Tax Me If You Can!
Optimal Nonlinear Income Tax between Competing Governments*

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Abstract

We investigate how potential tax-driven migrations modify the Mirrlees income tax schedule when two countries play Nash. The social objective is the maximin and preferences are quasilinear in income. Individuals differ both in skills and migration costs, which are continuously distributed. We derive the optimal marginal income tax rates at the equilibrium, extending the Diamond-Saez formula. The theory and numerical simulations on the US case show that the level and the slope of the semi-elasticity of migration on which we lack empirical evidence are crucial to derive the shape of optimal marginal income tax. Our simulations show that potential migrations result in a welfare drop between 0.4% and 5.3% for the worst-off and an average gain between 18.9% and 29.3% for the top 1%.

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I Introduction

The globalization process has not only made the mobility of capital easier. The transmission of ideas, meanings and values across national borders associated with the decrease in transportation costs has also reduced the barriers to international labor mobility. In this context, individuals are more likely to vote with their feet in response to high income taxes. This is in particular the case for highly skilled workers, as recently emphasized by Liebig, Puhani, and Sousa-Poza (2007) across Swiss cantons as well as by Kleven, Landais, and Saez (2013) and Kleven, Landais, Saez, and Schultz (2013) across European countries. Consequently, the possibility of tax-driven migrations appears as an important policy issue and must be taken into account as a salient constraint when thinking about the design of taxes and benefits affecting households.

The goal of this article is to cast light on this issue from the viewpoint of optimal tax theory. We investigate in what respects potential migrations affect the nonlinear income tax schedules that competing governments find optimal to implement in a Nash equilibrium. For this purpose, we consider the archetypal case of two countries between which individuals are free to move. We extend the model of Mirrlees (1971) to this setting and highlight the impact of potential migrations both analytically and through numerical simulations for the US economy. By assumption, taxes can only be conditioned on income and are levied according to the residence principle.

To represent migration responses to taxation in a realistic way, we introduce a distribution of migration costs at each skill level. Hence, every individual is characterized by three characteristics: her birthplace, her skill and the cost she would incur in case of migration, the last two being private information. As emphasized by Borjas (1999), “the migration costs probably vary among persons [but] the sign of the correlation between costs and (skills) is ambiguous”. This is why we do not make any assumption on the correlation between skills and migration costs. Individuals make decisions along two margins. The choice of taxable income operates on the intensive margin, whereas the location choice operates on the extensive margin. In accordance with Hicks’s idea, an individual decides to move abroad if her indirect utility in her home country is lower than her utility abroad net of her migration costs. To make the analysis more transparent, we assume away income effects on labor supply as in Diamond (1998) and consider the most redistributive social objective (maximin\(^1\)). By focusing on the maximin, we obtain the upper bound of the effects of migration.

Because of the combination of asymmetric information and potential migration, each government has to solve a self-selection problem with random participation à la Rochet and Stole (2002). Intuitively, each government faces a trade-off between three conflicting objectives: (i) redistributing incomes to achieve a fairer allocation of resources; (ii) limiting

\(^1\text{See Boadway and Jacquet (2008) for a study of the optimal tax scheme under the maximin in the absence of individual mobility.}\)
the variations of the tax liability with income to reduce marginal tax rates, thereby prevent distortions along the intensive margin; (iii) minimizing the distortions along the extensive margin to avoid a too large leakage of taxpayers. An additional term appears in the optimal marginal tax rate formula to take the third objective into account. This term depends on the semi-elasticity of migration, defined as the percentage change in the mass of taxpayers of a given skill level when their consumption is increased by one unit. Our main message is that the shape of the tax function depends on the slope of the semi-elasticity, which cannot be deduced from the slope of the elasticity. Our theoretical analysis calls for a change of focus in the empirical analysis: in an open economy, if one wants to say something about the shape of tax function, one needs to estimate the profile of the semi-elasticity of migration with respect to earning capacities. We now articulate this main message with the main findings of the paper.

We first characterize the best-response of each policymaker and obtain a simple formula for the optimal marginal tax rates. The usual optimal tax formula obtained by Diamond (1998), Piketty (1997), Saez (2001) for a closed economy is augmented by a “migration effect”. When the marginal tax rates are slightly increased on some income interval, everyone with larger income faces a lump-sum increase in taxes. This reduces the number of taxpayers in the given country. The magnitude of this new effect is proportional to the semi-elasticity of migration.

Second, we provide a full characterization of the overall shape of the tax function. When the semi-elasticity of migration is constant along the skill distribution, the tax function is increasing. This situation is for example obtained in a symmetric equilibrium when skills and migration costs are independently distributed, as assumed by Morelli, Yang, and Ye (2012) and Blumkin, Sadka, and Shem-Tov (2012). A similar profile is obtained when the semi-elasticity of migration is decreasing in skills, because for example of a constant elasticity of migration. When the semi-elasticity is increasing, the tax function may be either increasing, with positive marginal tax rates, or hump-shaped, with negative marginal tax rates in the upper part of the income distribution. A sufficient condition for the hump-shaped pattern is that the semi-elasticity becomes arbitrarily large in the upper part of the skill distribution. If this is the case, progressivity of the optimal tax schedule does not only collapse because of tax competition; the tax liability itself becomes strictly decreasing. There are then “middle-skilled” individuals who pay higher taxes than top-income earners. A situation that can be seen as a “curse of the middle-skilled” (Simula and Trannoy, 2010).

Third, we show that the slope is as important as the level of the semi-elasticity, even when one focuses on the upper part of the income distribution. To make this point, we consider three numerical approximations of the US economy which only differ by the profile of their migration responses. We calibrate the three of them in such a way that the average elasticity of migration within the top percentile is the same. We take this number from the study by Kleven, Landais, Saez, and Schultz (2013). However, we consider three different
plausible scenarios for the slope of the semi-elasticity. We obtain dramatically different optimal tax schedules. For example, an agent earning 2 millions of USD per year faces an average optimal tax rate of about 64% in the scenario with a decreasing semi-elasticity, 53% in the scenario with a constant semi-elasticity and 48% in the scenario with an increasing elasticity. In this latter scenario, the marginal tax rates become negative above 3 millions of annual income, so that the richest people do not pay the highest taxes, in absolute levels. Moreover, there are substantial welfare changes. The reduction in the well-being of the poorest ranges between 1% and 6% depending on the slope of the semi-elasticity. In contrast, the threat of migration increases the average welfare gain of the last centile by about 20% in all scenarios.

The article is organized as follows. Section II reviews the literature which is related to this paper. Section III sets up the model. Section IV derives the optimal tax formula in the Nash equilibrium. Section V shows how to sign the optimal marginal tax rates and provides some further analytical characterization of the whole tax function. Section VI uses numerical simulations to investigate the sensitivity of the tax function to the slope of the semi-elasticity of migration. Section VII concludes.

II Related Literature

In Mirrlees’s (1971) seminal paper, migrations are supposed to be impossible. However, Mirrlees emphasizes that this is an assumption one would rather not make because the threat of migration has probably a major influence on the degree of progressivity of actual tax systems. This is why Mirrlees (1982) considers the case where individuals choose to live in either of two regions. However, in this paper, incomes are exogenous. Wilson (1980,1982) allows for labor choices and derives a few general properties of the optimal tax schedules in an open economy. Osmundsen (1999) is the first to examine income taxation with type-dependent outside options. He studies how highly skilled individuals distribute their working time between two countries. However, there is no individual trade-off between consumption and effort along the intensive margin. Leite-Monteiro (1997) consider the case with differentiated lump-sum taxes. Huber (1999), Hamilton and Pestieau (2005), Piácer (2007) and Lipatov and Weichenrieder (2012) consider tax competition on nonlinear income tax schedules in the two-type model of Stiglitz (1982). However, the two-type setting rules out by assumption the possibility of countervailing incentives. This is one of the reason why Morelli, Yang, and Ye (2012) and Bierbrauer, Brett, and Weymark (2013) consider more than two types. Brewer, Saez, and Shephard (2010), Simula and Trannoy (2010, 2011) and Blumkin, Sadka, and Sham-Tov (2012) consider tax competition over nonlinear income tax schedules in a model with a continuous skill distribution. Thanks to the continuous population, it is possible to characterize and quantify the full income tax schedule. Brewer, Saez, and Shephard (2010) find that top marginal tax rates
should be strictly positive under a Pareto unbounded skill distribution and derive a simple formula to compute them. In contrast, Blumkin, Sadka, and Shem-Tov (2012) find that top marginal tax rates should be zero. This is because the first paper assumes that the elasticity of migration is constant in the upper part of the income distribution. This implies that the semi-elasticity is decreasing. Blumkin, Sadka, and Shem-Tov (2012) conversely assume that the skills and migration costs are independently distributed. This implies that the semi-elasticity of migration is constant and, thus, that the asymptotic elasticity of migration is infinite. So, the asymptotic marginal tax rate is zero. It is also the case in the framework considered by Bierbrauer, Brett, and Weymark (2013). Two utilitarian governments compete when labor is perfectly mobile whatever the skill level. They show that there does not exist equilibria in which individuals with the highest skill pay positive tax payments to either country. Finally, Simula and Trannoy (2010, 2011) assume that there is a single level of migration cost per skill level. There is thus a skill level below which the semi-elasticity of migration is zero and above which it is infinite. This is the reason why Simula and Trannoy (2010) find that marginal tax rates may be negative in the upper part of the income distribution. The present paper proposes a general framework that encompasses most of these previous studies.

III Model

We consider an economy consisting of two countries, indexed by \( i = A, B \). The same constant-return to scales technology is available in both countries. Each worker is characterized by three characteristics: her native country \( i \in \{ A, B \} \), her productivity (or skill) \( w \in [w_0, w_1] \), and the migration cost \( m \in \mathbb{R}^+ \) she supports if she decides to live abroad. Note that \( w_1 \) may be either finite or infinite and \( w_0 \) is non-negative. In addition, the empirical evidence that some people are immobile is captured by the possibility of infinitely large migration costs.\(^2\)

The migration cost corresponds to a loss in utility, due to various material and psychic costs of moving: application fees, transportation of persons and household’s goods, forgone earnings, costs of speaking a different language and adapting to another culture, costs of leaving one’s family and friends, etc.\(^3\) We do not make any restriction on the correlation between skills and migration costs. We simply consider that there is a distribution of migration costs for each possible skill level.

We denote by \( h_i(w) \) the continuous skill density in country \( i = A, B \) by \( H_i(w) \equiv \int_{w_0}^{w} h_i(x) \, dx \) the corresponding cumulative distribution function (CDF) and by \( N_i \) the size of the population. The size of \( t \) For each skill \( w \), \( g_i(m \mid w) \) denotes the conditional density of the migration cost and \( G_i(m \mid w) \equiv \int_{0}^{m} g_i(x \mid w) \, dx \) the conditional CDF. The initial joint density of \((m, w)\) is thus \( g_i(m \mid w) h_i(w) \) whilst \( G_i(m \mid w) h_i(w) \) is the mass of individuals of

\(^2\)We could assume that \( m \in [0, \bar{m}] \) but this would only complexify the analysis without changing the main insights.

\(^3\)Alternatively, the cost of migration can be regarded as the costs incurred by cross-border commuters, who still reside in their home country but work across the border.
skill $w$ with migration costs lower than $m$. Moreover, it is constrained to treat native and immigrant workers in the same way. Therefore, it can only condition transfers on earnings $y$ through an income tax function $T_i(\cdot)$. It is unable to base the tax on an individual’s skill level $w$, migration cost $m$, or native country.

**Individual Choices**

Every worker derives utility from consumption $c$, and disutility from effort and migration, if any. Effort captures the quantity as well as the intensity of labor supply. The choice of effort corresponds to an intensive margin and the migration choice to an extensive margin.

Let $v(y; w)$ be the disutility of a worker of skill $w$ to obtain pre-tax earnings $y \geq 0$ with $v'_y > 0 > v'_w$ and $v''_y > 0 > v''_w$. Let $\mathbb{1}$ be equal to 1 if she decides to migrate, and to zero otherwise. Individual preferences are described by the quasi-linear utility function:

$$c - v(y; w) - \mathbb{1} \cdot m.$$  

Note that the Spence-Mirrlees single-crossing condition holds because $v''_{yw} < 0$. The quasi-linearity in consumption implies that there is no income effect on taxable income and appears as a reasonable approximation. For example, Gruber and Saez (2002) estimate both income and substitution effects in the case of reported incomes, and find small and insignificant income effects. The cost of migration is introduced in the model as a monetary loss.

**Intensive Margin**

We focus on income tax competition under the residence principle. Everyone living in country $i$ is liable to an income tax $T_i(\cdot)$, which is solely based on earnings $y \geq 0$, and thus in particular independent of the native country. Because of the separability of the migration costs, two individuals living in the same country and having the same skill level choose the same gross income/consumption bundle, irrespective of their native country. Hence, a worker of skill $w$, who has chosen to work in country $i$, solves:

$$U_i(w) \equiv \max_y y - T_i(y) - v(y; w).$$  

We call $U_i(w)$ the gross utility of a worker of skill $w$ in country $i$. It is the net utility level for a native and the utility level absent migration cost for an immigrant. We call $Y_i(w)$ the solution to program (2) and $C_i(w) = Y_i(w) - T(Y_i(w))$ the consumption level of a worker of skill $w$ in country $i$. The first-order condition can be written as:

$$1 - T'_i(Y_i(w)) = v'_y(Y_i(w); w).$$  

---

4In several countries, highly skilled foreigners are eligible to specific tax cuts for a limited time duration. This is for example the case in Sweden and in Denmark. These exemptions are temporary.

5If (2) admits more than one solution, we make the tie-breaking assumption that individuals choose the one preferred by the government.
Differentiating (3), we obtain the elasticity of gross earnings with respect to the retention rate $1 - T'_i$,

$$
\epsilon_i (w) = \frac{1 - T'_i (Y_i(w))}{Y_i(w)} \frac{\partial Y_i(w)}{\partial (1 - T'_i (Y_i(w)))} = \frac{v'_y (Y_i(w); w)}{Y_i(w) v''_{yy}(Y_i(w); w)},
$$

and the elasticity of gross earnings with respect to productivity $w$:

$$
\kappa_i (w) = \frac{w}{Y_i(w)} \frac{\partial Y_i(w)}{\partial w} = - \frac{w v''_y (Y_i(w); w)}{Y_i(w) v''_{yy}(Y_i(w); w)}.
$$

**Migration Decisions**

A native of country $A$ of type $(w,m)$ gets utility $U_A(w)$ if she stays in $A$ and utility $U_B(w) - m$ if she relocates to $B$. She therefore emigrates if and only if: $m < U_B(w) - U_A(w)$.

Hence, among individuals of skill $w$ born in country $A$, the mass of emigrants is given by $G_A (U_B(w) - U_A(w)|w) h_A(w) N_A$ and the mass of agents staying in their native country by $(1 - G_A (U_B(w) - U_A(w)|w)) h_A(w) N_A$. Natives of country $B$ behave in a symmetric way.

Combining the migration decisions made by agents born in the two countries, we see that the mass of residents of skill $w$ in country $A$, denoted $\varphi_A (U_A(w) - U_B(w); w)$, depends on the difference in the gross utility levels $\Delta = U_A(w) - U_B(w)$, with:

$$
\varphi_i (\Delta; w) \equiv \begin{cases} 
  h_i(w) N_i + G_{-i}(\Delta|w) h_{-i}(w) N_{-i} & \text{when } \Delta \geq 0, \\
  (1 - G_{-i}(\Delta|w)) h_i(w) N_i & \text{when } \Delta \leq 0.
\end{cases}
$$

We impose the technical restriction that $g_A(0|w) h_A(w) N_A = g_B(0|w) h_B(w) N_B$ to ensure that $\varphi_i(\cdot; w)$ is differentiable. This restriction is automatically verified when $A$ and $B$ are symmetric or when there is a fixed cost of migration, implying $g_i(0|w) = 0$. We have:

$$
\frac{\partial \varphi_i(\cdot; w)}{\partial \Delta} = \begin{cases} 
  g_{-i}(\Delta|w) h_{-i}(w) N_{-i} & \text{when } \Delta \geq 0, \\
  g_i(-\Delta|w) h_i(w) N_i & \text{when } \Delta \leq 0.
\end{cases}
$$

Hence, $\varphi_i(\cdot; w)$ is increasing in the difference $\Delta$ in the gross utility levels. By symmetry, the mass of residents of skill $w$ in country $B$ is given by $\varphi_B (U_B(w) - U_A(w); w)$.

All the responses along the extensive margin can be summarized in terms of elasticity concepts. We define the semi-elasticity of migration in country $i$ as:

$$
\eta_i (\Delta_i(w); w) \equiv \frac{\partial \varphi(\Delta_i(w); w)}{\partial \Delta} \frac{1}{\varphi(\Delta_i(w); w)} \text{ with } \Delta_i(w) = U_i(w) - U_{-i}(w).
$$

Because of quasi-linearity in consumption, this semi-elasticity corresponds to the percentage change in the density of taxpayers with skill $w$ when their consumption $C_i(w)$ is increased at the margin. The elasticity of migration is defined as:

$$
\nu_i (\Delta_i(w); w) \equiv C_i(w) \times \eta(\Delta_i(w), w).
$$

**Governments**

In country $i = A,B$, a benevolent policymaker designs the tax system to maximize the welfare of the worst-off individuals. We chose a maximin criterion for several reasons. The
maximin tax policy is the most redistributive one, as it corresponds to an infinite aversion to income inequality. A first motivation is therefore to explore the domain of potential redistribution in the presence of tax competition. A second motivation is that in an open economy, there is no obvious way of specifying the set of agents whose welfare is to count (Blackorby, Bossert, and Donaldson 2005). The policymaker may care for the well-being of the natives, irrespective of their country of residence. Alternatively, it may only account for the well-being of the native taxpayers, or for that of all taxpayers irrespective of native country. As an economist, there is no reason to favor one of these criteria (Mirrlees 1982). In our framework and in a second-best setting, all these criteria are equivalent. This provides an additional reason for considering maximin governments. The budget constraint faced by country $i$’s government is:

$$\int_{w_0}^{w_1} T_i(Y(w)) \varphi_i(U_i(w) - U_{-i}(w); w) \, dw \geq E$$  \hspace{1cm} (9)$$

where $E \geq 0$ is an exogenous amount of public expenditures to finance.

### IV Optimal Tax Formula

Following Mirrlees (1971), the standard optimal income tax formula provides the optimal marginal tax rates that should be implemented in a closed economy (e.g., Atkinson and Stiglitz 1980, Diamond 1998, Saez 2001). From another perspective, these rates can also be seen as those that should be implemented by a supranational organization (“world welfare point of view” (Wilson 1982a) or in the presence of tax cooperation. In this section, we derive the optimal marginal tax rates when policymakers compete on a common pool of taxpayers. We investigate in which way this formula differs from the standard one.

**Best Responses**

We start with the characterization of each policymaker’s best response. Because a taxpayer interacts with only one policymaker at the same time, it is easy to show that the standard taxation principle holds. Hence, it is equivalent to choose a non-linear income tax, taking individual choices into account, or to directly select an allocation allocation satisfying the usual incentive-compatible constraints $C_i(w) - v(Y_i(w); w) \geq C_i(x) - v(Y_i(x); w)$ for every $(w, x) \in [w_0, w_1]^2$. Due to the single-crossing condition, these constraints are equivalent to:

$$U'_i(w) = -v'_w(Y_i(w); w),$$

$$Y_i(\cdot) \text{ non-decreasing.}$$

The best-response allocation of government $i$ to government $-i$ is therefore solution to:

$$\max_{U_i(w), Y_i(w)} U_i(w_0) \quad \text{s.t.} \quad U'_i(w) = -v'_w(Y_i(w); w) \quad \text{and} \quad \int_{w_0}^{w_1} (Y_i(w) - v(Y_i(w); w) - U_i(w)) \varphi_i(U_i(w) - U_{-i}(w); w) \, dw \geq E.$$
in which $U_{-i}(.)$ is given. To save on notations, we from now on drop the $i$-subscripts and denote the skill density of taxpayers and the semi-elasticity in the Nash equilibrium by $f^*(w) = q_i(U_i(w) - U_{-i}(w); w)$ and $\eta^*(w) = \eta_i(U_i(w) - U_{-i}(w); w)$ respectively.

### Nash Equilibria

In Appendix A.1 we derive the first-order conditions for (12) and rearrange them to obtain a characterization of the optimal marginal tax rates in a Nash equilibrium. We below provide an intuitive derivation based on the analysis of the effects of a small tax reform perturbation around the equilibrium.

**Proposition 1.** In a Nash equilibrium, the optimal marginal tax rates are:

$$
\frac{T'(Y(w))}{1 - T'(Y(w))} = \frac{\alpha(w)}{\varepsilon(w)} \frac{X(w)}{w f^*(w)},
$$

with

$$
X(w) = \int_w^{\hat{w}} [1 - \eta^*(x) T(Y(x))] f^*(x) \, dx.
$$

Our optimal tax formula (13) differs from the one derived by Piketty (1997), Diamond (1998) and Saez (2001) for a closed economy in two ways: on the one hand, the mass of taxpayers $f^*(\cdot)$ naturally replaces the initial density of skills and, on the other hand, $\eta^*(\cdot) T(Y(\cdot))$ appears in the expectation term $X(w)$.

Proposition 1 and all other results hold in the absence of symmetry. The symmetric case where the two countries are identical $(N_A = N_B, h_A(\cdot) = h_B(\cdot) = h(\cdot)$ and $g_A(\cdot|w) = g_B(\cdot|w) = g(\cdot|w))$ is however particularly interesting. Indeed, both countries then implement the same policy, which implies $U_A(w) = U_B(w)$. Then, in the equilibrium, no one actually moves but the tax policies differ from the closed-economy ones because of the threat of migration. The skill density of taxpayers $f^*(\cdot)$ is therefore equal to the exogenous skill density $h(\cdot)$ whilst (7) implies that the semi-elasticity of migration reduces to the structural parameter $g(0|\cdot)$. Obviously, if $g(0|w) \equiv 0$ for all skill levels, the optimal fiscal policy coincides with the optimal tax policy in a closed economy. For instance, this is the case when migration costs include a fixed-cost component. However, in practice, countries are asymmetric and the semi-elasticity is positive as long as the difference in utility in the two countries is larger than the lower bound of the support of the distribution of migration costs. The main difference is that for asymmetric countries the mass of taxpayers $f^*(\cdot)$ and the semi-elasticity of migration $\eta^*(\cdot)$ are both endogenous.

### Interpretation

We now give an intuitive proof which in particular clarifies the economic interpretation of $X(w)$. To this aim, we investigate the effects of a small tax reform in a unilaterally-deviating country: the marginal tax rate $T'(Y(w))$ is uniformly increased by a small amount $\Delta$ on a small interval $[Y(w) - \delta, Y(w)]$ as shown in Figure 1. Hence, tax liabilities above $Y(w)$ are uniformly increased by $\Delta \delta$. This gives rise to the following effects.
First, an agent with earnings in \([Y_i(w) - \delta, Y_i(w)]\) responds to the rise in the marginal tax rate by a substitution effect. From (4), the latter reduces her taxable income by:

\[
dY(w) = \frac{Y(w)}{1 - T'(Y(w))} \epsilon(w) \Delta.
\]

This decreases the taxes she pays by an amount:

\[
dT(Y(w)) = T'(Y(w)) dY(w) = \frac{T'(Y(w))}{1 - T'(Y(w))} Y(w) \epsilon(w) \Delta.
\]

Taxpayers with income in \([Y_i(w) - \delta, Y_i(w)]\) have a skill level within the interval \([w - \delta_w, w]\) of the skill distribution. From [5], the widths \(\delta\) and \(\delta_w\) of the two intervals are related through:

\[
\delta_w = \frac{w}{Y(w)} \frac{1}{\alpha(w)} \delta.
\]

The mass of taxpayers whose earnings are in the interval \([Y_i(w) - \delta, Y_i(w)]\) being \(\delta_w f^*(w)\), the total substitution effect is equal to:

\[
dT(Y(w)) \delta_w f^*(w) = \frac{T'(Y(w))}{1 - T'(Y(w))} \frac{\epsilon(w)}{\alpha(w)} w f^*(w) \Delta \delta.
\]

Second, every individual with skill \(x\) above \(w\) faces a lump-sum increase \(\Delta \delta\) in her tax liability. In the absence of migration responses, this mechanically increases collected taxes from those \(x\)-individuals by \(f^*(x) \Delta \delta\). This is referred to as the “mechanical” effect in the literature. However, an additional effect takes place in the present open-economy setting. The reason is that the unilateral rise in tax liability reduces the gross utility in the deviating country, compared to its competitor. Consequently, the number of emigrants increases or the number of immigrants decreases. From [7], the number of taxpayers with skill \(x\) decreases by \(\eta^*(x) f^*(x) \Delta \delta\), and thus collected taxes are reduced by:

\[
\eta^*(x) T(Y(x)) f^*(x) \Delta \delta.
\]

We define the tax liability effect \(X(w) \delta \Delta\) as the sum of the mechanical and migration effects for all skill levels \(x\) above \(w\), where \(X(w)\) – defined in [14] – is the intensity of the tax liability effects for all skill levels above \(w\).
The unilateral deviation we consider cannot induce any first-order effect on the tax revenues of the deviating country; otherwise the policy in the deviating country would not be a best response. This implies that the substitution effect \[15\] must be offset by the tax liability effect \[X(w) \delta \Delta\]. We thus obtain Proposition 1’s formula.

An alternative way of writing formula \(13\) illuminates the relationship between the marginal and the average optimal tax rates. Using the definition of the elasticity of migration, we obtain:

\[
\frac{T'(Y(w))}{1 - T'(Y(w))} = \frac{\alpha(w)}{\epsilon(w)} \frac{1 - F(w)}{w f(w)} \left[ 1 - \mathbb{E}_f \left( \frac{T(Y(x))}{Y(x) - T(Y(x))} v_0(x) \mid x \geq w \right) \right]. \tag{17}
\]

This alternative way of writing the optimal tax rate formula shows that the new “migration factor” makes the link between the marginal tax rate at a given \(w\) and the mean of the average tax rates above this \(w\). More precisely, it corresponds to the weighted mean of the average tax rates \[\frac{T(Y(x))}{Y(x) - T(Y(x))}\] weighted by the elasticity of migration \(v_0(x)\), for everyone with productivity \(x\) above \(w\). The reason is that migration choices are basically driven by average tax rates, instead of the marginal tax rates.

V The Profile of the Optimal Marginal Tax Rates

It is trivial to show that the optimal marginal tax rate is equal to zero at the top if skills are bounded from above. We also find that the optimal marginal tax rate at the bottom is non negative. Our contribution is to characterize the overall shape of the tax function, and thus of the entire profile of the optimal marginal tax rates.

The second-best solution is potentially complicated because it takes both the intensive labor supply decisions and the location choices into account. To derive qualitative properties, we follow the method developed by Jacquet, Lehmann, and Van der Linden (2013) and start by considering the same problem as in the second best, except that skills \(w\) are common knowledge (migration costs \(m\) remain private information). We call this benchmark the Tiebout best, as a tribute to Tiebout’s seminal introduction of migration issues in the field of public finance.

The “Tiebout Best” as a Useful Benchmark

In the Tiebout best, each government faces the same program as in the second best but without the incentive-compatibility constraint \(10\):

\[
\max_{U_i(w), Y_i(w)} U_i(w_0) \quad \text{s.t.} \quad \int_{w_0}^{w_1} (Y_i(w) - v(Y_i(w); w) - U_i(w)) \varphi(U_i(w) - U_{-i}(w); w) \, dw \geq E, \tag{18}
\]

The first-order condition with respect to gross earnings \(v'(Y(w); w) = 1\) highlights the fact that there is no need to implement distortionary taxes given that skills \(w\) are observable.

\[\text{ Einstein's formula for } X(w) = 0 \text{ according to } (13). \]
Therefore, a set of skill-specific lump-sum transfers $\tilde{T}_i(w)$ decentralizes the Tiebout best. We now consider the optimality condition with respect to $U(w)$. Because preferences are quasilinear in consumption, increasing utility $U(w)$ by one unit for a given $Y(w)$ amounts to giving one extra unit of consumption, i.e. to decreasing $T_i(w)$ by one unit. In the policymaker’s program, the only effect of such a change is to tighten the budget constraint. In the Tiebout best, the $f^*(w)$ workers’ taxes are reduced by one unit. However, the number of taxpayers with skill $w$ increases by $\eta^*(w) f^*(w)$ according to (7). In the Tiebout best, the negative migration effect of an increase in tax liability fully offsets the positive mechanical effect, implying:

$$\tilde{T}(w) = \frac{1}{\eta^*(w)}.$$  

(19)

The tax liability $\tilde{T}_i(w)$ required from the residents with skill $w > w_0$ is equal to the inverse of their semi-elasticity of migration $\eta^*_i(w)$. The least productive individuals receive a transfer determined by the government’s budget constraint. Therefore, the optimal tax function is discontinuous at $w = w_0$, as illustrated in Figures 2 – 5. We can alternatively express the best response of country $i$’s policymaker using the elasticity of migration instead of the semi-elasticity. We recover the formula derived by Mirrlees (1982):

$$\frac{\tilde{T}_i(w)}{Y_i(w) - \tilde{T}_i(w)} = \frac{1}{\nu(\Delta_i; w)}.$$  

(20)

Combining best responses, we easily obtain the following characterization for the Nash equilibrium in the Tiebout best. We state it as a proposition because it provides a benchmark to sign second-best optimal marginal tax rates.

**Proposition 2.** In a Nash equilibrium equilibrium, the Tiebout-best tax liabilities are given by (19) for every $w > w_0$, with an upwards jump discontinuity at $w_0$.

### Signing Optimal Marginal Tax Rates

The Tiebout-best tax schedule provides insights into the second-best solution, where both skills and migration costs are private information. Using (19), Equation (14) can be rewritten as:

$$X(w) = \int_w^\infty \left[ \tilde{T}(x) - T(Y(x)) \right] \eta^*(x) f^*(x) \, dx.$$  

(21)

We see that the tax level effect $X(w)$ is the weighted sum of the difference between the Tiebout-best tax liabilities and second-best tax liabilities for all skill levels $x$ above $w$. The weights are given by the product of the semi-elasticity of migration and the skill density, i.e. by the mass of pivotal individuals of skill $w$, who are indifferent between migrating or not. In the Tiebout best, the mechanical and migration effects of a change in tax liabilities cancel out. Therefore, the Tiebout-best tax schedule defines a target for the policymaker in the second best, where distortions along the intensive margin have also to be minimized. The second-best solution thus proceeds from the reconciliation of three underlying forces:
i) maximizing the welfare of the worst-off; ii) being as close as possible to the Tiebout-best tax liability to limit the distortions stemming from the migration responses; iii) being as flat as possible to mitigate the distortions coming from the intensive margin. These three goals cannot be pursued independently because of the incentive constraints \((10)\). The following proposition is established in Appendix \(A.2\) but we below provide graphs that cast light on the main intuitions. We consider the case of purely redistributive tax policies \((E = 0)\). This implies that the laissez-faire policy with \(T(Y) \equiv 0\) is feasible. Therefore, in a best response, each government must choose a policy for which the least skilled individuals are not worse-off than in the laissez faire.

**Proposition 3.** Let \(E = 0\). In a Nash equilibrium:

i) if \(\eta^\prime = 0\), marginal tax rates are positive \(T'(Y(w)) > 0\) for \(w \in (w_0, w_1)\);

ii) if \(\eta^\prime < 0\), marginal tax rates are positive \(T'(Y(w)) > 0\) for \(w \in (w_0, w_1)\);

iii) if \(\eta^\prime > 0\), then the marginal tax rates are either

(a) positive \(T'(Y(w)) \geq 0\) for \(w \in (w_0, w_1)\);

(b) or there exists a threshold \(\bar{w} \in [w_0, w_1]\) such that \(T'(Y(w)) \geq 0\) for \(w \in (w_0, \bar{w})\) and \(T'(Y(w)) < 0\) for \(w \in (\bar{w}, w_1)\).

iv) if \(\eta^\prime(w) > 0\) and \(\lim_{w_1 \to \infty} \eta^*(w) = \infty\), then there exists a threshold \(\bar{w} \in (w_0, w_1)\) below which \(T'(Y(w)) > 0\) and above which \(T'(Y(w)) < 0\).

This proposition casts light on the part played by the slope of the semi-elasticity of migration. It considers the three natural benchmarks that come to mind when thinking about it. First, the costs of migration may be independent of \(w\) as in Blumkin, Sadka, and Shem-Tov (2012) and Morelli, Yang, and Ye (2012), implying a constant semi-elasticity in a symmetric equilibrium. This makes sense, in particular, if most relocation costs are material (moving costs, flight tickets, etc.). Second, one might want to consider a constant elasticity of migration, as in Brewer, Saez, and Shephard (2010) and Piketty and Saez (2012). In this case, the semi-elasticity must be decreasing: if everyone receives one extra unit of consumption in country \(i\), then the relative increase in the number of taxpayers becomes smaller for more skilled individuals. Third, the costs of migration may be decreasing in \(w\). This seems to be supported by the empirical evidence that highly skilled are more likely to emigrate than low skilled (Docquier and Marfouk, 2006). This suggests that the semi-elasticity of migration may be increasing in skills. A special case is investigated in Simula and Trannoy (2010,2011), with a semi-elasticity equal to zero up to a threshold and infinite above.

The case of a constant semi-elasticity of migration is illustrated in Figure 2. The dashed line represents the “Tiebout target” given by Equation \((19)\). It consists of a constant tax level,
equal to at $1/\eta^* > 0$ for all $w > w_0$ and redistributes the obtained collected taxes to workers of skill $w_0$. It is therefore negative at $w_0$ and then jumps upwards to a positive value $1/\eta^* > 0$ for every $w > w_0$. The solid line corresponds to the Nash-equilibrium tax schedule in the second best. A flat tax schedule, with $T(Y(w)) \equiv 1/\eta^*(w)$, would maximize tax revenues and avoid distortions along the intensive margin. It would however not benefit to workers of skill $w_0$. Actually, the laissez faire policy where $T(Y(w)) \equiv 0$ would provide workers of skill $w_0$ with a higher utility level. Consequently, the best compromise is achieved by a tax schedule that is continuously increasing over the whole skill distribution, from a negative value – so that workers of skill $w_0$ receive a net transfer – to positive values that converge to the Tiebout target $1/\eta^*$ from below. In particular, implementing a negative marginal tax rate at a given $w$ would just make the tax liabilities of the less skilled individuals further away from the Tiebout target, thereby reducing the transfer to the $w_0$-individuals.

![Figure 2: Constant Semi-Elasticity of Migration](image)

The case of a decreasing semi-elasticity of migration is illustrated in Figure 3. The Tiebout target is thus increasing above $w_0$. This reinforces the rationale for having an increasing tax schedule over the whole skill distribution in the second best.

![Figure 3: Decreasing Semi-Elasticity of Migration](image)
The case of an increasing semi-elasticity of migration is illustrated in Figure 4. The Tiebout target is now decreasing for $w > w_0$. To provide the workers of skill $w_0$ with a net transfer, the tax schedule must be negative at $w_0$. It then increases to get closer to the Tiebout target. This is why marginal tax rates must be positive in the lower part of the skill distribution. As shown in Figure 4, two cases are possible for larger $w$. In case a), the tax schedule is always slowly increasing, to get closer to the Tiebout target, as skill increases. The optimal marginal tax rates are therefore always positive. In case b), the Tiebout target is so decreasing than once the optimal tax schedule becomes close enough to the Tiebout target, it becomes decreasing in skills so as to remain close enough to the target.

When the semi-elasticity of migration tends to infinity, the target converges to 0 as skill goes up. Consequently, the optimal tax schedule cannot remain below the target and only case b) can occur, as illustrated in Figure 5.
Asymptotic Properties

First, the studies by Brewer, Saez, and Shephard (2010) and Piketty and Saez (2012) can be recovered as special cases of our analysis. The latter look at the asymptotic marginal tax rate given potential migration. They assume that the elasticity of migration is constant, equal to $\nu$. From Equation (8), a constant elasticity of migration is a special case of a decreasing semi-elasticity, because $C(w)$ must be non-decreasing in the second best. They also assume that the elasticities $\varepsilon(w), \alpha(w)$ converge asymptotically to $\varepsilon$ and $\alpha$ respectively. They finally assume that the distribution of skills is Pareto in its upper part, so that $(wf^*(w))/(\alpha(w)(1 - F^*(w)))$ asymptotically converges to $k$. Making skill $w$ tends to infinity in the optimal tax formula (17), we retrieve their formula for the optimal asymptotic marginal tax rate:

$$T'(Y(\infty)) = \frac{1}{1 + k\varepsilon + \nu}. \quad (22)$$

We see that the asymptotic marginal tax rate is then strictly positive. For example, if $k = 1.5$, $\varepsilon = 0.25$ and $\nu = 0.25$, we obtain $T'(Y(\infty)) = 61.5\%$ instead of $72.7\%$ in the absence of migration responses. Note that when migration costs and skills are independently distributed and the skill distribution is unbounded, as assumed by Blumkin, Sadka, and Shem-Tov (2012), the elasticity of migration tends to infinity according to (8). In this case, the asymptotic optimal marginal tax rate is equal to zero. The result of a zero asymptotic marginal tax obtained by Blumkin, Sadka, and Shem-Tov (2012) is thus a limiting case of Piketty and Saez (2012).

Second, one may wonder whether the optimal tax schedule must converge asymptotically to the Tiebout target, as suggested in Figure 2 for the case of a constant elasticity of migration.\footnote{In this case, when the skill distribution is unbounded, Blumkin, Sadka, and Shem-Tov (2012) show that the tax liability converges to the Tiebout target (that they call the “Laffer tax”) when the skill increases to infinity.} We can however provide counter-examples where this is not the case. For instance, when the skill distribution is unbounded and approximated by a Pareto distribution, and when the elasticity of migration converges asymptotically to a constant value $v_0$, the optimal tax schedule converges to an asymptote that increases at a slope given by the optimal asymptotic marginal tax rate provided by Piketty’s and Saez’s (2012) formula. Conversely, the Tiebout target is given by (20). The Tiebout target therefore converges to an asymptote that increases at a pace $1/(1 + v_0)$, which is larger than the asymptotic optimal marginal tax rate. The two schedules must therefore diverge when the skill level tend to infinity.

Discussion

Proposition 3 shows that the slope of the semi-elasticity of migration is crucial to derive the shape of optimal income tax. According to (8), even under the plausible case where the elasticity of migration is increasing over the skill distribution, the semi-elasticity may be either

\footnote{By L'Hôpital's rule, $\lim_{w \to \infty} \frac{T(Y(w))}{Y(w)} = \lim_{w \to \infty} \frac{T'(Y(w))}{1 - T'(Y(w))}$.}
decreasing or increasing, depending on whether the elasticity of migration is increasing at a lower or higher pace than consumption. In the former case, the optimal tax schedule is increasing and the optimal marginal tax rates are positive everywhere. In the latter case, the optimal tax schedule may be hump-shaped and optimal marginal tax rates may be negative in the upper part of the skill distribution. Therefore, the qualitative features of the optimal tax schedule may be very different, even with a similar elasticity of migration in the upper part of the skill distribution. This point will be emphasized by the numerical simulations of the next section.

One may wonder why this is the slope of the semi-elasticity of migration and not that of the elasticity that matters in Proposition. This is because the distortions along the intensive margin depend on whether marginal tax rates are positive or negative, i.e. on whether the optimal tax liability is increasing or decreasing. Consequently, the second-best optimal tax schedule inherits the qualitative properties of the Tiebout-best solution, in which tax liabilities are equal to the inverse of the semi-elasticity of migration. We see that in order to clarify how migrations affect the optimal tax schedule, it is not sufficient to use an empirical strategy that only estimates the level of the migration response, as estimated by Liebig, Puhani, and Sousa-Poza (2007), Kleven, Landais, and Saez (2013) or Kleven, Landais, Saez, and Schultz (2013). Our theoretical analysis thus calls for a change of focus in the empirical analysis: in an open economy, one needs to also estimate the profile of the semi-elasticity of migration with respect to earning capacities.

VI Numerical Simulations

This section provides numerical simulations of the equilibrium optimal tax schedule that competing policymakers should implement. One of our objectives is to emphasize the part played by the slope of the semi-elasticity of migration. In particular, we will show that the marginal tax rates faced by rich individuals may be quite sensitive to the overall shape of the semi-elasticity.

Calibration

We calibrate a symmetric equilibrium. However, our simulations can be seen as providing the best response of the US in an asymmetric equilibrium for which the equilibrium values would correspond to the chosen values for the semi-elasticity. We use the distribution of weakly earnings for singles without children in 2007 (CPS data) to recover the skill distribution \( f^*(w) \), using the workers’ first-order condition. We compute annual earnings \( Y \) and then proceed by inversion to find the value of \( w \), assuming an approximation of the federal and local income tax in 2007 (See Appendix B, Tables 1 and 2). Following Diamond (1998) and Saez (2001), we correct for top coding by extending the obtained es-

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10The Mathematica program used for the simulations is available upon request.
timation with a Pareto distribution of coefficient 1.59. The disutility of effort is given by \( v(y; w) = (y/w)^{1+1/\epsilon} \). This specification implies a constant elasticity of gross earnings with respect to the retention rate \( \epsilon \), as in Diamond (1998) and Saez (2001). In a recent survey, Saez, Slemrod, and Giertz (2012) conclude that “the best available estimates range from 0.12 to 0.4” in the United States. We use a central value, \( \epsilon = 0.25 \). Public expenditures \( E \) are kept at their initial level $18,157, which corresponds to 33.2% of the total gross earnings of single without children. Our calibration provides a very good approximation of the top of the income distribution as described by Alvaredo, Atkinson, Piketty, and Saez (2013). In the absence of migration responses, we find that the top 0.1%, top 1%, top 5% and top 10% of the population respectively get 6.3%, 17.6%, 33.7% and 44.5% of total income. The corresponding numbers in the World Top Income Database are 8.2%, 18.3%, 33.8% and 45.7%.

The semi-elasticity of migration is a key parameter in our computations. Even though the potential impact of income taxation on migration choices has been extensively discussed in the theoretical literature since Tiebout’s (1956) seminal contribution, there are still few empirical results. Kleven, Landais, and Saez (2013) study tax-induced mobility of football players in Europe and find substantial mobility elasticities. More specifically, the mobility of domestic players with respect to domestic tax rate is rather small around 0.15, but the mobility of foreign players is much larger, around 1. Kleven, Landais, Saez, and Schultz (2013) confirm that these large estimates apply to the broader market of highly skilled foreign workers and not only to football players. They find an elasticity above 1 in Denmark. In a given country, the number of foreigners at the stop is relatively small. Hence, these findings would translate into a global elasticity at the top of at most 0.25 for most countries (see Piketty and Saez 2012).

![Figure 6: Elasticity of Migration by Fractile of the Actual Earnings Distribution. Case 1 (Red), Case 2 (Purple - dotted) and Case 3 (Blue - dashed)](image)

As far as we know, there are no empirical studies regarding the possible shape of the elasticity or semi-elasticity of migration. We therefore investigate three possible scenarios,
as shown in Figures 6 and 7. In each of them, the average elasticity in the actual economy top 1% of the population is equal to 0.25. In the first scenario, the semi-elasticity is constant up to the top centile and then decreasing in such a way that the elasticity of migration is constant within the top centile. In the second scenario, the semi-elasticity is constant throughout the whole skill distribution. In the third scenario, the semi-elasticity is zero up to the top centile and then increasing. Note that, in the three scenarios, the elasticity of migration is non-decreasing along the skill distribution whilst the semi-elasticity of migration is constant across the bottom 99% of the skill distribution. The average elasticity in the population is higher in the first scenario (0.028) than in the second (0.013) and third (0.003) ones.

**Optimal Tax Liabilities**

The optimal tax liabilities in the equilibrium are shown in Figure 8. The x-axis represents annual gross earnings and the y-axis total taxes paid, both expressed in millions of US dollars. In addition to the three scenarios presented above, we added the tax liabilities that would be chosen in a closed economy or in the presence of tax coordination (black curve). We observe that the threat of migration implies a non-negligible decrease in the total taxes paid by top income earners. Even though the average elasticity of migration is the same for the top 1% of income earners in the three scenarios, we observe significant differences due to variations in the shape of the semi-elasticity of migration.

In the first case, the tax function is close to being linear for high-income earners and remains close to the closed-economy benchmark. In the second case, the tax function is more concave for large incomes, but remains increasing. In the third case, the tax function becomes decreasing around $Y = 2.9$ millions. In particular, the richest people are not those paying the largest taxes. It is very striking that the largest difference in tax liabilities is observed in the third case which yet exhibits the lowest average elasticity of migration over the
total population. This illustrates the fact that the profile of the semi-elasticity of migration within the top centile, on which we lack empirical evidence, has a much stronger impact on the optimal tax schedule than the average elasticity of migration within the bottom 99% of the population.

**Optimal Average and Marginal Tax Rates**

The effect of fiscal competition on tax progressivity is emphasized in Figure 9, which shows the average tax rate. The tax policy is progressive in case 1, but strongly regressive in the two other cases. The average tax rates faced by rich people differ significantly in the three scenarios. For example, the average tax rate for households with $5 millions of annual earnings is about 65% in case 1, 36% in case 2 and 18% in case 3.

Figure 9: Optimal Average Tax Rates. Autarky (Bold), Case 1 (Red), Case 2 (Purple - dotted) and Case 3 (Blue - dashed).

Figure 10 casts light on the differences in the optimal marginal tax rates. We see that differences in the slope of the semi-elasticity of migration may translate into large differences in marginal tax rates for high-income earners. Consequently, our numerical results put the
stress on the need for empirical studies on the slope of the semi-elasticity of migration, in addition to its level.

The Winners and Losers from Tax Competition

Figure 11 displays the relative change of utility $U(w)$ compared to autarky for the entire population. The right-hand side zooms in on the top 1%. As expected, the least productive workers loose and the more productive workers gain from tax competition. However, the magnitudes of the gains and losses strongly differ across the three scenarios.

In the first case, the losers are below the 6\textsuperscript{th} decile. The maximin objective is reduced by only 0.4% while individuals in the top 1\% benefit from an average gain of 18.9\%. The welfare analysis is more striking in cases 2 or 3, where only the top 8\% and the top 2\% benefit
from tax competition respectively. Moreover, the gains and losses are then much larger in absolute value: the worst-off households loose 3.1% in case 2 and 5.3% in case 3, while the top 1% agents gain 28.5% and 29.3% on average respectively. This reflects the differences in tax liabilities displayed in Figure\textsuperscript{8}. Once again, it is striking that the largest loses and gains are obtained in the third scenario, in which there is no migration response at all in the bottom 99% but an increasing semi-elasticity within the top 1%. Our simulations show that tax competition leads to booming inequalities, because of substantial tax cuts and welfare gains in the top 1% of the population.

VII Concluding Comments

This paper characterizes the nonlinear income tax schedules that competing Rawlsian governments should implement when individuals with private information on skills and migration costs decide where to live and how much to work. First, we obtain an optimality rule in which a migration term comes in addition to the standard formula obtained by Diamond (1998) for a closed economy. Second, we show that the optimal tax schedule for top income earners not only depends on the intensity of the migration response of this population, which has been estimated by Liebig, Puhani, and Sousa-Poza (2007), Kleven, Landais, and Saez (2013) and Kleven, Landais, Saez, and Schultz (2013), but also on the way in which the semi-elasticity of migration varies along the skill distribution. If the latter is constant or decreasing, optimal marginal tax rates are positive. Conversely, marginal tax rates may be negative if the semi-elasticity of migration is increasing along the skill distribution. To illustrate the sensitivity of marginal tax rates to the slope, we numerically compare three economies that are identical in all aspects, including the average elasticity of migration among the top percentile of the distribution, except that they differ in term of the slope of the semi-elasticity of migration along the skill distribution. Given our calibration, optimal marginal tax rates for high income earners are positive and around 65% when the semi-elasticity is increasing, but are negative over 3 millions of USD in the scenario with an increasing semi-elasticity of migration.

Therefore, it is not sufficient to estimate the elasticity of migration. The level as well as the slope of the semi-elasticity of migration are crucial to derive the shape of optimal marginal income tax, even for high income earners. The empirical specification (2) of Kleven, Landais, Saez, and Schultz (2013) does not allow to recover the slope of the semi-elasticity. Another specification with additional terms should be estimated.

Different conclusions can be drawn from our results. From a first perspective, the uncertainty about the profile of the semi-elasticity of migration may justify very low, and maybe even negative, marginal tax rates for the top 1% of the income earners. This may partly explain why OECD countries were reducing their top marginal tax rates before the financial crisis of 2007. From a second perspective, the potential consequences of mobility might be
so substantial in terms of redistribution that governments might want to hinder migration. For example, departure taxes have recently been implemented in Australia, Bangladesh, Canada, Netherlands and South Africa. Finally, from a third viewpoint, the problem is not globalization per se but the lack of cooperation between national tax authorities.

A first possibility is an agreement among national policymakers resulting in the implementation of supranational taxes, for example at the EU level. A second possibility relies on the exchange of information between Nation states. Thanks to this exchange, the policymaker of a given country would be able to levy taxes on its citizens living abroad, as implemented by the United States. Indeed, in a citizenship-based income tax system, moving abroad would not change the tax schedule an individual faces, so that the distortions due to tax competition would vanish. There has been some advances in the direction of a better exchange of information between tax authorities. For example, the OECD Global Forum Working Group on Effective Exchange of Information was created in 2002 and contains two models of agreements against harmful tax practices. However, these agreements remain for the moment non-binding and are extremely incomplete.

A Proofs

A.1 Proposition 1

We use the dual problem to characterize best response allocations:

\[
\begin{align*}
\max_{\mathcal{U}_i(w), Y_i(w)} & \int_{w_0}^{w_1} (Y_i(w) - v(Y_i(w); w) - \mathcal{U}_i(w)) \varphi_i(\mathcal{U}_i(w) - \mathcal{U}_{-i}(w); w) \, dw \\
\text{s.t.} & \quad \mathcal{U}_i(w) = -\psi'_i(Y_i(w); w) \quad \text{and} \quad \mathcal{U}_i(w_0) \geq \mathcal{U}_i(w_0),
\end{align*}
\]

(23)
in which \(\mathcal{U}_i(w_0)\) is given. We adopt a first-order approach by assuming that the monotonicity constraint is slack. We further assume that \(Y(\cdot)\) is differentiable. Denoting \(q(\cdot)\) the co-state variable, the Hamiltonian associated to Problem (23) is:

\[
\mathcal{H}(\mathcal{U}_i, Y_i, q; w) \equiv [Y_i - v(Y_i; w) - \mathcal{U}_i|_1 \varphi_i(\mathcal{U}_i - \mathcal{U}_{-i}; w) - q(w) \psi'_i(Y_i; w)].
\]

Using Pontryagin’s principle, the first-order conditions for a maximum are:

\[
1 - v'_Y(Y_i(w); w) = \frac{q(w)}{\varphi_i(\Delta_i(w); w)} \psi''_{yw}(Y_i(w); w),
\]

(24)

\[
q'(w) = \{1 - \varphi_i(Y_i(w) - \mathcal{U}_i(w)) \eta_i(\Delta_i(w); w) \} \varphi_i(\Delta_i(w); w),
\]

(25)

\[
q(w_1) = 0 \quad \text{when} \quad w_1 < \infty \quad \text{and} \quad q(w_1) \to 0 \quad \text{when} \quad w_1 \to \infty,
\]

(26)

\[
q(w_0) \leq 0.
\]

(27)

Integrating Equation (25) between \(w\) and \(w_1\) and using the transversality condition (26), we obtain:

\[
q(w) = -\int_{w_0}^{w_1} [1 - \eta^*(x) T(Y(x))] f^*(x) \, dx.
\]

(28)

Defining \(X(w) = -q(w)\) leads to (14). Equation (24) can be rewritten as:

\[
1 - v'_Y(Y(w); w) = -\frac{X(w)}{f^*(w)} \psi''_{yw}(Y(w); w)
\]

(29)

Dividing (5) by (4) and making use of (3), we get

\[
\psi''_{yw}(Y(w); w) = -\frac{a(w)}{c(w)} \frac{1 - T'(Y(w))}{w}.
\]

Plugging (5) and the latter equation into (29) leads to (13).
A.2 Proposition\(^3\)

From (13), \(T'(Y(w))\) has the same sign as the tax level effect \(X(w)\). The transversality condition (27) is equivalent to \(X(w_0) \geq 0\), while (26) is equivalent to \(\lim_{w \to w_1} X(w) = 0\). From (14), the derivative of \(X(w)\) is

\[
X'(w) = \left[ T(Y(w)) - \frac{1}{\eta^*(w)} \right] \eta^*(w) f^*(w)
\]  

(30)

We now turn to the proofs of the different parts of Proposition\(^3\).

i) \(\eta^*(w)\) is constant and equal to \(\eta^*\)

We successively show that any configuration but \(T'(Y(w)) < 1/\eta^*\) for all \(w \in (w_0, w_1)\) contradicts at least one transversality conditions (26) or (27). We start by establishing the following Lemmas.

**Lemma 1.** Assume that for any \(w \in [w_0, w_1]\), \(\eta_0(w) \leq 0\) and assume there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) \leq 0\) and \(T(Y(\tilde{w})) > 1/\eta^*(\tilde{w})\). Then \(X(w_0) < 0\), so the transversality condition (27) is violated.

**Proof** As \(T'(Y(w)) = Y(w) - C(w)\) and \(\eta(w)\) are continuous functions of \(w\), there exists by continuity an open interval around \(\tilde{w}\) where \(T(Y(w)) > 1/\eta^*(w)\). Let \(w^* \in (w_0, \tilde{w})\) be the lowest bound of this interval. Then either \(w^* = w_0\) or \(T(Y(w^*)) = 1/\eta^*(w^*)\). Moreover, for all \(w \in (w^*, \tilde{w})\), one has that \(T(Y(w)) > 1/\eta^*(w)\), thereby \(X'(w) > 0\), according to (30). Hence, one has that \(X(w) < X(\tilde{w}) \leq 0\), thereby \(T'(Y(w)) < 0\) for all \(w \in [w^*, \tilde{w}]\). Consequently, \(T(Y(w^*)) > T(Y(\tilde{w})) \geq 1/\eta^*(\tilde{w}) \geq 1/\eta^*(w^*)\). So, one must have \(w^* = w_0\). Finally, we get \(X(w^*) = X(w_0) < 0\), which contradicts the transversality condition (27). \(\square\)

**Lemma 2.** Assume that for any \(w \in [w_0, w_1]\), \(\eta_0(w) \leq 0\) and assume there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) \leq 0\) and \(T(Y(\tilde{w})) < 1/\eta^*(\tilde{w})\). Then \(X(w_1) < 0\), so the transversality condition (27) is violated.

**Proof** As \(T(Y(w)) = Y(w) - C(w)\) and \(\eta(w)\) are continuous functions of \(w\), there exists by continuity an open interval around \(\tilde{w}\) where \(T(Y(w)) < 1/\eta^*(w)\). Let \(w^* \in (\tilde{w}, w_1]\) be the highest bound of this interval. Then either \(w^* = w_1\) or \(T(Y(w^*)) = 1/\eta^*(w^*)\). Moreover, for all \(w \in [\tilde{w}, w^*)\), one has that \(T(Y(w)) < 1/\eta^*(w)\), thereby \(X'(w) < 0\), according to (30). Hence, one has that \(X(w) < X(\tilde{w}) \leq 0\), thereby \(T'(Y(w)) < 0\) for all \(w \in (\tilde{w}, w^*)\). Consequently, \(T(Y(w^*)) < T(Y(\tilde{w})) \leq 1/\eta^*(\tilde{w}) \leq 1/\eta^*(w^*)\). So, one must have \(w^* = w_1\). Finally, we get \(X(w^*) = X(w_1) < 0\), which contradicts the transversality condition (26). \(\square\)

From Lemmas\(^1\) and \(^2\), it is not possible to have \(T'(Y(\tilde{w})) \leq 0\) and \(T'(Y(\tilde{w})) \neq 1/\eta^*(\tilde{w})\), otherwise one of the transversality conditions is violated. Assume there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) \leq 0\) and \(T(Y(\tilde{w})) = 1/\eta_0(\tilde{w})\). By continuity there exists \(\varepsilon > 0\) such that \(T'(Y(\tilde{w} - \varepsilon)) < 0\) and \(T(Y(\tilde{w} - \varepsilon)) > 1/\eta^*\), in which case, Lemma\(^1\) applies.

Last, assume there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) < 0\) and \(T(Y(\tilde{w})) = 1/\eta^*(\tilde{w})\). According to the Cauchy-Lipschitz theorem (equivalently, the Picard-Lindelöf theorem), the differential system of equations in \(U(w)\) and \(X(w)\) defined by (10) and (30) (and including \(\tilde{w}\) to express \(Y(w)\) as a function of \(X(w)\)) with initial condition that corresponds to \(T'(Y(\tilde{w})) = X(\tilde{w}) = 0\) and \(T(Y(\tilde{w})) = 1/\eta^*(\tilde{w})\) admits a single solution where for all \(w\) \(X(w) \equiv 0\) and \(T(.) = 1/\eta^*\). From \(^9\), such a solution provides excess budget resources when \(E\) is assumed nil and provides less utility level than the laissez-faire policy where \(T(.) = 0\).

Consequently, any case where \(T'(Y(\tilde{w})) \leq 0\) for \(w \in (w_0, w_1)\) leads to the violation of at least one of the transversality conditions, which ends the proof of Part i) of Proposition\(^3\).

ii) \(\eta^*(w)\) is decreasing

If there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) \leq 0\) and \(T(Y(\tilde{w})) > 1/\eta^*(\tilde{w})\), Lemma\(^1\) applies. If there exists a skill level \(\tilde{w} \in (w_0, w_1)\) such that \(T'(Y(\tilde{w})) \leq 0\) and...
Moreover, according to Lemma 3, one must have $T(Y(\bar{w})) < 1/\eta^*(\bar{w})$, Lemma 2 applies. Finally, if there exists a skill level $\bar{w} \in (w_0, w_1)$ such that $T'(Y(\bar{w})) \leq 0$ and $T(Y(\bar{w})) = 1/\eta^*(\bar{w})$, then function $w \mapsto T(Y(w)) - 1/\eta^*(w)$ is non-positive and admits a negative derivative at $\bar{w}$, as $\eta''(\cdot) < 0$. Hence, there exists $\bar{w} > \bar{w}$ such that $T(Y(w)) < 1/\eta^*(w)$, thereby $X'(w) < 0$ for all $w \in (\bar{w}, \bar{w})$. Consequently, $X'(\bar{w}) < 0$ (equivalently $T'(Y(\bar{w})) < 1/\eta^*(\bar{w})$) and $X(\bar{w}) = 0$ (equivalently $T'(Y(\bar{w})) < 0$, in which case Lemma 2 applies at $\bar{w}$.

Consequently, any case where $T'(Y(\bar{w})) \leq 0$ for $w \in (w_0, w_1)$ leads to the violation of at least one of the transversality conditions, which ends the proof of Part ii) of Proposition 3.

iii) $\eta^*(w)$ is increasing

We first show the following Lemma.

Lemma 3. Assume that for any $w \in [w_0, w_1]$, $\eta'_0(w) > 0$ and assume there exists a skill level $\bar{w} \in (w_0, w_1)$ such that $T'(Y(\bar{w})) \geq 0$ and $T(Y(\bar{w})) \geq 1/\eta^*(\bar{w})$. Then, $X(w_1) > 0$, so the transversality condition (23) is violated.

Proof. We first show that we can assume that $T(Y(\bar{w})) > 1/\eta^*(\bar{w})$ without any loss of generality. Assume that $T(Y(\bar{w})) = 1/\eta^*(\bar{w})$ and $T'(Y(\bar{w})) \geq 0$. As $\eta'_0(\cdot) > 0$, function $w \mapsto T(Y(w)) - 1/\eta^*(w)$ is non-negative and admits a positive derivative at $\bar{w}$. Hence, there exists $\bar{w} > \bar{w}$ such that $T(Y(w)) > 1/\eta^*(w)$, thereby $X'(w) > 0$ for all $w \in (\bar{w}, \bar{w}]$. Consequently, $X'(\bar{w}) > 0$ (equivalently $T'(Y(\bar{w})) > 1/\eta^*(\bar{w})$) and $X(\bar{w}) > 0$ (equivalently $T'(Y(\bar{w})) > 0$).

Consider now that $T'(Y(\bar{w})) \geq 0$ and $T(Y(\bar{w})) > 1/\eta^*(\bar{w})$. As $T(Y(w)) = Y(w) - C(w)$ and $\eta(w)$ are continuous functions of $w$, there exists by continuity an open interval around $\bar{w}$ where $T(Y(w)) > 1/\eta^*(w)$. Let $w^* \in (\bar{w}, w_1]$ be the highest bound of this interval. Then either $w^* = w_1$ or $T(Y(w^*)) = 1/\eta^*(w^*)$. Moreover, for all $w \in \{w^*, \bar{w}^\}$, one has that $T(Y(w)) > 1/\eta^*(w)$, thereby $X'(w) > 0$, according to (30). Hence, one has that $X(w) > X(\bar{w}) \geq 0$, thereby $T'(Y(w)) > 0$ for all $w \in (w^*, \bar{w})$. Consequently, $T(Y(w^*)) > T(Y(\bar{w})) > 1/\eta^*(\bar{w}) > 1/\eta^*(w^*)$. So, one must have $w^* = w_1$. Finally, we get $X(w^*) = X(w_1) > 0$, which contradicts the transversality condition (23). □

Assume by contradiction that $X(w_0) = 0$ and there exists a skill level $\bar{w} \in (w_0, w_1)$ such that $T'(Y(\bar{w})) \geq 0$ and $T(Y(\bar{w})) < 1/\eta^*(\bar{w})$. As $T(Y(w)) = Y(w) - C(w)$ and $\eta(w)$ are continuous functions of $w$, there exists by continuity an open interval around $\bar{w}$ where $T(Y(w)) < 1/\eta^*(w)$. Let $w^* \in [w_0, \bar{w}]$ be the lowest bound of this interval. Then either $w^* = w_0$ or $T(Y(w^*)) = 1/\eta^*(w^*)$. Moreover, for all $w \in \{w^*, \bar{w}^\}$, one has that $T(Y(w)) < 1/\eta_0(w)$, thereby $X'(w) > 0$, according to (30). Hence, one has that $X(w) > X(\bar{w}) \geq 0$, thereby $T'(Y(w)) > 0$ for all $w \in (w^*, \bar{w})$. Consequently, $T(Y(w^*)) < T(Y(\bar{w})) < 1/\eta^*(\bar{w}) < 1/\eta^*(w^*)$. So, one must have $w^* = w_0$. Finally, we get $X(w^*) = X(w_0) > 0$, which contradicts the presumption that there exists a skill level $\bar{w} \in (w_0, w_1)$ such that $T'(Y(\bar{w})) \geq 0$ and $T(Y(\bar{w})) < 1/\eta^*(\bar{w})$.

Consequently, if $X(w_0) = 0$, we must have $T'(Y(w)) < 0$ for all $w \in (w_0, w_1)$. Using (2) and the assumption that $E = 0$, this implies that $T(Y(w_0)) > T(Y(\bar{w}))$. Hence such policy provides less utility to workers of skill $w_0$ than the laissez faire policy $T(\cdot) = 0$, which contradicts the presumption that $X(w_0) = 0$.

We consider hereafter the case where $X(w_0) > 0$. There thus exists $\bar{w} \in (w_0, w_1]$ such that $X(w) \geq 0$ (equivalently $T'(Y(w)) \geq 0$) for $w \leq \bar{w}$ and either $\bar{w} = w_1$ or there exists $w_2 \in (\bar{w}, w_1]$ such that $X(w) < 0$ (equivalently $T'(Y(w)) < 0$) for all $w \in (\bar{w}, w_2)$ and $X(w) > 0$ in the neighborhood to the right of $w_2$.

If $\bar{w} < w_1$, either we have $w_2 = w_1$ or we must have $T'(Y(w_2)) = 0$ by continuity of $T'$. Moreover, according to Lemma 3, one must have $T(Y(w)) < 1/\eta^*(w)$ for all $w \in (\bar{w}, w_2)$, otherwise the transversality condition $X(w_1) = 0$ would be violated. Consequently, one has that $X'(w) < 0$ for all $w \in (\bar{w}, w_2)$, so one has that $0 = X(\bar{w}) < X(w_2)$. Hence we have that $T'(Y(w_2)) < 0$, implying that $w_2 = w_1$, which ends the proof of Part iii) of Proposition 3.
iv) $\eta^*(w)$ is increasing and tends to infinity

From case iii), either marginal tax rates are positive, or there exists a threshold above which marginal tax rate is negative. Assume by contradiction that marginal tax rates are positive. Then, the tax schedule is increasing. It must also be positive to clear the budget constraint. As the semi-elasticity of migration increases to infinity, there thus exists a skill level $\bar{w}$ at which $T'(Y(\bar{w})) \geq 0$ and $T'(Y(\bar{w})) > 1/\eta^*(\bar{w})$. Then the transversality condition is violated according to Lemma 3, which leads to the desired contradiction. So, marginal tax rate must be negative above some skill level.

B Numerical Simulations

We consider two symmetric countries in a symmetric Nash equilibrium, so that $f^*(\cdot)$ and $\eta^*(\cdot)$ are structural parameters. The simulation program consists in solving the differential system of Equations (10) and (29) in $U(w)$ and $X(w)$, using $T(Y(w)) = Y(w) - v(Y(w); w) - U(w)$ and (29), with the terminal condition $X(w_1) = 0$. The remaining terminal condition $U(w_1)$ is selected to clear the budget constraint. The algorithm actually solves this system by the Newton-Raphson method using a discrete grid of 25,001 skill levels over $[w_0, w_1]$.

We calibrate the skill distribution using the distribution of weekly earnings among singles without dependent extracted from CPS 2007. We multiply this weakly earnings by 52 to get annual earnings. Given the specified utility function $c - (y/w)^{1+1/\nu}$, we recover the skill level for each earnings observation from $[3]$, using an approximation of the federal income tax schedule for singles described in Table 1 and an approximation of the local income tax in California described in Table 2.

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<th>$7,550$</th>
<th>$30,650$</th>
<th>$74,200$</th>
<th>$154,800$</th>
<th>$336,500$</th>
</tr>
</thead>
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<td>10%</td>
<td>15%</td>
<td>25%</td>
<td>28%</td>
<td>33%</td>
<td>35%</td>
</tr>
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</table>

Table 1: Approximation of the Federal Income tax

<table>
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<tr>
<th>$0$</th>
<th>$13,251$</th>
<th>$31,397$</th>
<th>$40,473$</th>
<th>$50,090$</th>
<th>$59,166$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>2%</td>
<td>4%</td>
<td>6%</td>
<td>8%</td>
<td>9.3%</td>
</tr>
</tbody>
</table>

Table 2: Approximation of the local Income tax (California)

We use a Gaussian Kernel of bandwidth $1, 157.2$. We expand this estimated density by a Pareto density of the form $k w^{-(p+1)}$. The skill level at which the expansion occurs and the scale parameter $p$ are selected to insure that the density $f^*(\cdot)$ remains continuously differentiable. The truncation at $w_1$ implies that the ratio $(1 - F^*(w)) / (w f^*(w))$ is not constant at $1/p$ and instead tends to zero at $w_1$, despite the Pareto expansion. This the reason why we add at the highest point $w_1$ of the grid of skills a mass point whose weight is such that $(1 - F^*(w)) / (w f^*(w))$ is constant at $1/p$ in the upper part of the skill distribution. This lead us with an approximation of the current economy. Parameter $p$ is selected to 1.984 to get plausible values for the shares of total income earned by the top 1% of the population.

In each scenario, we calibrate the semi-elasticity of migration $g(0|w)$ such that the average of the elasticity of migration $(Y(w) - T_{Actual}(Y(w)))g(0|w)$ in this approximation of the current economy is 0.25 for the top 1% of the income distribution. In the scenario with a constant elasticity of migration, this is done such that for each skill level in the top 1%, the elasticity of migration is equal to 0.25. In the scenario with an increasing semi-elasticity of migration, we choose a quadratic - concave specification for the function $w \mapsto g(0|w)$ such that the semi-elasticity of migration is nil at the 99% percentile.

\[ \text{i.e., we approximate the solution of } a'(w) = \xi(a(w), w) \text{ by computing recursively } a(w_{i-1}) = a(w_i) + (w_{i-1} - w_i) \xi(a(w_i), w_i). \]
References


