Welfare Comparisons and Equivalence Scales

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Equivalence scales are used in both demand and welfare analysis. In demand analysis they permit us to pool data from households of different sizes, or, more generally, with different demographic profiles. In welfare analysis, they enable us to compare the well-being of such households, since they purport to answer questions of the form: "What expenditure level would make a family with three children as well off as it would be with two children and \$12,000?" Such welfare comparisons are generally thought to provide the rationale for different treatments of different family types in income tax or family allowance schedues, or in income maintenance programs.

In this paper we argue that the equivalence scales required for welfare comparisons are logically distinct from those which arise in demand analysis. The usual practice is to base welfare comparisons on equivalence scales estimated from observed differences in the consumption patterns of households with different numbers of children. This is illegitimate. The expenditure level required to make a three-child family as well off as it would be with two children and \$12,000 depends on how the family feels about children. Observed differences in the consumption patterns of two- and three-child families cannot even tell us whether the third child is regarded as a blessing or a curse.

In Section I we discuss the type of equivalence scale appropriate for demand analysis (conditional equivalence scales) and, in Section II, the type required to make welfare comparisons (unconditional equivalence scales). Conditional equivalence scales can be estimated from observed differences in the consumption patterns of households with different demographic profiles, but construction of unconditional equivalence scales requires more information than is contained in household consumption data. In Section III we discuss identification of a family's unconditional equivalence scale, while in Section IV we discuss the interpretation of welfare comparisons when families have different tastes. In Section V, the concluding section, we summarize our discussion of welfare comparisons of families with different demographic profiles and question whether such comparisons account for the widespread belief that different treatments of different family types are appropriate.

I. Demographic Variables in Demand Analysis: Conditional Equivalence Scales

In demand analysis, the "objects of choice" are consumption vectors X, and preferences over them depend on an assumed predetermined vector of demographic variables η; we call such a preference ordering "conditional." We denote the conditional preference ordering by $R(\eta)$ and interpret the statement "$X^a R(\eta^*) X^b$" to mean that the family finds X^a at least as good as X^b when its demographic profile is given by η^*. If each family takes its demographic profile as fixed when choosing its consumption pattern, demand analysis need never ask how it would choose between alternatives which differ with respect to the demographic variables; hence, conditional preferences are an appropriate foundation for demand analysis.[1]

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[1]By "family preferences" we mean the preferences of the adults in the family; preferences of children are ignored. For a family containing one adult, this notion of

In our 1978b paper, we examine a number of alternative ways to incorporate demographic variables into demand analysis by allowing some of the parameters of a demand system to be functions of demographic variables. These functions, which we call conditional equivalence scales, are usually estimated along with the parameters of the demand system by combining data from households with different demographic profiles. The alternatives to this procedure are (i) to analyze separately data from households with distinct demographic profiles or (ii) to combine data from households with different demographic profiles using conditional equivalence scales estimated from other data or specified a priori. The assumption that demand functions are independent of demographic variables, or that per capita consumption of each good is a function of per capita total expenditure, are examples of a priori specifications of conditional equivalence scales.

II. Demographic Variables in Welfare Analysis: Unconditional Equivalence Scales

In contrast to demand analysis, welfare analysis must compare the well-being of a family in alternative situations which differ with respect to its demographic profile as well as its consumption pattern. For example, we might ask whether a family with given tastes would prefer to have two children and \$12,000 or three children and \$13,000 at a particular set of goods' prices. The traditional approach to welfare comparisons ignores the fact that such comparisons cannot be based on conditional preferences but requires a conceptual framework in which preferences are defined over family size as well as goods.

family preferences is unambiguous, but for a family containing two adults, there is an aggregation problem unless the adults' preferences happen to coincide. We ignore this and assume that the notion of family preferences is well-defined. Basing welfare comparisons on family preferences means that we can only compare demographic profiles whose adult compositions are identical; thus, we cannot compare the welfare of a family consisting of one adult and two children with that of a family of two adults and two children.

In general, welfare analysis requires us to define the objects of choice to include not only the consumption vector, but also the demographic variables, which from the standpoint of conditional preferences are predetermined. We call an ordering over such an augmented set of alternatives an "unconditional preference ordering," and denote it by R: the statement "$(X^a, \eta^a)\ R\ (X^b, \eta^b)$" means that the family finds (X^a, η^a) at least as good as (X^b, η^b). The additional information contained in the unconditional preference ordering is irrelevant for demand analysis, but for welfare comparisons, it is indispensable.

Unconditional equivalence scales are index numbers which reflect the ratio of the expenditures required to attain a particular indifference curve under alternative demographic profiles.[2] Corresponding to the unconditional preference ordering R, we define the "unconditional expenditure function," $E[P, \eta, (P^o, \mu^o, \eta^o)]$, whose value is the minimum expenditure required to reach the indifference curve attained in the price-expenditure-demographic situation (P^o, μ^o, η^o), when the household faces prices P with the demographic profile η. The unconditional equivalence scale $I[(P^a, \eta^a), (P^b, \eta^b), (P^o, \mu^o, \eta^o)]$, is defined by

$$I[(P^a, \eta^a), (P^b, \eta^b), (P^o, \mu^o, \eta^o)] = \frac{E[P^a, \eta^a, (P^o, \mu^o, \eta^o)]}{E[P^b, \eta^b, (P^o, \mu^o, \eta^o)]} \tag{1}$$

If we let the base indifference curve correspond to the "reference situation" (P^b, μ^b, η^b), then the denominator is μ^b and the index is equal to $E[P^a, \eta^a, (P^b, \mu^b, \eta^b)]/\mu^b$. In our

[2]This corresponds to John Muellbauer's definition of equivalence scales as "budget deflators which are used to calculate the relative amounts of money two different types of households require in order to reach the same standard of living." However Muellbauer uses what we have called conditional equivalence scales to make welfare comparisons, and his paper provides numerous references to other studies which do so. We contend that such an approach is not valid because unconditional equivalence scales rather than conditional equivalence scales are required for welfare comparisons. Our objection to the use of conditional equivalence scales in welfare analysis does not depend on whether families can or do regulate their fertility.

example, such an index would show the percentage expenditure adjustment which would enable a family with three children to attain the same indifference curve it would attain with two children and \$12,000.[3]

We illustrate this with an unconditional preference ordering which is consistent with the familiar linear expenditure system (*LES*) conditional demand functions. Consider the direct utility function

$$(2)\quad W(X,\eta) = \prod_{k=1}^{n} (x_k - b_k^* - \beta_k \eta)^{a_k} + \phi(\eta); \quad \sum a_k = 1, \quad x_i - b_i^* - \beta_i \eta > 0$$

where η is the number of children in the family; in some very informal sense the function $\phi(\eta)$ represents the "direct" contribution of children to family utility. Substituting the conditional demand functions into this direct utility function yields a "mixed" indirect utility function whose arguments are P, μ, and η:

$$(3)\quad V(P,\mu,\eta) = (\mu - \sum p_k b_k^* - \eta \sum p_k \beta_k) \cdot \Pi (p_k)^{-a_k} (a_k)^{a_k} + \phi(\eta)$$

Solving for μ yields the unconditional expenditure function

$$(4)\quad E(P,\mu,\eta,s_o) = \sum p_k b_k^* + \eta \sum p_k \beta_k + [s_o - \phi(\eta)] \Pi (p_k)^{a_k} (a_k)^{-a_k}$$

where s_o is the value of the utility function (2) evaluated at any point on the base indifference curve. To find the unconditional equivalence scale evaluated at the base indifference curve corresponding to (P^b, μ^b, η^b), we divide the unconditional expenditure function evaluated at $s_o = V(P^b, \mu^b, \eta^b)$ by μ^b. This yields[4]

$$(5)\quad I[(P^a,\eta^a),(P^b,\eta^b),(P^b,\mu^b,\eta^b)] = \{\sum p_k^a b_k^* + \eta \sum p_k^a \beta_k + [(\mu^b - \sum p_k^b b_k^* - \eta^b \sum p_k^b \beta_k) \Pi (p_k^b)^{-a_k} (a_k)^{a_k} + \phi(\eta^b) - \phi(\eta^a)] \Pi (p_k^a)^{a_k} (a_k)^{-a_k}\} / \mu^b$$

III. Identification of Unconditional Equivalence Scales

Since the unconditional equivalence scale corresponding to $W(X,\eta)$ depends on the function $\phi(\eta)$, we must estimate this function. But if we interpret our data in terms of conditional choices (i.e., choices in which the number of children is taken to be fixed or predetermined) the function $\phi(\eta)$ is not identified.[5] All functions $\phi(\eta)$ imply the same conditional demand functions for goods, so information about how a family would reallocate its expenditure among consumption categories as the number of children varies is not sufficient to identify $\phi(\eta)$.

Of course if $\phi(\eta)$ were assumed to be a constant then it would not appear in (5) and the unconditional equivalence scale corresponding to $W(X,\eta)$ could be identified from conditional choices. This appears to be the assumption generally made, although not explicitly, in the literature on equivalence

[3]The conventional cost-of-living index holds the demographic profile fixed and compares the expenditure required to attain a particular indifference curve under alternative price regimes. Such an index can be interpreted as a "subindex" of the unconditional equivalence scale which is itself the "complete index." Pollak develops the theory of subindexes of the cost-of-living index. In this paper we are concerned with complete indexes, or at least with indexes complete enough to include the demographic variables. Subindexes (i.e., conventional cost of living indexes) can be constructed separately for each family type, but such indexes do not permit comparisons of families of different types.

[4]Notice that the unconditional preference ordering corresponding to the direct utility function $\hat{W}(X,\eta) = \Sigma a_k \log (x_k - b_k^* - \beta_k \eta) + \phi(\eta)$ is not the same as that corresponding to $W(X,\eta)$ and hence these two unconditional preference orderings yield distinct unconditional equivalence scales. However, both imply the same *LES* conditional demand functions and, hence, the same conditional equivalence scales.

[5]Whether a particular demographic variable should be treated as predetermined or an object of choice is not automatically resolved by the fact that the variable in question is controlled or chosen by the family, and, hence, could legitimately be treated as an object of choice. For purposes of demand analysis, it is useful to treat family size as predetermined and work with conditional demand functions. When we treat such choices as unconditional, estimation of (unconditional) preferences requires us to reconstruct the feasible set from which the choice was made. But estimation of unconditional preferences is a secondary issue for us. We are primarily concerned with drawing the distinction between conditional and unconditional preferences and arguing that the latter are required for welfare comparisons.

scales and welfare comparisons. However, this assumption has grossly implausible implications for unconditional preferences and unconditional choices involving family size. In particular, consider a "perfect contraceptive society"—one in which there are no economic costs or preference drawbacks associated with fertility regulation. If $\phi(\eta)$ is a constant and $\Sigma p_k \beta_k$ is positive, then the family will have no children; if $\phi(\eta)$ is a constant and $\Sigma p_k \beta_k$ is negative, the family will have as many children as it can. This follows immediately from the fact that when $\phi(\eta)$ is a constant the utility function (3) depends linearly on η.

Another illustration of the counterintuitive results that may occur when we make the transition from household consumption patterns to welfare conclusions by assuming that $\phi(\eta)$ is a constant is provided by the linear expenditure system estimated by the authors (1978a) using *U.K.* household budget data. The estimated conditional demand functions exhibit reasonable price and expenditure elasticities, and reasonable consumption responses to changes in family size. The estimated β's, however, are all negative so $\Sigma p_k \beta_k < 0$. Hence, when $\phi(\eta)$ is assumed to be constant, the unconditional expenditure function decreases with η, and the corresponding unconditional equivalence scale implies that large families need less money than small families to attain any fixed indifference curve.

If unconditional preferences cannot be recovered from conditional demand functions, how can they be discovered? For some demographic variables information about unconditional preferences is revealed by observable choice behavior. For example, in advanced industrial societies where deliberate choice of completed family size is the rule rather than the exception, an argument can be made for treating the observed consumption-family size configurations as observable unconditional choices, using them to infer unconditional preferences, and using these preferences to make welfare comparisons. Thus, in a perfect contraceptive society, if a family chooses to have three children and \$12,000 when it could have had two children and \$12,000, then a revealed preference argument implies that the family prefers the alternative it chose.[6] Other demographic variables (say, race) are not susceptible to deliberate control, while still others (say, the sex of a family's first child) may be moving from the uncontrollable to the controllable category. Unconditional preferences for demographic variables might also be obtained by analyzing responses to direct questions about preferences or hypothetical choices, although economists have traditionally been suspicious of this approach.[7]

IV. Welfare Comparisons with Taste Differences

Taste differences—that is, differences in families' unconditional preferences—substantially complicate welfare comparisons. There are two approaches. The first is to select a particular unconditional preference ordering as the appropriate base for welfare comparisons and proceed as before. The selection is trivial if a particular preference ordering is obviously appropriate as when all families have identical unconditional preferences. It is especially troublesome when systematic differences in preferences are associated with systematic differences in the demographic variables, as is the case with family size or other demographic variables over which families exercise partial or complete control. For some demographic variables it may be plausible to assume that families with different demographic profiles have the same unconditional preferences, or more precisely, that the distribution of unconditional preferences is independent of the distribution of demographic characteristics. But for demographic variables over which families exercise some deliberate control, this independence assumption is clearly unwarranted.

Suppose, for example, that some families

[6]Multiple births create special problems which we ignore. We interpret the \$12,000 as total expenditure on goods and ignore both the labor-leisure choice and the dependence of taxes on demographic variables.

[7]For an example of an equivalence scale constructed from responses to a questionnaire asking individuals what income level corresponds to such verbal evaluations as "good," "sufficient," "bad," etc., see Arie Kapteyn and Bernard van Praag.

have a strong desire for children while others have a weak desire for children. Then the expenditure required to make a family with three children as well off as it would be with two children and $12,000 depends on which unconditional preference ordering it has. Hence, the unconditional equivalence scale depends on which of the two unconditional preference orderings we select as the base. But neither selection compares the welfare levels of families with different tastes. Instead, they compare two situations (for example, three children, $13,000 vs. two children, $12,000) on the basis of a particular preference ordering—whichever one selected is the appropriate base for the comparison.

The second approach to welfare comparisons requires interpersonal (interfamily) comparisons of happiness or satisfaction. Technically, we need a mapping which associates with each indifference curve from one unconditional indifference map a corresponding curve on the other, so that the corresponding curves represent the same levels of happiness or satisfaction. Only if such a correspondence exists can we compare the welfare of families with different tastes in alternative situations—for example, strong desire for children, three children, $13,000 vs. weak desire for children, two children, $12,000.

V. Conclusion

The implications of our analysis of welfare comparisons and equivalence scales should be stated explicitly: 1) Even if all families have identical unconditional preferences, conditional equivalence scales estimated from observed differences in the consumption patterns of families with different demograpic profiles cannot be used to make welfare comparisons; for example, we cannot use such data to determine the amount needed to make families with three children as well off as those with two children and $12,000. Unconditional equivalence scales are required to make welfare comparisons. 2) If tastes vary systematically with demographic characteristics, then the construction of unconditional equivalence scales requires the selection of an appropriate base unconditional preference ordering; theory offers little guidance in making this selection, but there is no selection which permits us to compare the welfare of a family with a strong desire for children with that of one with a weak desire for children. Such comparisons require interpersonal or interfamily comparisons of welfare levels. The question of whether such comparisons are meaningful, and if so, how they can be made, is beyond the scope of this paper.

Our analysis suggests that it is very difficult to make welfare comparisons between families with different demographic profiles. But are comparisons of this sort the principal basis of the widespread belief that it is appropriate to treat different family types differently in income tax or family allowance schedules or in income maintenance programs? We think not. For example, differences in treatment might be justified in terms of effects on the children's present or future welfare, the effects on the children's future productivity, or the effect on the family's fertility.[8] Our analysis implies that differences in treatment cannot easily be justified by an appeal to equity or fairness if this is interpreted in terms of "family preferences" (i.e., the welfare of the adult members of the family). But the arguments one would advance to justify providing children in large families with consumption levels which society somehow establishes to be "socially adequate" are very different from those one would advance for making the adults in large and small families equally well-off. The problem of defining socially adequate consumption levels is a difficult one which has received virtually no attention from economists, in part because of the profession's unfortunate preoccupation with welfare comparisons and equivalence scales.[9]

[8]The relevance of these considerations and of welfare comparisons will vary from one policy question to another.

[9]There is no reason to think that conditional equivalence scales have any role to play in the determination of socially adequate consumption levels.

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