s labor supply is decreasing in the caring parameter \(\mu\). This follows from the fact that, at any \(\mu < 1/2\), there is an inefficiency in the chosen labor supplies. In particular, each partner works “too much” from the perspective of
the spouse. To see this, note that partner \( i \) values an increase in the earnings of partner \(-i\) at \( \mu v_{-i}^' (c_{-i}) \) whereas partner \(-i\) values it more highly at \( (1 - \mu) v_{-i}^' (c_{-i}) \). The closer \( \mu \) is to one-half, the more each partner “internalizes” this effect and hence chooses a lower level of market work.

We will also make a set of further assumptions which we impose directly on the labor supply functions rather than on the primitives as the corresponding assumptions on the primitives would be highly involved and contain difficult-to-interpret third derivatives of \( v(\cdot) \) and \( z(\cdot) \).

**Assumption 3** (Second Derivatives of Labor Supply).

1. Convexity of labor supply and earnings in unearned income:
   \[
   \frac{\partial^2 \ell_i (w_i, Y_i)}{\partial Y_i^2} \geq 0;
   \]

2. Positive cross-partial for earnings:
   \[
   \frac{\partial^2 \ell_i (w_i, Y_i)}{\partial Y_i \partial w_i} \geq \frac{1}{w_i} \frac{\partial \ell_i (w_i, Y_i)}{\partial Y_i}.
   \]

3. Concavity of labor supply in wage — decreasing labor supply elasticity:
   \[
   \frac{\partial^2 \ell_i (w_i, Y_i)}{\partial w_i^2} \leq \frac{(1 - \varepsilon_i (w_i, Y_i)) \varepsilon_i (w_i, Y_i)}{w_i^2 / \ell (w_i, Y_i)};
   \]

Part (i), which is equivalent to earnings being a weakly convex function of unearned income, is used below to demonstrate that, if one partner is making a transfer to the other, then the donor’s utility is a concave function of the transfer. Many common utility specifications, including the Cobb-Douglas and CES case (see below), satisfy this assumption through \( \ell_i (w_i, Y_i) \) being linear in \( Y_i \).

Part (ii) is equivalent to the cross-partial of the earnings function \( m(w, Y) \) being non-negative. This assumption is a sufficient (but not necessary) condition for the transfer given from partner \( i \) to \(-i\) being decreasing in the recipient’s wage when the recipient is working. This condition is satisfied with equality in the Cobb-Douglas case and holds in the CES case if and only if the elasticity of substitution exceeds unity.

Part (iii) says that the individual’s labour supply is a sufficiently concave function of the wage. The particular assumption is equivalent to the wage elasticity \( \varepsilon_i (w_i, Y_i) \) being decreasing in \( w_i \). This is used below to argue that the husband’s incentives for violence eventually diminish as the wife’s wage grows. The assumption is satisfied by Cobb-Douglas and CES preferences whenever \( Y_i \geq 0 \). Taken together, the assumptions also imply that the elasticity \( \varepsilon_i (w_i, Y_i) \) is increasing in unearned income.\(^8\)

2.3. The Transfer Decision

Each partner has two potential motives for transferring income to the spouse. First, to support the spouse’s consumption. Second, to induce the spouse to work less in the labor market. We will

\(^8\)To see this, note that differentiating (??) yields that \( w_i \frac{\partial \varepsilon_i}{\partial w_i} = \frac{\partial^2 \ell_i}{\partial w_i^2} w_i^2 + \frac{\partial \ell_i}{\partial w_i} \left( 1 - \frac{w_i}{\ell_i} \right) \).

\(^9\)Differentiating (??) yields that \( \ell_i \frac{\partial \varepsilon_i}{\partial Y_i} = \frac{\partial^2 m_i}{\partial Y_i \partial w_i} - \frac{\partial \ell_i}{\partial w_i} \left( 1 + \frac{\partial \ell_i}{\partial w_i} \frac{w_i}{\ell_i} \right) \) which is positive due to the cross-partial of the earnings function being positive, and \( \partial \ell_i / \partial Y_i < 0 \) and \( \partial \ell_i / \partial w_i > 0 \).
characterize here the transfer $t_i$ from partner $i$ to the spouse $-i$ under the assumption that $-i$ is not making any transfer back, i.e. that $t_{-i} = 0$. Below we will verify that, indeed, in equilibrium at most one partner will be making a positive transfer.

We will demonstrate that partner $i$’s choice of $t_i$, when viewed as a function of the spouse’s wage $w_{-i}$, has four “regimes”. First, in the low-wage regime, partner $i$ makes a pure “benevolent transfer”, denoted $t_i^0(w_i, w_{-i})$, and $-i$ strictly prefers not to work. Second, in the low-to-medium wage regime, partner $i$ makes a “crowding out” transfer, denoted $t_i^1(w_i, w_{-i})$, and $-i$ just prefers not to work. Third, in the medium-to-high wage regime, partner $i$ makes a positive transfer, denoted $t_i^2(w_i, w_{-i})$, and $-i$ works some strictly positive amount of time. Finally, in high wage regime, partner $i$ does not make a transfer and $-i$ works some strictly positive amount of time. The following proposition formalizes partner $i$’s transfer behavior:

**Proposition 1** (Equilibrium Transfer Choices). For given $w_i > 0$ there exist three critical wages for the spouse, $w^k_{-i}(w_i), k = 0, 1, 2$, ranked increasingly in $k$ and each strictly increasing in $w_i$, such that:

1. When $w_{-i} < w^0_{-i}(w_i)$, partner $i$ makes a (“benevolent”) transfer $t_i^* = t_i^0(w_i, w_{-i})$, which is strictly increasing in $w_i$ and independent of $w_{-i}$, and $-i$ (strictly) fully specializes in household production, $\ell^*_{-i} = 0$.
2. When $w_{-i} \in [w^0_{-i}(w_i), w^1_{-i}(w_i)]$, partner $i$ makes a (“crowding-out”) transfer $t_i^* = t_i^1(w_i, w_{-i})$, which is independent of $w_i$ and strictly increasing in $w_{-i}$, and $-i$ (just) fully specializes in household production, $\ell^*_{-i} = 0$.
3. When $w_{-i} \in [w^1_{-i}(w_i), w^2_{-i}(w_i)]$, partner $i$ makes an (“interior”) transfer $t_i^* = t_i^2(w_i, w_{-i})$, which is strictly increasing in $w_i$ and strictly decreasing $w_{-i}$, and $-i$ works positive hours, $\ell^*_{-i} > 0$.
4. When $w_{-i} \geq w^2_{-i}(w_i)$, partner $i$ makes no transfer $t_i^* = 0$, and $-i$ works positive hours, $\ell^*_{-i} > 0$.

The benevolent transfer $t_i^0(w_i, w_{-i})$ equalizes the marginal utility of each partner’s consumption as viewed from the perspective of $i$’s preferences under the assumption that the spouse has no earnings,

$$ (1 - \mu) v'_i(c_i) = \mu v'_{-i}(c_{-i}). \quad (10) $$

This is an equilibrium when the spouse chooses not to work when provided with the benevolent transfer. Indeed, the first critical wage, $w^0_{-i}(w_i)$, is defined as the highest $w_{-i}$ at which $-i$ prefers not to work upon receiving $t_i^0(w_i, w_{-i})$.

Whenever $w_{-i} > w^0_{-i}(w_i)$, $-i$ would choose to work some strictly positive hours if provided with $t_i^0(w_i, w_{-i})$. For a range of $w_{-i}$, up to a second critical wage $w^1_{-i}(w_i)$, partner $i$ then increases the transfer $t_i$ so as to (just) ensure that the spouse chooses not to work. Thus, the “crowding out” transfer $t_i^1(w_i, w_{-i})$ is characterized by the first order condition for $-i$’s labor supply (??) holding with equality at $\ell_{-i} = 0$. As an increase in $w_{-i}$ strengthens $-i$’s labor supply incentives, it also increases $t_i^1(w_i, w_{-i})$. As the spouse’s wage increases further, it eventually no longer becomes optimal for partner $i$ to completely crowd out the spouse’s labor supply. However, the transfer chosen by partner $i$ even in this case will be aimed in part at reducing the spouse’s labor supply. The first order condition characterizing the “interior” $t_i^2(w_i, w_{-i})$ transfer is

$$ (1 - \mu) v'_i(c_i) = v'_{-i}(c_{-i}) \left[ \mu - (1 - 2\mu) \frac{\partial m_{-i}}{\partial Y_{-i}} \right]. \quad (11) $
Note that $t^2_i(w_i, w_{-i})$ contains both a benevolent aspect and a labor supply reducing aspect. The two remaining critical wages, $w^1_{-i}(w_i)$ and $w^2_{-i}(w_i)$, are characterized by the recipient’s labor supply at the interior transfer going down to zero and the interior transfer itself going down to zero, respectively.

Figure ?? illustrates the transfer made by partner $i$ as a function of the spouse’s wage $w_{-i}$ for a given own wage $w_i$. Note in particular how the transfer is a non-monotonic function of $w_{-i}$.

The hatched line in Figure ?? shows the effect of an increase in the own wage $w_i$. From Proposition ?? we know that an increase in $w_i$ increases both the benevolent transfer $t^0_i(w_i, w_{-i})$ and the interior transfer $t^1_i(w_i, w_{-i})$ which in turn increases all three critical wages, $w^k_{-i}(w_i)$, $k = 0, 1, 2$. As the figure illustrates, this implies that the equilibrium transfer made by partner $i$, denoted $t^*_i(w_i, w_{-i})$ is (weakly) increasing in the own wage $w_i$.

**Lemma 1** (Effect of Own Wage on Transfers). The equilibrium transfer $t^*_i(w_i, w_{-i}) > 0$ is weakly increasing in $w_i$.

In characterizing the transfer made by partner $i$ we have assumed that no simultaneous transfer was made from the spouse back to partner $i$. This can be easily verified.

**Lemma 2** (No Simultaneous Transfers). If partner $i$ makes a positive equilibrium transfer, $t^*_i(w_i, w_{-i}) > 0$, then the spouse $-i$ strictly prefers not to make a transfer, $t^*_{-i}(w_{-i}, w_i) = 0$.

Indeed, the proof of Lemma ?? demonstrates that, for any given $w_i$ there will exist a range of spousal wages $w_{-i}$ such that both $i$ and $-i$ choose not to make any transfer in equilibrium. Hence in a population of couples with a distribution of wage-profile types, equilibrium transfers will be zero for a positive measure of couples. In the limiting case of complete caring ($\mu = 1/2$)
the measure of couples who make no transfers reduces to zero as, in the limit, the partners agree on the consumption allocation and this allocation will, generically, not coincide with the partners’ income profile.

To summarize, if one partner makes an equilibrium transfer to the spouse, he/she does so to support the spouse’s private consumption and to influence the spouse’s time allocation away from market work. We now ask whether and when the husband has an incentive to resort to an additional means of influence, namely instrumental violence.

2.4. Incentives for Domestic Violence

We identify economic incentives for instrumental violence with the husband’s equilibrium utility being locally decreasing in the wife’s earnings capacity. The following result, which is interpreted after its statement, reveals that the risk of domestic violence is present when the economic roles within the partnership, in a sense, “hang in the balance”.

**Proposition 2** (Economic Incentives for Instrumental Violence). For a given $w_h$ there exists a critical wage $w^*_s(w_h)$ such that the husband has an equilibrium incentive for instrumental violence when $w_s \in (w^0_s(w_h), w^*_s(w_h))$. The critical wage $w^*_s(w_h)$ strictly exceeds $w^1_s(w_h)$ and is weakly increasing in $w_h$.

The proposition states that the husband’s economic incentives for abuse kick in when his benevolent transfer is not sufficient to induce the wife to fully specialize in household production, i.e. when $w_s > w^0_s(w_h)$, and it continues up to some level of the wife’s wage at which she is working in equilibrium. The result has a simple intuition. For any $w_s \leq w^0_s(w_h)$, the husband makes the “benevolent transfer” $t^*_h = t^0_h(w_h, w_s)$, which is sufficient to induce the wife to fully specialize in household production. Her wage is low enough to be, in effect, irrelevant and any further reduction in $w_s$ would leave the equilibrium entirely unchanged. Thus, the relationship will remain violence free.

Consider then the case where $w_s \in (w^0_s(w_h), w^1_s(w_h))$, implying that the husband chooses the crowding out transfer $t^*_h = t^1_h(w_h, w_s)$. In this case, the husband is making a transfer which exceeds the one he would voluntarily make, but the wife is still not working in equilibrium. The husband’s equilibrium utility would be increased by a reduction in the wife’s wage as this would allow him to reduce his transfer towards his benevolent transfer. Thus, he has an economic incentive for abuse.

Finally, consider the case where $w_s > w^1_s(w_h)$ so that, in equilibrium, the wife works and the husband either makes the “interior” transfer $t^*_h(w_h, w_s)$ or no transfer at all. In this case, the impact of a marginal increase in the wife’s wage on the husband’s equilibrium utility can be written as

$$v'_s(c^*_s)\ell^*_s [\mu - (1 - 2\mu) \varepsilon_s(w_s, t^*_h)],$$

and it can be demonstrated that this expression is strictly negative as $w_s$ approaches $w^1_s(w_h)$ from above (see proof of Proposition 2). This establishes that incentives for violence will also be present for some couples where the wife works. The expression in (??) is negative if and only if

$$\mu < \frac{\varepsilon_s(w_s, t^*_h)}{[1 + 2\varepsilon_s(w_s, t^*_h)]}.$$  

As the wife’s wage increases, her labour supply responsiveness decreases, thus decreasing the right
hand side. Instead, for a high enough wife’s wage, his equilibrium utility will be increasing in her wage as he benefits, through caring, from her higher earnings.

Note that in characterizing the husband’s incentives for abuse, we have ignored any potential transfer from the wife to the husband. However, it should be clear enough that any transfers from the wife to the husband will weaken the husband’s abuse incentives as we know from Lemma ?? that the transfer that the wife makes to the husband will be increasing in her wage. Hence by interfering with her earnings capacity, the husband would also reduce the transfer that he obtains from her.

Finally, note that economic incentives for instrumental violence obtain from the inefficiency in public good provision associated with incomplete caring. The following result shows that, when partners behave nearly “completely cooperative”, then they will abstain from domestic violence altogether:

**Proposition 3** (Household Mode of Behavior and Instrumental Violence). In the limit with complete caring, \( \mu \rightarrow \frac{1}{2} \), the set of wages profiles \( w_s \in (w_{h0}(w_h), w_{s*}(w_h)) \) at which the husband has equilibrium incentives for instrumental violence reduces to the empty set.

While this result is highly intuitive, it is at same time in stark contrast to the insights provided by exchange theories of domestic violence. Indeed, our framework implies that, with complete caring, couples pursue a common objective and the household operates at an agreed point on the Pareto frontier. In this cooperative-like setup, an act of violence which reduces the wife’s earnings capacity would then simply reduce the utility possibility set and would hence never increase the husband’s equilibrium utility. In exchange theories, by contrast, acts of violence occur when they increase the utility possibility set and consequently become part of a Pareto-improving trade (involving compensating side-payments) between spouses.

### 3. The Effect of Welfare Policy

Most of the existing theoretical literature on domestic violence does not explicitly model labor supply and the targeting by abusers of the victim’s economic activities, which explains why it tends to be silent on the consequences of welfare policy. An advantage of our theoretical structure is that we can get a clear sense of the margins where we may see a change in the incidence of domestic violence when a government intervenes with families. In this section, we thus use our framework to explore the consequences of welfare policy for the incidence of domestic violence. Here we will consider two very simple examples of policy interventions: a wage subsidy policy and a flat-rate benefit policy, with either policy potentially being gender-specific.

Consider first a wage subsidy policy and let \( \sigma_i \geq 0 \) denote the subsidy rate that applies to partner \( i, i = h, s \). The effective wage for partner \( i \) is hence \( \tilde{w}_i = (1 + \sigma_i) w_i \), and we refer to \( w_i \) as the individual’s *primary wage*. We are interested in understanding how a wage subsidy offered to either gender affects the range of wife’s primary wages at which the husband has economic incentives.

The analysis of wage subsidies is much simplified by the insight that what matters for violence incentives are the partners’ effective wages. This immediately implies that a wage subsidy provided

\[^{10}\text{When the husband is making the interior transfer } t_h^* (w_h, w_s) \text{ this effect is further reinforced by this transfer decreasing in } w_s.\]
to women will shift downwards the set of wives’ primary wages at which the husband has violence incentives. For example, a woman whose primary wage is low enough that she, in the absence of a wage subsidy, would choose not to work at the husband’s benevolent transfer may find that, once provided with a wage subsidy, she would prefer to work given the same transfer. Hence a wage subsidy provided to women may pull some low wage women into the violence region. Conversely, a woman whose primary wage would not be high enough to put her beyond the risk of violence, may find that a positive wage subsidy increases her effective wage enough to do so.

Similarly, since the boundaries of the violence region, \( w_0^h(w_h) \) and \( w^*_s(w_h) \), are increasing in the husband’s wage, it follows that a wage subsidy provided to the husband shifts upwards the set of wives’ primary wages at which the husband has violence incentives. Hence, a wage subsidy provided to the husband will reduce violence against some low wage women who, with the husband’s subsidy, will choose not to work at his benevolent transfer. Conversely, by reducing the women’s relative wage, a wage subsidy provided to males, will put into the violence region some women whose wages would otherwise have been sufficiently large to put them beyond the risk of violence.

**Proposition 4 (Effect of Wage Subsidies).**

1. A wage subsidy given to the husband, \( \sigma_h > 0 \), increases both the lower- and the upper bound, \( w_0^h(w_h) \) and \( w^*_s(w_h) \), of the set of wage profiles at which the husband has equilibrium incentives for instrumental violence.

2. A wage subsidy given to the wife, \( \sigma_s > 0 \), decreases both the lower- and the upper bound, \( w_0^h(w_h) \) and \( w^*_s(w_h) \), of the set of wage profiles at which the husband has equilibrium incentives for instrumental violence.

While the effect of wage subsidies are highly intuitive, the effect of flat-rate benefits are perhaps somewhat more surprising. So far we have focused on the case where the unearned income for each partner was zero, \( y_i = 0 \) (\( i = h, s \)). We will now look at how a marginal increase, starting from zero, in either partner’s unearned income affects the equilibrium in general and violence incentives in particular, and will refer to this as the effect of an *introduction* of a gender-specific flat rate benefit. A crucial aspect for understanding the impact of flat rate benefits on violence incentives is whether, in equilibrium, there is a positive transfer from either partner to the spouse. If, in equilibrium, either partner is making a positive transfer, a local “income pooling property” applies:

**Lemma 3 (Income Pooling).** If, in equilibrium, partner \( i \) is making a positive transfer to the spouse, \( t^*_i > 0 \), then \( \partial t^*_i / \partial y_i - \partial t^*_i / \partial y_{-i} = 1 \).

The key implication of local income pooling is that, locally (i.e. for small variations), only total household unearned income, \( y_h + y_s \), matters for the equilibrium outcome. This in turn implies that the effect of the introduction of a flat-rate benefit is necessarily the same irrespective of which partner it is given to.

Consider then first the effect of the introduction of a flat-rate benefit on the lower bound on the set of wife’s wages at which the husband has incentives for violence, \( w_0^s(w_h) \). At the wage profile \((w_h, w_0^s(w_h))\) the husband is making a positive (benevolent) equilibrium transfer (see Proposition ??), and hence there is local income pooling. Moreover, as intuition suggests, providing the husband with a flat rate benefit will make him increase his benevolent transfer, which in turn will expand the set of wife’s wages at which she will have no incentive to work, thus raising the lower bound \( w_0^s(w_h) \) on the violence region. Due to income pooling, the effect of the introduction of a flat rate benefit provided to the wife is identical.
Consider next the impact of a small flat benefit on the upper bound on the set of wife’s wages at which the husband has incentives for violence, \( w^*_s (w_h) \). If \( w^*_s (w_h) < w^2_s (w_h) \) then the husband is making a positive (interior) transfer at the wage profile \((w_h, w^*_s (w_h))\) (see Proposition 5), and hence there is again local income pooling. If the husband is provided with small flat benefit he will increase his transfer. The indirect effect of this additional unearned income accruing to the wife is to increase her labour supply wage responsiveness, which enhances the husband’s violence incentives, thus increasing \( w^*_s (w_h) \). From local income pooling, we know that the effect of a flat rate benefit to the wife is the same. If \( w^*_s (w_h) < w^2_s (w_h) \), then the husband is not making any transfer at the wage profile \((w_h, w^*_s (w_h))\). In this case a flat benefit provided to the husband has no impact on \( w^*_s (w_h) \). In contrast, a flat benefit provided to the wife directly increases her wage responsiveness, giving the husband stronger violence incentives, thus again increasing \( w^*_s (w_h) \).

**Proposition 5** (Effect of Flat Rate Benefits).

1. The introduction of a flat rate benefit \( y_i \), provided either to the husband or to the wife, equally strictly increases the lower bound \( w^0_s (w_h) \) of the set of wife’s wages at which the husband has incentives for violence.
2. If \( w^*_s (w_h) < w^2_s (w_h) \), the introduction of a flat rate benefit \( y_i \), provided either to the husband or to the wife, equally strictly increases the upper bound \( w^*_s (w_h) \) of the set of wife’s wages at which the husband has incentives for violence.
3. If \( w^*_s (w_h) > w^2_s (w_h) \), the introduction of a flat rate benefit \( y_s \) provided to the wife strictly increases \( w^*_s (w_h) \) whereas a corresponding flat rate benefit \( y_h \) provided to the husband has no effect on \( w^*_s (w_h) \).

Our results, so far, suggest that welfare policies may shift the incidence of domestic violence without necessarily reducing it: wage subsidies to women shift the incidence of domestic violence downwards in the female wage distribution. Conversely, unconditional family cash benefits shift the incidence of domestic violence upwards in the female wage distribution. Each policy in isolation may, therefore, have undesirable effects on the risk of violence for some women. These observations permit us to deduce what type of policy mix would, according to the model, reduce the incidence of domestic violence. Such a policy mix would require: (1) cash benefits to couples in the low-wage and low-to-medium wage regime, which would allow males to prevent the inefficiency of non-cooperative equilibrium by means of a benevolent transfer instead of instrumental violence; and (2) wage subsidies to women in the medium-to-high wage regime, which would reinforce the importance of female earnings for the household, and so induce males to abstain from violent behavior.

4. A Cobb-Douglas Example

In this section we provide a simple Cobb-Douglas example which, in addition to illustrating the model, allows us to highlight a few more of its features. Hence we let

\[
    u_i(c_i, Q) = \ln c_i + 2\alpha \ln Q \quad \text{and} \quad Q(q_h, q_s) = q_h^{1/2} q_s^{1/2}.
\]

In this specification we include the parameter \( \alpha > 0 \) as a measure of the relative importance of the household produced good.
A feature of the Cobb-Douglas specification (with zero exogenous unearned incomes) is that the nature of the household equilibrium depends only on the relative wages $w_s/w_h$, not on the wage levels. In particular, maintaining the assumption that $y_h = y_s = 0$, the critical wages partitioning the transfer regimes can all be written in the form

$$w^k_{-i}(w_i) = C^k(\mu, \alpha) w_i,$$

with

$$C^0(\mu, \alpha) = \frac{\alpha \mu}{(\alpha + 1)(1 - \mu)}$$

$$C^1(\mu, \alpha) = \frac{\alpha (\alpha + \mu)}{3\alpha (1 - \mu) - \mu + \alpha^2 + 1}$$

$$C^2(\mu, \alpha) = \frac{(\alpha + \mu)}{(1 + \alpha - \mu)}.$$  

(16)

In a similar way, the upper boundary on the set of wife’s wages at which the husband has incentives for violence can be written in the form

$$w^*_{s}(w_h) = C^*(\alpha) w_h$$

with

$$C^*(\alpha) = \frac{\alpha}{\alpha + 1}.$$  

(17)

As (??) indicates, in the Cobb-Douglas example, the upper bound on the set of wife’s relative wages at which the husband has incentives for violence does not actually depend on $\mu$ but depends positively on $\alpha$. Note that $C^2(\mu, \alpha) > C^*(\alpha)$. Hence in the Cobb-Douglas case, violence always coexist with positive transfers from the husband to the wife. This result follows from the fact that, with Cobb-Douglas preferences and with $y_s = 0$, the wife’s labour supply responsiveness, $\varepsilon_s(w_s, t_h)$, drops to zero when she does not receive a transfer from the husband. Hence the husband cannot influence the wife’s labour supply by attacking her wage unless he also make a positive transfer.

Figure ??(a) illustrates how the space of wage-profiles is partitioned into regions with different

Figure 2: Illustration of the equilibrium.
and the wife just chooses not to work. At nature/direction of transfers and labour supply status for the case of $\alpha = 1$. At $w_s < w_h^2(w_h)$ the husband makes the “benevolent” transfer $t_h^1(w_h, w_s)$ and the wife strictly prefers not to work. At $w_s \in (w_h^1(w_h), w_h^1(w_h))$ the husband makes the “crowding out” transfer $t_h^1(w_h, w_s)$ and the wife just chooses not to work. At $w_s \in (w_h^1(w_h), w_h^2(w_h))$ the husband makes an “interior” transfer $t_h^2(w_h, w_s)$ and the wife works positive hours. At higher wife’s wages, the husband does make any transfer to the wife. As, in this example, preferences are symmetric across the two genders, corresponding regions apply when the wife’s wage exceeds the husbands.

Figure 3(b) illustrates in the same example the set of wage profiles at which the husband has violence incentives. The figure illustrates how incentives for violence obtain for a set of wage profiles around those where the wife enters the labour market.

The Cobb-Douglas example can also be used to illustrate the impact of caring and of the relative importance of household produced goods. Figure 3(a) illustrates the three critical relative wages $C^k(\mu, \alpha), k = 0, 1, 2$ as well as $C^*(\alpha)$ as a function of the caring parameter $\mu$ when $\alpha = 1$. The example illustrates how an increase in $\mu$ decreases the set of wage profiles $(w_h, w_s)$ at which the husband has equilibrium violence incentives. The example also illustrates how the incentives for violence disappear when $\mu$ approaches 1/2. In the limit where $\mu = 1/2$ the couple has a common objective and the household operates on the Pareto frontier. An act of violence which reduces $w_s$ would then simply reduce the utility possibility set and would hence never increase the husband’s equilibrium utility.

Figure 3(b) illustrates the same critical relative wage functions, now as a function of $\alpha$ given $\mu = 1/4$. This figure highlights how an increase in the relative importance of household production, as parameterized by $\alpha$, tends to increase the incentives for violence. This reflects how, in the current model, violence obtains as a result of disagreements between the two partners regarding the allocation of time in the presence of household production: if the importance of household produced goods diminish, so do the incentives for violence.

Above we outlined some general results regarding the effect of wage subsidies and flat-rate benefits on the incidence of instrumental violence. In line with the result that only relative wages matter in the Cobb-Douglas, it is only the relative subsidy rate that matters for violence incentives.\footnote{In particular in the presence of wage subsidies $\sigma_h$ and $\sigma_s$, each of the critical relative wages in (15) and (16) are...}

Figure 3: Comparative statics.
Figure 4: Comparative statics.

Figure 4(a) illustrates the effect of a 50 percent relative subsidy to women \( (\sigma_s = 1/2, \sigma_h = 0) \). A corresponding relative wage subsidy to men would shift the violence region in the opposite direction.

Figure 4(b) illustrates the effect of a small flat rate benefit. Noting that \( w_s^*(w_h) < w_h^*(w_h) \) holds in the Cobb-Douglas case, it follows that the effect of the introduction of a flat rate benefit is the same irrespective of to which partner it is provided.\(^{12}\) In line with Proposition 4 the figure illustrates how a small benefit \( y_i > 0 \) (provided to either partner) shifts the violence region upwards.

5. Discussion and Conclusion

We have proposed a theoretical framework which revolves around the notion of domestic violence being instrumental and directly related towards women’s economic activity. At the heart of the proposed model is an intra-household disagreement about the allocation of time between market work and home production. We have shown that this mechanism generates a non-monotonic relationship between the gender wage gap and the incidence of domestic violence. With a low relative wage, it is individually rational for a woman—being supported financially by her partner—to fully specialize at home. The woman’s chosen allocation of time coincides with her partner’s preferred choice, and the relationship stays violence free. With an intermediate relative wage, it is privately rational for a woman to look for work and enter employment (at the level of financial

rescaled by a factor \( (1 + \sigma_h) / (1 + \sigma_s) \).

\(^{12}\)It should be noted that the analysis above was for the case case of \( y_i = 0 \) in order to ensure that someone in the household always chooses to work in equilibrium and to ensure that a partner’s benevolent transfer to the spouse will be positive. It was for the same reason that the claims in Proposition 4 focused on the introduction of a “small” flat benefit starting from zero unearned income, that is, the derivatives of the equilibrium with respect to \( y_i \) taken at the point where \( y_i = 0 \). Figure 4 is drawn from a small strictly positive amount of unearned income, but ignores the possibility of both partners not working and of a zero benevolent transfers. Hence the figure should be considered only as illustrative and it should be recognized that it is not entirely complete for couples with wage profiles where one or both partners have very low wages.
support that her husband would voluntarily provide her with), but the husband’s preferred choice is for her to stay specialized in household production. While he will increase his financial support in order to induce her not to enter the labour force, he also has an incentive to directly target her labour market opportunities and hence resorts to violence. Finally, with a high relative wage, a woman’s earned income is too important for a male partner to interfere with her earnings capacity. Overall, therefore, our model predicts that domestic violence is associated with a woman’s labor supply being contentious and economic roles in the family hanging in the balance. This result has an important implication for the impact of employment on domestic violence as well: women who are not working at all do not necessarily face a higher risk domestic violence than those engaged in paid employment. Quite to the contrary, female participation in the labor force may be a trigger for instrumental partner violence.

In addition to providing a novel explanation for domestic violence, our analysis also serves to highlight some policy dilemmas. Specifically, our results suggest that policies aimed at improving the economic situation of families may shift the incidence of domestic violence without necessarily reducing it. For example, unconditional family cash benefits shift the incidence of violence upwards in the female wage distribution, while wages subsidies to women may have the opposite effect. In order to reduce domestic violence, therefore, different types of households have to be targeted with different types of policy measures. This underlines the complexities policy makers have to negotiate when facing the problem of domestic violence.

It is important to acknowledge that this paper is a purely theoretical exercise. In general, we know precious little about the empirical importance of the channels identified by our theory. Recent empirical research has examined the basic prediction of economic exchange theories of domestic violence, namely that the risk of domestic violence against women decreases with women’s relative wages. Exploiting exogenous changes in the demand for labor in female-dominated industries, Aizer (2010) provides support for a negative causal relationship between the female-male wage gap and domestic violence. Her findings are entirely consistent with the current theoretical model. However, Aizer’s analysis focuses on local average effects and so may be missing potentially non-monotonic effects of the gender wage gap on domestic violence. Our framework puts forward the empirical hypothesis that the risk of domestic violence is inversely U-shaped in the female-male wage gap at the level of the household. Turning to the association between employment and domestic violence, the empirical evidence is mixed (Tauchen and Witte, 1995; Macmillan and Gartner, 1999; Rodriguez et al., 2001; Gibson et al., 2005). However, it is interesting to note that, in developing countries, women in paid employment are frequently more likely to be subjected to domestic violence than those who are not in the labor force. For example, 27 percent of working women in India report ever having experienced domestic violence, compared with less than 15 percent of those who do not work (Kishor and Johnson, 2004, p. 28, Table 3.1.1). Likewise, in Peru, roughly 46 percent of women engaged in paid employment report ever having experienced domestic violence, while 36 percent of nonworking women report same. Similar observations can also be made in several other Latin American countries. While these empirical patterns cannot be easily explained with a model of household bargaining that incorporates preference-based violence, they are consistent with the notion of domestic violence being instrumental and directly related to women’s labor market activity.

There are several directions in which our analysis could be expanded. From a theoretical perspective, it would be interesting to embed the present framework in a model where women have the option of quitting abusive relationships and use the unifying model to examine how the internal
organization of the family and outside options interact in shaping the incidence of domestic violence. From an empirical perspective, it would be interesting to use survey data on domestic violence to deduce some of the key parameters of our model and to generate quantitative predictions from the estimated parameter values. This would allow us to get a feel for the empirical relevance of the non-monotonic relationship between the gender wage gap and the incidence of abuse as predicted by the theory. These and other issues are important and challenging topics for future research. In the meantime, our analysis demonstrates the value of going beyond preference-based bargaining models of domestic violence and to incorporate instrumental incentives into economic explanations of spousal violence against women.

References
Appendix

Proof of Proposition ??: The proposition is proven through a series of lemmas. First, define the “benevolent” transfer, $t^0_i(w_i, w_{-i})$, from partner $i$ to $-i$ implicitly as the solution to the following equation:

$$(1 - \mu) v'_i \left( m_i \left( w_i, y_i - t^0_i(w_i, w_{-i}) \right) + y_i - t^0_i(w_i, w_{-i}) \right) = \mu v'_{-i} \left( y_{-i} + t^0_i(w_i, w_{-i}) \right).$$

(A1)

Lemma A.1. The benevolent transfer $t^0_i(w_i, w_{-i})$ is (i) a uniquely identified continuous function with $t^0_i(w_i, w_{-i}) \in (0, w_i)$, (ii) strictly increasing in $w_i$ and independent of $w_{-i}$.

Proof. Both sides of (??) are continuous functions of $t_i$. Using (??) it follows that the left hand side is strictly increasing in $t_i$. The right hand side is strictly decreasing in $t_i$. Moreover, as $t_i$ approaches either 0 the right hand side approaches infinity whereas if $t_i$ approaches $w_i$, the left hand side approaches infinity. Hence (??) must have a unique solution in the interval $(0, w_i)$. The effect of an increase in $w_i$ follows from the fact that $\partial m_i/\partial w_i > 0$. Independence of $w_{-i}$ follows from the fact that it does not feature directly in (??).

Next, define the “crowding out” transfer, $t^1_i(w_i, w_{-i})$, from partner $i$ to $-i$ implicitly as the transfer at which $-i$ just chooses zero labour supply:

$v'_{-i} \left( y_{-i} + t^1_i(w_i, w_{-i}) \right) = \frac{1}{w_{-i}(1 - \mu)}.$

(A2)

Lemma A.2. The crowding out transfer $t^1_i(w_i, w_{-i})$ is (i) a uniquely identified continuous function, (ii) strictly increasing in $w_{-i}$, independent of $w_i$, (iii) approaches 0 as $w_{-i}$ approaches 0.

Proof. Uniqueness and continuity follow immediately from differentiability and strict concavity of $v_{-i}(\cdot)$. Monotonicity in $w_{-i}$ follows from strict concavity of $v_{-i}(\cdot)$. Independence of $w_i$ follows from the fact that $w_i$ does not feature directly in (??). Finally, note that as $w_{-i} \to 0^+$, any positive transfer will strictly induce $-i$ not to work. Hence $t^1_i(w_i, w_{-i})$ (which is the smallest transfer at which $-i$ chooses not to work) must approach zero.

For large enough $w_{-i}$, the crowding out transfer will not be “affordable” to $i$. Hence we implicitly define $w^\text{max}_{-i}(w_i)$ as the $w_{-i}$ at which the crowding out transfer corresponds to $i$’s maximum earnings.

$t^1_i(w_i, w^\text{max}_{-i}(w_i)) = w_i.$

(A3)

From the properties of $t^1_i(w_i, w_{-i})$ in Lemma ?? it follows that $w^\text{max}_{-i}(w_i)$ is uniquely identified and strictly increasing in $w_i$.

We next define $w^0_i(w_i)$ as $w_{-i}$ which equates the benevolent and the crowding out transfer,

$t^0_i(w_i, w^0_i(w_i)) = t^1_i(w_i, w^0_i(w_i)).$

(A4)

Lemma A.3. The critical wage $w^0_i(w_i)$ is a uniquely identified continuous and strictly increasing function of $w_i$, and $w_{-i} < (>) w^0_i(w_i)$ implies $t^0_i(w_i, w_{-i}) > (<) t^1_i(w_i, w_{-i})$.

Proof. Immediate from the properties of $t^0_i(w_i, w_{-i})$ and $t^1_i(w_i, w_{-i})$ given in Lemma ?? and ??.

In order to analyze the transfer decision it is useful to write $-i$’s utility as a (continuous) function of $t_i$ in the following way:

$$U_i(t_i; w_i, w_{-i}) \equiv (1 - \mu) v_i \left( m_i \left( w_i, y_i - t_i \right) + y_i - t_i \right) + \mu v_{-i} \left( m_{-i} \left( w_{-i}, y_{-i} + t_i \right) + y_{-i} + t_i \right) + z \left( \frac{1 - m_i \left( w_i, y_i - t_i \right)}{w_i} - \frac{1 - m_{-i} \left( w_{-i}, y_{-i} + t_i \right)}{w_{-i}} \right).$$

(A5)
rememering that for any $t_i \geq t^*_i (w_i, w_{-i})$ the recipient, $-i$, chooses zero labor supply. We can now establish strict concavity of $-i$’s utility in $t_i$.

**Lemma A.4.** $U_i (t_i; w_i, w_{-i})$ is strictly a strictly concave function of $t_i$ with a “downward kink” at $t_i = t^*_i (w_i, w_{-i})$.

**Proof.** Differentiating (22) with respect to $t_i$ and evaluating at some $t_i < t^*_i (w_i, w_{-i})$ yields, after substituting using (23),

$$\frac{\partial U_i (t_i; w_i, w_{-i})}{\partial t_i} = -(1 - \mu) v'_i (c_i) + v''_i (c_{-i}) \left[ \mu - (1 - 2\mu) \frac{\partial m_{-i}}{\partial Y_{-i}} \right].$$

(A6)

When evaluating at some $t_i > t^*_i (w_i, w_{-i})$, the same expression holds but with $\partial m_{-i}/\partial Y_{-i} = 0$ as $-i$ strictly prefers not to work at such a transfer. At $t_i = t^*_i (w_i, w_{-i})$, the derivative (22) does not exist. However, the left and the right derivatives both exist and are given by (22), with and without the $\partial m_{-i}/\partial Y_{-i}$ term included respectively. Since $\partial m_{-i}/\partial Y_{-i} < 0$ (see eq. (22)), the left derivative exceeds the right derivative.

Differentiating a second time and evaluating at some $t_i < t^*_i (w_i, w_{-i})$ yields

$$\frac{\partial^2 U_i (t_i; w_i, w_{-i})}{\partial t_i^2} = (1 - \mu) v''_i (c_i) \left( 1 + \frac{\partial m_i}{\partial Y_i} \right) + \frac{\partial m_{-i}}{\partial Y_{-i}} \left( 1 + \frac{\partial m_{-i}}{\partial Y_{-i}} \right) \left[ \mu - (1 - 2\mu) \frac{\partial m_{-i}}{\partial Y_{-i}} \right] - v''_i (c_{-i}) (1 - 2\mu) \frac{\partial^2 m_{-i}}{\partial Y_{-i}^2}.$$  

(A7)

When evaluating at some $t_i > t^*_i (w_i, w_{-i})$, the same expression holds but with $\partial m_{-i}/\partial Y_{-i} = 0$ etc. It follows from (22) and Assumption 22, that (22) is strictly negative at any $t_i \neq t^*_i (w_i, w_{-i})$.

From Lemma 22 it follows that $i$’s optimal transfer (if positive) is either characterized by $t_i \neq t^*_i (w_i, w_{-i})$ and (22) being equal to zero, or by $t_i = t^*_i (w_i, w_{-i})$ and (22) being strictly positive at all $t_i < t^*_i (w_i, w_{-i})$ and strictly negative at all $t_i > t^*_i (w_i, w_{-i})$. For future reference it is also useful to note that, when evaluated at some $t_i < t^*_i (w_i, w_{-i})$,

$$\frac{\partial^2 U_i (t_i; w_i, w_{-i})}{\partial t_i \partial w_i} = v''_i (c_{-i}) \frac{\partial m_{-i}}{\partial Y_{-i}} \left[ \mu - (1 - 2\mu) \frac{\partial m_{-i}}{\partial Y_{-i}} \right] - v''_i (c_{-i}) (1 - 2\mu) \frac{\partial^2 m_{-i}}{\partial Y_{-i} \partial w_i} < 0,$$

(A8)

where the inequality follows from Assumption 22. When evaluated at some $t_i > t^*_i (w_i, w_{-i})$ the expression in (22) is identically equal to zero as $-i$ then strictly prefer to not work. Note also that

$$\frac{\partial^2 U_i (t_i; w_i, w_{-i})}{\partial t_i \partial w_i} = -(1 - \mu) v''_i (c_i) \frac{\partial m_i}{\partial w_i} > 0.$$  

(A9)

**Lemma A.5.** At $w_{-i} \leq w^0_{-i} (w_i)$, $i$’s optimal transfer is the benevolent transfer $t^0_i (w_i, w_{-i})$.

**Proof.** Suppose that $w_{-i} \leq w^0_{-i} (w_i)$ and $i$ chooses $t_i = t^0_i (w_i, w_{-i})$. Since $t^0_i (w_i, w_{-i}) \geq t^*_i (w_i, w_{-i})$ (see Lemma 22), it follows from the definition of the benevolent transfer in (22) that $\partial U_i/\partial t_i$ given by (22) equals zero at this transfer, thus confirming that it is an optimal choice for $i$.

Next we define implicitly a second critical recipient’s wage. To this end, consider the left derivative of (22) with respect to $t_i$, evaluated at the limit point $t_i = t^*_i (w_i, w_{-i})$,

$$\frac{\partial U_i (t^*_i (w_i, w_{-i}); w_i, w_{-i})}{\partial t_i} = 0,$$

(A10)

where we note that the left derivative is given by (22) with the inclusion of the $\partial m_{-i}/\partial Y_{-i}$ term. The second critical wage, denoted $w^*_{-i} (w_i)$, is implicitly defined as the $w_{-i}$ at which (22) equals zero.
Lemma A.6. The critical wage \( w_{-i}^1(w_i) \) (i) is a uniquely identified continuous and strictly increasing function of \( w_i \), and (ii) is strictly larger than \( w_{-i}^0(w_i) \).

Proof. Evaluating at \( w_{-i} \leq w_{-i}^0(w_i) \), and using that, at any such recipient’s wage, \( t_i^1(w_i, w_{-i}) \leq t_i^0(w_i, w_{-i}) \), one can easily verify that the left hand side of (??) is positive. For large enough \( w_{-i} \), (??) will, on the other hand, be negative. To see this, note that when \( w_{-i} \) approaches \( w_{-i}^\max(w_i) \), \( t_i^1(w_i, w_{-i}) \) approaches \( w_i \), implying that \( i \)'s consumption approaches zero. Given continuity of the left hand side of (??) in \( w_{-i} \), at least one solution to the equation in (??) exists, and any solution strictly exceeds \( w_{-i}^0(w_i) \).

To demonstrate uniqueness of \( w_{-i}^1(w_i) \), note that (??) is strictly decreasing in \( w_{-i} \), both directly (see eq. ??), and indirectly via \( t_i^1(w_i, w_{-i}) \) being increasing in \( w_{-i} \) (Lemma ??) and concavity of \( U_i \) in \( t_i \) (Lemma ??). Finally, that \( w_{-i}^1(w_i) \) is increasing in \( w_i \) follows from simple comparative statics using (??) and recalling that \( t_i^1(w_i, w_{-i}) \) is independent of \( w_i \) (Lemma ??). \[ \Box \]

Lemma A.7. At \( w_{-i} \in (w_{-i}^0(w_i), w_{-i}^1(w_i)) \), \( i \)'s optimal transfer is the crowding-out transfer \( t_i^1(w_i, w_{-i}) \).

Proof. For \( w_{-i} \in (w_{-i}^0(w_i), w_{-i}^1(w_i)) \) we now have that the right derivative of \( U_i(t_i; w_i, w_{-i}) \) with respect to \( t_i \) evaluated at \( t_i = t_i^1(w_i, w_{-i}) \) is strictly negative (since, for such \( w_{-i} \), \( t_i^1(w_i, w_{-i}) > t_i^0(w_i, w_{-i}) \)) while the left derivative is strictly positive. Hence, by global concavity of \( U_i(t_i; w_i, w_{-i}) \) in \( t_i \) (Lemma ??) it follows that \( t_i^1(w_i, w_{-i}) \) is \( i \)'s optimal transfer. \[ \Box \]

We define a third critical recipient’s wage, denoted \( w_{-i}^2(w_i) \), as the smallest \( w_{-i} \) at which \( i \) chooses to make a zero transfer. Specifically, we define \( w_{-i}^2(w_i) \) implicitly through the following equation,

\[
\frac{\partial U_i(0; w_i, w_{-i}^2(w_i))}{\partial t_i} = 0. \tag{A11}
\]

Lemma A.8. The critical wage \( w_{-i}^2(w_i) \) (i) is a uniquely identified continuous and strictly increasing function of \( w_i \), and (ii) is strictly larger than \( w_{-i}^1(w_i) \).

Proof. From Lemmas ?? and ??, we know that for any \( w_{-i} \leq w_{-i}^1(w_i) \), \( i \)'s optimal transfer is either \( t_i^1(w_i, w_{-i}) \) or, if \( w_{-i} \leq w_{-i}^0(w_i) \), \( t_i^0(w_i, w_{-i}) \) which then exceeds \( t_i^1(w_i, w_{-i}) \). Hence for any \( w_{-i} \leq w_{-i}^1(w_i) \), \( i \)'s optimal transfer is strictly positive, implying that \( \partial U_i(0; w_i, w_{-i})/\partial t_i \) is strictly positive at any such \( w_{-i} \). Next we note that \( \partial U_i(0; w_i, w_{-i})/\partial t_i \) is strictly decreasing in \( w_{-i} \) as \( -i \)'s earnings are increasing in \( w_{-i} \) and due to Assumption ?? (and, under natural conditions, for large enough \( w_{-i} \) the derivative becomes negative). This demonstrates that \( w_{-i}^2(w_i) \) is unique and exceeds \( w_{-i}^1(w_i) \). That \( w_{-i}^2(w_i) \) increases in \( w_i \) follows from (??). \[ \Box \]

Lemma A.9. At \( w_{-i} \in (w_{-i}^1(w_i), w_{-i}^2(w_i)) \), \( i \)'s optimal transfer is an “interior” transfer \( t_i^2(w_i, w_{-i}) \), where \( t_i^2(w_i, w_{-i}) \in (0, t_i^1(w_i, w_{-i})) \) and is strictly increasing in \( w_i \) and strictly decreasing in \( w_{-i} \). At \( w_{-i} \geq w_{-i}^2(w_i) \), \( i \)'s optimal transfer is the zero transfer.

Proof. For \( w_{-i} > w_{-i}^1(w_i) \), the left derivative of \( U_i(t_i; w_i, w_{-i}) \) with respect to \( t_i \) evaluated at \( t_i = t_i^1(w_i, w_{-i}) \) is strictly negative, implying that \( i \)'s optimal transfer is given by some \( t_i < t_i^1(w_i, w_{-i}) \) (at which the recipient works some positive amount of time). The optimal transfer in this case is either strictly positive and characterized by the derivative in (??) being equal to zero, or equal to zero with the derivative in (??) being non-positive at \( t_i = 0 \). Per definition, \( w_{-i}^2(w_i) \) is the unique recipient’s wage above which the derivative of \( U_i(t_i; w_i, w_{-i}) \) with respect to \( t_i \) evaluated at \( t_i = 0 \) is negative. We denote the strictly positive transfer made at \( w_{-i} \in (w_{-i}^1(w_i), w_{-i}^2(w_i)) \) by \( t_i^2(w_i, w_{-i}) \). Monotonicity of \( t_i^2(w_i, w_{-i}) \) in \( w_i \) and \( w_{-i} \) then follows immediately from (??) and (??). \[ \Box \]

Proposition ?? follows in a straightforward manner, using Lemmas ?? to ??.
Proof of Lemma ???. Suppose that, in equilibrium, partner $i$ is making a positive transfer to partner $-i$. It then follows that
\[(1 - \mu) v'_i(c_i) \leq v'_i(c_{-i}) \left[ \mu - (1 - 2\mu) \frac{\partial m}{\partial Y_{-i}} \right]. \tag{A12}\]
To see this, note that if $i$ makes an interior transfer $t_i^0(w_i, w_{-i})$ to $-i$, then (??) holds with equality as it is the first order condition characterizing the interior transfer. If $i$ makes a benevolent transfer $t_i^1(w_i, w_{-i})$ to $-i$, then (??) holds with equality, but with $\partial m_{-i}/\partial Y_{-i} = 0$ as $-i$ strictly prefers not to work. Finally, if $i$ is making the crowding out transfer $t_i^1(w_i, w_{-i})$ to $-i$, then (??) holds with inequality as the left derivative of $U_i$ with respect $t_i$ at $t_i^1(w_i, w_{-i})$ is positive (see proof of Proposition ??). If partner $-i$ is then also making a positive transfer to $i$, the same inequality with $i$ and $-i$ interchanged also holds. Combining the two inequalities to eliminate the marginal utilities yields that
\[(1 - \mu)^2 \leq \left[ \mu - (1 - 2\mu) \frac{\partial m}{\partial Y_{-i}} \right] \times \left[ \mu - (1 - 2\mu) \frac{\partial m_{-i}}{\partial Y_{-i}} \right]. \tag{A13}\]
However, this is a contradiction since the right hand side is, for any $\mu \in [0,1/2)$ and $\left( \frac{\partial m}{\partial Y_{-i}}, \frac{\partial m_{-i}}{\partial Y_{-i}} \right) \in (-1,0) \times (-1,0)$, in the open interval $\left( \mu^2, (1-\mu)^2 \right)$.

Proof of Proposition ???. Let $U_i^*(w_i, w_{-i})$ denote the equilibrium utility of partner $i$. At any $w_s \leq w_i^0(w_h)$, the husband makes the “benevolent transfer” $t_i^0(w_h, w_s)$ which is independent of $w_s$, and the wife strictly prefers not to work in equilibrium. A marginal reduction $w_s$ does not affect any aspect of the equilibrium, and hence, in particular, does not increase the husband’s equilibrium utility $U_h^*(w_h, w_s)$.

At any $w_s \in (w_i^0(w_h), w_i^1(w_h))$ the husband chooses the “crowding-out transfer” $t_i^1(w_h, w_s)$ and, in equilibrium, the wife just chooses not to work. At such a $w_s$ it follows that
\[\frac{\partial U_i^*(w_i, w_s)}{\partial w_s} = - [(1 - \mu) v'_h(c'_h) - \mu v'_s(c'_s)] \frac{\partial t_i^1(w_h, w_s)}{\partial w_s} < 0, \tag{A14}\]
where the sign follows from the fact that, as $w_s > w_i^0(w_h)$, $t_i^1(w_h, w_s) > t_i^0(w_h, w_s)$ (see Lemma ??), implying that the term in brackets is positive.

At any wage $w_s \in (w_i^1(w_h), w_i^2(w_h))$ the husband chooses the “interior transfer” $t_i^2(w_h, w_s)$ and, in equilibrium, the wife is working some positive amount of time in the labour market. At such a $w_s$ it can be shown, after simplifying using the first order conditions for labour supplies and the characterization of the “interior” transfer, that
\[\frac{\partial U_h^*(w_h, w_s)}{\partial w_s} = \mu v'_s(c'_s) \left[ w_s \frac{\partial \ell^*_s}{\partial w_s} + \ell^*_s \right] - v'_s(q'_s) \frac{\partial \ell^*_s}{\partial w_s}. \tag{A15}\]
Substituting for $\ell^*_s(q'_s)$ in the final term using the first order condition for the wife’s labour supply and collecting terms, this can be rewritten as
\[\frac{\partial U_h^*(w_h, w_s)}{\partial w_s} = \mu v'_s(c'_s) \ell^*_s \left\{ \mu - (1 - 2\mu) \varepsilon_s(w_s, y_s + t^*_h) \right\}. \tag{A16}\]

It should be noted that the husband’s incentives for violence are “smooth” at $w_s = w_i^1(w_h)$: taking the limit of (??) as $w_s \rightarrow w_i^1(w_h)^-$ and the limit of (??) as $w_s \rightarrow w_i^1(w_h)^+$, and using simple comparative statics, yields that
\[\lim_{w_s \searrow w_i^1(w_h)} \frac{\partial U_h^*(w_h, w_s)}{\partial w_s} = \lim_{w_s \nearrow w_i^1(w_h)} \frac{\partial U_h^*(w_h, w_s)}{\partial w_s} = -v'_s(c'_s) (1 - 2\mu) w_h \frac{\partial \ell^*_s}{\partial w_s} < 0. \tag{A17}\]
Thus the husband’s incentives for violence extend to some $w_s$ at which the wife works in equilibrium.
Moreover, equation (??) holds also at any \( w_* \geq w_*^2 (w_h) \), at which the husband makes no transfer to the wife and the wife works in equilibrium.

From (??) it follows that, for \( w_* > w_*^1 (w_h) \), the husband has equilibrium incentives for violence until

\[
\frac{\mu}{1 - 2\mu} = \varepsilon_s \left( w_*^s (w_h), y_s + t_h^* (w_h, w_*^w (w_h)) \right). \tag{A18}
\]

Note, by Assumption ??, \( \varepsilon_s \) directly decreases in \( w_* \) and increases in \( Y_s \). Moreover, as \( w_* > w_*^1 (w_h) \), the husband is either making the interior transfer \( t_h^* (w_h, w_*^w) \), which by Lemma ?? is decreasing in \( w_* \), or a zero transfer. If the husband is making the interior transfer, then an increase in \( w_* \) further leads to a reduction in \( \varepsilon_s \) through the reduction in the wife’s total unearned income. Hence the right-hand side of (??) is a decreasing function of \( w_* \), implying that if, given \( w_h \), it first fails at some \( w_*^s (w_h) > w_*^i (w_h) \), it also fails at any \( w_*^i > w_*^s (w_h) \).

Finally, to see that \( w_*^s (w_h) \) weakly increases in \( w_h \), note that \( w_h \) only enters (??) via the husband’s interior transfer. If \( w_*^s (w_h) > w_*^2 (w_h) \), then the husband is making zero transfer at \( w_* = w_*^s (w_h) \) in which case marginal variations in \( w_* \) has no impact on the critical wage \( w_*^1 (w_h) \). If \( w_* \in (w_*^1 (w_h), w_*^2 (w_h)) \), then the husband is making a positive transfer at \( w_* = w_*^s (w_h) \) which, by Lemma ??, is increasing in \( w_h \). A marginal increase in \( w_* \) then increases \( Y_s \) via the husband’s equilibrium transfer, and thus increases \( \varepsilon_s \) and \( w_*^s (w_h) \).

Proof of Proposition ??: Consider the allocation problem

\[
\max_{(c, c_i)} \left\{ \frac{1}{2} [v_h (c_h) + v_s (c_s)] + z (1 - \ell_h, 1 - \ell_s) |c_h + c_s = w_h \ell_h + w_s \ell_s + y_h + y_s \right\} \tag{A19}
\]

the first order conditions for which imply

\[
v_h' (c_h) = v_s' (c_s) \tag{A20}
\]

where the latter condition holds with equality when \( \ell_i > 0 \). The solution to the above problem has a “reservation wage property”. In particular, the wife does not work at the solution to (??) if and only if \( w_* \leq w_*^s (w_h) \) for some \( w_*^s (w_h) \).

The non-cooperative equilibrium in the limiting case where \( \mu \to 1/2 \) corresponds to the solution to (??). Note in particular, that the characterization of both the “benevolent” transfer and the “interior” transfer reduce to the “equalized marginal utility” condition (??), and the labour supply condition (??) corresponds to the limit of (??). (As will be clear shortly, in the limit, “crowding out” transfers will not occur). Recalling that \( w_*^0 (w_h) \) is defined as the highest \( w_* \) wage at which the wife chooses not to work at the husband’s benevolent transfer, it follows that \( w_*^0 (w_h) \to w_*^s (w_h) \) as \( \mu \to 1/2 \). Consider then \( w_*^s (w_h) \) defined as the highest wife’s wage at which \( \partial U_h^*/\partial w_* \) given in (??) is strictly negative. Note however that, in the limit as \( \mu \to 1/2, \partial U_h^*/\partial w_* \to v_*' (c_*') \ell_*^2 /2 \) which is thus positive for any \( w_* \) at which the wife is working. Hence it follows that \( w_*^0 (w_h) \to w_*^s (w_h) \) as \( \mu \to 1/2 \). The value region \( (w_*^0 (w_h), w_*^s (w_h)) \) thus reduces to the empty set as \( \mu \to 1/2 \).

Proof of Proposition ??: Extend the notation to include the subsidy rates \( \sigma_i, i = h, s \) in the notation for the lower and upper bound on the interval of wife’s wages at which the husband has equilibrium incentives for violence, \( w_*^0 (w_1; \sigma_h, \sigma_s) \) and \( w_*^s (w_h; \sigma_h, \sigma_s) \). What matters for the equilibrium outcome are the partners’ “effective wages” \( \tilde{w}_i \equiv (1 + \sigma_i) w_i, i = h, s \). This implies that the following two equations hold as identities

\[
w_*^0 (w_h, \sigma_h, \sigma_s) = \frac{w_*^0 (w_h (1 + \sigma_h), 0, 0)}{(1 + \sigma_s)} \quad \text{and} \quad w_*^s (w_h, \sigma_h, \sigma_s) = \frac{w_*^s (w_h (1 + \sigma_h), 0, 0)}{(1 + \sigma_s)}. \tag{A22}
\]
It immediately follows that both critical wages are decreasing in $\sigma_s$. To see that both critical wages are also increasing in $\sigma_h$ recall that both $w^0_s(w_h)$ and $w^*_s(w_h)$ are increasing in $w_h$ (see Lemma ?? and Proposition ??).

Proof of Lemma ?? Recall that $Y_i \equiv y_i - t_i + t_{-i}$ denotes $i$'s total unearned income (inclusive of transfers). If partner $i$ is making an equilibrium transfer $t_i^* > 0$ to $-i$ (implying that $t_{-i}^* = 0$, see Lemma ??), then individual $i$ is effectively choosing, on the margin, the intra-household allocation of total unearned income subject to $Y_i + Y_{-i} = y_i + y_{-i}$. As $y_i$ and $y_{-i}$ only feature in the form of their sum it follows that $\partial Y_i^*/\partial y_i = \partial Y_{-i}^*/\partial y_{-i}$, or equivalently, $1 - \partial t_i^*/\partial y_i = -\partial t_i^*/\partial y_{-i}$. Note that this holds irrespective of whether $i$'s transfer is “benevolent”, “crowding-out”, or “interior”.

Proof of Proposition ?? Using the definition of $w^0_s(w_h)$ in (?) and differentiating with respect to unearned income for partner $k$ yields
\[
\frac{\partial t^0_h}{\partial w_s} \frac{\partial w^0_s(w_h)}{\partial y_k} + \frac{\partial t^0_h}{\partial y_k} = \frac{\partial t^1_h}{\partial w_s} \frac{\partial w^0_s(w_h)}{\partial y_k} + \frac{\partial t^1_h}{\partial y_k}.
\] (A23)

When $k = h$, the first term on the left hand side and the second term on the right hand side are, by Proposition ??, both equal to zero. Hence
\[
\frac{\partial w^0_s(w_h)}{\partial y_h} = \frac{\partial t^0_h}{\partial y_h} > 0,
\] (A24)

where the sign follows from simple comparative statics which shows that $t^0_h(w_h, w_s)$ strictly increases in $y_h$ and, from Proposition ??, by local income pooling, $\partial w^0_s(w_h)/\partial y_s = \partial w^0_s(w_h)/\partial y_h$.

When $w^*_s(w_h) < w^2_s(w_h)$ the upper bound $w^*_s(w_h)$ is implicitly defined by (??). Differentiating with respect to the wife's unearned income yields, after using Lemma ??,
\[
\frac{\partial w^*_s(w_h)}{\partial y_s} = -\frac{\partial^2}{\partial y_s^2} \frac{\partial^2}{\partial w_s^2} + \frac{\partial^2}{\partial y_s^2} > 0,
\] (A25)

where we note that the denominator is strictly negative by Assumption ?? and Proposition ??, and simple comparative statics show that the numerator is strictly positive. By local income pooling the introduction of $y_h$ must have the same positive effect on $w^*_s$ as $y_s$.

When $w^*_s(w_h) > w^2_s(w_h)$ equation (??) reduces to $\mu/(1 - \mu) = \varepsilon_s(w^*_s(w_s), y_s)$. As $y_h$ does not feature in this equation it immediately follows that a (small) $y_h$ has no impact on $w^*_s(w_h)$. Moreover, by Assumption ?? the labour supply elasticity is decreasing in the own wage and increasing in unearned income, which together imply that $w^*_s(w_h)$ increases in $y_s$. 

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