Wage discount vs. premium for the network search in the labor market

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Abstract

In this paper we deal with the question whether using social contacts or "formal" methods (such as application to advertisements) results in higher expected wage for a worker searching for job. Empirically, this question has produced contradictory evidences. In our model we show that one feature of the arrival process of new offers can be crucial for the relation of expected wages: the correlation between the quality of the job an employee holds and the quality of offers she might hear about. If this correlation is sufficiently high, the network search gives higher expected wages. The critical value of this parameter negatively depends on the arrival rate of offers, the probability that an offer is of good type, positively depends on the job destruction rate and the connectivity of the social network. Further, we show that the Rawlsian welfare is the highest while the utilitarian welfare might be the lowest when the network search gives the highest wage premium.

1 Introduction

There is vast evidence regarding the importance of social networks in the labor market: it is widely accepted that around 50% of the workers obtain jobs through their personal contacts. The role played by social contacts in the job search is at least twofold: providing information about the existence and characteristics of vacancies for the worker, providing information about the unobserved abilities of the worker for the employer (job referrals). Network seach is usually seen as a method which gives higher arrival rate of offers, less cost of search, higher rate of acceptance of offers, longer tenure and higher job satisfaction for the worker compared to the formal methods such

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as applying to advertisements (see Granovetter (1995)). This also implies that the network search not only results in lower unemployment duration but also provides a better match of attributes of the two sides of the market.

Although this latter statement suggests a higher expected wage for the network search compared to formal methods, the empirical literature does not confirm the advantage of social contacts in this respect. Pellizziari (2009) finds a mixed cross-country evidence: for Austria, Belgium and the Netherlands he estimates a wage premium of the network search, while for the Greece, Italy, Portugal, United Kingdom he finds a wage discount.

Theoretically, the job referral model of Montgomery (1991) predicts a positive association between wages and network search. In his model workers differ in their ability to work. The ability of currently employed workers is known for the employer as it has been revealed during the tenure of the job, while the ability of new applicants is unobserved. Assuming that high ability workers tend to be connected together, the model predicts that employers use their current workers as referees to contract high ability workers who earn high wages while low ability workers are hired mostly on the anonymous market.

Kugler (2003) predicts wage premium for the network search as well, though based on the idea that hiring through network decreases the efficiency wages paid by the firm, because referees caring about their own reputation monitor the workers they have recommended. Firms can choose between hiring through social contacts and formal methods, and also between paying efficiency wages or bargained wages according to the Nash bargaining outcome in which case the firm is exposed to the shirking behavior of the employees. It is also assumed that referrals provide candidates at a lower rate than the formal search. In equilibrium, firms offering 'good' high-paying jobs use referral hiring and pay efficiency wages while firms in low-paying sectors use formal methods and pay bargained wages. Kugler confirms the prediction of the model using estimates from industry and micro-level data from the United States.

On the other hand, Bentolila et al. (2009) finds negative association between network search and wages. In their model, network search provides offers at a higher rate than formal search, but the offers received from social contacts might not exactly be of the type which fits the chararteristics of the worker. Workers have incentives to accept offers coming earlier and in this way sacrificing their productivity for higher immediate earnings. This establishes a wage discount for the network search which is confirmed by the authors using data from the US and the European Union. However, they restrict their sample to workers younger than 35 years old who have found job thourgh their friends and relatives and not through their professional contacts which already goes in the direction of their assumption that contacts often provide offers of different "professions".

The motivation of our paper is based on the presented mixed evidences: our aim is to provide a simple model which is able to account for both wage premiums and discounts of the network search. Relying on the mentioned theoretical models the ambiguity in question could be accounted for saying that in the different countries the social network plays a different role in the labor markets. However, we build a model which is able to predict in which cases should we expect to observe wage discount or premium whitout changing the role of social networks: assuming that contacts provide information about the vacancies themselves do not want to occupy.

In our model agents are either unemployed or employed in one of two status: low wage or high wage employed. Hence, we assume that individuals are identical (ie. homogeneous labor assumption), but they might be paid differently depending on which kind of offer arrives to them. [1] Constantly, new job offers appear and job separations happen. An incoming offer is of low wage with a certain constant probability and of high wage with the complementary one. Unemployed agents take any offer they are aware of, low wage employed agents take only high wage offers, assuming that there is on-the-job search in the labor market. If unneeded offers arrive to employed agents, they pass them only to those social contacts who are interested in taking that offer. For example, if a high wage employed is aware of a low wage job, she passes it to her unemployed contacts but if she has a high wage job, to low wage employed as well. We analyse the case of regular graphs where every agent has the same number of neighbors but the analysis readily extends to networks of any kind of degree distribution.

The key feature of our model is that we assume that there is a correlation between the current wage of an employed and the quality of the offer she might know about. We keep constant the "number" of incoming high wage offers but we assume that high wage offers are "distributed" to high wage employed with a higher probability than to low wage employed. We are not aware of explicit empirical evidence of this assumption, but casual observation suggests that employed agents tend to know about new offers of similar characteristics to their jobs: either vacancies of their own firm or some related firms.

Our main results show that the expected wage of the network search is higher than that of the formal search when the mentioned correlation is high while we observe a wage discount for the network search if this correlation is low. By increasing the correlation, high wage offers tend to go to agents who will pass them, hence the wage distribution of offers received from social contacts puts higher weight on the high wage jobs. The critical value of this correlation, where the two methods result in the same expected wage, is decreasing in the employment rate and in the high wage employment rate. This means that if the arrival rate of new offers is low or the

[1] There are wide empirical evidences that similar workers of all important attributes might be differently paid in different jobs. See Mortensen (2003).

wage distribution of the new offers puts higher probability on the low wage jobs, then the critical correlation increases and we are more likely to observe wage discount of the network search. Moreover, this threshold is increasing in the average connectivity of the social network. We also show that the utilitarian welfare of the society is the lowest when the network search gives the highest expected wage: when the correlation is perfect, the number of lost offers is the highest in the society. On the other hand, the unemployment rate is the highest in this case.

these results shed some new light on why we observe cross-country differences in the relationship of the expected wage of the two search methods: we attribute these variances to the differences in the extent of the mentioned correlation, in state of the economy which determines how many and what type of jobs are created and destroyed, and in the connectivity of the social network.

Our model is close to the works of Calvó-Armengol and Jackson (2004, 2007), referred as CAJ (2004) or (2007) in the sequel. CAJ (2004) assumes that offers can travel only one step in the network: if among the direct neighbors of an employed agent there are no unemployed, the offer is lost (as it is also assumed in Calvo-Armengol and Zenou (2005)). They consider only homogeneous wages, so agents are either employed or unemployed. CAJ (2007) is more general regarding the assumptions about the transmission of information by the social networks, and they consider arbitrary many possible wages. Our model is the direct extension of CAJ (2004) to the heterogeneous wage case without modifying the assumption about the information transmission. On the other hand, we assume that there exist a correlation between the wage status of an employed agent and the quality of the offer she might hear about, and we exploit the consequences of the degree of this correlation for the expected wages. CAJ (2004) assumes that each agent has the same probability of hearing about an offer, while CAJ (2007) admits any arbitrary differences in these probabilities, but without connecting them to the actual status of the individuals or analysing the question of expected wages.

Other paper dealing with the issue of expected wages of the different search methods is Mortensen and Vishwanath (1994) where they show that the network search provides wage premium. They analyse a wage posting game which gives rise to a wage offer distribution associated to the distribution from which formal search draws an offer. They assume that the network provides offers from the upper tail of this distribution by which employment ex-post realizes. This supposes that an employed agent transmits a job offer of exactly the same wage as his own earned wage. Our model does not use this assumption, higher wage employed also transmit offers which are of lower wage than theirs.

Regarding the analysed network structures, Calvó-Armengol and Zenou (2005) examines the case of regular graphs and finds that connectivity has a non-monotonic effect on the unemployment rate: after a critical value adding more links increases the unemployment rate. Ioannides and Soetevent (2006) generalizes their model to random graphs with any arbitrary degree distribution and finds that in this case the mentioned non-monotonicity does not hold. However, these models deal with the case of homogeneous wages hence do not contribute to the literature on the expected wages of the different search methods. In our paper, we investigate both types of network structures and show that in both cases an increase in connectivity decreases the unemployment rate.

2 Model

In our economy N agents are placed on an undirected graph defined by the NxN adjacency matrix G where the element $g_{ij} = 1$ if individual i and j are directly connected and otherwise $g_{ij} = 0$. We have that $g_{ij} = g_{ji}$.

In the economy constantly new vacancies are open up and job separations happen because of fluctuations of the economic production unexplained by our model. At a rate α a new offer arrives to the economy. Jobs can be of two types: either low wage jobs or high wage jobs. Low wage jobs pay w_L, high wage jobs w_H, being $w_H>w_L$. Unemployed agent assumed to earn zero (or anything less than w_L). Given the arrival of an offer, it can be of low wage with probability p_L and of high wage with the complementary probability $p_H = 1 - p_L$. Job separation happens at a rate βe, where e is the employment rate in the economy. Hence we assume that if the employment rate is lower, job separation happens less often. This corresponds to the idea that the economy needs to meet certain demands, so if the employment rate is already low, further dismissals have low probability.

Upon arrival of any type of offer, with probability u an unemployed agent receives a new employment opportunity where u is the unemployment rate. In this case the unemployed agent takes the offer and becomes employed either of low or high wage. With probability e the offer goes to an employed agent. We assume that high wage employed might have higher probability to hear about high wage offers, and low wage employed about low wage offers, since we think that it is likely that workers are aware of vacancies that have similar characteristics to their own job. Hence, we suppose that a high wage offer goes to a randomly chosen high employed with probability:

$$h(e_H, e, \lambda) \equiv \frac{e\lambda e_H}{\lambda e_H + e_L}$$

where e_H is the high wage employed rate, e_L is the low wage employed rate and λ is a parameter which measures the extent of the mentioned "correlation" between the type of the job and the vacancy. If $\lambda = 1$, there is no bias toward the high wage agents. If $\lambda \to \infty$, high wage workers become aware of high wage jobs with probability e: low wage agents don't hear about this kind of vacancies. If $\lambda = 0$, we have the reverse: high wage employed do not hear about high wage jobs. Hence, if a high wage offer arrives, it goes to a randomly chosen low wage worker with probability $\frac{ee_L}{\lambda e_H + e_L}$. If a low wage offer arrives we again have that it might go to a low wage agent with higher probability than to a high wage agent. Table 1 summarizes these probabilities.

We can look at the same arrival process from an individual point of view. We define $a(e) = \frac{\alpha}{\alpha + \beta e}$, the probability that given an event happens, it is an arrival of a new offer and $b(e) = \frac{\beta}{\alpha + \beta e}$, the probability that given the occurence of an event, it is a job separation. If any offer arrives, with probability u we choose an unemployed, randomly out of the uN unemployed agents. Hence, an unemployed has $\frac{a(e)p_L}{N}$ chance to hear about a low wage offer and $\frac{a(e)p_H}{N}$ chance to learn about a high wage one. If a high wage vacancy opens up, with probability $\frac{e\lambda e_H}{\lambda e_H + e_L}$ we choose a high wage agent randomly out of the $e_H N$ such individuals. Hence, a high wage worker has $a(e)p_H \frac{e\lambda}{(\lambda e_H + e_L)N}$ chance to hear about a good offer which is above the average probability $(\frac{a(e)p_H}{N})$ if $\lambda > 1$. Consequently, a low wage agent has less than $\frac{a(e)p_H}{N}$ chance to hear about an offer of high salary. For a low wage vacancy we have the reverse: low wage agents have above average chance to hear about it.

The idea behind this formulation is the following. For an unemployed to learn about an offer "directly from the process" (compared to social contacts) is a matter of formal job search: opening newspapers, applying directly to the companies which advertise opportunities. Doing formal seach, the unemployed is facing the objective arrival process of offers and getting aware of an offer which is a random draw from the offer distribution. Hence, she has $a(e)p_L$ probability to get aware of a low wage vacancy and $a(e)p_H$ to get aware of a high wage one. [2] On the other hand, we assume that for an employed individual to directly learn about an opportunity means to hear about a vacancy at their own firm or some related firms. Hence we suppose that a worker has higher probability to learn about a vacancy which has the same characteristics as her own job and less than average probability to hear about a job of dissimilar attributes (if $\lambda > 1$).

Unemployed agents accept any offer they learn about. Note that in our model this is entirely rational: unemployed agents have no incentives to deny low wage opportunities in order to accept a high wage offer later. This is because the acceptance of a low wage offer does not decrease the probability to find a high wage job.

Low wage employed individuals accept only high wage offers and forward low wage offers to their social contacts. They choose one of their unemployed contacts at random. In case they do not have anyone, the offer is lost. High wage workers cannot upgrade their situations, so they forward any offer they become aware of. If they hear about a low wage offer, they pass it to one unemployed contact at random. While if they learn of a high wage offer, they also consider the low

[2] Note that Mortensen and Vishwanath (1994) uses a similar assumption: formal search gives an offer from the overall wage offer distribution.

Given that a low wage offer arrives, it goes to a randomly chosen:	
unemployed	u
low wage employed	$\frac{e\lambda e_L}{\lambda e_L+e_H}$
high wage employed	$\frac{ee_H}{\lambda e_L+e_H}$
Given that a high wage offer arrives, it goes to a randomly chosen:	
unemployed	u
low wage employed	$\frac{ee_L}{\lambda e_H+e_L}$
high wage employed	$\frac{e\lambda e_H}{\lambda e_H+e_L}$

Tab. 1: Distribution of arriving offers over the different types of agents

wage friends: choose one contact at random from the pool of unemployed and low wage employed workers.

Hence the wage distribution of vacancies one unemployed can learn of by network search is determined by the composition of her neighborhood: having more high wage employed contacts increases the probability to get a high wage offer. This is even more valid if we increase the value of the "correlation" λ, since in this case good offers arrive more often to high wage agents. This idea is known as "social resource theory" in Sociology which states that returns to informal methods depends on the status of the contact person (see Lin et al. (1981)).

Our main question is how the expected wage of the network search relates to the expected wage of the formal search in this model. We associate formal search to the "direct" arrival of offers: an individual obtains a job if she learns it directly "by the arrival process". On the other hand, the network search means that the individual has learned about an opportunity through one of her neighbors in the network. We aim to analyse how the wages from these two sources relate to each other for an average person in the society. Note that in the case of network search, concentrating on the average hides the heterogeneity existing in the neighborhood of the individuals. However, we analyse this because we would like to relate our results to the empirical literature which actually estimates the average relationships.

2.1 Markov process representation

In this section, we represent our model as a Markov process with state space $\{U, L, H\}^N$: each of the N agents can be in one of the states of unemployment (U), low wage employment (L) or high wage employment (H). There are two types of events that can occur, offer arrival and job separation, which take place after some amount of time exponentially distributed, with parameter

α and βe, respectively, where e is the employment rate in the economy. Note that the transition probabilities depend on the current state of the system not only because the employment rate influences the relative frequency of offer arrivals and job separation but also because the distribution of the new offers over the agents depend on it. Moreover, the state change of an agent is influenced by the state of their neighbors.

Given that the transition probabilities are complicated, we are not going to analyse directly the generator matrix and the limit distribution of this process. We just mention that a limit distribution exists and write up the transition probabilities of a typical individual in state $s_i \in \{U, L, H\}$ to some state $s_j \in \{U, L, H\}$. Based on this latter and using some simplifying assumptions, we construct the laws of motion of the rates u, e_L and e_H, the unemployment rate, low wage employment rate and high wage employment rate.

Note that a limit distribution exists since every states communicate with each other. This is because the job separation happens completely at random: any employed has some probability to loose her job. On the other hand, any unemployed can get any type of offer directly from the arrival process and for any finite λ, low wage employed can directly hear about a high wage job. Later we run simulations of the process on regular random graphs and we use the existence of the limit distribution to calculate some statistics of the process.

To construct the mentioned transition probabilities we look at the moments upon which the state of the system changes. As already mentioned, we define $a(e) = \frac{\alpha}{\alpha+\beta e}$, the rate that a vacancy is opened and $b(e) = \frac{\beta}{\alpha+\beta e}$, the rate that a job separation happens. If employment is high, job separation is relatively more frequent than the arrival of new offers ($a'(e) < 0, b'(e) > 0$). Table 2 describes the transition probabilities between the states for individual i. We denote N_i^S, the set of neighbors of i in state $S \in \{U, L, H\}$, ie. $N_i^S := \{j | g_{ij} = 1 \wedge s_j = S\}$. η_i^S is the cardinality of this set. Further, we denote the number of neighbors in state U or L as $\eta_i^{U+L} = |\{j | g_{ij} = 1 \wedge (s_j = U \vee s_J = L)\}|$.

The interpretations of these probabilities are the following. An unemployed obtains a low wage job if it arrives directly to her. A low wage offer arrives with probability $a(e)p_L$, we give this offer to a randomly chosen unemployed with probability u, hence the probability that we choose a given unemployed is $1/Nu$. The other possibility is that the offer arrives either to one of her low wage employed contacts or to one of her high wage employed contacts. This happens if we give the low wage offer to a high wage worker for example and this agent is exactly the one who is connected to our given unemployed. The latter has higher probability if the unemployed has more high wage contacts. The employed contacts having information should choose exactly her from the set of unemployed neighbors of theirs at random.

An unemployed can occupy a high wage job exactly the same way: either directly or in-

Transition	Probability
U → U	$1 - A - B$
U → L	$A \equiv \frac{a(e)p_L u}{Nu} + \sum_{j\in N_i^L} a(e)p_L \frac{e\lambda e_L}{\lambda e_L + e_H}\frac{1}{Ne_L}\frac{1}{\eta_j^U} + \sum_{j\in N_i^H} a(e)p_L \frac{ee_H}{\lambda e_L + e_H}\frac{1}{Ne_H}\frac{1}{\eta_j^U}$
U → H	$B \equiv \frac{a(e)p_H u}{Nu} + \sum_{j\in N_i^H} a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}}$
L → U	$\frac{b(e)}{Ne}$
L → L	$1 - \frac{b(e)}{Ne} - C$
L → H	$C \equiv a(e)p_H \frac{ee_L}{\lambda e_H + e_L}\frac{1}{Ne_L} + \sum_{j\in N_i^H} a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}}$
H → U	$\frac{b(e)}{Ne}$
H → L	0
H → H	$1 - \frac{b(e)}{Ne}$

Tab. 2:

directly. The difference is that high wage offer can come only from high wage contacts since low wage employed take these jobs themselves. On the other hand, for high wage offers there is a higher competition: they are possibly passed to low wage employed contacts as well. With the complementary probability of these two events the unemployed remains in her situation.

A given low wage employed becomes unemployed if exactly her job is destroyed. She might upgrade to a high wage job: this either happens through direct offer arrival or through their high wage neighbors. With the complementary probability nothing changes.

A high wage agent either looses her job or remains in the actual situation.

Based on the transition probabilities we can constract the law of motions of the unemployment rate and the two types of employment rates by aggregating the possible movements over the agents.

$$\dot{u} = Ne_L\frac{b(e)}{Ne} + Ne_H\frac{b(e)}{Ne} - Nu\frac{a(e)p_L u}{Nu} - \sum_{i\in N^U}\sum_{j\in N_i^L} a(e)p_L\frac{e\lambda e_L}{\lambda e_L + e_H}\frac{1}{Ne_L}\frac{1}{\eta_j^U} - \sum_{i\in N^U}\sum_{j\in N_i^H} a(e)p_L\frac{ee_H}{\lambda e_L + e_H}\frac{1}{Ne_H}\frac{1}{\eta_j^U} - Nu\frac{a(e)p_H u}{Nu} - \sum_{i\in N^U}\sum_{j\in N_i^H} a(e)p_H\frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}} \quad (1)$$

where N^U is the set of unemployed agents.

$$\dot{e_L} = Nu\frac{a(e)p_L u}{Nu} + \sum_{i\in N^U}\sum_{j\in N_i^L} a(e)p_L \frac{e\lambda e_L}{\lambda e_L + e_H}\frac{1}{Ne_L}\frac{1}{\eta_j^U} + \sum_{j\in N_i^H} a(e)p_L \frac{ee_H}{\lambda e_L + e_H}\frac{1}{Ne_H}\frac{1}{\eta_j^U} -$$

$$Ne_L a(e)p_H \frac{ee_L}{\lambda e_H + e_L}\frac{1}{Ne_L} - \sum_{i\in N^L}\sum_{j\in N_i^H} a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}} - Ne_L\frac{b(e)}{Ne} \quad (2)$$

$$\dot{e_H} = Nu\frac{a(e)p_H u}{Nu} + \sum_{i\in N^U}\sum_{j\in N_i^H} a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}} +$$

$$Ne_L a(e)p_H \frac{ee_L}{\lambda e_H + e_L}\frac{1}{Ne_L} + \sum_{i\in N^L}\sum_{j\in N_i^H} a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}} - Ne_H\frac{b(e)}{Ne} \quad (3)$$

Obviously, we have that $u + e_L + e_H = 1$ and $\dot{u} + \dot{e_L} + \dot{e_H} = 0$.

To analyse this system in an analytical way we apply some further assumptions:

Assumption 2.1: The network is a regular random graph, ie. every agent has the same number of links k. These k links are drawn randomly using the so called configuration model (See Vega-Redondo (2007)).

Assumption 2.2: Homogeneous mixing: the probability that a neighbor of an agent of any type is of a given type is equal to the population share of that type. For example, the probability that a neighbor of a high wage worker or an unemployed holds a high wage job is exactly the same and is equal to e_H. This assumption holds if at each period we had redrawn the state of each agent using the actual population shares u, e_L, e_H as probabilities to be in states U, L, H, respectively.

The first assumption we will relax later on in the paper. The second assumption is a strong one, but without this we cannot analyse the system because of the heterogeneity in the individual neighborhoods with respect to the type composition. Instead of analysing the original process, hence we analyse a mean-field version of it: we write up the law of motion of the expected movement of the system when the population gets large and the heterogeneity in the neighborhoods is averaged out. We acompany this analysis with simulations of the original process without the second assumption. In the simulations we expect to have correlations between the types of the connected agents which is assumed away by the mean-field analysis. However, we will see that our main results regarding the expected wages is going to hold in the same way in the mean-field analysis as in the simulations.

2.2 Mean-field representation

In this section we derive the simplified system using the two mentioned assumptions. Applying the homogeneous mixing assumption, we basically restrict our attention to a single individual who moves between the states based on the average transition probabilities. We concentrate on the equations of u and e_H.

In u we can make the following simplifications:

$$\sum_{i\in N^U}\sum_{j\in N_i^L} a(e)p_L\frac{e\lambda e_L}{\lambda e_L+e_H}\frac{1}{Ne_L}\frac{1}{\eta_j^U}-\sum_{i\in N^U}\sum_{j\in N_i^H} a(e)p_L\frac{ee_H}{\lambda e_L+e_H}\frac{1}{Ne_H}\frac{1}{\eta_j^U}=$$

$$Nuke_L a(e)p_L\frac{e\lambda e_L}{\lambda e_L+e_H}\frac{1}{Ne_L}\sum_{i=0}^{k-1}\binom{k-1}{i}\frac{1}{i+1}u^i(1-u)^{k-i-1}-Nuke_H a(e)p_L\frac{ee_H}{\lambda e_L+e_H}\frac{1}{Ne_H}\sum_{i=0}^{k-1}\binom{k-1}{i}\frac{1}{i+1}u^i(1-u)^{k-}$$

$$Nuke_L a(e)p_L\frac{e\lambda e_L}{\lambda e_L+e_H}\frac{1}{Ne_L}\frac{1-(1-u)^k}{uk}-Nuke_H a(e)p_L\frac{ee_H}{\lambda e_L+e_H}\frac{1}{Ne_H}\frac{1-(1-u)^k}{uk}=$$

$$a(e)p_L e(1-e^k) \tag{4}$$

First, the number of unemployed agents N^u is equal to Nu. Each unemployed has on average ke_L low wage employed neighbors. $\frac{1}{\eta^U}$ is the probability that a given low employed having an offer passes the information exactly to a given unemployed contact among the η^U unemployed contacts. We need to take the average of this probability. In the second line we use that the number of unemployed agents follows a binomial distribution $B(k,u)$ and that at each η^U the probability that a given unemployed is chosen is $\frac{1}{\eta^U}$. We have the same derivations for the case when a high wage employed learn about a low wage job and passes the information.

On the other hand, in the case when a high wage agent gets aware of a high wage offer and passes it to a given unemployed chosen from the set of unemployed and low wage employed agents, we have that:

$$\sum_{i\in N^U}\sum_{j\in N_i^H} a(e)p_H\frac{e\lambda e_H}{\lambda e_H+e_L}\frac{1}{Ne_H}\frac{1}{\eta_j^{U+L}}=$$

$$Nuke_H a(e)p_H\frac{e\lambda e_H}{\lambda e_H+e_L}\frac{1}{Ne_H}\sum_{i=0}^{k-1}\binom{k-1}{i}\frac{1}{i+1}(u+e_L)^i(1-(u+e_L))^{k-i-1}=$$

$$Nuke_H a(e)p_H\frac{e\lambda e_H}{\lambda e_H+e_L}\frac{1}{Ne_H}\frac{1-e_H^k}{k(1-e_H)}=a(e)p_H\frac{e\lambda e_H}{\lambda e_H+e_L}\frac{u(1-e_H^k)}{1-e_H} \tag{5}$$

Applying these derivations we arrive to the following differential equation:

$$\dot{u} = b(e) - a(e)u - a(e)p_L e(1 - e^k) - a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \frac{u(1 - e_H^k)}{1 - e_H} \tag{6}$$

Applying similar derivations to the law of motion of the high wage employment rate e_H and making simplifications, we arrive to the equation:

$$\dot{e_H} = a(e)p_H \left(1 - \frac{e\lambda e_H}{\lambda e_H + e_L} e_H^k\right) - b(e)\frac{e_H}{e} \tag{7}$$

3 Results

In this section we provide some results regarding the expected wages of the different search methods and the welfare of the society. We begin with a proposition saying that the dynamical system defined by the mean-field equations has a one stationary state (u^*, e_H^*) which is a global attractor and we show how the position of this state changes with the parameters of the model. To this end it is convenient to change the variables of the system to e and e_H because in this way we can use that $e \geq e_H$. We also substitute the functions $a(e)$ and $b(e)$ by their definitions and we multiply the system by $(\alpha + \beta e)$ which does not change the sign of $\dot{e}$ and $\dot{e_H}$. Hence the steady state of the system is defined by the following equations:

$$\begin{aligned}\dot{e} = -\beta e^* + \alpha(1 - e^*) + \alpha p_L e^*(1 - e^k) + \alpha p_H \frac{e^*\lambda e_H^*}{\lambda e_H^* + e^* - e_H^*} \frac{(1 - e^*)(1 - e_H^k)}{1 - e_H^*} = \\ -(\beta + \alpha p_H)e^* + \alpha + \alpha p_L e^{k+1} + \alpha p_H \frac{e^*\lambda e_H^*}{\lambda e_H^* + e^* - e_H^*} \frac{(1 - e^*)(1 - e_H^k)}{1 - e_H^*} \equiv f(e_H, e) = 0\end{aligned} \tag{8}$$

$$\dot{e_H} = \alpha p_H \left(1 - \frac{e^*\lambda e_H^*}{\lambda e_H^* + e_L^*} e_H^k\right) - \beta e_H^* \equiv g(e_H, e) = 0 \tag{9}$$

Proposition 3.1: For any $0 < \alpha, 0 < \beta, 0 < p_H < 1, \alpha p_H < \beta$, there exists a unique stationary state of the dynamics defined by (8) and (9) where $0 < e^* < 1$ and $0 < e_H^* < 1$. This stationary state is a global attractor. e^* and e_H^* increase in k, α, decrease in β. As λ increases e^* increases as well, while e_H^* decreases. e_H^* is increasing in p_H, while the effect of p_H on e^* is ambiguous.

Proof State space of the dynamics defined by (8) and (9) is $\Gamma := \{(e, e_H) : 0 \leq e \leq 1, 0 \leq e \leq 1, e_H \leq e\}$.

First, we can see that the loci $f(e_H, e) = 0$ and $g(e_H, e) = 0$, as defined in (8) and (9) respectively, cross only once. We have that $g(e_H, 0) = \frac{\alpha p_H}{\beta}$ and when $\lambda > 1$, $\frac{\partial g(e_H, e)}{\partial e} < 0$, while for

$\lambda \leq 1$, $\frac{\partial g(e_H,e)}{\partial e} \geq 0$. This is because, $\frac{\partial h}{\partial e} \geq 0$ when $\lambda \geq 1$ and $\frac{\partial h}{\partial e} < 0$ when $\lambda < 1$. $\frac{\partial h}{\partial e_H} \geq 0$ and $\frac{\partial h}{\partial \lambda} > 0$ where $h(e_H, e, \lambda) = \frac{e\lambda e_H}{\lambda e_H + e_L}$

We also know that $\frac{\partial g(e_H,e)}{\partial e_H} \leq 0$. Hence, for $\lambda > 1$ as we increase e, e_H has to decrease to have $\dot{e_H} = 0$ (see Figure 3). For $\lambda \leq 1$ as we increase e, e_H has to increase to have $\dot{e_H} = 0$ (see Figure 3). So, in both cases we have a monotone change. On the other hand, we have that it is impossible that $g(0, e) = 0$, so the curve cannot cross the x-axis in the plane (e, eH)

Looking at $f(e_H, e) = 0$, we have that $f(0, e) = -e(\beta + \alpha p_H) - \alpha p_L e^{k+1} + \alpha$ which is monotone decreasing in e, and is positive at $e = 0$ and negative at $e = 1$, hence we have a root of this function for some $e \in (0, 1)$. Observe that $\frac{\partial f}{\partial e_H} \geq 0$ (given that h is increasing in e_H and that $\frac{1-e_H^k}{1-e_H}$ is increasing as well), and $\frac{\partial f}{\partial e} \leq 0$ (even though the last term might be increasing in e, the increase is smaller than the decrease in the first term). Hence, as we increase e_H from zero, we need to increase e as well to stay on the curve $\dot{e} = 0$. We also have that $f(e_H, 1)$ is never zero, so the locus does not cross the line $e = 1$

This argument characterizes a single cross of the loci $\dot{e} = 0$ and $\dot{e_H} = 0$ as it is shown in Figure 3 and Figure 3. The cross cannot be on the boundary of the state space because $e^* \neq e_H^*$ since this would imply that $e_L^* = 0$ which cannot be when $p_L > 0$.

Now, it is easy to see that this steady state is a global attractor. If e_H is higher (lower) for a given e than the value defined by $\dot{e} = 0$, we have that e increases (decreases). If e_H is higher (lower) for a given e than the value defined by $\dot{e_H} = 0$, we have that e_H decreases (increases). Hence, the out of the equilibrium dynamics is such that the system converges to the equilibrium, as it is shown in the figures.

Now we turn to the comparative statics results. If k or α increases, the $\dot{e_H} = 0$ curve moves up, while the $\dot{e} = 0$ locus shifts to the right, hence we have that the equilibrium values of e, e_H are higher than before. If β increases, the $\dot{e_H} = 0$ curve moves upward, while the $\dot{e} = 0$ locus moves to the left, the equilibrium quantities decrease. If λ increases, the $\dot{e} = 0$ locus moves to the right, while the $\dot{e_H} = 0$ locus shifts downward. This determines a new equilibrium at a higher level of e and a lower level of e_H. When p_H increases, the locus $\dot{e_H} = 0$ moves upward, the new equilibrium value of e_H is higher than the previous one. By an increase in p_H, we cannot tell what happens to $\dot{e}$: for example, take the derivative of $f(e_H, e)$ when $\lambda \to \infty$, the highest value of the derivative wrt λ:

$$\frac{\partial f(e_H, e)}{\partial p_H} = ae\left(-1 + e^k + \frac{(1-e)(1-e_H^k)}{1-e_H}\right) = ae\frac{(1-e)(1-e_H^k) + (e^k-1)(1-e_H)}{1-e_H}$$
$$= ae\frac{e_H(ee_H^{k-1} - e^k) + e_H - e + e^k - e_H^k}{1-e_H}$$

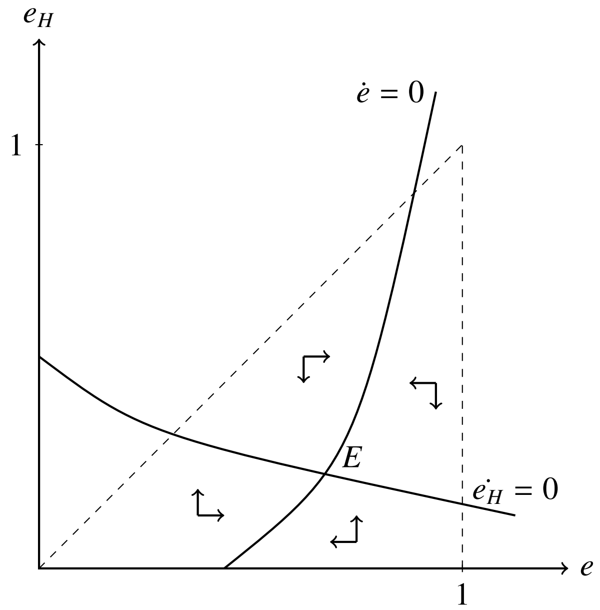


Fig. 1: Phase diagram $\lambda > 1$

this can be positive or negative. Hence, we cannot asses the final effect of p_H on the employment level. ∎

The unique steady state of the system u^*, e_L^*, e_H^* corresponds to the unique invariant distribution of the Markov process. The invariant distribution determines the probability that an agent is in one of the states $\{U, L, H\}$, using these probabilities we can derive the unique unemployment and employment rates.

As the number of neighbors k increases, the efficiency of the arrival process increases since there is higher probability that an employed agent having an unneeded offer finds someone in the neighborhood who is interested in taking it. Hence the employment level increases, part of this is realized in the higher number of high wage employed. Note that Calvó-Armengol and Zenou (2005) obtained different result on regular graphs: there is a critical value of k which minimizes the unemployment rate, however, having higher degree does not support employment. The reason is that in their model offers arrive at some rate simultaneously to different workers which gives rise to congestion of offers in case of high connectivity: many offers are passed to the same person who make use only one of them and does not transmit the other ones further (since they assume that offers travel one step in the network). On the contrary, in our model offers arrive sequentially so this possibility is excluded, higher connectivity facilitates the placement of unneeded offers. [3]

[3] Note that Ioannides and Soetevent (2006) also challenges their non-monotonicity result: assuming arbitrary degree distribution they obtain that higher average connectivity helps the employment

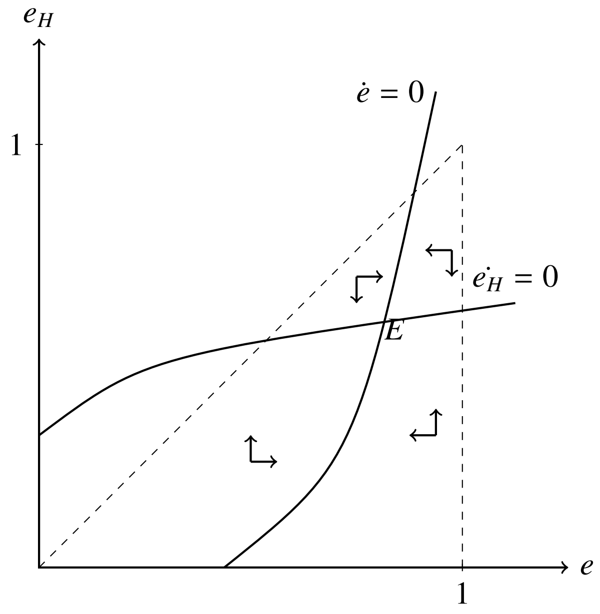


Fig. 2: Phase diagram $\lambda < 1$

Obviously, when the arrival rate of offers α rises or the frequency of job separations β diminishes, the (high wage) employment level increases.

If there are more high wage jobs arriving (p_H goes up), there will be more high wage employed agents. Concerning the effect on the total employment we have two forces to consider. First a negative one: since there are more (less) high (low) wage offers coming in, higher percentage of the new vacancies serve actually the improvement of the low wage agents and not the job finding of the unemployed. Second a positive effect: since there will be more high wage agents who pass all offers they get, unemployed agents have more chance to hear about a vacancy through social contacts. In general, we do not know which effect dominates the other.

When λ rises, we give low wage offers to low wage employed more frequently than to high wage employed. This does not affect the employment rate since these offers are passed anyway to unemployed agents. On the other hand, we more often give high wage offers to high wage employed than to low wage employed. This has a positive effect on the employment because in this way more high wage offers are going to be used by unemployed who get it from their high wage contacts and less such offer will be used to improve the situation of low wage agents. On the other hand, this implies a lower rate of high wage employment since some part of these offers will be lost when there is nobody among the contacts who will need it. This does not happen when the high wage offers arrive directly to the low wage agents. However, this effect should be very small, since the probability that a high employed has exlusively neighbors of similar status is e_H^k.

3.1 Wages

In this section, we present the main results of our paper: depending on the value of the correlation parameter λ our model predicts wage premium or discount of the network search. There exists a critical value of this parameter when the two methods results in the same wage and we show how this threshold depends on the other parameters of the model.

We associate the expected wage of the formal search to the average wage of the offers arriving directly to an agent who can make use of that offer. On the other hand, the expected wage of the network search is defined as the average wage of vacancies agents get aware of through their neighbors. In both cases we consider the conditional expectation upon arrival of a useful offer. In this way we "normalise" the expectations with the probability that such an offer arrives which is different in the case of the two search methods making the comparison more difficult.

We choose this option in order to measure the same quantity what is used in the empirical literature. The estimation of the expected wages of the search methods is based on the information obtained from employed agents who answer the kind of question "By what means were you first informed about your current job?". Individuals are provided with possible answers containing the social contacts (family, friends etc.) or some options describing formal methods (by answering adverts in newspapers, TV, radio, employment agencies, direct application to firms etc.) Based on this answer two groups of individuals is established and the average wage within these two groups is computed, controlling for the characteristics of the employee and the job etc. Hence, these wages are conditioned on the employment status of the individual: on the fact that the worker had accepted the offer which means that it was an improvement to do it compared to his/her previous employment status. Moreover, the studies do not differentiate according to the previous status of the individual: we use the average over all workers independently how much they earned previously or whether they were employed or not before they accepted the offer.

First, we write up the conditional expected wage for the direct arrival:

$$E(\text{direct } w|\text{useful offer arrives}) = \frac{\sum_{i\in N^U}\left(\frac{a(e)p_L u}{Nu}w_L + \frac{a(e)p_H u}{Nu}w_H\right) + \sum_{i\in N^L} a(e)p_H \frac{ee_L}{(\lambda e_H+e_L)Ne_L}w_H}{\sum_{i\in N^U}\left(\frac{a(e)p_L u}{Nu} + \frac{a(e)p_H u}{Nu}\right) + \sum_{i\in N^L} a(e)p_H \frac{ee_L}{(\lambda e_H+e_L)Ne_L}} \tag{10}$$

Useful direct offer arrives if any offer reaches an unemployed or a high wage offer goes to a low wage employed. We devide the probabilities by the probability of the condition: an useful offer arrives directly to some agent. Applying our two assumptions (regular graphs and homogeneous mixing) we get the following expression:

$$E(\text{direct } w|\text{useful offer arrives}) = \frac{w_L p_L u + w_H p_H (u + \frac{ee_L}{\lambda e_H + e_L})}{p_L u + p_H (u + \frac{ee_L}{\lambda e_H + e_L})} \tag{11}$$

Next, we turn to the indirect channel: an useful offer arrives through this if a low or high wage offer reaches an unemployed or if a high wage offer is passed to a low wage employed. High offers are forwarded only by high wage workers while low offers by any employed. We have the following in the general case:

$$E(\text{indirect } w|\text{useful offer arrives}) =$$
$$\frac{\sum_{i\in N^U}\left(\sum_{j\in N_i^L} a(e)p_L\frac{e\lambda e_L}{(\lambda e_L+e_H)Ne_L\eta_j^U} + \sum_{j\in N_i^H} a(e)p_L\frac{ee_H}{(\lambda e_L+e_H)Ne_H\eta_j^U}\right)w_L + \sum_{i\in N^{U+L}}\sum_{j\in N_i^H} a(e)p_K\frac{e\lambda e_H}{(\lambda e_H+e_L)Ne_H\eta_j^{U+L}}w_H}{\sum_{i\in N^U}\left(\sum_{j\in N_i^L} a(e)p_L\frac{e\lambda e_L}{(\lambda e_L+e_H)Ne_L\eta_j^U} + \sum_{j\in N_i^H} a(e)p_L\frac{ee_H}{(\lambda e_L+e_H)Ne_H\eta_j^U}\right) + \sum_{i\in N^{U+L}}\sum_{j\in N_i^H} a(e)p_K\frac{e\lambda e_H}{(\lambda e_H+e_L)Ne_H\eta_j^{U+L}}} \tag{12}$$

This simplifies to the following in the mean-field case for regular graphs:

$$E(\text{indirect } w|\text{useful offer arrives}) =$$
$$\frac{w_L p_L e(1-e^k) + w_H p_H \frac{e\lambda e_H}{\lambda e_H+e_L}(1-e_H^k)}{p_L e(1-e^k) + p_H\frac{e\lambda e_H}{\lambda e_H+e_L}(1-e_H^k} \tag{13}$$

In the following result we relate these two expected wages to each other.

Proposition 3.2: There exists a critical value of λ such that for $\forall\lambda < \bar{\lambda}$, the expected wage of a direct offer is higher than the expected wage of the network search, while for $\forall\lambda \geq \bar{\lambda}$ the reverse is the case. $\bar{\lambda}$ is decreasing in α and p_H while is increasing in β.

Proof INCOMPLETE!!! First, we can see that for $\lambda = 1$ we have that the direct wage is higher, while for $\lambda = \infty$, the indirect one. For $\lambda = 1$, $\frac{e\lambda e_H}{\lambda e_H + e_L} = e_H$. To compare the two expected wages, we can relate the ratio of the probabilities of high and low wage offer arrival (direct-indirect):

$$\frac{p_H(1-e_H)}{(1-p_H)(1-e)} - \frac{p_H e_H(1-e_H^k)}{(1-p_H)e(1-e^k)} \geq 0$$

That is:

$$\frac{(1-e^k)e}{1-e} - \frac{(1-e_H^k)e_H}{1-e_H} \geq 0$$

This difference is indeed positive given that the function $g(x) \equiv \frac{(1-x^k)x}{1-x}$ is increasing in $x \in [0, 1]$.

Note that for $\lambda = 0$, we have the same relation.

For $\lambda = \infty$, $\frac{e\lambda e_H}{\lambda e_H + e_L} = e$. Here we can show that the weight on the high wage in the indirect expectation is higher than in the direct expectation:

$$\frac{p_H(1 - e_H^k)}{(1 - e^k)(1 - p_H) + p_H(1 - e_H^k)} > p_H$$

That is:

$$p_H(1 - e_H^k) > p_H(1 - e^k) + p_H^2(e^k - e_H^k)$$

This is true given that $p_H > p_H^2$ if $p_H < 1$.

To establish that there exists a threshold value of the correlation λ, we need to show that the direct expected wage decreases in it and the indirect wage increases. We again concentrate on the fraction between the weights on the high wage and on the low wage. In case of the formal search we have:

$$\frac{p_H\left(u + \frac{ee_L}{\lambda e_H + e_L}\right)}{p_L u}$$

which is clearly decreasing in λ. In case of the indirect wage:

$$\frac{p_H \frac{e\lambda e_H}{\lambda e_H + e_L}(1 - e_H^k)}{p_L e(1 - e^k)}$$

which is increasing in λ.

The critical value can be expressed from the equation where the ratio between the weight on the high wage and the weight on the low wage is equal for the two wages:

$$\frac{p_H(1 - e_H)^k \frac{e\lambda e_H}{\lambda e_H + e_L}}{p_L e(1 - e^k)} = \frac{p_H\left(u + \frac{ee_L}{\lambda e_H + e_L}\right)}{p_L u}$$

After rearranging and expressing λ, we have the following:

$$\bar{\lambda} = \frac{(e - e_H)(1 - e^k)}{e_H(1 - e)(e^k - e_H^k)} \tag{14}$$

Regarding the effect of parameters on the critical value, we can see that $\bar{\lambda}$ is decreasing in e and e_H. First, we have that as these employment rates go to 1, the threshold takes it's minimal value: 1:

$$\lim_{e_H \to 1} \lim_{e \to 1} \bar{\lambda} = \lim_{e_H \to 1} \lim_{e \to 1} \frac{(e - e_H)(1 - e^k)}{e_H(1 - e)(e^k - e_H^k)} = \lim_{e_H \to 1} \frac{k(1 - e_H)}{e_H - e_H^k} = 1$$

Second, as e_H approaches to 0, the threshold goes to infinity:

$$\lim_{e_H \to 0} \bar{\lambda} = \lim_{e_H \to 1} \frac{(e - e_H)(1 - e^k)}{e_H(1 - e)(e^k - e_H^k)} = \infty$$

On the other hand, we have that the function should go in a monotone way from 1 to infinity as the high wage employment rate decreases. To see this we take the derivative of $\bar{\lambda}$ wrt e_H and we have that this cannot be zero:

$$\frac{\partial \bar{\lambda}}{\partial e_H} = \frac{(e^k - 1)(-e^{1+k} + e_H^k(e + ek - e_H k))}{(e - 1)e_H^2(e^k - e_H^k)^2}$$

This can be zero only if $-e^{1+k} + e_H^k(e + ek - e_H k) = 0$. After rearranging and taking logarithm:

$$\log e + k(\log e - \log e_H) \neq \log(e + k(e - e_H))$$

Hence, the function has no critical point, so it has to go in a monotone way between 1 and infinity. As an illustration, we plot the threshold λ for the case when $k = 6$. So we have that as α or p_H increases, the critical λ decreases since by Proposition 1 e_H is increasing in these parameters. ∎

The intuition behind this result is the following: when we increase λ the information about high wage vacancies more often goes to high wage employed who pass that offer. Since the share of the forwarded low wage offers does not change, low and high wage employed pass them in the same way, the expected wage of the network search increases. On the other hand by higher λ, low wage employed less often get high wage offers in the direct way, hence the weight put on the high wage in the expected wage of the formal search decreases. By the correlation between status and the quality of offer we channel the high wage offers to individuals who actually pass them to their contacts. As the fraction of high wage employed decreases (keeping fixed the total employment rate), in order to keep the expected wage of the network search at the same level, we need to give the high wage offers more systematically to them because they forward these offers.

Our proposition sheds new light why and when we observe wage premium or discount for the network seach compared to the formal methods. First, we suggest that the network search provides wage premium in cases when the status of an individual and the quality of the vacancies she might hear about exhibits high correlation. Second, the requiered correlation depends on the actual state of the economy: in "bad times", when less vacancies open up or the wage distribution of new offers puts lower weight on the well-payed jobs or there is more job destruction, we need a higher correlation, hence we are less likely to observe a wage premium of the network search and more likely to have wage discount. On the other hand, Figure 4 indicates that the critical correlation level is higher when the connectivity of the network is higher as well and the effect of connectivity is higher when the frequency of job arrivals is low.

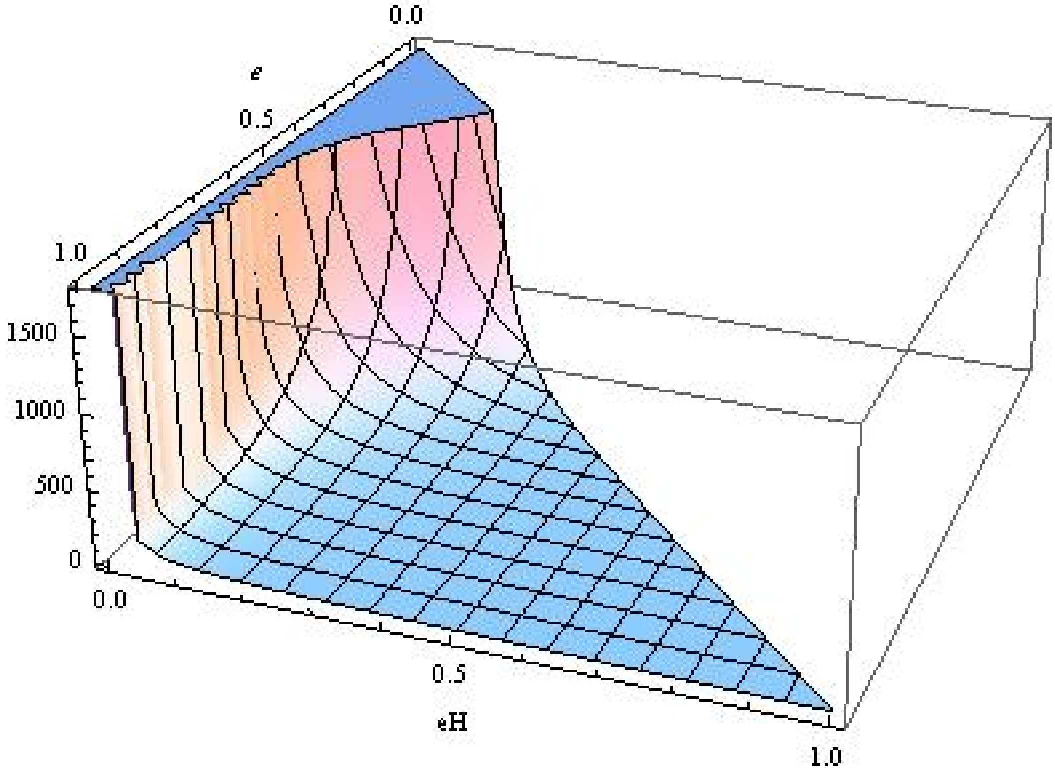


Fig. 3: Critical value of λ for $k = 6$ (taking into account that $e \geq e_H$)

3.2 Welfare

We also look at how the welfare of the society is related to the wage premium/discount of the network search. We can define the welfare of the society as the total amount of wages earned (utilitarian welfare): $e_L w_L + e_H w_H$. Or we can minimize the share of the individuals who are in the worst position, u (Rawlsian welfare). On the other hand, we can also describe the welfare of the society using the expected number of lost offers as a measure of societal efficiency:

$$a(e)p_L e(1 - e^k) + a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L} e_H^k$$

It is easy to see that the number of lost offers is minimized by $\lambda = 0$: when the highest share of the high wage offers goes directly to the low wage employed who take them. In this case we have the highest wage discount of the network search. On the other hand, recalling that the employment level increases in λ, we have that the Rawlsian welfare is the highest in the other extreme case, when the network search gives the highest wage premium ($\lambda = \infty$).

Regarding the utilitarian welfare, as we increase the correlation parameter λ, the employment level $(e_L + e_H)$ increases but the share of high wage employed decreases. The overall effect on

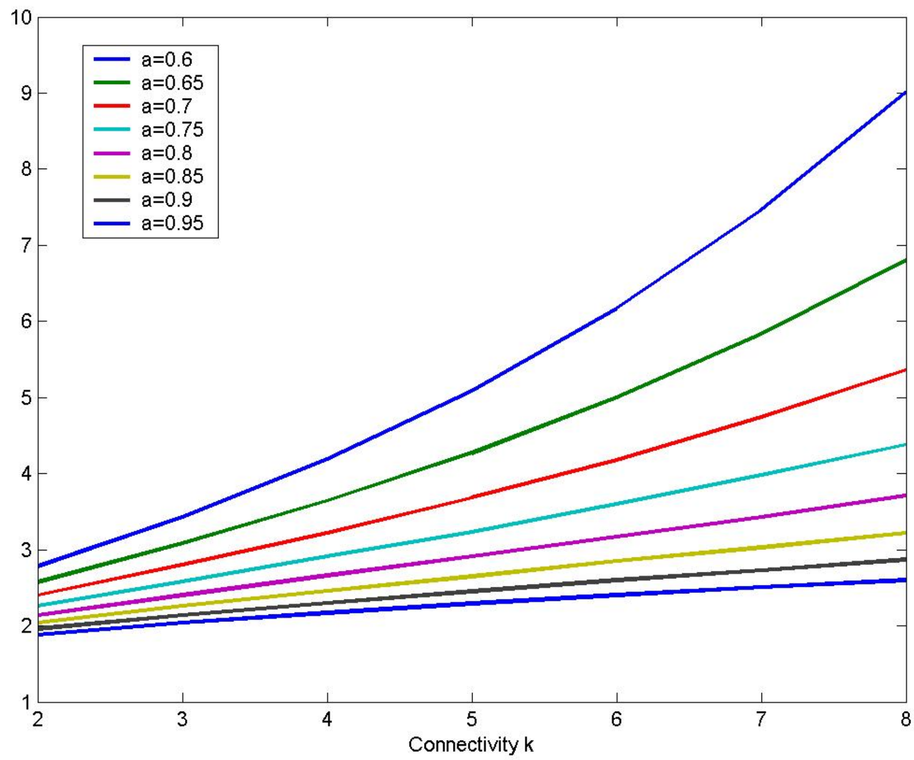


Fig. 4: The critical value of the correlation as a function of connectivity for different values of the job arrival probability - numerical solution of the mean-field system, $p_H = 0.5, \beta = 0.5$.

welfare depends on the ratio between the high wage and the low wage ($\bar{w_H} \equiv w_H/w_L$). If this fraction is small, we have that the utilitarian welfare is the highest when the network search gives the highest expected wage ($\lambda = \infty$). On the other hand, when the ratio is high we have a contradiction between the Rawlsian and the utilitarian welfare. The utilitarian welfare is equal to $e + e_H(\bar{w_H} - 1)$. If $\bar{w_H} = 1$, the welfare is equal to the employment level, hence increasing in λ. While for higher values of $\bar{w_H}$, the increase in the employment level is outweighted by the decrease in the ratio of high wage employed.

On the other hand, when there are more vacancies opened (α is higher), the (high wage) employment rate rises, so the critical value of the correlation decreases, we are more likely to observe wage premium and the utilitarian and Rawlsian welfare get higher. Higher job destruction has the opposite effect. If we increase the connectivity of the graph k, the critical correlation increases as well, making less likely to observe network premium. However, the total employment rate and the share of high wage employed rises, guaranteing higher Rawlsian and utilitarian welfare.

Hence, we conclude that in our model the welfare of the society need not be the highest when the network search gives a wage premium. In this way the expected reward of the network search need not be connected to the welfare of the society even though the higher arrival rate of offers, often associated to the social contacts, would indicate that this should be the case.

4 Computational analysis

Appart from the presented mean-field analysis we have also simulated the described labor market turnover process on regular graphs. This is needed since so far we have assumed homogeneous mixing which discloses any correlation in the state of the connected individuals. However, we can expect the existence of such correlations in the original version of the model (without making our assumptions) since connected agents pass offers to each other which makes them more likely to share the same situation.

Our simulation strategy is based on the existence of the invariant distribution of the Markov process: we simulate the model for sufficiently long time and use the fact that the proportional time spent in each state is equal to the corresponding probability in the invariant distribution (when the time goes to infinity). Hence, we can obtain the long-run value of different statistics derived from the invariant distribution by taking the time average of them using the data recorded during the simulation. To be sure that we have chosen a sufficiently long period of time, we run the simulation for $2T$ periods and we compute the average statistics after period T and $2T$. Based on the fact that the invariant distribution is independent of the initial conditions, the averages taken at the two point of time should coincide. In our simulations, this happens after 60000 periods.

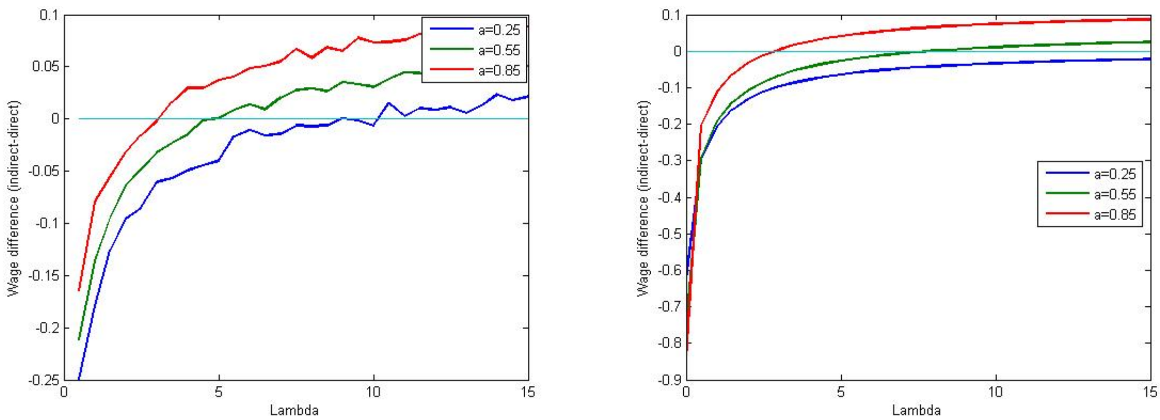
We record the unemployment rate, the low and high wage employment rates, whether actually a new offer arrived and whether this has been taken by some agent (or lost), the type of the offer that has been taken and the way (direct or indirect) it has reached the individual who has taken it. Corresponding to the mean-field analysis, we simulate the model on regular graphs, ie. when every agent has the same number of links. The graph is a random network: we use the configuration model to generate the actual network for the simulation. To exclude the effect of any arbitrary realization of this network generation process, we take a sample of 30 runs, ie. we run the model on 30 different networks and take the average of the outcomes in this sample. Initially each agent has 1/3 probability to be in each of the states. This has no effect on the simulation as the invariant distribution is independent of the initial conditions. After we run the process for 60000 periods which actually guaranties that we have run the model for long enough.

The baseline values of the parameters are the following: $\alpha = 0.85$, $\beta = 0.5$, $p_H = 0.5$, $k = 6$. λ has been changed between 0 and 15 by step 0.5.

Our simulations confirm all the results presented using the mean-field approach: the presence of correlation in the state of connected agents does not influence our results regarding the relation of the expected wages of the search methods and the effect of the parameters.

The following graphs present some results regarding the effect of the correlation (λ) on the expected wage of the search methods and the unemployment and employment rates. We also analyse how is the interaction between this correlation and the other parameters of the model. Next to the simulation results we plot the numerical solution of the mean-field system of differential equations applying the same parameter values.

Figure 5 shows that the wage difference between indirect and direct arrival is a concave function in the correlation parameter λ which has only one intersection with the axis determining the critical value of λ. We also can see that this threshold is decreasing in the arrival rate of offers.



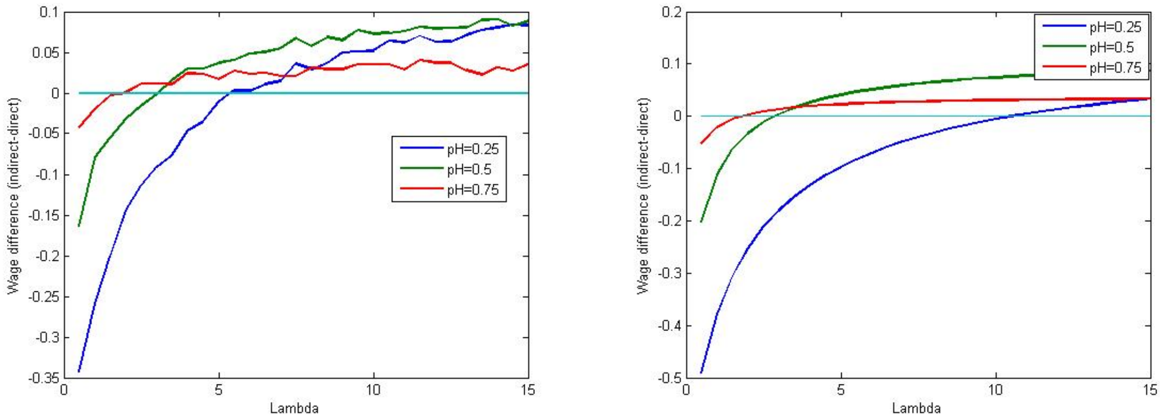
(a) Simulation (b) Mean-field

Fig. 5: Wage difference (indirect-direct) as a function of the correlation ($\lambda \in [0, 15]$) for different values of the job arrival rate ($\alpha \in \{0.25, 0.55, 0.85\}$), blue, green, red lines, respectively.

Figure 6 shows that as the share of high wage offers in the arriving vacancies is lower, the critical value of λ increases.

Figure 7 confirms that the threshold correlation $\bar{\lambda}$ is increasing in the connectivity k. Note that we have small differences between the critical values because the arrival rate of offers is taken to be high as it was indicated by the graph 4. As we can see, the pattern of the wage difference as a function of λ is robust to the parameter changes.

Figure 8 shows that the employment rate is increasing while the high wage employment is decreasing in λ, though for $\lambda > 1$ these changes are of small scale. We also can observe that with the arrival rate of offers α the employment increases but this is due to the high wage employment,



(a) Simulation

(b) Mean-field

Fig. 6: Wage difference (indirect-direct) as a function of the correlation ($\lambda \in [0, 15]$) for different values of the job arrival probability of the high wage offer ($p_H \in \{0.25, 0.5, 0.75\}$), blue, green, red lines, respectively.

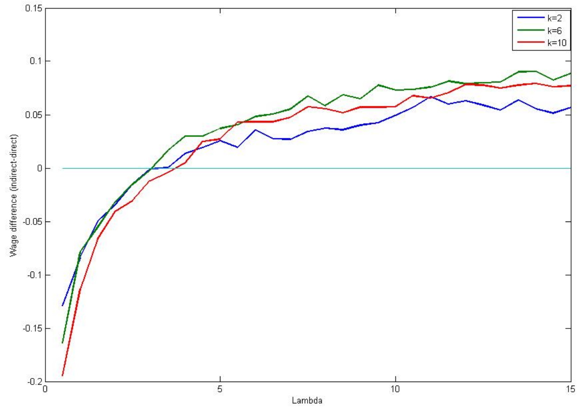
the low wage employment rate does not rise. This is because the high wage offers are more useful: many low wage employed upgrade their situation.

Finally, the number of lost offers is shown in Figure 9. We can see that it is increasing in λ: more high wage offers are allocated to high wage employed agents who pass these offers which comes with the possibility of not finding anyone who is interested in that job. On the other hand we observe that as the arrival probability of offers increases, the fraction of lost offers increases as well: there are more high wage employed agents. For the same reason, as the probability of an offer being of high quality increases, there are more lost offers. Higher connectivity obviously means less lost offers.

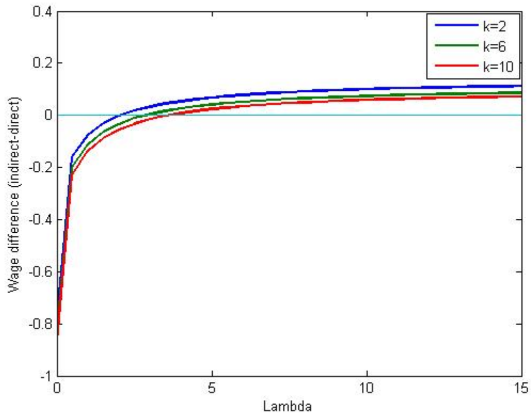
5 Extensions

5.1 Graphs with heterogeneous degree distribution

In this section, we look at the situation when individuals can be different with respect to their degree: we consider general random graphs instead of regular graphs, so we abandon assumption 2.1. In this case the network is defined by an exogenous degree distribution, $P(k)$ is the probability that an individual has k degrees, $k \in [0, \infty)$. Our mean-field equations do not change significantly in this case. The system is defined by the following two differential equations:



(a) Simulation



(b) Mean-field

Fig. 7: Wage difference (indirect-direct) as a function of the correlation ($\lambda \in [0, 15]$) for different values of the connectivity offer ($k \in \{2, 6, 10\}$), blue, green, red line, respectively.

$$\dot{u} = b(e) - a(e)u - a(e)p_L e \sum_k P(k)(1 - e^k) - a(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_k P(k) \frac{u(1 - e_H^k)}{1 - e_H} \quad (15)$$

$$\dot{e_H} = a(e)p_H \sum_k P(k) \left(1 - \frac{e\lambda e_H}{\lambda e_H + e_L} e_H^k\right) - b(e)\frac{e_H}{e} \quad (16)$$

Here we again suppose homogeneous mixing which in this case also means that there is independence between being unemployed, for example, and the degree of an individual.

Having general random graphs does not change much on our analysis, the effects of the parameters on the steady state values e^*, e_H^* remains the same. Now, a rise in the connectivity

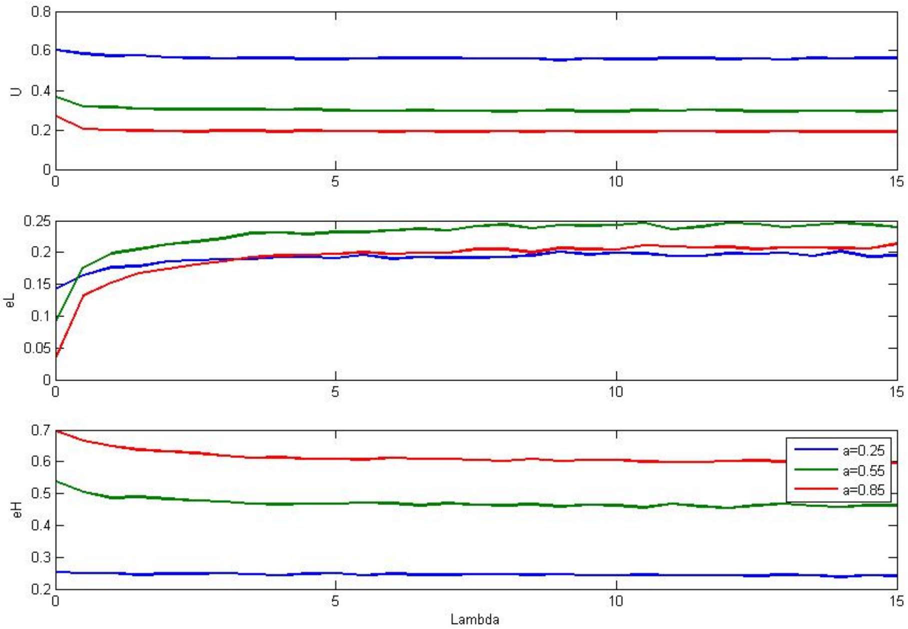


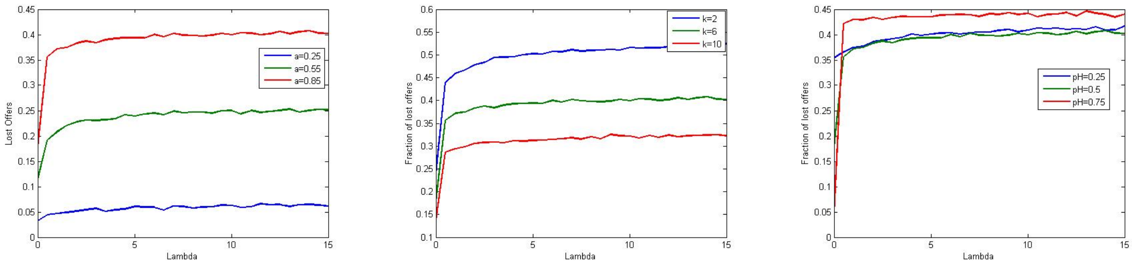
Fig. 8: Unemployment rate, low wage employment rate and high wage employment rate as a function of $\lambda \in [0, 15]$ for different values of the job arrival probability $\alpha \in \{0.25, 0.55, 0.85\}$, blue, green,, red lines, respectively.

of the graph can be represented as a first order stochastic shift in the degree distribution which increases these employment rates, just as in the regular case.

The expected wage of the formal search does not change while that of the network search is defined as follows:

$$E(\text{indirect } w|\text{useful offer arrives}) = \frac{w_L p_L e \sum_k P(k)(1-e^k) + w_H p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_k P(k)(1-e_H^k)}{p_L e \sum_k P(k)(1-e^k) + p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_k P(k)(1-e_H^k} \tag{17}$$

Our conclusions regarding the wage difference between the two methods do not change going from regular graphs to networks with heterogeneous degree distribution. In the same way as before, it can be shown that for zero or negative correlation ($\lambda = 1$ or $\lambda = 0$) the expected wage of formal search is higher. While for perfect positive correlation the network search yield higher expected wage. It is also true that there is a threshold value in the correlation where the two methods give the same expected wages. The parameters of the model influence this threshold in



(a) Effect of $a \in \{0.25, 0.55, 0.85\}$, red, green, blue line respectively.

(b) Effect of $k \in \{2, 6, 10\}$, blue, green, red line, respectively.

(c) Effect of $p_H \in \{0.25, 0.5, 0.75\}$, blue, green, red line, respectively.

Fig. 9: Fraction of lost offers as a function of $\lambda \in [0, 15]$ for different values of the other parameters

the same way. For example, Figure 10 indicates that, taking a Poisson degree distribution, as the average degree increases, the critical value of λ increases as well.

Instead of solving for one average unemployment rate and employment rates we can define the corresponding variables for each classes of individuals with a given degree k. Then we allow the different agents to have different unemployment rates according to their degree: we do not assume independence between state and degree. In this case, u_k denotes the fraction of unemployed agents given that the individual has degree k. $e_{L,k}$ and $e_{H,k}$ are defined in a similar way. We have the following identities: $u = \sum_k P(k)u_k$, $e_L = \sum_k P(k)e_{L,k}$, $e_H = \sum_k P(k)e_{H,k}$, $u_k + e_{L,k} + e_{H,k} = 1$.

We can write up the law of motion of these quantities as follows.

$$\dot{u_k} = b(e)\frac{e_k}{e} - a(e)u_k - u_k k a(e) p_L e \sum_l \zeta(l)\frac{1-(1-\hat{u})^l}{\hat{u}l} - u_k k a(e) p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_l \zeta(l)\frac{1-\hat{e_H}^l}{(1-\hat{e_H})l} \quad (18)$$

where $\zeta(l)$ is the degree distribution of a neighboring node, given by:

$$\zeta(l) = \frac{lP(l)}{\sum_k P(k)k}$$

$\hat{u} \equiv \sum_k \zeta(k)u_k$ represents the average unemployment rate of a neighboring node (similarly for the other rates). Note that the degree distribution of a neighboring node differs from the degree distribution because the former is conditional on the fact that one link leads to the given agent. Hence, the average unemployment rate of a neighboring node is different as well. Here we take into account that the employed agent who passes an offer to an unemployed have degree l according to the neighboring degree distribution and that each of her neighbors (who are competitors for the same information) are unemployed with probability $\hat{u}$.

The low of motion of the fraction of high wage employed with degree k is given by:

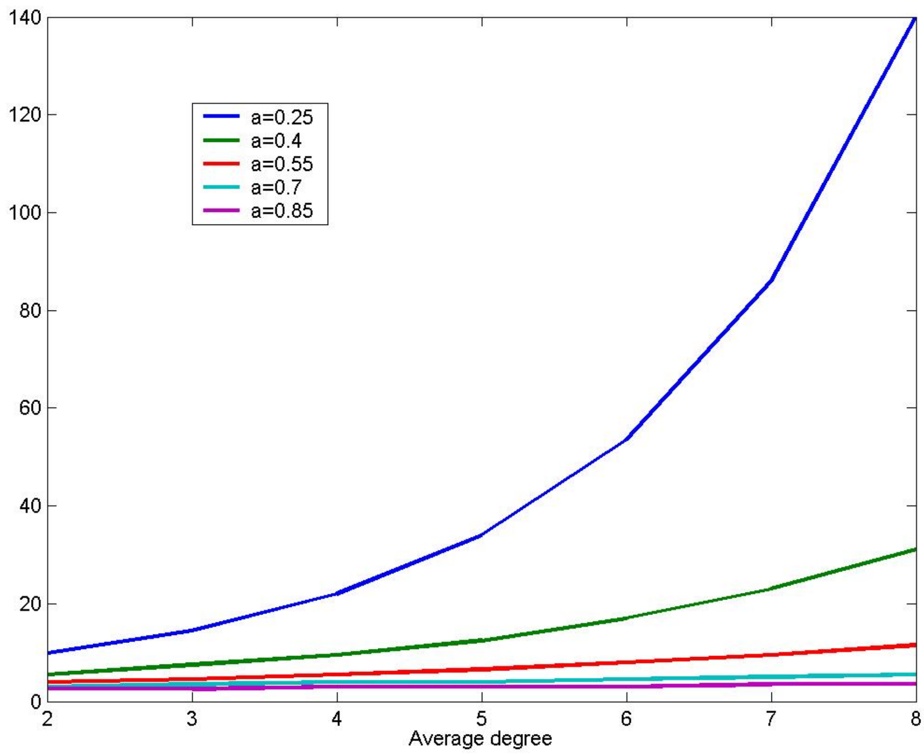


Fig. 10: The critical value of the correlation as a function of average degree for different values of the job arrival probability α, Poisson case

$$\dot{e_{H,k}} = a(e)p_H\left(u_k + \frac{ee_{L,k}}{\lambda e_H + e_L}\right) + (u_k + e_{L,k})ka(e)p_H\frac{e\lambda e_H}{\lambda e_H + e_L}\sum_l \zeta(l)\frac{1 - \hat{e_H}^l}{(1 - \hat{e_H})l} - b(e)\frac{e_{H,k}}{e} \quad (19)$$

By similar arguments as used in the proof of Proposition 3.1, it can be seen that the unemployment rate of a high degree agent is lower than that of a low degree agent, while the former has higher probability to hold a high wage job. This implies that agents with higher degree have higher utilitarian or Rawlsian welfare. We also can see that the effect of degree is increasing in the correlation represented by λ, since the technology $\frac{e\lambda e_H}{\lambda e_H + e_L}$ which directs the high offers to high wage agents positively depends on λ.

It is an interesting question whether the threshold value of the correlation is different for agents of different degrees. Obviously, the expected wage of the formal search is independent of degree. We can write up the expected wage of the network search for an individual of degree k in the following way:

$$\frac{ka(e)p_L e \sum_l \zeta(l)\frac{1-\hat{e}^l}{\hat{u}l} w_L + ka(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_l \zeta(l)\frac{1-\hat{e_H}^l}{(1-\hat{e_H})l} w_H}{ka(e)p_L e \sum_l \zeta(l)\frac{1-\hat{e}^l}{\hat{u}l} + ka(e)p_H \frac{e\lambda e_H}{\lambda e_H + e_L} \sum_l \zeta(l)\frac{1-\hat{e_H}^l}{(1-\hat{e_H})l}}$$

We can see on this expression that the degree k drops out, hence the expected wage of the network search is the same for all agent in the society independently of degree. This implies that the threshold of the correlation is independent of degree as well.

5.2 Heterogeneous labor

In future work, we would like to introduce different types of workers and offers, let say type "A" and type "B". Type "A" workers have higher probability to learn about type "A" offers (our correlation assumption). Both types of workers are able to fill any job, however, holding a job of the own type gives a higher wage for the worker. We take into account that agents of similar type tend to be connected in the network, ie. the network exhibits homophily. In this model we still expect to get the same result regarding the expected wages: high correlation between the type of a worker and the type of the job he/she might hear about implies a wage premium for the network search. However, in this version this effect also depends on the degree of homophily. We investigate the effect of this latter to the critical value of the correlation λ.

6 Conclusions

We analyzed the relationship between expected wages of two job search methods: using social contacts and formal methods. Previous empirical (and theoretical) literature have found contradictory evidences regarding this relation: Pellizziari (2009) has obtained wide cross-country differences, Kugler (2003) has estimated wage premium for the network search while Bentolila et al. (2009) wage discount. In our model, we have seen that social contacts provide higher expected wage if there is a high correlation between the status of an employed individual and the quality of the job offer she might hear about. The critical value of this correlation depends on the parameters of the arrival process and the connectivity of the social network. We suggest that in "bad times", when less vacancy opens, the new offers have lower status or the job destruction rate is high, the correlation has to be higher than in "good times" to obtain wage premium of the network search. We believe that this "business cycle" argument can add to the explanation of the observed variances in the relative wages of the two search methods.

On the other hand, we obtain that the Rawlsian welfare of the society is the highest when the network search gives the highest wage premium. In contrast, the utilitarian welfare might be the highest in the opposite case, when the formal search gives the highest wage premium. This also suggest that exclusively looking at unemployment rate can be misleading regarding the welfare consequences of the presence of social networks in the labor markets: in a context of heterogenous wages we might get different conclusions.

In future research, we plan to extend the model to take into account different occupations and the fact that individuals of the same occupation tend to be connected, ie. the social network exhibits homophily. We would like to investigate how this latter phenomenon influences the expected wages of the search methods.

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