Taxation and Corporate Choices in Credit Markets with Asymmetric Information

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Abstract

In this theoretical paper we extend the literature on tax effects on corporations’ choices in two directions. First we analyze corporate and dividend tax effects when not only equity issues and retained profits but also debt finance is allowed. Second, we introduce asymmetric information between lenders and firms: this gives rise to credit rationing. Finally, we study how the rationing problem is affected by taxation.

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1 Introduction

The dispute between Old and New View has lasted for almost thirty years. After so much time, a winner has not yet emerged: the empirical evidence shows that neither the former nor the latter can explain the effects of dividend taxation on companies’ choices.\(^1\) Curiously, the analysis of dividend taxation on real decisions is based on the comparison between two possible sources of finance: equity issues and retained profits. Debt is however almost always disregarded.\(^2\) Debt finance under asymmetric information is another important

\(^1\)Well-known references for the Old view are: Harberger (1962), Feldstein (1970) and Poterba and Summers (1985). The New view is dealt with by King (1974), Auerbach (1979) and Bradford (1981). To our knowledge, there is only one exception that deal with debt explicitly: the empirical paper by Auerbach and Hassett (2003). In their article they estimate the determinants of the dividend/assets ratio and find a negative coefficient on initial debt. They argue that this is consistent with their expectation that a higher debt level discourages firms from additional borrowing and forces them to find alternative sources of funds. Moreover, they find that: i) firms with weak capital-market access — with no bond rating or analysts on record — are more sensitive in their dividend decisions than are other firms; ii) large firms with high bond ratings and several analysts following them, have dividends which are more responsive to cash needs. However, with greater access to borrowing than the typical firm likely to issue shares, these large firms may be relying on debt, rather than external equity, as a source of funds; iii) firms with a high probability of new issues tend to be those in between the two extremes, and also to exhibit dividend behavior more consistent with the traditional view of dividend taxation, with dividends substantially more responsive to cash flow than to investment.

\(^2\)In their survey, Hanlon and Heitzman (2010) deal with the dispute between different views of dividend taxation. In a footnote they state that: "this discussion does not touch on the more complicated issues. Extant Theory typically ignores what happens when firms can finance with debt. Because projects are evaluated based on their net present value using a discount rate that factors in capital structure, a project financed with new debt is implicitly financed with a combination of debt and internal equity. A firm with existing debt that raises new equity may be closer to the traditional view. But because dividend taxation does not affect the cost of debt, at least directly, the investing decision of firms with more debt are likely to be less sensitive to dividend taxation. This general proposition is not surprising if firms with extreme debt levels are compared. Further, the
issue that has been widely discussed by financial economists \cite{JensenMecklin1986} but almost always disregarded by tax economists.\footnote{Two notable exceptions are Boadway and Keen (2006) and Minelli and Modica (2009).}

Given this twofold lack, we build a theoretical framework aimed at analyzing the effects of both corporate and dividend taxation when debt finance is allowed (along with equity issues and retained profits) and information is asymmetric. In particular, we focus on risk-neutral firms that can decide whether to undertake a risky investment and how to finance it. Firms are either cash-rich and cash-poor, depending on whether they have enough retained profits to finance the whole investment. When information is symmetric, i.e., when competitive lenders can observe firms’ returns, debt turns out to be the preferred source because of interest deductibility. We then turn to a second-best framework where, more realistically, lenders cannot observe firms’ expected returns. We assume that there are two types of firms: bad firms, with lower success probability and expected returns and good firms, with higher success probability and expected returns. We first focus on cash-rich firms and find that at the equilibrium, only bad firms are offered the first best financial agreement with full debt finance; good firms are instead credit-rationed, in that the amount of debt they receive is lower than the first-best one. This is a separating equilibrium, since the two types of firms sign different contracts, and it is unique in our framework.

Dividend taxation does not affect corporate choices when firms are cash-rich, according to the New View. This is confirmed in our framework since second-best equilibrium contracts depend only on the corporate tax rate. Interestingly, increasing the corporate tax rate does not affect bad firms’ contract, while it reduces rationing for good firms since interest deductibility is enhanced and external debt becomes even more attractive. However, the separating equilibrium disappears if the fraction of good firms is relatively high since a pooling contract (i.e. a financial contract accepted by both bad and good firms) is now preferred by all firms, thus representing a profitable deviation. We show that a higher corporate tax rate may exacerbate the problem of equilibrium non-existence because it reduces the maximum fraction of good firms below which an equilibrium exists. The reason is still related to the increase in interest deductibility: because of this increased tax benefit, good firms are more attracted by contracts with both higher debt and interest rate. Since bad firms are always fully debt financed, the difference between good firms’ contracts and bad firms’ ones is now lower. This means that, in terms of optimal debt finance, good firms’ choices are closer to bad firms’ ones and therefore it is easier that a lender offers a pooling contract. Relative to the optimal separating contract, good firms, choosing a pooling one, are now better off. We conclude that any tax authority aiming at minimizing credit rationing to good firms should trade-off the positive effect of corporate taxation on the amount of debt with the negative one related to the nonexistence of equilibrium.

We plan to extend our analysis to cash-poor firms and expect the dividend tax rate can play a role in affecting the equilibrium outcome, as suggested by the Old View.

This article is related to two streams of literature. The first deals with the effects of dividend taxation on companies’ strategies. This literature directly focuses on the dispute between Old and New View. As pointed out by Auerbach and Hassett (2003) however, "there is no reason to argue that one view is “correct” and another is “incorrect.” What matters for tax policy decisions is the relative importance of the different views, in terms of the extent to which dividend taxes affect the cost of capital". Given these results, scholars

\footnote{Two notable exceptions are Boadway and Keen (2006) and Minelli and Modica (2009).}
have been trying to depart from previous work, by designing alternative and complementary models. For instance, Bernheim (1991) proposed a signaling theory. Chetty and Saez (2007) and Gordon and Dietz (2006) applied an agency theory. We depart from previous work by introducing a low-cost source of finance: debt.

The second stream of research we refer to deals with the effects of taxation on capital structure. In doing so we mainly refer to Gordon (2010) who analyses three different models: an Agency-Cost, a Signaling and a Trade-Off model of debt finance. He explores the implications of these models and then concludes that the existing empirical evidence is more supportive of a “lemons” model in which lack of information about the viability of borrowing firms inhibits use of debt. It is worth noting that Gordon (2019) does not investigates to what extent taxation affects borrowing contracts and under what conditions a separating equilibrium may emerge.

The structure of the paper is as follows. The model is laid out in Section 2. Section 3 describes both investment and financial choices under symmetric information. Section 4 extends the model by introducing asymmetric information between firms and lenders. Section 5 discusses tax policy implications.

2 The Model

In this section we introduce a two-period model where a representative risk-neutral firm decides whether to make a risky investment and how to finance it. Investment size is exogenously given and, for simplicity, normalized to 1. The firm has cash holdings equal to $M \geq 0$ and can decide to issue equity $E \in [0, 1]$ and/or to borrow an amount of resources equal to $B \in [0, 1]$ from external lenders.

The timing of events is as follows:

**Time 0** The firm pays out dividends

$$D = \tilde{M} - M$$

and decides whether to invest the amount

$$M + E + B = 1$$

in the project.

**Time 1** The project yields either $A$ with probability $p \in (0, 1)$ or $a < A$ with probability $(1 - p)$. Therefore the firm’s expected return is equal to

$$R = pA + (1 - p)a.$$  

When returns accrue, the firm repays the money to the lenders if it has cash enough and then pays out dividends, if any.

Let us next introduce two assumptions regarding the state of nature.

**Assumption 1** The bad-state return $a$ belongs to interval $[-1, r]$, where $r > 0$ is the risk-free interest rate.

**Assumption 2** The good-state return $A$ is weakly higher than $\bar{A} = \frac{r - (1 - p)a}{p}$. 
Assumption 1 states that bad-state gross return $1 + a$ can be neither less than zero nor higher than $1 + r$. This means that, in the worst case, a "black swan" event (with returns less than $-100\%$) is excluded. The upper bound holds by definition: a state of nature is bad if it ensures returns that are less than the opportunity cost $r$. Inequality $A \geq A$ in Assumption 2 can be rewritten as $R \geq r$ given (3): this means that the investment project is creditworthy for any $B \in [0, 1]$.

In conclusion, we propose the following

**Definition 1** A firm is defined cash-rich if $M \geq 1 - \frac{1+a}{1+r}$ and cash-poor if $M < 1 - \frac{1+a}{1+r}$, where $\frac{1+a}{1+r} \in [0, 1]$. 

### 2.1 Lenders

Let us assume the existence of a capital market with at least two homogeneous risk-neutral lenders competing à la Bertrand. Borrowing is regulated through a standard debt contract: if the firm is not able to repay, the lenders seize the firm’s assets and become shareholders. Let $\rho$ denote the interest rate on debt, hence a debt contract is characterized by the pair $\{\rho, B\}$. Since lenders operate in a Bertrand environment, the optimal interest rate $\rho$ is determined by their break-even condition.

### 2.2 Taxation

Let us next introduce taxation. By assumption, a firm’s profits, when distributed, are taxed both at the corporate and the personal level. We thus denote $t_c \in (0, 1)$ and $t_d \in (0, 1)$ as corporate and dividend tax rate, respectively. Given that, retained profit $M$ (which has already been taxed at the corporate level) is subject to dividend taxation at time 1.

The after-tax net present value of a firm is calculated as the summation between the investment cost at time 0 and the present value of gross return at time 1. More generally, suppose 1 unit of dividends paid out by the firm is worth $(1 - t_d)$ to the shareholder after personal dividend taxation. If the firm retains $M$ units of profit, the investment project’s cost borne by the shareholder at time 0 is $-(1 - t_d)M$. At time 0, firm’s investment is given by retained profits plus equity:

$$-(1 - t_d)M - E. \quad (4)$$

At time 1, the firm’s expected value depends on its ability to repay the debt, which is in turn affected by the amount $B$ of borrowing: two cases are considered.

**Case 1** The inequality

$$(1 + \rho) B \leq 1 + a \quad (5)$$

holds: debt contract $(\rho, B)$ is such that gross cost of borrowing, $(1 + \rho)B$, is weakly lower than the before-tax gross value of the project when returns are minimum, $(1 + a)$. In this case the firm can always repay debt. Accordingly, default does not occur and the lenders’ break-even condition writes as

$$p(1 + \rho)B + (1 - p)(1 + \rho)B = (1 + r)B \quad (6)$$

or, equivalently,

$$\rho = r,$$
where \((1 + r) B\) is the lenders’ opportunity cost of lending the amount \(B\). The firm’s expected value at time 1 is therefore given by
\[
(1 - t_d) [(1 - t_c) (R - rB) + M] + E + B - B,
\]
which is the gross production function minus debt costs \(rB\). Summing up (4) and (7) and taking into account the time discount factor \(\frac{1}{1+r}\) yields the firm’s net present value (henceforth NPV):
\[
-(1 - t_d) M - E + \frac{(1 - t_d) [(1 - t_c) (R - rB) + M] + E}{1 + r}.
\]

**Case 2** The inequalities
\[(1 + a) < (1 + \rho) B \leq 1 + A\] (9)
hold: debt contract \((\rho, B)\) is such that gross costs of borrowing are higher than the firm’s before-tax gross value in the bad scenario, but weakly lower than the corresponding value in the good scenario. Here default occurs with probability \((1 - p)\), in which case the lenders become shareholders. Accordingly, their break-even condition writes as follows:
\[
p (1 + \rho) B + (1 - p) (1 + a) = (1 + r) B.
\]
In this case the firm’s NPV takes into account the positive probability of bankruptcy and writes as
\[
(1 - t_d) M - E + p \frac{(1 - t_d) [(1 - t_c) (A - \rho_0 B) + M] + E}{1 + r} + (1 - p) \times 0,
\]
where \(\rho_0\) solves lenders’ break-even condition (10):
\[
\rho_0 \equiv \frac{RB - (1 - p) (a + M + E)}{pB}.
\]
It is worth noting that (9) requires \(\rho\) to exceed \(r\) in order to satisfy (10): lenders require a risk premium when a positive probability of default arises.\(^4\)

The lenders’ break-even conditions (6) and (10) are unaffected by taxation because we make the following

**Assumption 3** Lenders are tax-exempt.

Assumption 3 is introduced for two reasons. Firstly, the tax burden on capital income is relatively low or even close to zero in most cases. Secondly, as pointed out by Cooper and Nyborg (2006), under default risk, the evaluation of debt "depends upon the tax position of insolvent firms and the tax treatment of debt write-downs" (p. 366). In particular, they refer to the tax treatment of debt write-downs in the USA: according to the law, no tax liability is accounted for if the insolvent firm’s liabilities exceed its assets. Since we aim to focus on a real default case rather than a debt restructuring one, we use Assumption 3.\(^5\)

\(^4\)Case \((1 + \rho) B > 1 + A\) is ruled out by Assumption 2. Indeed \((1 + \rho) B \leq 1 + A\) can be rewritten as
\[
\left(1 + \frac{RB - (1 - p)(a + M + E)}{pB}\right) B \leq 1 + A
\]
after substituting (12). The LHS of the above inequality is maximum for \(B = 1\). Plugging \(B = 1\) into (a) yields
\[
1 + r - \frac{(1 - p)a}{p} \leq 1 + A
\]
which is equivalent to Assumption 2.

\(^5\)Notice that the tax treatment of debt write-downs is similar in many other countries.
### 3 A Firm’s problem

Let us next study the problem of a firm which maximizes its \( NPV \) by choosing the opportune combination of self-finance \( M \), internal equity \( E \) and debt \( B \). Of course if \( \max NPV < 0 \), no investment is made. Let us first focus on self- and equity finance. We will then turn to debt.

#### 3.1 Shareholders’ funds

**Full Self-financing.** If the firm relies entirely on cash holdings to finance the project, equity and debt are nought. By replacing \( E = B = 0 \) into (2) gives \( M = 1 \). Full-self financing strategy is feasible if \( \bar{M} \geq 1 \): in this case, the firm pays out dividends \( D = \bar{M} - 1 \) at time 0, according to (1). Since debt is zero, no default occurs. Substituting conditions \( E = B = 0 \) into (8) thus gives:

\[
NPV_M \equiv - (1 - t_d) + \frac{(1 - t_d)(1 - t_c)R + 1}{1 + r}, \tag{13}
\]

Expression (13) can be rearranged as \( \frac{1 - t_d}{1 + r} [(1 - t_c)R - r] \). As can be seen, the sign of such a value does not depend on \( t_d \). As we know, this result is referred to as the "new view" theory, in that it relies on the assumption that investment is financed by retained profit. The intuition behind Proposition 4.2 is straightforward. Under self-finance, dividend taxation has a two-fold effect for shareholders. On the one hand, they save dividend taxes at time 0. On the other hand, they will pay the dividend tax at time 0. So, there is a tax saving at time 0 and a tax burden at time 1: their net effect is null and therefore dividend taxation does not matter.

**Full Equity-financing.** Under full equity finance, we have: \( E = 1 \) and \( B = M = 0 \). Hence, from (1), at time 0 dividend payouts equal cash holding \( D = \bar{M} \). Substituting \( E = 1 \) and \( B = M = 0 \) into (8) and rearranging gives:

\[
NPV_E \equiv -1 + \frac{(1 - t_d)(1 - t_c)R + 1}{1 + r}. \tag{14}
\]

Contrary to the full self-financing case, the sign of (14) does depend on \( t_d \), as one can easily check by rearranging it as \( \frac{(1-t_d)(1-t_c)R-r}{1+r} \). An investment decision of an equity-financed firm is, therefore, affected by the dividend taxation. This result is in line with the Old View according to which the marginal source of funds is new equity, and thus dividend taxation distorts both dividend and investment decisions.

It is worth noting that \( NPV_M > NPV_E \) for any \( t_d > 0 \): due to dividend taxation, self-financing is preferred to equity.

#### 3.2 Debt finance

Unlike self- and equity finance, debt may lead to default. For this reason, we first analyze a no default scenario, characterized by inequality (5). Then we focus on a potential default scenario, described by (9).

**No Default.** Suppose (5) holds: in this case no default occurs. Lenders’ break-even condition is \( \rho = r \)

\[\text{Computations concerning the firm’s problem in absence of default are in Appendix A.1.}\]
and the firm’s problem is
\[
\max_{E, B} \left\{ - (1 - t_d) (1 - E - B) - E + \frac{(1 - t_d) [(1 - t_c) (R - rB) + (1 - E - B)] + E}{1 + r} \right\} \tag{15}
\]
\[
s.t. \quad B \leq \frac{1 + a}{1 + r},
\]
where (15) is (8) after substituting \( M = 1 - E - B \) from (2) and \( B \leq \frac{1 + a}{1 + r} \) is (5) after dividing by \((1 + r)\); recall that \( \frac{1 + a}{1 + r} \in [0, 1] \) under Assumption 1.

**Lemma 1** NPV (15) is increasing in \( B \) and decreasing in \( E \) for any admissible value of parameters \( r, t_c \) and \( t_d \).

The above result hinges upon the deductibility of interest expenses, which makes debt less costly than other sources in absence of default. Accordingly, the firm is induced to maximize \( B \):
\[
B^* = \frac{1 + a}{1 + r} \leq 1. \tag{16}
\]
Due to its higher cost, however, equity is as low as possible. Hence the optimal amount of self-financing is given by
\[
M^* = \min \left\{ \tilde{M}, 1 - \frac{1 + a}{1 + r} \right\}. \tag{17}
\]
Substituting \( B^* \) and \( M^* \) into (2) yields optimal equity
\[
E^* = 1 - \frac{1 + a}{1 + r} - M^*. \tag{18}
\]
Since level of \( E^* \) depends upon \( \tilde{M} \) according to (17), we have two kinds of firm.

(i) A cash-rich firm \((\tilde{M} \geq 1 - \frac{1 + a}{1 + r})\) sets \( M^* = 1 - \frac{1 + a}{1 + r} \) according to (17), pays out dividends \( D^* = \tilde{M} - M^* \) and issues no equity, \( E^* = 0 \) according to (18). Plugging \( B^* = \frac{1 + a}{1 + r} \) and \( E^* = 0 \) into (15) and rearranging gives the optimal firm’s NPV without default:
\[
NPV_{B, M} \equiv \frac{(1 - t_d) (1 - t_c)}{1 + r} \left[ (R - r) - \frac{r t_c}{1 - t_c} \left( 1 - \frac{1 + a}{1 + r} \right) \right]. \tag{19}
\]

(ii) A cash-poor firm \((\tilde{M} < 1 - \frac{1 + a}{1 + r})\) has not enough cash to invest. In this case \( M^* = \tilde{M} \) and \( D^* = 0 \) according to (17) and (1), respectively, and \( E^* = 1 - \frac{1 + a}{1 + r} - \tilde{M} \). Substituting \( B^* = \frac{1 + a}{1 + r} \) and \( E^* = 1 - \frac{1 + a}{1 + r} - \tilde{M} \) into (15) gives
\[
NPV_{B, M, E} \equiv \frac{(1 - t_d) (1 - t_c)}{1 + r} \left[ (R - r) - \frac{r t_c}{1 - t_c} \left( 1 - \frac{1 + a}{1 + r} \right) - \frac{r t_d}{(1 - t_d) (1 - t_c)} \left( 1 - \frac{1 + a}{1 + r} - \tilde{M} \right) \right]. \tag{20}
\]
By virtue of Lemma 1, the above expression is strictly lower than (19): indeed, the last term in square brackets is the cost due to the absence of tax benefits when issuing equity.\(^7\)

\(^7\)If the opportunity cost of shareholders’ funds were deductible, however, this cost would vanish.
Default risk.\footnote{Computations concerning the firm’s problem when default is possible are in Appendix A.2.} Let us now focus on the interval \( \frac{1 + a}{1 + r} < B \leq 1 \): since \( 1 + a < (1 + r)B \) and \( \rho > r \), the firm goes bankrupt with probability \((1 - p)\). Given the threat of default the interest rate on debt is \( \rho_0 \) and depends the amount of borrowing. Indeed

\[
\frac{\partial \rho_0}{\partial B} = \frac{(1 - p)(1 + a)}{pB^2} > 0;
\]

the higher the debt level \( B \in (\frac{1 + a}{1 + r}, 1] \), the higher the interest rate \( \rho_0 \) charged by lenders in order to break-even.

When debt is an available source of finance, a firm’s problem is one of choosing the optimal combination of debt, equity and self-finance, by maximizing (11). In symbols:

\[
\max_{E, B} \left\{ - (1 - t_d) (1 - E - B) - E + p \frac{(1 - t_d) [(1 - t_c) (A - \rho_0 B) + (1 - E - B)] + E}{1 + r} \right\}
\]

s.t. \( \frac{1 + a}{1 + r} < B \leq 1 \),

where (22) is (11) after substituting \( M = 1 - E - B \) from (2). Solving (22) gives the following:

Lemma 2 \( NPV \) (22) is increasing in \( B \) and decreasing in \( E \) for any \( r, t_c \) and \( t_d \).

Lemma 2 implies that the firm finances the entire investment with debt, \( B^* = 1 \), while equity is nought, \( E^* = 0 \), as well as the amount of cash holdings invested in the project, \( M^* = 0 \); finally, dividend payout is \( D^* = M \). Substituting \( B^* = 1 \), \( M^* = 0 \) and \( E^* = 0 \) into (12) yields

\[
\rho^* = \frac{r - (1 - p) a}{p},
\]

where \( \rho^* > r \) under Assumption 1. Substituting \( E^* = 0 \), \( B^* = 1 \) and \( \rho^* \) into (22), the optimal firm’s \( NPV \) under default risk is:

\[
NPV_B \equiv p \frac{(1 - t_d) (1 - t_c) (A - \rho^*)}{1 + r}.
\]

It is worth noting that \( \rho^* = A \) under Assumption 2; as a consequence \( NPV_B \geq 0 \), which rewrites as \( A \geq \rho^* \), holds true.

Finally, we state the following

Corollary 1 Only a cash-poor firm issues equity given that \( NPVs \) (15) and (22) decrease with \( E \).

3.3 Optimal financial policy

The firm compares the optimal \( NPV \) without default to the corresponding value with potential default, \( NPV_B \), in order to define its optimal funding policy. Since the former depends on the amount of firm’s cash holdings, we distinguish two cases: the firm is cash-rich, in which case optimal \( NPV \) without default is given by \( NPV_{B,M} \); the firm is cash-poor, in which case optimal \( NPV \) without default is given by \( NPV_{B,M,E} \).

As shown in Appendix A.3, the inequality \( NPV_{B,M} \leq NPV_B \) holds under Assumption 1. Moreover we have \( NPV_{B,M} = NPV_B \iff a = r \). As a consequence, the optimal financial policy for a cash-rich firm
is to rely completely on debt, \( B^* = 1 \). In this case, the optimal NPV is equal to \( NPV_B \). Similarly, when the focus is on a cash-poor firm, it is sufficient to recall that \( NPV_{B,M,E} < NPV_{B,M} \) to conclude that \( NPV_{B,M,E} < NPV_B \) for any admissible value of the parameters.

To sum up, we can write the following:

**Proposition 1** Both cash-rich and cash-poor firms prefer full debt finance:

\[
\{\rho^*, B^*\} = \left\{ \frac{r - (1 - p)a}{p}, 1 \right\}, \ E^* = M^* = 0 \text{ and } D^* = M.
\]

4 Hidden Information

4.1 Cash-rich firms

In this section we suppose that probability \( p \) associated to good outcome \( A \) is firms’ private information.

Moreover, we introduce two types of firms, which differ in their success probability: \( p \in \{p_L, p_H\} \) with \( 0 < p_L < p_H < 1 \). We let \( \lambda \ (1 - \lambda) \) be the fraction of type \( L \) (\( H \)) firms in the population endowed with success probability \( p_L \) (\( p_H \)). Expected return for firm \( i = L, H \) is therefore

\[
R_i = p_i A + (1 - p_i) a,
\]

with \( a \in [-1, r] \) and \( A \geq A \) according to Assumptions 1 and 2, respectively. It follows that \( R_H > R_L \), this is why we refer to \( L \)- (\( H \)-) firms as bad (good).

It is worth observing that asymmetric information on repayment probability \( p \) matters only in the default area, where \( \rho > r \) and \( B \in \left( \frac{1 + a}{1 + r}, 1 \right] \). Indeed, for lower values of \( B \) both types of firms repay with probability 1 or, equivalently, success probability is 1 for each type. Accordingly, we restrict our attention to the default area and study the pure-strategy subgame perfect Nash equilibria (SPNEs) of the following two-stage game:

1. \( N \geq 2 \) lenders compete à la Bertrand by simultaneously offering contracts to the firms; each contract is a pair \( \{\rho, B\} \), with \( \rho > r \) and \( B \in \left( \frac{1 + a}{1 + r}, 1 \right] \).

2. Afterward, firms of each type choose whether to accept a contract and, if so, which one.

In case of symmetric information, i.e. when \( p_i \) is observable, one can easily check that at the equilibrium competitive lenders earn zero profits by offering first-best type-dependent contracts to firms:

\[
\left\{ \rho^*_L = \frac{r - (1 - p_L)a}{p_L}; B^*_L = 1 \right\}
\]
to bad firms and

\[
\left\{ \rho^*_H = \frac{r - (1 - p_H)a}{p_H}; B^*_H = 1 \right\}
\]
to good firms, with \( \rho_L > \rho_H \). On the contrary, the above pair of contracts cannot be an equilibrium under asymmetric information: indeed, \( L \)-firms would take contract \( \{\rho^*_H, 1\} \), given \( \rho^*_L > \rho^*_H \), and the lenders would end up with negative expected profits.

In principle, three types of equilibria may arise:

\[
9\text{Recall that Assumption 1 ensures } NPV_B \geq 0.
\]
(i) pooling equilibria, when both bad and good firms accept the same contract;
(ii) separating equilibria, when the two types of firms sign different contracts;
(iii) rationing equilibria, when one type of firms sign a contract, while the other type sign no contract.

Invoking a Bertrand argument we first state that

**Claim 1** There is no equilibrium (pooling, separating or rationing) in which the lenders earn positive profits.

In what follows we focus on cash-rich firms, i.e. we restrict our attention to the New View: we plan to extend the analysis also to cash-poor firms.

We build on Mas Collel et al. (1995, ch. 13 D) by providing a graphical solution in plane \((\rho, B)\) to the game described above. We first check the existence of pooling equilibrium contract(s).

**Pooling equilibrium.** As a first step, we demonstrate that no pooling equilibrium contract exists. See Figure 1, where the firms are better-off (worse-off) when moving north-west (south-east) in plane \((\rho, B)\) since firms’ NPV \((11)\), with \(E = 0\) and \(M = 1 - B\), is increasing in \(B\) and decreasing in \(\rho\). By contrast, the lenders are better-off (worse-off) when moving south-east (north-west). Indeed, lenders’ expected profits \((10)\) can be rewritten as

\[
[p(1 + \rho) - (1 + r)]B + (1 - p)(1 + a),
\]

which increases with \(\rho\) and decreases with \(B\) since maximum value of \(p(1 + \rho) - (1 + r)\) is for \(\rho^* = \frac{r - (1 - p)a}{p}\) and equal to \(-(1 - p)(1 + a) < 0\).

Five curves are drawn in Figure 1. \(\overline{OL}\) and \(\overline{OH}\) are the lenders’ zero-profit curves for \(L\)- (\(H\)-) firms, respectively. In symbols

\[
p_i(1 + \rho)B + (1 - p_i)(1 + a) - (1 + r)B = 0
\]

Rearranging

\[
B[1 + r - p_i(1 + \rho)] = (1 - p_i)(1 + a)
\]

Solving by \(B\) yields

\[
B = \frac{(1 - p_i)(1 + a)}{1 + r - p_i(1 + \rho)}, \tag{25}
\]

Plugging \(p_L\) (\(p_H\)) into (25) gives \(\overline{OL}\) (\(\overline{OH}\)). The derivative of (25) w.r.t. to \(\rho\) is \(\frac{p_i(1-p_i)(1+a)}{(1+r-p_i(1+\rho))^2} > 0\): the above curve is thus increasing in \(\rho\), equal to \(\frac{1+a}{1+r}\) when \(\rho = r\) (contract \(O\)) and to 1 when \(\rho^* = \frac{r - (1 - p)a}{p}\) (contract \(L\) when \(p_i = p_L\) and contract \(H\) when \(p_i = p_H\)). In addition one can check that such a derivative increases with \(p\),

\[
\frac{\partial}{\partial p_i}\frac{p_i(1-p_i)(1+a)}{(1+r-p_i(1+\rho))^2} = (1 + a) \frac{1 + r - p_i (1 + 2r - p_i)}{(1 + r - p_i (1 + \rho))^3} > 0,
\]

hence \(\overline{OL}\) is flatter than \(\overline{OH}\): given \(p_L < p_H\), an increase in \(\rho\) impacts less on bad firms’ NPV; as a consequence, their indifference curve is flatter.

\(^{10}\)In Figure 1 and all subsequent ones parameters take the following values: \(p_L = .95; p_H = .99; r = .05; a = -.1; t_c = .3; \lambda = 0.5\).
Instead, $OP$ is the lenders’ zero-profit curve for pooling contracts $\{\rho, B\}$ that attract both types of firms. In symbols

$$\lambda [p_L (1 + \rho) B + (1 - p_L) (1 + \rho) - (1 + r) B] + (1 - \lambda) [p_H (1 + \rho) B + (1 - p_H) (1 + \rho) - (1 + r) B] = 0$$

Solving by $B$ yields

$$B = \frac{\lambda (1 - p_L) (1 + a) + (1 - \lambda)(1 - p_H) (1 + a)}{\lambda [(1 + r) - p_L (1 + \rho)] + (1 - \lambda) [(1 + r) - p_H (1 + \rho)]}. \quad (26)$$

Such a curve, which we refer to also as pooled break-even curve, boils down to $OL$ when all firms are bad, $\lambda = 1$, and to $OH$ when all firms are good, $\lambda = 0$. Being a linear combination between $OL$ and $OH$, also $OP$ is increasing in $\rho$, steeper (flatter) than $OL$ ($OH$), equal to $\frac{1 + a}{1 + r}$ for $\rho = r$, i.e. contract $O \equiv \left( r, \frac{1 + a}{1 + r} \right)$, and to 1 for

$$\rho_P = \frac{(1 + r) - (\lambda (1 - p_L) + (1 - \lambda)(1 - p_H)) (1 + a) - \lambda p_L - (1 - \lambda) p_H}{(\lambda p_L + (1 - \lambda) p_H),}$$

i.e. contract $P \equiv (\rho_P, 1)$.

Finally, $OP^m$ and $OP^p$ are $L$- ($H$-) firms’ indifference curve, respectively, through contract $Q \equiv (\rho_Q, B_Q)$ lying on the pooled break-even line $OP$, with $\rho_Q < \rho_P$ and $B_Q < 1$. The equation of $OP^m$ is

$$B_L = \frac{B_Q \left[1 + r - p_L \left(1 + (1 - t_c) \rho_Q\right)\right]}{1 + r - p_L \left[1 + \rho (1 - t_c)\right]}.$$  

The equation of $OP^p$ is

$$B_H = \frac{B_Q \left[1 + r - p_H \left(1 + (1 - t_c) \rho_Q\right)\right]}{1 + r - p_H \left[1 + \rho (1 - t_c)\right]}.$$  

**Figure 1. No pooling equilibrium exists** (1)

Suppose a pooling equilibrium contract exists: it then must lie on the pooled break-even curve $OP$. Indeed, any pooling contract at north-west of the pooled break-even line yields negative profits, which
cannot be an equilibrium; similarly, there is no pooling contract lying at south-east of the pooled break-even line, otherwise banks would earn strictly positive profits, which is ruled out by Claim 1.

Assume now all lenders offer pooling contract $Q$. Such a contract cannot be an equilibrium, since at least a lender may offer another pooling contract $Q' \equiv (\rho_{Q'}, B_{Q'})$ belonging to area $QPP'$, with $\rho_Q < \rho_{Q'}$ and $B_Q < B_{Q'} \leq 1$: contract $Q'$ would be preferred by both types of firms and would give positive profits. As a consequence, the only candidate pooling equilibrium contract is $P \equiv (\rho_P, 1)$. However, neither $P$ can be a pooling equilibrium. To prove it we draw two additional curves in Figure 2: $LPP$ and $HPP$ are $L$-firms’ and $H$-firms’ indifference curves through contract $P$.

If all lenders offer $P$ to both types, at least a lender has the following profitable deviation: proposing a rationing contract $B$, which is accepted only by $H$-firms and yields positive profits, because it lies in area $GFP$. We can correctly conclude that no pooling equilibrium contract exists. The reason is as follows. Good firms’ indifference curve is steeper than bad firms’ one, hence good firms accept to be more credit-rationed than bad firms in order to pay a lower interest rate. Accordingly, it is always possible to find a profitable deviation, such as contract $B$ by moving from $P$ to the south-east cone $GFP$, a region with both lower debt and interest rate.

**Figure 2. No pooling equilibrium exists (2)**

**Separating equilibrium.** We now check the existence of separating equilibrium contract(s). Recall that when first-best contracts $L \equiv \{\rho_L^*, 1\}$ and $H \equiv \{\rho_H^*, 1\}$ are offered, the lenders incur losses since $L$-firms choose the latter agreement. To prevent this unpleasant behavior, the lenders might offer (i) $L$ to bad firms and a worse contract than $H$ to good firms, (ii) a better contract than $L$ to bad firms and $H$ to good firms (iii) a worse contract than $H$ to good firms and a better contract than $L$ to bad firms. Yet, options (ii) and (iii) are not viable since $L$ is bad firms’ first-best contract. Moreover, at a separating equilibrium (if any) the lenders earn zero profits on each per-type contract according to Claim 1. As a consequence, the unique separating equilibrium is the pair of contracts $L \equiv (\rho_L^*, 1)$ to bad firms and $S$ to good firms, where $S$ is given by the intersection between $L\overline{L}$, the $L$-firms’ indifference curve through $L$, and $\overline{OH}$, the lenders’ zero-profit curves for $H$-firms: see Figure 3. Contract $L$ is the unique contract offered to bad firms at a separating equilibrium because of Bertrand competitive pressure. Similarly, contract $S$ is unique because
any contract lying on $\overline{OS}$ would be undercut by rival lenders and any contract lying on $\overline{SH}$ would be chosen also by $L$-firms. Contract $S$ is given by the following system:

$B = \frac{1+r-p_L[1+p_L^*(1-t_c)]}{1+r-p_H^*[1+p(1-t_c)]} \quad (\Pi L)$,

$B = \frac{1+r-p_L[1+p(1-t_c)]}{(1+r)p_H(1+p)} \quad (\Pi H)$.

It is easy to check that both types of firms’ participation constraints are not active at the separating equilibrium $L, S$. because all firms get higher profits than those they would obtain under the best outside option. Indeed, (cash-rich) firms’ outside option coincides with (13), the NPV in case of self-financing: this is indeed the best mode in absence of external debt since (13) > (14). Yet, (13) is lower than (19), what they get when signing contract $O \equiv \left\{ r, \frac{1+a}{1+r} \right\}$, which, in turn, is lower than bad firms’ NPV in $L$ and good firms’ NPV in $S$.

![Figure 3. Separating equilibrium](image)

Interestingly, the unique separating equilibrium $L, S$ cannot be sustained if fraction $\lambda$ of bad firms reduces. Indeed, in such a case the pooled break-even curve $\overline{OP}$ rotates leftward and intersects curve $PH''$, the $H$-firms’ indifference curve through contract $S$, at a point $M$ with $B_M < 1$: see Figure 4, where $\overline{OP}$ is depicted for $\lambda = .2$. Indeed, in such a case the following profitable deviation is available when all lenders are offering contracts $L, S$: proposing contract $D$, which lies in area $MPH''$, being therefore preferred by both types and yielding positive profits.

To obtain the minimum fraction $\lambda$ of $L$-firms such that a separating equilibrium exists, it is sufficient to compute $\lambda$ such that the pooled break-even line $\overline{OH''}$ intersects $PH''$ at $B = 1$. We have

$\lambda_{\text{min}} = \frac{p_H t_c (1+r-p_H)}{(1+r)(p_H-p_L)+p_H t_c (1+r-p_H)}$.

**Rationing equilibrium.** Finally, we study the existence of rationing equilibrium contract. Suppose such an equilibrium contract (if any) is accepted only by $H$-firms. Then, given Bertrand competition, it must be contract $H \equiv \{p_H^*, 1\}$. Yet, we have already proved that $H$ would be chosen also by $L$-firms. It follows that $H$ cannot be a rationing equilibrium. Suppose now that the rationing equilibrium contract is accepted
only by \( L \)-firms. Then, given Bertrand competition, it must be contract \( L = \{ \rho_L^*, 1 \} \). Yet, if all lenders offer \( L \), at least one lender has the following profitable deviation. If \( \lambda < \lambda_{\text{min}} \), then it can offer the pooling contract \( D \). If \( \lambda \geq \lambda_{\text{min}} \) it can propose \( L \) and a second contract \( S' \) with \( \rho_{S'} = \rho_S \) and \( B_{S'} = B_S - \varepsilon \) (see Figure 3): contract \( S' \) is signed only by \( H \)-firms and yields positive profits. We can correctly conclude that no rationing equilibrium contract exists. The intuition is as follows. In order to have a rationing equilibrium, there should not exist any other contract (whether pooling or separating) respecting both good firms’ participation constraint - giving a weakly higher \( NPV \) than the one in (13) - and the lenders’ zero-profit conditions. Yet, at least both pooling contract \( D \) and the couple of separating contracts \( L \) and \( S' \) give good firms more than (13).

![Figure 4. No separating equilibrium exists](image)

We are now able to sum up our findings.

**Proposition 2** If \( \lambda \geq \lambda_{\text{min}} \) the unique SPNE of the two-stage game played by lenders and firms is a separating equilibrium, where bad firms sign their first best contract \( L = \{ \rho_L^*, 1 \} \) and good firms sign contract \( S = \{ \rho_S, B_S \} \) with

\[
\rho_S = \frac{r (p_H - p_L) (1 + a) + t_c (1 + r - p_H) (r - a + a p_L)}{(p_H - p_L) (1 + a) + t_c [p_H (r - a) + p_L (1 + a - p_H)]} < \rho_H^* \tag{27}
\]

and

\[
B_S = \frac{(1 - p_H) (1 + a)}{(1 + r) - p_H (1 + \rho_S)} < 1. \tag{28}
\]

If \( \lambda < \lambda_{\text{min}} \) the above game has no SPNEs in pure strategies.

To explain the result of Proposition 2, we focus on pooling contract \( P = \{ \rho_P, 1 \} \) lying on the pooled break-even curve \( OP \) with \( B = 1 \). One can easily check that \( \rho_P \) augments with \( \lambda \) because a competitive lender anticipates that each potential client in the economy is, with increasing probability, a bad firm. Instead, when few bad firms are present in the economy, i.e. \( \lambda < \lambda_{\text{min}} \), then \( \rho_P \) is relatively low. Accordingly, good firms turn out to prefer contract \( P \) to the separating contract \( S \), with the effect that no separating SPNE exists.
4.2 Cash-poor firms

To be written.

5 Comparative statics

5.1 Cash-rich firms

In this section we aim at understanding how corporate tax rate $t_c$ affects contracts $L$ and $S$ at the separating SPNE described in Proposition 2.

Suppose $t_c$ increases. First notice that lenders’ zero-profit curves in (25) and (26) are not affected by $t_c$: this is due to Assumption 3, according to which the lenders are tax-exempt. By contrast, firms’ indifference curves depend on the level of corporate tax rate $t_c$. More exactly, the derivative of the $L$-firms’ indifference curve through $L$ (curve $LL$ in Figures 3 and 4) w.r.t. $t_c$ is

$$\frac{\partial}{\partial t_c} \left\{ \frac{1+r-p_L[1+\rho'_L(1-t_c)]}{1+r-p_L[1+\rho(1-t_c)]} \right\} = \frac{p_L (\rho^*_L - \rho) [(1+r) - p_L]}{(1+r) - p_L [1+\rho (1-t_c)]^2},$$

the above expression is positive since $(\rho^*_L - \rho) \geq 0$. Taking into account that $L$-firms’ equilibrium contract $L \equiv \{\rho^*_L; 1\}$ is unaffected by $t_c$, we can state that their indifference curve through $L$ rotates rightward as $t_c$ increases: see Figure 5, where $H''P''H''$ is $H$-firms’ indifference curve through their separating contract $S''$.

As a consequence, the higher $t_c$, the lower the credit rationing in that $B_S$ augments. In symbols

$$\frac{\partial \rho_S}{\partial t_c} = \frac{(p_H - p_L) (1-p_H) (r-a) (1+a) [(1+r) - p_L]}{p_H [1+t_c (r-p_L)] - p_L (1-t_c) + ap_H (1-t_c) - ap_L (1-t_c)} > 0$$

and

$$\frac{\partial B_S}{\partial t_c} = \frac{p_H (p_H - p_L) (r-a) [(1+r) - p_L]}{p_H (1+r - p_L t_c) - p_L (1-t_c) (1+r)} > 0.$$

\[\text{Figure 5. Comparative statics on } t_c\]

\[11\text{In Figure 5 thick curves are obtained with parameters used for Figure 1 to 4, while thin curves are drawn after setting } t_c = .5.\]
Yet, a deeper analysis shows that a trade-off appears as \( t_c \) rises in that a stronger condition on \( \lambda_{\text{min}} \) is required in order to have the separating SPNE. Indeed, since also \( H \)-type firms’ indifference curve rotates rightward, contract \( H'' \equiv (\rho_{H''}, 1) \) shifts to the right, given \( \frac{\partial \rho_{H''}}{\partial t_c} > 0 \), as shown in (29) below:

\[
\frac{\partial \rho_{H''}}{\partial t_c} = \frac{1 + r - p_H - B_S (1 + r - p_H (1 + (1 - t_c) \rho_S))}{p_H (1 - t_c)} = \frac{1}{p_H} (p_H - p_L)^2 (r - a) (1 + r) [(1 + r) - p_H] [p_H (1 + r - p_L t_c) - p_L (1 - t_c) (1 + r)]^2 > 0.
\]

Hence, given that \( \lambda_{\text{min}} \) is indirectly derived from the level of \( \rho_{H''} \), a higher \( \rho_{H''} \) implies a stronger parametric condition on \( \lambda \). As a matter of fact,

\[
\frac{\partial (\lambda_{\text{min}})}{\partial t_c} = \frac{p_H (p_H - p_L) (1 + r) (1 + r - p_H)}{(p_H + t_c) - p_L (1 + r) - p_H^2} > 0
\]

which means that heavier corporate tax burden makes less likely the existence of the separating SPNE. In Figure 5 contracts \( L, S \) are a separating equilibrium for a given \( t_c \). If \( t_c \) increases, contracts \( L, S'' \) are a candidate separating equilibrium. Unfortunately, this is not an equilibrium since a profitable deviation is available when all lenders are offering contracts \( L, S'' \): proposing a contract lying in area \( M'PH'' \), which is preferred by both types and yields positive profits.

According to the above analysis and taking into account (30), any tax authority aiming at minimizing the level of credit rationing to good firms should therefore set \( t_c \) at the maximum level compatible with the existence of a separating equilibrium. In symbols, the tax authority should solve the following problem:

\[
\max_{t_c} \lambda_{\text{min}} \equiv \frac{p_H t_c (1 + r - p_H)}{(1 + r) (p_H - p_L) + p_H t_c (1 + r - p_H)}
\]

s.t. \( \lambda_{\text{min}} \leq \lambda \),

where \( \lambda \) is the actual fraction of bad firms in the economy. Solution to (31) is \( \lambda_{\text{min}} = \lambda \). Solving the equality by \( t_c \) yields the rationing-minimizing corporate tax rate:

\[
t^*_c = \frac{\lambda (1 + r) (p_H - p_L)}{(1 - \lambda) (1 + r - p_H) p_H}.
\]

**Proposition 3** The rationing-minimizing corporate tax rate is the highest value of \( t_c \) compatible with the existence of a separating SPNE. In symbols

\[
t^*_c = \frac{\lambda (1 + r) (p_H - p_L)}{(1 - \lambda) (1 + r - p_H) p_H},
\]

which solves problem (31).

**5.2 Cash-poor firms**

To be written.

**6 Conclusion**

To be written.
A Debt finance

A.1 No default

Proof of Lemma 1. \( NPV \) (15) can be rewritten as

\[
(1 - E - B) \left( - (1 - t_d) + \frac{(1 - t_d) ((1 - t_c) R + 1)}{1 + r} \right) + \\
E \left( -1 + \frac{(1 - t_d) (1 - t_c) R + 1}{1 + r} \right) + B \left( \frac{(1 - t_d) (1 - t_c) (R - r)}{1 + r} \right).
\]

(32)

Let

\[
- (1 - t_d) + \frac{(1 - t_d) ((1 - t_c) R + 1)}{1 + r} = x,
\]

\[
-1 + \frac{(1 - t_d) (1 - t_c) R + 1}{1 + r} = y,
\]

\[
\frac{(1 - t_d) (1 - t_c) (R - r)}{1 + r} = z.
\]

One can check that \( z - x = \frac{r}{1 + r} (1 - t_d) t_c > 0 \) and \( x - y = t_d r \frac{1}{1 + r} > 0 \). It follows that \( z > y \) and (32) rewrites as

\[
(1 - E - B) x + Ey + Bz,
\]

(33)

with \( y < x < z \). Notice that \( \frac{\partial (33)}{\partial E} = y - x < 0 \) and \( \frac{\partial (33)}{\partial B} = z - x > 0 \): (33) is increasing in \( B \) and decreasing in \( E \).

Cash-rich firm. Plugging \( B = \frac{1 + a}{1 + r} \) and \( E = 0 \) into (32) yields

\[
\left( 1 - \frac{1 + a}{1 + r} \right) \left( - (1 - t_d) + \frac{(1 - t_d) ((1 - t_c) R + 1)}{1 + r} \right) + \frac{1 + a}{1 + r} \left( \frac{(1 - t_d) (1 - t_c) (R - r)}{1 + r} \right).
\]

After rearrangement:

\[
\frac{1}{1 + r} (R (1 - t_c) - r) + \frac{rt_c (1 - t_d)}{(r + 1)^2} (1 + a),
\]

or

\[
\frac{(1 - t_d) (1 - t_c)}{1 + r} \left[ (R - r) - \frac{rt_c}{1 - t_c} \left( 1 - \frac{1 + a}{1 + r} \right) \right],
\]

which is (19) in the text. Taking into account (3), (19) rewrites as

\[
\frac{(1 - t_d) (1 - t_c)}{1 + r} \left[ (pA + (1 - p) a - r) - \frac{rt_c}{1 - t_c} \left( 1 - \frac{1 + a}{1 + r} \right) \right].
\]

(34)

Solving \( NPV_{B,M} \geq 0 \) by \( A \) yields

\[
pA \geq \frac{rt_c}{1 - t_c} \left( 1 - \frac{1 + a}{1 + r} \right) - (1 - p) a + r,
\]

\[
pA \geq r \frac{(1 + r - (1 + a) t_c)}{(1 - t_c) (1 + r)} - (1 - p) a,
\]

\[
A \geq \frac{r (1 + r - t_c) - a (rt_c + (1 - t_c) (1 + r) (1 - p))}{p (1 - t_c) (1 + r)} \equiv A_{B,M}.
\]
Cash-poor firm. Plugging \( B = \frac{1 + a}{1 + r} \) and \( E = 1 - \frac{1 + a}{1 + r} - \bar{M} \) into (32) yields
\[
\bar{M} \left( - (1 - t_d) + \frac{(1 - t_d) ((1 - t_c) R + 1)}{1 + r} \right) + \\
(1 - \frac{1 + a}{1 + r} - \bar{M}) \left( -1 + \frac{(1 - t_d) (1 - t_c) R + 1}{1 + r} + \frac{1 + a}{1 + r} \left( \frac{(1 - t_d) (1 - t_c) (R - r)}{1 + r} \right) \right)
\]
Rearranging gives (20) in the text. Solving \( NPV_{B,M,E} \geq 0 \) by \( A \) yields
\[
pA \geq - (1 - p) a + r + \frac{r t_c}{(1 - t_c)} \left( 1 - \frac{1 + a}{1 + r} \right) + \frac{r t_d}{(1 - t_d) (1 - t_c)} \left( 1 - \frac{1 + a}{1 + r} - \bar{M} \right) \]
\[
A \geq - \frac{1 - p a}{p} + r + \frac{r t_c}{p (1 - t_c)} \left( r - a \right) + \frac{r t_d}{p ((1 - t_d) (1 - t_c))} \left( r - a \right) - \frac{r t_d}{p ((1 - t_d) (1 - t_c))} \bar{M}
\]

A.2 Default risk

Proof of Lemma 2. \( NPV \) (22) can be rearranged as
\[
(1 - E - B) \left( - (1 - t_d) + p \frac{(1 - t_d) ((1 - t_c) A + 1)}{1 + r} \right) + \\
E \left( -1 + p \frac{(1 - t_d) (1 - t_c) A + 1}{1 + r} \right) + B \left( p \frac{(1 - t_d) (1 - t_c) (A - \rho_0)}{1 + r} \right)
\]
Let
\[
- (1 - t_d) + p \frac{(1 - t_d) ((1 - t_c) A + 1)}{1 + r} = X \\
-1 + p \frac{(1 - t_d) (1 - t_c) A + 1}{1 + r} = Y \\
p \frac{(1 - t_d) (1 - t_c) (A - \rho_0)}{1 + r} = Z
\]
We compute
\[
X - Y = (1 + r - p) \frac{t_d}{1 + r} > 0
\]
\[
Z - Y = \frac{1 + r - p - p \rho_0 (1 - t_d) (1 - t_c)}{1 + r}
\]
\[
Z - X = \left[ 1 + r - p - p \rho_0 (1 - t_c) \right] (1 - t_d)
\]
We study the sign of \( Z - X \), taking into account that \( Z - X > 0 \Rightarrow Z - Y > 0 \). \( Z - X \) is minimum for \( \rho = \rho^* \), where \( \rho^* \) is given by (23), and equal to
\[
\frac{1 - t_d}{1 + r} \left\{ 1 + a (1 - t_c) \right\} (1 - p) + rt_c \}
\]
after rearrangement. We can hence rewrite (35) as
\[
(1 - E - B) X + EY + BZ
\]
with \( Y < X < Z \). We compute: \( \frac{\partial (36)}{\partial E} = Y - X < 0 \) and \( \frac{\partial (36)}{\partial B} = Z - X > 0 \). The firm’s \( NPV \) is hence decreasing in \( E \) and increasing in \( B \).

We finally compute
\[
\rho^* = \frac{r - (1 - p) a}{p} \geq r;
\]
this rewrites as \( a \leq r \), which holds true under Assumption 1. \( \frac{r - (1 - p) a}{p} \)
A.3 Optimal funding policy

We focus on a cash-rich firm and solve for a inequality $NPV_{B,M} \leq NPV_B$, i.e. $(34) \leq (24)$. Taking into account $(3)$, $(24)$ can be written as

$$\frac{(1-t_d)(1-t_c)[pA+(1-p)a-r]}{1+r}. \quad (37)$$

We can write

$$\frac{1-t_d}{1+r}\{[pA+(1-p)a](1-t_c)-r\} + \frac{rt_c(1-t_d)}{(r+1)^2}(1+a) \leq \frac{(1-t_d)(1-t_c)[pA+(1-p)a-r]}{1+r}$$

or, equivalently

$$[pA+(1-p)a](1-t_c)-r + \frac{rt_c}{1+r}(1+a) \leq (1-t_c)[pA+(1-p)a-r]$$

Rearranging

$$-r + \frac{rt_c}{1+r}(1+a) \leq -(1-t_c)r$$

$$\frac{rt_c}{1+r}(1+a) \leq rt_c$$

Simplifying, inequality $(34) \leq (37)$ rewrites as $a \leq r$, which is fulfilled under Assumption 1.

B Hidden information

Computations on MRS. The derivative of $(25)$ w.r.t. to $\rho_i$ is

$$(1+a)\frac{1+r-p_i(1+2r-\rho_i)}{[1+r-p_i(1+\rho_i)]^3}.$$  

To show that the above value is positive note first that $1+r-p_i(1+2r-\rho_i)$ is increasing in $\rho_i$, hence minimum for $\rho_i = r$ and equal to $(1-p_i)(1+r) > 0$. Instead $1+r-p_i(1+\rho_i)$ is decreasing in $\rho_i$, hence minimum for $\rho_i = \rho_i^*$ and equal to $1+r-p_i\left(1+\frac{r-(1-p_i)a}{p_i}\right)$; rearranging gives $(1-p_i)(1+a) > 0$.

Let

$$B_i = \frac{1+r-p_i[1+\rho_i^*(1-t_c)]}{[1+r-p_i[1+\rho_i(1-t_c)]]}, \quad i = L,H$$

be $i$–firms’ indifference curve passing through their first-best contracts.

The derivative of $(38)$ w.r.t. to $\rho_i$ is

$$p_i(1-t_c)\frac{1+r-p_i[1+\rho_i^*(1-t_c)]}{[1+r-p_i[1+\rho_i(1-t_c)]]^2} > 0.$$  

To show that the above value is positive remark that $1+r-p_i[1+\rho_i^*(1-t_c)]$ rewrites as

$$1+r-p_i\left(1+\frac{r-(1-p_i)a}{p_i}\right)(1-t_c).$$

Rearranging yields $(1-p_i)(1+a(1-t_c)) + rt_c > 0$.

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We also have
\[
\frac{\partial}{\partial p_i} \left( p_i (1 - t_c) \frac{1 + r - p_i \left[ 1 + \rho_i^* (1 - t_c) \right]}{1 + r - p_i \left[ 1 + \rho_i (1 - t_c) \right]} \right)^2 = \frac{(1 - t_c) (1 + r) (1 + r - p_i (1 + (2 \rho_i^* - \rho_i) (1 - t_c)))}{(1 + r - p_i (1 + \rho_i (1 - t_c)))^3}.
\]

To show that the above value is positive note first that
\[
0 < 1 + r - p_i (1 + \rho_i) < 1 + r - p_i (1 + \rho_i (1 - t_c)) .
\]

Moreover, \(1 + r - p_i (1 + (2 \rho_i^* - \rho_i) (1 - t_c))\) rewrites as
\[
1 + r - p_i - (2r - 2 (1 - p_i) a - p_i \rho_i) (1 - t_c).
\]

The above value is increasing in \(\rho_i\), hence minimum for \(\rho_i = r\) and equal to
\[
1 + r - p_i - (2 (1 - p_i) (r - a) + p_i r) (1 - t_c).
\]

The above value is increasing in \(t_c\), hence minimum for \(t_c = 0\) and equal to
\[
1 - p_i (1 - 2 (r - a)) + r (1 - p_i).
\]

The above value is decreasing in \(a\), hence minimum for \(a = r\) and equal to
\[
(1 + r) (1 - p_i) > 0.
\]

**Contract S.** \(S\) is given by the following system:
\[
\begin{align*}
B &= \frac{1 + r - p_L \left[ (1 + r - p_L) a (1 - t_c) \right]}{(1 - p_H) (1 + a) (1 + \rho (1 - t_c))} \quad (\text{UL}) \\
B &= \frac{1 + r - p_L \left[ (1 + r - p_L) a (1 - t_c) \right]}{(1 + r - p_H) (1 + \rho (1 - t_c))} \quad (\text{OH})
\end{align*}
\]

Solving \(\frac{1 + r - p_L \left[ (1 + r - p_L) a (1 - t_c) \right]}{(1 + r - p_L) (1 + \rho (1 - t_c))}\) by \(\rho\) yields
\[
\rho_S = \frac{r (p_H - p_L) (1 + a) + t_c (1 + r - p_H) (r - a + a p_L)}{(p_H - p_L) (1 + a) + t_c [p_H (r - a) + p_L (1 + a - p_H)]}.
\]

It follows that \(B_S = \frac{(1 - p_H) (1 + a)}{(1 + r - p_H) (1 + \rho_S)}\).

**PIT curve.** Its equation is given by
\[
- (1 - t_d) (1 - B) + p_H \frac{(1 - t_d) [(1 - t_c) (A - pB) + (1 - B)]}{1 + r} = - (1 - t_d) (1 - B_S) + p_H \frac{(1 - t_d) [(1 - t_c) (A - \rho_S B_S) + (1 - B_S)]}{1 + r}
\]

Solving by \(B\) yields
\[
B = \frac{B_S \left[ 1 + r - p_H (1 + (1 - t_c) \rho_S) \right]}{1 + r - p_H \left[ 1 + \rho (1 - t_c) \right]}.
\]
We finally compute contract $H'' \equiv (\rho_{H''}, 1)$, where $\rho_{H''}$ is given by

$$1 = \frac{B_S [1 + r - p_H (1 + (1 - t_c) \rho_S)]}{1 + r - p_H [1 + \rho (1 - t_c)]}.$$ 

Solving by $\rho$ yields

$$\rho_{H''} = \frac{1 + r - p_H - B_S (1 + r - p_H (1 + (1 - t_c) \rho_S))}{p_H (1 - t_c)}.$$

**Minimum $\lambda$.** Recall that plugging $B = 1$ into the pooled break-even curve yields

$$\rho_P = \frac{(1 + r) - (\lambda (1 - p_L) + (1 - \lambda) (1 - p_H)) (1 + a) - \lambda p_L - (1 - \lambda) p_H}{(\lambda p_L + (1 - \lambda) p_H)}.$$

We then solve by $\lambda$ equation $\rho_{H''} = \rho_P$:

$$\frac{1 + r - p_H - B_S (1 + r - p_H (1 + (1 - t_c) \rho_S))}{p_H (1 - t_c)} = \frac{(1 + r) - (\lambda (1 - p_L) + (1 - \lambda) (1 - p_H)) (1 + a) - \lambda p_L - (1 - \lambda) p_H}{(\lambda p_L + (1 - \lambda) p_H)}$$

and get

$$\lambda_{\text{min}} = \frac{p_H t_c (1 + r - p_H)}{(1 + r) (p_H - p_L) + p_H t_c (1 + r - p_H)}$$

**C Comparative statics**

We solve by $t_c$

$$\frac{p_H t_c (1 + r - p_H)}{(1 + r) (p_H - p_L) + p_H t_c (1 + r - p_H)} = \lambda$$

and get

$$t_c^* = \frac{\lambda (1 + r) (p_H - p_L)}{(1 - \lambda) (1 + r - p_H) p_H}.$$

Comparative statics:

$$\frac{\partial t_c^*}{\partial \lambda} = \frac{(p_H - p_L) (1 + r)}{p_H (1 - \lambda)^2 (1 + r - p_H)} > 0.$$ 

$$\frac{\partial t_c^*}{\partial r} = -\frac{\lambda (p_H - p_L)}{(1 - \lambda) (1 + r - p_H)^2} < 0.$$ 

$$\frac{\partial t_c^*}{\partial p_H} = \frac{\lambda (1 + r) p_L (1 + r) + p_H (p_H - 2p_L)}{p_H^2 (1 - \lambda) (1 + r - p_H)^2}.$$ 

$$\frac{\partial t_c^*}{\partial p_L} = -\frac{\lambda (1 + r)}{p_H (1 - \lambda) (1 + r - p_H)} < 0.$$
References


