General Equilibrium in Monopolistic Competition Beyond the CES

E. Zhelobodko  S. Kokovin
M. Parenti       J.-F. Thisse
Examples of “weird” results

• markups and market prices are independent from the mass of firms
• firms’ size is independent from market size
• prices and outputs are independent of the spatial distribution of demand
Plan of the paper

• 1. the model
• 2. the homogenous firm case
• 3. cost heterogeneity
• 4. quality heterogeneity
• 5. international pricing
The model
Additive preferences

\[
\max_{x(\cdot)} U \equiv \int_0^N u(x_i) \, di \quad \text{s.t.} \quad \int_0^N p_i x_i \, di = E
\]

Houthaker (1960) - Spence (1976)
CES

\[ u(x) = x^\rho \]

CARA

\[ u(x) = 1 - \exp(-\alpha x) \]
Assume that $u$ is strictly increasing and concave

**relative love for variety (RLV)**

$$r_u(x_i) \equiv -\frac{x_iu''(x_i)}{u'(x_i)} > 0$$
Variable Elasticity of Substitution

along the diagonal in the quantity space ($x_i = x$)

$$r_u (x) = \frac{1}{\sigma (x)}$$

→ the RLV is the inverse of the elasticity of substitution
• A higher consumption of the differentiated product makes consumers’ love for variety stronger when the RLV is increasing, whereas love for variety gets weaker when the RLV is decreasing.
Demand

• Consumer’s FOC:

\[ u'(x_i) = \lambda p_i \]

\( \lambda \) is the marginal utility of income
along the diagonal, the elasticity of substitution is equal to the price-elasticity

\[ \varepsilon_i(p_i) = \frac{1}{r_u[x_i(p_i)]]} \]
Producer

\[ \pi(x_i; x(\cdot), E) = \left[ \frac{u'(x_i)}{\lambda(\cdot)} - c \right] Lx_i - f \]

a maximizer exists if the inverse demand is not too convex
Homogeneous firms
Short-run equilibrium

The mass $N$ of firms is given:

at a symmetric equilibrium

\[
\bar{p} = \frac{c}{1 - r_u(E/N\bar{p})} \quad \bar{x} = \frac{E}{N\bar{p}}
\]

\[
\bar{M} \equiv \frac{\bar{p} - c}{\bar{p}} = r_u(\bar{x})
\]
• **Proposition 1.** There exists a *unique* and *symmetric* short-run equilibrium. Furthermore, when the RLV *increases* (resp., *decreases*) with $x$, then the equilibrium price *decreases* (resp., *increases*) with $N$. The equilibrium price is *independent* of the mass of firms if and only if the utility is given by a *CES*
The equilibrium consumption of each variety always decreases with $N$ because consumers spend their income over a wider range of varieties.
→ well-behaved utility functions may generate price-decreasing as well as price-increasing competition
\[ u_1(x) = 2\sqrt{x + 1} - 2 \]

the RLV increases with \( x \)

\[ u_2(x) = 2\sqrt{x} + x \]

the RLV decreases with \( x \)
How can we interpret a VES within the Lancasterian framework of product differentiation?
The long-run equilibrium

the zero-profit condition $\rightarrow (\bar{p} - c)L\bar{x} = f$

• Proposition 2. Every symmetric long-run equilibrium satisfy the following two conditions:

$$\bar{M} = ru \left[ \frac{1}{c\bar{L}} \left( \frac{1}{\bar{M}} - 1 \right) \right] \quad \bar{N} = \bar{L}E\bar{M}$$
**Proposition 3.** The impact of the relative market size on the long-run equilibrium:

<table>
<thead>
<tr>
<th></th>
<th>$r'_u(x) &gt; 0$</th>
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<tbody>
<tr>
<td>$\mathcal{E}_{\bar{p}/\bar{L}} \equiv \frac{\bar{L}}{\bar{p}} \frac{d\bar{p}}{d\bar{L}}$</td>
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Proposition 4. The impact of the marginal cost on the long-run equilibrium:

<table>
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<tr>
<th>Term</th>
<th>Condition 1</th>
<th>Condition 2</th>
<th>Condition 3</th>
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<td>( r'_u(x) &gt; 0 )</td>
<td>( 0 &lt; \mathcal{E}_{\tilde{p}/c} &lt; 1 )</td>
<td>( \mathcal{E}_{\tilde{p}/c} = 1 )</td>
<td>( 1 &lt; \mathcal{E}_{\tilde{p}/c} )</td>
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<td>( \mathcal{E}_{\tilde{p}/c} \equiv \frac{c}{\tilde{p}} \frac{d\tilde{p}}{dc} )</td>
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• For any symmetric long-run equilibrium, there exists a CES model that yields the same market outcome. However, to estimate a model with cross-section or panel dataset, data heterogeneity stemming from variations in the underlying structural parameters is needed.
• Once we allow for such variations, Proposition 3 tells us that the corresponding elasticity of substitution also changes. This has the following implication: *it is likely to be meaningless to assume that the elasticity of substitution is the same across space and/or time*
Multi-sector economy

\[
\max \mathcal{U} \equiv U(X, A) = U\left[ \int_0^N u(x_i) di, A \right]
\]
• Proposition 5. In a two-sector economy, the long-run equilibrium price, consumption and production vary with market size and cost parameters as in Proposition 3. Furthermore, if

\[ 0 \leq \frac{p}{E} \cdot \frac{\partial E}{\partial p} < 1 \quad \frac{N}{E} \cdot \frac{\partial E}{\partial N} < 1 \]

holds, the equilibrium mass of varieties increases with the market size.
Cost heterogeneity
\[
\max_{x_c(.)} \mathcal{U} \equiv N \int_0^c u(x_c) d\Gamma(c) \quad \text{s.t.} \quad N \int_0^c p_c x_c d\Gamma(c) = 1
\]

\[
\bar{M}_c = r_u(\bar{x}_c) = \frac{1}{\sigma(\bar{x}_c)}
\]

a firm's markup **increases** (decreases) with its degree of efficiency when the RLV is **increasing** (decreasing)
Proposition 6. When firms are heterogeneous in cost, the long-run equilibrium cutoff cost and prices vary with the relative market size as follows:

<table>
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<tr>
<th>Condition</th>
<th>$r_u'(x) &gt; 0$</th>
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<td>$c_{\tilde{L}} \equiv \frac{\tilde{L}}{\bar{c}} \frac{d\bar{c}}{d\tilde{L}}$</td>
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<td>$p_{c\tilde{L}} \equiv \frac{\tilde{L}}{\bar{p}_c} \frac{d\bar{p}_c}{d\tilde{L}}$</td>
<td>$\forall c \leq \bar{c}$</td>
<td>$\mathcal{E}<em>{p</em>{c\tilde{L}}} &lt; 0$</td>
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Quality heterogeneity
\[
\max_{x_s(\cdot)} \mathcal{U} \equiv \int_S u(sx_s) d\Psi(s) \quad \text{s.t.} \quad \int_S p_s x_s d\Psi(s) = 1
\]

**quality-adjusted cost**

\[
c(s) = \frac{C(s)}{s}
\]
Proposition 7. When firms are heterogeneous in cost and quality, we have:

| $c'(s) > 0$ | $\bar{c}(s) > 0$ | $\bar{M}(s)$ ↓ | $\mathcal{E}_{\bar{M}(s)/\bar{L}} < 0$ |
| $c'(s) < 0$ | $\bar{c}(s) < 0$ | $\bar{M}(s)$ ↑ | $\mathcal{E}_{\bar{M}(s)/\bar{L}} < 0$ |

$\Rightarrow \tilde{S} = [s_{\text{min}}, \bar{s}]$  \hspace{1cm} $\mathcal{E}_{\tilde{s}/\bar{L}} < 0$  \hspace{1cm} $\mathcal{E}_{\bar{c}/\bar{L}} < 0$

$\Rightarrow \tilde{S} = [\bar{s}, s_{\text{max}}]$  \hspace{1cm} $\mathcal{E}_{\tilde{s}/\bar{L}} > 0$  \hspace{1cm} $\mathcal{E}_{\bar{c}/\bar{L}} < 0$
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The average efficiency corrected for quality increases in the price-decreasing case and decreases in the price-increasing one.
When \( C(s) \) is strictly convex (concave), firms with higher (lower) quality charge lower (higher) markups in the price-decreasing case, but higher (lower) markups in the price-increasing case.
• The above proposition does not support the widespread idea that developed countries should necessarily aim to produce high-quality products to insulate their workers from competition with emerging countries. What matters for firms to survive on the international marketplace is the level of their quality-adjusted cost within the product range
Quadratic utility
\[ u(x_i, X) = x_i - \frac{\gamma}{2} x_i^2 - x_i X \]

The demand for a variety now depends on two aggregate statistics: \( \lambda \) and \( X \)

\[ r_u = \frac{\gamma x_i}{1 - \gamma x_i - X} \]

which increases with \( x_i \)
\[ \bar{p}_1(N) = \frac{(1+c)N + 2\gamma + \sqrt{(1-c)^2 N^2 + 4\gamma N + 4\gamma^2}}{2N} \]

- **OTT**: quadratic nested into a quasi-linear utility

\[ \bar{p}_2(N) = \frac{\gamma}{2\gamma + N} + \frac{\gamma + N}{2\gamma + N} c \]

- both prices decrease with \( N \)
  \[ \rightarrow \text{price-decreasing competition} \]
An application to international pricing
• **Proposition 8.** *In the long-run equilibrium with two-way trade, firms’ pricing is described as follows:*

<table>
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<td><strong>Dumping</strong></td>
<td></td>
<td></td>
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<tr>
<td>$p^{HF} &lt; \tau p^{HH}$</td>
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Concluding remarks

1. “Minor” changes in the utility may have “major” consequences

2. Tweaking the CES is not the best strategy

3. What about non-additive preferences

4. There is plenty of food for thought
Thank you for your attention