Bargaining and Networks in a Gas Bilateral Oligopoly

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Abstract

In the context of international gas markets, we investigate the interaction between price formation and communication networks in a bilateral duopoly with heterogeneous buyers. Given a particular buyers-sellers network graph, prices are formed as the outcome of dynamic decentralized negotiations among traders. We characterize, for any network structure, the full set of sub-game perfect Nash equilibria in pure and stationary strategies (PSSPNE) of the non-cooperative bargaining game with random order of proposals and simultaneous responses. Depending on the inter-temporal discount factor and the dispersion of reservation values across buyers, negotiations may lead, even in a completely connected buyers-sellers network, to multiple equilibria, coexistence of different prices, delays in trade and inefficient allocations. The endogenous bargaining power of each trader as a function of her position in the communication network is derived by comparing traders’ payoffs across networks. Network formation equilibria and experimental evidence are discussed.

Keywords: Non-cooperative bargaining; buyer-seller networks; thin markets; gas markets.

JEL classification: D43, D85, L71, L95.

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1 Motivation

On January 2009, during the coldest days of the year, Europe stared at what resembled the sequel of a scene already on stage three years before. Alike on 1st January 2006, 2009 New Year’s celebrations in western Europe were shadowed by the news that gas extracting company Gazprom, backed by the Russian government, began cutting off gas in pipelines to Ukraine. European Union countries were concerned about such an exacerbation of Ukraine-Russia conflict specially because about 80 percent of Russian gas exports to Western Europe were actually made through Ukraine.

Clearly Gazprom’s decision to reduce pressure in the pipelines did not help Russia and Ukraine to reach a compromise in their on-going negotiations over the revision of both the price of supplied gas and the transit fee. At the contrary, until January 19th, after many European countries saw an immediate drop in the supply of gas, Russia accused Ukraine to siphon off gas and Ukraine accused Russia to undersupply gas and falsely accuse of siphoning.

It was only on January 19th, after the intervention of the European Union, that the on-going negotiations between Ukraine and Russia finally reached an equilibrium: premiers Putin and Tymoshenko agreed that, during 2009, Ukraine would pay for the russian gas a price 20 percent cheaper than the one charged to European Union countries, a figure estimated at 360 US dollars for 1000 cubic metres, lying in between the initial offers of 450 and 250 US dollars, by Russia and Ukraine, respectively.

Even though the European Union officially welcomed the reached agreement in the negotiations as a positive factor of stability, many European countries still remain worringly concerned about the substantial problem still unsolved, namely the heavy dependency from Russia for the gas supplies. In fact, several other times in the past Gazprom and Russia have used exclusive access to distribution networks as a threat to enhance their bargaining power during negotiations over the gas price to be charged to European countries.

Actually, looking at the material web of gas pipelines connecting Russia to western european countries, one may believe a commonly quoted idea: Gazprom strategically builds
gas networks in order not only to enhance its own bargaining position in negotiations, but also to weaken the power of countries - with large domestic gas consumption - which are either reluctant to accept its price conditions, or attracted by the possibility to diversify the portfolio of gas suppliers.

For instance, it is often argued that Gazprom signed with E.On Ruhr and Basf a partnership in Germany to build a direct gas pipeline - called North Stream - in the Northern Sea, in order to by-pass Ukraine, Belarus and Poland. It can also be observed how effectively Gazprom has managed to exploit the inability of European countries to set up a common energy purchasing agency, and even to exacerbate competition between European states, by deriving country-specific branches from the main pipelines and by signing up individual contracts. Moreover, even very recently\(^1\) it has been claimed that the main reason beyond the agreement signed by Gazprom and ENI to build in partnership the South Stream pipeline from Russia to Bulgaria under the Black Sea, is Gazprom’s aim at blocking any other project - such as the EU-funded Nabucco and Transcaspian projects - of gas networks directly connecting Europe to alternative gas-extracting countries, like Turkmenistan, Azerbaijan, Uzbekistan or Iran\(^2\).

How the negotiations may be affected by the shape of network architectures is indeed one central issue still to be investigated in modelling gas markets. Clearly this is also of more general interest, as, for instance, in any network infrastructure a company owning (or having the exclusive right to build) an important branch of the network passing in its country and directed to other domestic markets has clearly a better bargaining position than a terminal node of a foreign-owned pipeline. However, the interrelation between negotiations and networks becomes a much more serious issue in the specific case of the gas industry. This is due to two further features of the international gas markets.

\(^1\)See The Economist, 26th January 2008.  
\(^2\)The European Union Nabucco project, in fact, is meant to bring gas from the Central and Caspian Asia to Europe through the Balkans, and would be the only pipeline from the region that does not cross Russian territory. This would be give Europe the only hope of more diversified gas supplies. In order to be effectively working, gas must arrive to Nabucco from either a trans-Caspian pipeline (the Transcaspian project), which is however blocked by Russia, or through Iran, a solution which is strongly opposed by USA. If Gazprom alternative pipeline South Stream were ever built, it would certainly make Nabucco uneconomic and would convince European Union to definitely abandon it.
Firstly, the one of gas is not a fully competitive market, but, at the contrary, shows all the salient characteristics of a *bilateral oligopoly*: a thin market where a very limited number of traders on both sides are likely to strategically affect both the formation of price and the choice of their trading partners. In fact, a handful of largest extracting countries sell natural or liquefied gas to few major buying companies, mostly behaving as national distributing monopolies. According to the latest statistics on gas world import-export (IEA, 2006, 2007, 2008), Russia, Algeria, Canada, Norway, Qatar, Indonesia, Malaysia, Turkmenistan and Iran alone represent almost 80 percent of the world export, while two thirds of gas imports are concentrated in the purchases of national companies from less than a dozen of countries: Japan, South Korea, China, India, United States, France, Germany, Italy and Spain.

The bilateral concentration of the international gas market is even higher if the existence of a global network of exclusive or primary partnership relationships is taken into account, which is further shaping gas trades within macro-regional areas: Japan, for instance, buys more than half of its liquefied imported gas from Indonesia and Malaysia; France relies on gas from Algeria for 80 percent of its import; from Russia is imported more than 90 percent of the gas consumed by Finland, Slovakia, Bulgaria, Lithuania and Czech Republic, and between 50 and 75 percent of the gas consumed by Greece, Austria, Hungary and Poland; Italy depend almost exclusively on gas supplies by Russia and Algeria.

Secondly, and consequently, in the international gas markets, prices are not simply reflecting the daily trading in an organized financial institution, but are the outcome of bilateral contracts and of, possibly intricate, decentralized negotiations. Most south-european countries, for instance, depend almost exclusively on gas supplies by just two national extracting companies, the russian *Gazprom* and the algerian *Sonatrach*, with which they bargain bilateral contracts specifying trading prices and conditions.

It seems interesting to explore at which extent the negotiations depend on the shape of the distribution network: *what are the interrelations between a trader’s bargaining power*
The issue can be very intricate\textsuperscript{3}. Here we just focus on the simpler case of a small buyers-sellers network with heterogeneous traders, in which fully decentralized negotiations take place. It can be seen as an exploratory analysis: of course, there remains a lot more to investigate.

\section{Discussion of modelling issues and related literature}

\textit{Bilateral oligopolies} are characterized by a small number of traders on each side. Being both sides of the market concentrated and endowed by market power, both buyers and sellers are able to affect the prices at which they trade. Furthermore, due to the absence of serious searching costs, traders in such thin markets do not act anonymously and are usually able to affect to some extent the choice of their trading partners.

Examples of bilateral oligopolies, beyond the case of the international gas market, can be found in some of the basic commodities markets - such as the ones for the coffee, tobacco, hazelnuts\textsuperscript{4} - of some minerals, and, above all, in estimated 90 percent of the intermediate goods markets: just to name some, the aerospace, aircrafts\textsuperscript{5} and shipping\textsuperscript{6} industries, the

\textsuperscript{3}For a fairly updated survey of the insights to gas markets from bargaining and network models, see Galizzi (2006).

\textsuperscript{4}In the market for hazelnuts, about 60 percent of the orders come from Ferrero - an Italian food company famous for producing Nutella - while a centralized agricultural agency of Turkey represents more than half of the supply.

\textsuperscript{5}The US-based Boeing and the pan-European consortium Airbus are, in fact, accounting for substantially two thirds of the world supply of aircrafts. Contrary to what it may be believed, the demand side is also extremely concentrated: the market leader US-based ILFC, with about half of the demand share, and a company of the General Electric group, infact, buy almost 70 percent of the world production of aircrafts and, then, sign long-term leasing contracts with most the airlines world-wide.

\textsuperscript{6}In the segment of the cruises, for instance, the shipping industry is substantially an oligopoly of three main producers facing three big buyers. In fact, about 98 percent of the supply side is represented by the Italian Fincantieri, with 44.8 percent of the market, the Norwegian Aker, which, with a share of 29.6 percent, also controls the former Alstom ship-building activities in France, and the German Meyer Werft with the 23.2 percent of the market. What is left of the market is then covered by a fringe of small companies. On the other hand, also the demand side of the market is highly concentrated, since most the purchases come from three cruising companies: from the bigger to the smaller market share, the US-based Carnival (also controlling Costa, Cunard, P\&O Cruises and Princess Cruises), the American Royal Caribbean and the Swiss-Italian MSC Cruises. Interestingly, moreover, the buyers tend to strategically differentiate their portfolio of business partners more than the producers do: for instance, Fincantieri builds almost exclusively for Carnival, Aker sells to Royal Caribbean the ships produced in Finland and to MSC Cruises the ones build in France, while Meyer Werft is the only supplier working for both Carnival and Royal Caribbean.
gigantic-size mechanical and electro-mechanical engineering, the infrastructural plants, the defence or pharmaceutical hi-tech.

As few pioneering studies (Björnerstedt and Stennek, 2004, Hendricks and McAfee, 2005), have recently pointed out, it is very unlikely that the traders on any side of a bilateral oligopoly may behave as price-takers. Rather, it seems reasonable to think at the price formation as the outcome of a complex of negotiations among traders. The mentioned studies have argued that bilateral oligopolies may be reduced to a collection of many bilateral monopolies: the prices, thus, may emerge as the outcome of many simultaneous Nash-bargaining cooperative solutions, or of many simultaneous bilateral negotiations each involving an exogenously matched pair of one seller and one buyer.

In this paper, on the contrary, we explore an alternative approach, by focusing on non-cooperative interdependent bargaining solutions. The aim of this work, in particular, is to investigate the role of communication networks on endogenous price formation in a bilateral oligopoly.

In the literature on non-cooperative bargaining in decentralized markets, in fact, it is traditionally assumed that buyers and sellers are pair-wise matched through some random procedure, and that the order in which agents can make or respond to price offers is exogenously given. However, as Chatterjee and Dutta (1998) observe, while these assumptions are acceptable when modelling large anonymous markets, they are less appropriate in small markets (often called thin markets) where the search costs are usually low, and, particularly when agents are heterogeneous, traders may have interest in choosing their partner.

Chatterjee and Dutta (1998) provides an important insight into the role of competition for trading partners on the price prevailing in a thin market. They investigate three models of interdependent bargaining among two identical sellers and two heterogeneous buyers. All the models are based on a bargaining procedure with alternating offers between sellers and buyers, and differ just as the communication structure is concerned. In particular, the

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\(^7\)In a very different context, moreover, examples of so-called thin markets may emerge every time the stocks or derivatives markets are systematically characterized by a restricted number of traders.
strategic interaction among traders is cast on three exogenously designed frames where offers are, respectively, public, privately targeted but publicly known or, finally, privately targeted and secret. The equilibria of the negotiation game typically imply multiple prices and delay.

The analysis of Chatterjee and Dutta (1998) raises two interesting, closely related, research questions.

The first concerns the opportunity of modelling negotiations in thin markets and bilateral oligopolies with an alternating order of proposers. Clearly, alternating offers is the most natural specification for any exclusive bilateral bargaining. On the other hand, the hypothesis of a random order of proposals has been typically adopted for the analysis of bargaining in large decentralized markets (see for instance Gale, 1986; Osborne and Rubinstein, 1990; De Fraja and Sakovics, 2001) in the specific sense that, at any instant of time, either side of the market may, equally likely, be entitled to make proposals. In fact, such a stylized mechanism is usually justified in view of the fact that it mimicks the neutral anonymity of markets and enables to draw a direct comparison with the outcome of a Walrasian competitive framework.

Here, in contrast, we argue that the most peculiar features exhibited by bilateral oligopolies are the negligible searching frictions and the sheer role played by the identity of each individual trader. Therefore, it is difficult to reject the conjecture that the traders themselves, rather than the sides of the market, should be endowed by an \textit{ex ante} identical ability to strategically affect price formation. Therefore, in our thin market we imagine that, at any instant of time, any individual trader is equally likely to start a negotiation, by being selected to announce a proposal to the counterparts on the opposite side of the market.

The second question opened up by Chatterjee and Dutta (1998) is the investigation of whether and how the communication structure can affect strategic negotiations among traders. In fact, any possible set of communication restrictions can equivalently be thought as a network of potential links among agents: the existence of a communication link enables
a pair of agents to negotiate.

The existence of physical infrastructural networks, altogether with their shape, in fact, play a crucial role in the distribution of bargaining power and in the feasibility of the implementation of trades among companies in international gas markets. Furthermore, most the intermediate markets are endowed with an immaterial web of communication, reputation and trust links which is very likely to affect business relationships and negotiations.

Therefore, we aim at drawing a preliminary picture of the interrelations among bargaining in thin markets with heterogeneous traders and specific architectures of the buyers-sellers networks.

The issue of endogenous formation of trading links has been tackled by Kranton and Minehart (2001). On the other hand, sound descriptions of the negotiations’ outcome given a fixed network structure has been provided by the works of Calvò-Armengol (2001, 2002, 2003a, 2003b) and Corominas-Bosch (2004). From this perspective, our work is at the crossroads between these two approaches, as concerns the case of decentralized thin markets. With respect to the first work, our paper introduces an explicit analysis of a structured bargaining process with interdependent strategic negotiations. With respect to those in the second group, on the other hand, our work contributes to extend the analysis of the interaction between network architectures and negotiations to the case of markets with heterogeneous traders and fully decentralized bargaining procedures, beyond the specific case of alternating offers with identical traders.

We study a simple model of endogenous price formation in a thin market where trading is restricted by the shape of the formed bipartite networks. In particular, we consider completely decentralized negotiations with random order of proposers in the simplest case of a bilateral duopoly with impatient traders and heterogeneous buyers: trade of a homogeneous asset between a seller and a buyer is possible only when a link is present between them.

The rest of the paper is organised as follows. Section 3 is a description of the model.
In Section 4 we fully characterize the equilibria of the negotiations game within any fixed network structure. Section 5 contains a comparison of the bargaining position of each trader across networks and a discussion of our results.

3 The Model

3.1 The market

In our bilateral duopoly two identical sellers, \( S_1 \) and \( S_2 \), own one identical indivisible asset - such as a gas bundle. Both sellers have the same null reservation value for the asset. We can think of them as the national exporting companies from two major gas extracting countries (for instance, the russian Gazprom and the algerian Sonatrach) endowed by comparable industrial strength in terms of financial means, extracting volumes, market shares and so on. We will refer to sellers as females.

In the bilateral duopoly there are two heterogeneous buyers, \( B_1 \) and \( B_2 \), each of whom demands one single asset. The buyers’ valuations are \( v_1 = 1 \) and \( v_2 = \lambda \), respectively, with \( 1 > \lambda > 0 \). Analogously, we can think at them as two gas purchasing and distributing companies which are (almost) monopolists in two asymmetric national final markets (say, ENI in Italy, and Gaz de France in France or E.On in Germany)\(^8\). In the following, we will refer to buyers as males, and to \( B_1 \) and \( B_2 \) as the strong and the weak buyer, respectively.

We assume that all the valuations are common knowledge. Also, we assume that traders are impatient and discount their future payoffs at a common discount rate \( \delta \in (0,1) \). Thus, if one unit of the good is exchanged in period \( t \) between the buyer \( i \) and the seller \( j \) at the price \( p \), then the payoff of the buyer will be \( \delta^{t-1} (v_i - p) \) and the payoff of the seller \( \delta^{t-1} p \).

The prices at which the goods are exchanged if trade takes place, are exclusively determined by endogenous bargaining among the players. In particular, we assume that all traders in the thin market negotiate according to a public offers bargaining procedure\(^9\).

\(^8\)Alternatively, as two asymmetric competitors in a domestic market (say, ENI, and Edison or A2A in Italy).

\(^9\)
with random order of proposers. Moreover it is assumed there is no possibility of price discrimination.

The key feature of the model, however, is that trade may only take place between a buyer and a seller who are directly linked to each other. That is when an agent $i$ on one side of the market has to respond to a price offer from traders belonging to the opposite side, he - or she - may only accept or reject a proposal from $j$ such that $g_{ij} = 1$, where $g_{ij}$ denotes the existence of a link among agents $i$ and $j$. Analogous restrictions hold for proposal of price offers by agent $i$, which are intended to be directed exclusively to counterparts $j$ such that $g_{ij} = 1$. We denote with $L(i)$ the set of traders on the opposite side of the market linked with agent $i$. A network is said to be connected if there exists a path among any possible pair of traders.

It is immediate to see that in our thin market, only seven non-empty network architectures can emerge (see Figure 1). They are the exclusive trade networks, where each agent on any side of the market is linked only with a single partner (a) - which nests all networks where just one buyer-seller pair is exclusively connected; the supply-short-side networks where only one seller is linked to both buyers (b); the demand-short-side networks where one, either strong or weak, buyer is connected to both the sellers (c-d); the two asymmetrically connected structures where either the weak buyer (asymmetric weak network, e) or the strong buyer (asymmetric strong network, f) is linked with both sellers, while the other buyer is connected only with one exclusive partner; and, finally, the complete connected bipartite graph (g).

We now describe more in detail a model for the bargaining process.

### 3.2 The negotiations

In the negotiation stage, at every round $t \in \{1, 2, \ldots\}$, one trader is randomly selected to propose offers: each trader, independently of history of play, is selected with equal probability $\frac{1}{n}$, where $n = 1, \ldots, 4$, is the number of traders still active in the market.$^9$

$^9$Traders are considered active as long as there are still pairs of linked partners bargaining in the thin market.
Any round of the negotiation stage is composed by two phases. First the *price-offer phase* takes place: the agent who has been selected - say buyer $B_2$ - announces the price, $p_{B_2} \in [0, \lambda]$ he is willing to pay for one unit of the asset, from any linked seller $j = S_1, S_2 \in L(B_2)$, i.e. such that $g_{B_2j} = 1$.

Thereafter, the *price-response phase* occurs. Each seller $j = S_1, S_2 \in L(B_2)$ responds, simultaneously and independently, to any price offer from $B_2$. A response is simply either acceptance or rejection of the buyer’s latest announced offer $p_{B_2}$. In modelling individual strategic choice in the response game, we also assume the tie-breaking hypothesis by which, if any trader is perfectly indifferent about accepting or rejecting an offer, she (he) accepts it.

Therefore, if *just one* trader on the opposite side - namely $S_1$ - is in fact *linked* with $B_2$, the response phase reduces to an individual decision whether to accept or reject $p_{B_2}$ as in a standard bilateral negotiation.
If, at the contrary, both traders on the opposite side - say $S_1$ and $S_2$ - are indeed linked with the proposer, the response phase is modelled as a $2 \times 2$ simultaneous moves games. In such a $2 \times 2$ simultaneous moves game, sellers can end out in one of the four following situations: either one seller accepts $p_{B_2}$ while the other rejects it; or they both accept $p_{B_2}$; or, finally, both reject it.

First, if just one of the linked sellers accepts offer from $B_2$, she is matched with the weak buyer to trade at $p_{B_2}$, while the strong buyer and the remaining seller just enter a new round of negotiations if they are linked together.

In fact, our framework implies that, once a buyer and a seller leave the market after trading, all the links connecting them with any of the other traders are immediately removed by the bipartite graph. If, after such a removal, only isolated agents remain in the market, they all get automatically zero payoffs and the game ends.

If, at the contrary, a connected pair remains at the end of period $t$, then in period $t+1$ they enter a further round of the negotiation stage. Such a procedure is repeated so long as there are connected traders in the market.

It is worthwhile to underline a consequence of our bargaining procedure. Once just a single buyer and a single seller trade and leave the market, if the two remaining traders are linked each other, the subsequent negotiation stage reduces to a standard bilateral bargaining with random order of proposers. Therefore, from the following bilateral trade, the remaining players expect a surplus of approximately $\frac{1}{2}$ each in case the strong buyer is still in the market, or, alternatively, a surplus of $\frac{3}{2}$ whenever the weak buyer is, which correspond to the Binmore-Rubinstein bilateral bargaining payoffs.

If, on the contrary, both sellers $j = S_1, S_2 \in L(B_2)$ accept $B_2$’s offer of $p_{B_2}$, they then access a random tie-breaking selection to sort out who is going to trade with $B_2$: any of them is randomly picked with $\frac{1}{2}$ probability and matched to trade with $B_2$. As above, the strong buyer and the seller who has not been selected in the tie-break just enter a new round of negotiations only if they are linked together. If, at the contrary, they are not linked together, they are forced to leave the market with zero payoffs.

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Finally, if both sellers reject $p_{B_2}$ all traders access a further round of negotiations with a new selection of the player entitled to make offers.

Which of the above situations occurs only depends upon which Nash equilibrium of the $2 \times 2$ simultaneous moves game is reached. In general, traders’ optimal response correspondences are such that in theory any outcome can be supported as a Nash equilibrium of the response game.

A trader’s best response function depends upon her (his) continuation payoff and on her outside option in case of a rejection (or a random tie-break) as depending on her position in the graph.

In fact, each outcome of the $2 \times 2$ simultaneous moves game can occur within a specific set of conditions on the level of the announced price $p_{B_2}$ in terms of the responders’ continuation payoffs and outside options and, ultimately, of the values assumed by the primitive parameters $\delta$ and $\lambda$.

Thus, the different levels of the announced price $p_{B_2}$ imply the occurrence of different, possibly multiple, Nash equilibria of the response game. Hence, given any behaviour in the response game, it is then possible to move back to the price offer phase and to work out the proposer’s optimal choice. In general, if proposer is a buyer (seller), the optimal choice will be the lowest (highest) price offer implying a Nash equilibrium in which at least one of the traders on the opposite side accepts that price in the subsequent response phase, provided it can guarantee at least the proposer’s continuation payoff.

\[11\] By looking at the specific best response functions in the response games, sometimes restrictions may be ordered in a mutually exclusive way so that a given level of the announced price may be corresponding to one and just one Nash equilibrium in the response game. Some other times, however, multiple equilibria for a given price offer may arise in the response game. Typically, it may be the case that $B_2$ proposes an offer which is compatible with more than one set of conditions on mutual best responses: thus, for instance, two alternative Nash equilibria may coexist, one where both sellers accept $p_{B_2}$, the other where both sellers reject it. In such a case we will always provide a full characterization of all the resulting multiple equilibria in the response game.

\[12\] Moreover, it is possible that there are levels of the announced price $p_{B_2}$ for which no pure-strategies Nash equilibrium can be guaranteed in the response game. In such a case, we need to specify what happens in the negotiations game. Although the finiteness of the game clearly ensures the existence of a mixed strategies equilibrium, our exclusive focus on pure strategies equilibria implies we need to describe what follows any node at which a proposal does not match any set of conditions for a pure-strategies Nash equilibrium. We assume that in such a case traders just enter a further bargaining stage with a new draw of the proposer. Therefore, in such a case all traders just get their own continuation payoffs.
3.3 The solution

The negotiation game is solved given a fixed network structure. In particular, the negotiation game is an infinite horizon dynamic game of complete and imperfect information: players’ payoff functions are common knowledge and, although, at each move in the game, the players know the full history of the play thus far, the price-response phases in the negotiation stage are simultaneous-moves games. Therefore we will solve the negotiations game for its subgame-perfect Nash equilibria using backward induction.

Given the overall complexity of the present game, we will only focus on the subgame-perfect Nash equilibria in pure and stationary strategies (PSSPN equilibria).\(^{13}\)

4 Bargaining in Networks

Here we solve for the sequential bargaining game between the traders in the bilateral oligopoly, given the existence of a fixed bipartite network structure.

In the following we will start describing the negotiations game in the case one single pair of traders is linked, then gradually moving, through more connected bipartite graphs in which traders can still be isolated or asymmetrically connected, up to the complete network where any pair of traders is linked together.

Notice that, by a standard argument by theory of infinite horizon dynamic games of complete information (see for instance Osbourne and Rubinstein, 1990, Fudenberg and Tirole, 1991. Muthoo, 1999), a stationary dynamic game may be fully characterized by describing any of its strategically equivalent subgames.\(^ {14}\)

\(^{13}\) Thus, we will only consider equilibria where traders adopt pure strategies at every move, and whose strategies exclusively depend on the number of traders still active in the market and on which phase of the negotiation stage the players are: any trader always proposes the same price at every equivalent node where he or she has to make an offer, and he or she always behaves in the same way whenever facing identical proposals in the price-response phase.

\(^{14}\) Also notice that, within any network where the strong buyer is not isolated, there can not be PSSPN equilibria in which the bargaining process keeps on going on forever. Indeed, as the discounted payoffs of all the traders would be zero in such a case, there is certainly a profitable deviation at least by the strong buyer. In fact, whenever he is selected to make an offer, \(B_1\) can always propose a price equal to \(\delta\), which, being the highest price both sellers may ever gain in the following rounds, will be immediately accepted in the subsequent response phase. In turn, \(\delta < 1\) ensures the strong buyer a strictly positive payoff, and then a profitable deviation from the perpetual disagreement situation.
In particular, define $S_i$-games the subgames of the original game of negotiations among
the traders in a given network, starting whenever the seller $S_i$ is randomly selected to
make offers. Analogously define $B_i$-games the subgames of the original game that start
when buyer $B_i$ is randomly selected to make offers. Hence, being for any given network
structure, all the $S_i$-games and all the $B_i$-games strategically equivalent by the stationarity
hypothesis, the analysis of the PSSPN equilibria in the original overall game perfectly
 corresponds to the investigation of the PSSPN equilibria in any of the $S_1$-games, $S_2$-
games, $B_1$-games and $B_2$-games.

In the following sections, we will provide a full description of the equilibria for each
network structure. For two cases we also provide proofs in the Appendix. The remaining
proofs follow from analogous arguments and are therefore omitted for the sake of brevity.
They are available on request from the author.

4.1 The Exclusive-trade network

The exclusive-trade network (a) corresponds to a market where two separate pairs of
traders negotiate in a mutually exclusive partnership: for instance, Gazprom has an exclu-
sive partnership with ENI, while Sonatrach deals with Gaz de France only. In particular,
imagine that seller $S_1$ is linked with the strong buyer only, while $S_2$ is exclusively con-
 nected with the weak buyer. Although, in our framework each trader, at any round, has
an identical $\frac{1}{4}$ probability to make offers, this situation corresponds to a case where two
parallel bilateral negotiations are taking place simultaneously and independently, since
any trader has just a potential partner to trade with. Not surprisingly, the equilibrium
outcome of the bargaining process within each pair of traders is equivalent to the one of
a pairwise Binmore-Rubinstein negotiation with random order of proposers.

Proposition 1 For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there
exists a unique PSSPN equilibrium of the negotiation game in the exclusive trade network
such that:

- buyer $B_1$ proposes $S_1$ a price $p^*_B = \delta V(S_1)$, which is accepted by $S_1$. 

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• buyer $B_2$ proposes $S_2$ a price $p_{B_2}^* = \delta V(S_2)$, which is accepted by $S_2$.

• seller $S_1$ proposes $B_1$ a price $p_{S_1}^* = 1 - \delta W(B_1)$, which is accepted by $B_1$.

• seller $S_2$ proposes $B_2$ a price $p_{S_2}^* = \lambda - \delta W(B_2)$, which is accepted by $B_2$.

Traders’ expected continuation payoffs by entering a new stage of the negotiations game are

$$\begin{align*}
W(B_1) &= \frac{1 + \delta}{4} \\
W(B_2) &= \frac{(1 + \delta)\lambda}{4} \\
V(S_1) &= \frac{1 + \delta}{4} \\
V(S_2) &= \frac{(1 + \delta)\lambda}{4}
\end{align*}$$

Proof. In the Appendix. ■

Notice that in the limit case as $\delta \to 1$, the expected payoffs approach the values $W(B_1) \to V(S_1) \to \frac{1}{2}$, $W(B_2) \to V(S_2) \to \frac{\lambda}{2}$. Also notice that the exclusive trade network naturally nests two other subgraphs. In fact, it is possible to characterize the corresponding PSSPN equilibria of the negotiations game within the strong- and the weak-couple network configurations\textsuperscript{15}.

4.2 The Supply-short-side network

In the supply-short-side networks (b) only one seller $S_i$ with $i = 1, 2$ is linked to both buyers while $S_{i-1}$ is an isolated trader. This network captures all the market configurations where an exclusive seller is able to extract a significant extent of surplus from two competing buyers. The leading market position of Gazprom, which provides gas to all the European countries, is perhaps the best example.

A peculiar trait of our bargaining framework is that, as long as some not isolated agents still remain in the market, any trader is entitled with an identical probability to propose offers. Hence, in such a communication network - alike in the (c) and (d) below

\textsuperscript{15}By the former we mean a market in which only the strong buyer and a seller - say $S_1$ - are connected, while the weak buyer and the remaining seller - $S_2$ - are isolated traders. The latter case is the analogous graph where $S_2$ is connected with the weak buyer while the remaining traders are isolated. The corresponding equilibria are described in the Appendix.
agents incur delays in trade with at least $\frac{1}{4}$ probability, namely, at least after any $S_{-i}$ selection.

Another peculiar feature of the present supply-short-side structure is that the buyers are in a symmetric position concerning the number of accessible connections: both buyers can access an identical number of partners. Thus, it is with no lack of generality that henceforth we assume that the strong buyer’s continuation payoff does not differ from the weak buyer’s by more than some upper bound, equal to the original difference in their reservation prices. This is expressed by condition

$$\delta [W(B_1) - W(B_1)] \leq 1 - \lambda,$$

by which the discounted value of the difference in the buyers’ expected continuation payoffs can not exceed the relative distance between their primitive reservation prices.$^{16}$

Finally, given the symmetry across sellers, we characterize the equilibrium offers within a supply-short-side network where $S_2$ is linked with both buyers while $S_1$ is isolated, like in $(b)$, to easily extend the corresponding findings to the reverse positions of the sellers. As reported in the Appendix, we show that three equilibria are possible in the supply-short-side network.

**Proposition 2** For any discount rate $\delta \in (0, 1)$ there exist three PSSPN equilibria SS1, SS2 and SS3 of the negotiation game in the supply-short-side network such that:

- **buyer $B_1$, for any $\lambda \in (0, 1)$, proposes $S_2$ a price $p_{B_1}^* = \delta V(S_2)$ which is accepted by $S_2$ (equilibria SS1, SS2 and SS3).**

- **buyer $B_2$**

  - **for any $\lambda \geq \hat{\lambda}$, proposes seller $S_2$ a price $p_{B_2}^* = \delta V(S_2)$ which is accepted by seller $S_2$ (equilibria SS1 and SS2);**

$^{16}$It is worthwhile to clarify that we are not actually forcing the equilibrium payoffs to satisfy this property. Rather we assume it at the beginning of our analysis and we then check it a posteriori, selecting the equilibria whose computed continuation values actually satisfy it.
– for any $\lambda < \hat{\lambda}$, proposes seller $S_2$ any unacceptable price $p'_{B_2} < \delta V(S_2)$ which is rejected by seller $S_2$ (equilibrium SS3).

• for any offer proposed by $S_1$, the expected payoffs for the traders are just their continuation payoffs (equilibria SS1, SS2 and SS3).

• seller $S_2$

  – for any $\lambda < \bar{\lambda}$, proposes both buyers a price $p^*_{S_2} = 1 - \delta W(B_1)$ which is accepted only by $B_1$ (equilibria SS1 and SS3);
  – for any $\lambda \geq \bar{\lambda}$, proposes both buyers a price $p^*_{S_2} = \lambda$ which is accepted by both buyers.

The expected continuation payoffs in the three equilibria and the values they approach in the limit case as $\delta \rightarrow 1$ are reported in Figures 4 and 7 in the Appendix.

Notice that, in equilibrium SS1, for intermediate values $\hat{\lambda} \leq \lambda < \bar{\lambda}$, the connected seller is able to exploit most the surplus from the negotiation, leaving both buyers with payoffs close to zero. On the other hand, in equilibrium SS2, for sufficiently high values $\lambda \geq \bar{\lambda}$, the connected seller is not able to fully exploit all the potential surplus from the trade, and she can just appropriate from negotiations at most the weak buyer’s reservation price, leaving the strong buyer with a positive surplus tending to half the difference between the reservation prices. Finally, in equilibrium SS3, for sufficiently low values $\lambda < \hat{\lambda}$, the weak buyer decides to make unacceptable offer to avoid paying excessively onerous prices. Thus, when asymmetry among buyers is particularly sharp, the weak buyer chooses to abstain from active trading so that negotiations mimic in fact bilateral bargaining among the seller and the strong buyer only: the seller’s continuation payoff is so high compared to $\lambda$ that the weak buyer is better off by choosing to not compete with the strong buyer.

### 4.3 The $B_1$-short-side and $B_2$-short-side networks

In the $B_1$-short-side networks (c) only the strong buyer $B_1$ is linked to both sellers while the weak buyer is an isolated trader. This trading network captures all the market struc-
tures where an exclusive large purchaser is naturally endowed by the power to exploit the existing competition between two homogeneous sellers.\footnote{The case may be probably thought as the secret ambition by a national incumbent gas-distributor, such as ENI when it illustrates its plans to trasform Italy into a gas distribution hub for the Mediterranean Sea.} There are two equilibria in the negotiations within this network.

**Proposition 3** For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there exist two PSSPN equilibria ($B_1$-SA and $B_1$-SR) of the negotiation game in the $B_1$-short-side network such that:

- **buyer $B_1$**
  - proposes the sellers a price, $p^*_{B_1} = 0$, accepted by both sellers (equilibrium $B_1$-SA);
  - proposes the sellers a price $p^*_{B_1} = \min \{\delta V(S_1), \delta V(S_2)\}$ accepted by both sellers (equilibrium $B_1$-SR).

- for any offer proposed by $B_2$, the expected payoffs for the traders are just their continuation payoffs.

- **seller $S_1$** proposes the strong buyer a price $p^*_{S_1} = 1 - \delta W(B_1)$ which is accepted by $B_1$.

- **seller $S_2$** proposes the strong buyer a price $p^*_{S_2} = 1 - \delta W(B_1)$ which is accepted by $B_1$.

The expected continuation payoffs in the two equilibria and the values they approach in the limit case as $\delta \rightarrow 1$ are reported in Figures 4 and 7 in the Appendix. Notice that in both equilibria the strong buyer is able to extract all the potential surplus from the trade. The corresponding equilibria for the $B_2$-short-side network follow from analogous arguments.
4.4 The Asymmetric Weak Network

We now investigate the equilibrium prices and outcomes from the negotiations within network that are asymmetrically connected as the buyers are concerned, such as (e). In particular, we consider here the network where the weak buyer is connected to both sellers, while the strong buyer is only linked to seller $S_2$. As a consequence, both the strong buyer and seller $S_1$ are exclusively connected with, respectively, $S_2$ and $B_2$.\footnote{Clearly the negotiation game within such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes by switching $S_1$ with $S_2$.}

Alike for the previous networks (and for the complete network characterized below) where the buyers are in a symmetric position as the number of accessible connections is regarded, also in the present asymmetric weak network we may think at $\delta [W(B_1) - W(B_1)] \leq 1 - \lambda$, as a convenient restriction. In fact, if condition (1) is a reasonable restriction on buyers’ expected continuation payoffs for any symmetric structure, it must a fortiori be a necessary feature of an asymmetric weak network: in the latter case the payoff advantage by $B_1$ is even smaller as the weak buyer is endowed with a relatively stronger bargaining position concerning the number of accessible partners. An analogous consideration on the supposedly better trading opportunities by traders in more connected nodes justifies a corresponding assumption on the sellers’ continuation payoffs, $V(S_2) \geq V(S_1)$.

We now provide a full description of the unique equilibrium in the asymmetric weak network.

**Proposition 4** For any discount rate $\delta \in (0, 1)$ and reservation price $\lambda \in (0, 1)$, there exists a unique PSSPN equilibrium AW of the negotiation game in the asymmetric weak network such that:

- $B_1$ proposes $S_2$ a price $p_{B_1}^* = \delta V(S_2)$ which, in the response phase, is accepted by $S_2$
- $B_2$ proposes both sellers a price $p_{B_2}^* = \delta V(S_1)$ which is accepted only by $S_1$
- $S_1$ proposes $B_2$ a price $p_{S_1}^* = \lambda - \delta W(B_2)$, which is accepted by $B_2$
- $S_2$ proposes both buyers a price $p_{S_2}^* = 1 - \delta W(B_1)$ which is accepted only by $B_1$.\footnote{Clearly the negotiation game within such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes by switching $S_1$ with $S_2$.}
The expected continuation payoffs of the equilibrium and the values they approach in the limit case as \( \delta \to 1 \) are reported in Figures 5 and 8 in the Appendix.

Notice that the weak buyer is never able to take advantage of his most connected location in order to get better trading opportunities than the strong buyer.

4.5 The Asymmetric Strong Network

We now consider the network \((f)\) where the strong buyer is connected to both sellers, while the weak buyer is only linked to seller \(S_2\). As a consequence, both the weak buyer and seller \(S_1\) are exclusively connected with, respectively, \(S_2\) and \(B_1\). This market configuration fits very well the case of most domestic gas markets, where the incumbent is usually endowed by a wider set of energetic sources than the smaller competitors.

It is immediately reckoned that the previous arguments in favour of condition (1) as a neutral restriction on buyers’ continuation payoffs are no longer valid when moving to the asymmetric strong network. In fact, in this case, \(B_1\) reinforces his original strength due to the higher reservation value with the trading advantages conveyed by his central position in the graph. Thus, it should be argued that the strong buyer may expect a surplus from the trade well beyond any upper bound implied by condition (1). Therefore, in the following analysis, we will separately consider both the case where \(1 - \delta W(B_1) \geq \lambda - \delta W(B_2)\) and the one in which \(1 - \delta W(B_1) < \lambda - \delta W(B_2)\).

We now provide a full description of the final equilibria.

**Proposition 5** There exist six PSSPN equilibria (AS1, AS2, AS3, AS4, AS5, and AS6) of the negotiation game in the asymmetric strong network such that:

- **buyer \(B_1\)**

  - for any \(\delta \in \left[\frac{s}{\delta}, 1\right)\) and any \(\underline{\lambda} \leq \lambda \leq \underline{\lambda} \) (equilibrium AS1); and for any \(\delta \in \left[\frac{s}{\delta}, 1\right)\) and any \(\underline{\lambda} \leq \lambda \leq \underline{\lambda} \) (equilibrium AS2), proposes both sellers a price \(p_{B_1}^* = \frac{5\lambda}{2}\)

  which is accepted by both \(S_1\) and \(S_2\);

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19 Again the negotiation game in such a network is strategically equivalent to the one taking place in the homologous graph obtained re-labelling the nodes after having switched \(S_1\) with \(S_2\).
– for any $\delta \in \left(0, \frac{\pi}{2}\right]$ and any $\tilde{\lambda} \leq \lambda \leq \overline{\lambda}$ (equilibrium AS3); for any $\delta \in (0, 1)$ and for any $\lambda \geq \tilde{\lambda}$ (equilibrium AS4); and for any $\delta \in (0, 1)$ and for any $\overline{\lambda} \leq \lambda \leq \tilde{\lambda}$, or for any $\delta \in \left[\tilde{\delta}, 1\right)$ and any $\overline{\lambda} \leq \lambda \leq \tilde{\lambda}$ (equilibrium AS6), proposes both sellers a price $p_{B_1}^* = \delta V(S_1)$ which is accepted only by $S_1$, while $S_2$ rejects it;

– for any $\delta \in \left[\tilde{\delta}, 1\right)$ and any $\tilde{\lambda} \leq \lambda \leq \tilde{\lambda}$ (equilibrium AS5) proposes both sellers a price $p_{B_1}^* = \frac{\delta \lambda}{2}$ which is accepted by both sellers, while they both accept any other offer $p_{B_1} \in \left[\min \{\delta V(S_1), \delta V(S_2)\}, \frac{\delta}{2}\right]$.

- buyer $B_2$, for any $\delta \in (0, 1)$ and any $\lambda \in (0, 1)$, proposes $S_2$ a price $p_{B_2}^* = \delta V(S_2)$ which is accepted by $S_2$

- seller $S_1$, for any $\delta \in (0, 1)$ and any $\lambda \in (0, 1)$, proposes $B_1$ a price $p_{S_1}^* = 1 - \delta W(B_1)$, which is accepted by $B_1$

- seller $S_2$

  – for any $\delta \in \left[\frac{\pi}{2}, 1\right)$ and any $\frac{\lambda}{\overline{\lambda}} \leq \lambda \leq \overline{\lambda}$ (equilibrium AS1), proposes both buyers a price $p_{S_2}^* = 1 - \delta W(B_1)$ which is accepted by $B_1$ only, while is rejected by $B_2$;

  – for any $\delta \in \left[\frac{\pi}{2}, 1\right)$ and any $\lambda \leq \lambda \leq 1$ (equilibrium AS2); and for any $\delta \in \left(0, \frac{\pi}{2}\right]$ and any $\lambda \leq \lambda \leq \overline{\lambda}$ (equilibrium AS3), proposes both buyers a price $p_{S_2}^* = 1 - \frac{\delta}{2}$ which is accepted by both buyers;

  – for any $\delta \in (0, 1)$ and for any $\lambda \geq \tilde{\lambda}$ (equilibrium AS4), proposes both buyers a price $p_{S_2}^* = \lambda - \delta W(B_2)$ which is accepted only by $B_2$, while $B_1$ rejects it;

  – for any $\delta \in \left[\tilde{\delta}, 1\right)$ and any $\tilde{\lambda} \leq \lambda \leq \tilde{\lambda}$ (equilibrium AS5); and for any $\delta \in (0, 1)$ and for any $\overline{\lambda} \leq \lambda \leq \overline{\lambda}$, or for any $\delta \in \left[\tilde{\delta}, 1\right)$ and any $\overline{\lambda} \leq \lambda \leq \tilde{\lambda}$ (equilibrium AS6), proposes both buyers a price $p_{S_2}^* = 1 - \delta W(B_1)$ which is accepted by both buyers, while they both reject any other offer $p_{S_2} \in \left[1 - \delta W(B_1), \min \{\lambda, 1 - \frac{\delta}{2}\}\right)$.
The expected continuation payoffs of the six equilibria and the values they approach in the limit case as $\delta \to 1$ are reported in Figures 5 and 8 in the Appendix.

It can be noticed that the negotiations within an asymmetric strong network show a rich multiplicity of equilibria. In fact, for low levels of $\lambda$ equilibrium $AS_1$ is defined. Medium values of $\lambda$ are instead cover, in different ranges, by equilibria $AS_5$, $AS_2$ and $AS_6$. For high levels of $\lambda$ equilibrium $AS_3$ exists, while for extremely high values, equilibrium $AS_4$ is defined.

Furthermore, it can be seen that, in equilibrium offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for all but one equilibria, the only exception being $AS_4$. As a consequence, in some equilibria, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market without trading at all. In the former case, moreover, different prices usually form in the thin market. Therefore, inefficiency, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out in an asymmetric strong network.

4.6 The Complete Network

In the complete bipartite network ($g$), each buyer is connected with both the sellers. The existence of links between all the possible buyer-seller pairs enables the exploitation of any potential trade in the bilateral oligopoly.

The case of a bilateral duopoly connected by a complete bipartite graph corresponds to the market structure studied by the public offers model in Chatterjee and Dutta (1998). In fact, our model of negotiations differs from the latter only in two, though crucial, aspects. First, while in Chatterjee and Dutta (1998) the bargaining procedure follows an alternating order of proposers between the supply and the demand side, in our model the negotiation entails a random order of proposers with identical odds for any trader.

Second, in our model there is an explicit formalization of the strategic interaction occurring between traders of the same side of the market competing when responding to
an offer. In fact, unlike Chatterjee and Dutta (1998), we explicitly model a simultaneous moves 2 × 2 game between buyers (sellers) in the price response phase.

Furthermore, our model of negotiations in such a case may be seen as an extension of the model by Corominas-Bosch (2004) to the case of heterogeneous buyers and random selection of traders (rather than sides of the market).

Our model of negotiations among traders in such a complete bipartite graph implies that after a single buyer and a single seller have been matched to trade and left the market, the two remaining traders have always the chance to stay in the market to carry on further negotiations. In fact they automatically access a standard bilateral bargaining with random order of proposers, whose PSSPN equilibrium payoffs are the ones described by a standard Binmore-Rubinstein model.20

Alike for the above networks where the buyers are in a symmetric position concerning the number of accessible connections, also in the complete connected graph we will take advantage of condition (1), \( \delta [W(B_1) - W(B_1)] \leq 1 - \lambda \), by which the discounted value of the difference in the buyers’ expected continuation payoffs can never exceed the relative distance between their primitive reservation prices.

Three equilibrium outcomes can emerge.21

**Proposition 6** There exist three PSSPN equilibria (C1, C2 and C3) of the negotiation game in the complete network such that:

- **buyer** \( B_1 \)

  - for any \( \delta \in (0, 1) \) and any \( \lambda \geq \bar{\lambda} \) (equilibrium C1), proposes both sellers a price \( p_{B_1}^1 = \delta V(S_1) \) which is accepted by \( S_1 \):

20If the strong buyer is still in the market, each of the remaining traders expects from the following bargaining rounds a surplus of \( \frac{1}{2} \), alternatively, whenever the weak buyer is the one left, they expect a surplus of \( \frac{\lambda}{2} \).

21For an I-type equilibrium arising when \( B_2 \) has been selected to make offers in which continuation payoffs are such that \( V(S_2) \leq V(S_1) \). There exist three other equilibria corresponding to the case a II-equilibrium arises when the weak buyer has been selected to make offers in which continuation payoffs are such that \( V(S_2) \leq V(S_1) \). Such perfectly symmetric equilibria are immediately obtained by switching the labelling for the two sellers. Note that the payoffs for the two buyers remain unaltered.
– for any \( \delta \in (0,1) \) and any \( \tilde{\lambda} \leq \lambda \leq \bar{\lambda} \) (equilibrium C2), proposes both sellers a price \( p_{B_1}^* = \frac{\delta\lambda}{2} \) which is accepted by both \( S_1 \) and \( S_2 \), who would accept any offer in the range \( [\frac{\delta\lambda}{2}, \delta V(S_1)] \);

– for any \( \delta \in \left[ \frac{\delta}{2}, 1 \right) \) and any \( \tilde{\lambda} \leq \lambda \leq \bar{\lambda} \) (equilibrium C3), proposes both sellers a price \( p_{B_1}^* = \delta V(S_1) \) which is accepted by both \( S_1 \) and \( S_2 \), who would reject any offer in the range \( [\frac{\delta\lambda}{2}, \delta V(S_1)] \).

• buyer \( B_2 \), for any \( \delta \in (0,1) \) and any \( \lambda \in (0,1) \), proposes both sellers a price \( p_{B_2}^* = \delta V(S_1) \) which is accepted by \( S_1 \)

• seller \( S_1 \), for any \( \delta \in (0,1) \) and any \( \lambda \in (0,1) \), proposes both buyers a price \( p_{S_1}^* = 1 - \delta W(B_1) \), which is accepted by \( B_1 \)

• seller \( S_2 \), for any \( \delta \in (0,1) \) and any \( \lambda \in (0,1) \), proposes both buyers a price \( p_{S_2}^* = 1 - \delta W(B_1) \) which is accepted by \( B_1 \)

The expected continuation payoffs of the three equilibria and the values they approach in the limit case as \( \delta \rightarrow 1 \) are reported in Figures 6 and 7 in the Appendix.

5 Comparisons across different networks

In this section, by comparing traders’ equilibrium payoffs across networks, we draw some considerations on the impact of network structures on the negotiations in a gas bilateral duopoly.

5.1 Equilibria

In order to carry on some comparisons we first need to be able to rank the different equilibria for any network in some sensible way. In fact, we can order all the possible equilibria across different networks in a \((0,1)\) square box having the weak buyer’s reservation price \( \lambda \) on its horizontal axis and the common discount factor \( \delta \) on its vertical one. It is then possible to draw all the \((\lambda, \delta)\) regions where any equilibrium for a given network is defined
Figure 2: Comparisons between equilibria across networks

and thus see which equilibria are indeed comparable across different architectures. The only drawback of such procedure is that the final graphic representation turns out to be truly cumbersome. However, for providing an intuitive representation of our main qualitative findings here we draw an overall picture of the ranking of equilibria across networks according to the values of the weak buyer’s reservation price $\lambda$, for a fixed level of the discount factor $\delta = 0.85$. Qualitative results remain unaltered for any other value of the impatience rate.

Some network configurations present a single equilibrium for all values of $\lambda$. This is the case not only for the exclusive trade network - and the nested strong and weak couple architectures - but also for the asymmetric weak architecture. On the other hand, while
both the $B_1$ and $B_2$-short side structures show two coexisting equilibria, the supply-short-side network presents one equilibrium for high values of $\lambda$, one for intermediate levels and one for relatively low values of $\lambda$.

Similar is the case of the complete network where only equilibrium $C1$ is defined for high values of $\lambda$, equilibrium $C2$ for medium-high levels, and both equilibria $C2$ and $C3$ exist for medium-low values of $\lambda$.

Even richer is the asymmetric strong network. In fact, for relatively low levels of $\lambda$ equilibrium $AS1$ is defined. Medium values of $\lambda$ are instead cover, in different ranges, by equilibria $AS5$, $AS2$ and $AS6$. For high levels of $\lambda$ equilibrium $AS3$ exists, while for extremely high values, equilibrium $AS4$ is defined.

Some general considerations are in order. By looking at the most salient features of the equilibria, it can be reckoned that offers are often accepted by both traders on a side of the market, so that a random tie-break takes place. This is the case for both equilibria in the $B_1$ and $B_2$-short side architectures, for two out of three equilibria in the supply-short-side network, for all but one equilibrium in the asymmetric strong network (excluding $AS4$) and even in two of the three equilibria in the complete network. In some of such equilibria, then, trade occurs with delay, while in other equilibria, with half probability the least connected traders end up leaving the market with no trading. In the former case, moreover, different prices usually form in the thin market. Therefore, inefficiency, both in terms of delay in trade and of impossibility to achieve full exploitation of all the potential surplus from trade, can not be ruled out from our equilibria.

Moreover, comparisons are possible across equilibria for network structures defined within compatible values of the primitive parameters $\delta$ and $\lambda$. In the following, we compare the equilibrium payoffs of the traders across compatible equilibria in different networks. As the primary interest of the paper lies in the investigation of buyers’ bargaining power and our model is in fact symmetric between sellers, we have limited our comparisons to the payoffs of $B_1$ and $B_2$.

There are two main conjectures that interesting to confirming or to reject in view of
direct comparisons. First, it may be argued that gas purchaser $B_2$, who is clearly in weaker original conditions to start negotiations, if embedded in favourable network configurations, in theory should be able to counterbalance, at some extent, the natural advantage of the strong buyer. To seek confirmation of such a guess, one should look at the expected equilibrium payoffs in a given network structure to compare the surplus experienced by the two buyers.

It is immediate to check, however, that such intuitive guess is rejected by the model’s predicted payoffs. In fact, the only network architectures where $B_2$ is unambiguously better off than the strong buyer are the obvious cases of the weak-couple and the $B_2$-short side network. While it cannot surprise that the strong buyer indeed experiences sistematically higher surplus in an asymmetric strong network, this does sound less obvious for the other two salient connected networks. However, direct comparisons clearly show that the weak buyer is always worse off than $B_1$ even in the unique equilibrium of the asymmetric weak structure. Moreover, it turns out that also in a complete network the strong buyer is always strictly better off than the weak\textsuperscript{22}.

The second conjecture is related to the surplus of a given buyer across different networks. In fact, it may be argued that any buyer should always be in a better trading position whenever he is located in a more connected node than the competing buyer. In other words, intuition may suggest that the strong buyer would manage to extract better trading opportunities from being not only in a complete or asymmetric strong network rather than in an asymmetric weak, but also in an asymmetric strong rather than a complete structure. In fact, being connected with more potential partners than the competitor should enable a player to enjoy better trading conditions.

To confirm or discard such a conjecture we need to compare the payoffs for each buyers across different network architectures.

\textsuperscript{22}One exception is the case of values of $\lambda$ so high to approach the limit case $\lambda \rightarrow 1$ of symmetric buyers. In such a case, the relative counterbalance of $B_2$’s surplus seems to be due not only to the closeness of the reservation prices, but also to the fact that in the $C1$ equilibrium the weak buyer accesses bilateral negotiations more often than $B_1$. 

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5.2 Strong buyer

We start with the strong buyer, and by direct computations and numerical simulations, we obtain some results of interest. First, clearly, the highest surplus experienced by the strong buyer is the one attainable in a $B_1$-short-side network, while the worst is the equilibrium payoff in a $B_2$-short-side network. Secondly, the surplus faced within a strong couple network is equivalent, for very low values of $\lambda$, to the equilibrium payoff from the $SS3$ equilibrium payoff in a supply-short-side network. Third, the equilibrium surplus earned by the strong buyer within the exclusive trade exactly corresponds to the one gained in equilibrium within the asymmetric weak network. Moreover, by direct comparisons, it turns out that such surplus is always strictly lower than what the strong buyer can obtain in equilibrium from bargaining either in an asymmetric strong, or in a complete, so that the following holds:

$$\Pi (B_1)_{ET} \equiv \Pi (B_1)_{AW} < \{ \Pi (B_1)_{AS} , \Pi (B_1)_{C} \} \leq \Pi (B_1)_{B_1-Short}$$

This confirms that any connected bipartite graph makes $B_1$ strictly better off than within an exclusive bilateral negotiation. There exists a natural incentive for the strong buyer to avoid locking in an exclusive partnership and to prefer to be embedded into more connected architectures.

Moreover, as intuition would suggest, it turns out that $B_1$ always enjoy strictly higher surplus in an exclusive trade network than in any equilibrium of the supply-short-side architecture, unless for low values of $\delta$ when $\lambda$ is extremely low (SS2). It can be checked that, a fortiori, any from the complete, the asymmetric strong and the asymmetric weak network ensures to $B_1$ an equilibrium surplus at least as good as the supply-short-side structure:

$$\Pi (B_1)_{Supply-Short} \leq \Pi (B_1)_{ET} \equiv \Pi (B_1)_{AW} < \{ \Pi (B_1)_{AS} , \Pi (B_1)_{C} \}$$

Finally, it is possible to directly compare and rank the equilibrium payoffs conveyed
to $B_1$ by the completely connected graphs.

First, it turns out that both in an asymmetric strong and in a complete network, $B_1$ gets equilibrium payoffs never lower than in asymmetric weak. More precisely, the strong buyer is always strictly better off in an asymmetric strong network, unless when $\lambda$ is extremely high, in which case he earns the same surplus in both architectures.

Furthermore, the second conjecture can be directly tested. Direct computations reject the hypothesis that the strong buyer is always better off in an asymmetric strong network in which he has more trading links than the weak competitor. Indeed, while for $\lambda$ high enough $B_1$ is unambiguously better off within an asymmetric strong network, this is no longer true for lower values of the weak buyer’s reservation price. At the contrary, while, for low values of $\lambda$, comparisons among payoffs are not possible since there are no defined equilibria for the asymmetric strong and the complete networks, for intermediate levels of $\lambda$, the strong buyer is strictly better off within a complete network.

To shed some light on this, perhaps surprising, result, we provide a tentative explanation. In fact, consider an asymmetric strong network where the weak buyer is exclusively linked with seller $S_2$. Intuitively, the fact that is linked with an exclusive relationship with the weak buyer provides seller $S_2$ a safe outside option she can always rely on, in the sense that, whenever the weak buyer is selected to make offers, $S_2$ benefits from having an exclusive partnership with $B_2$ in terms of high trading prices. Hence, the existence of such alternative trading opportunity implies that, when bargaining with $B_1$, seller $S_2$ would never accept any proposal making her worse off with respect to such outside option.

In other words, the possibility of exclusive dealing with $B_2$ indirectly provides a lower bound for competition between the two sellers when fighting for serving the strong buyer. In fact, it may be expected that $S_2$ would never accept from the strong buyer any price below a proposal making her indifferent to what she can get from the weak buyer, and that $S_2$ would never exert any competitive pressure below that threshold. Also $S_1$ has no interest in proposing the strong buyer something more favourable than $S_2$’s outside option. Thus, both sellers have no incentives to compete too fiercely for the strong buyer,
by proposing prices below what $S_2$ can get from the weak buyer. The existence of such implicit lower bound for sellers’ competition clearly hurts the strong buyer, as he is not able to extract larger trading surplus from negotiations. This is because, when making offers to $B_1$, both sellers are likely to ask something comparable to what $S_2$ can get from the weak buyer.

Therefore, to avoid being hurt by such price floor limit to competition, the strong buyer may be better off in a complete network. In fact, as long as $B_2$’s reservation price is relatively low, $B_1$ prefers that the weak buyer takes part into negotiations from a fully connected, rather than in a less central node. From this point of view, it seems that the strongest competing purchaser may prefer a market structure where communication and trading opportunities are less constrained to one with protected exclusive partnerships. Asymmetry across reservation prices is sharp enough to guarantee that the strong buyer gets larger surplus than in an asymmetric strong network anyway. This result seems counterintuitive, though, and is susceptible of interesting regulation policy implications.\textsuperscript{23}

There is a limit, however, to such $B_1$’s preference towards the complete network. In fact, as $\lambda$ increases, buyers become more similar in terms of attractiveness for the sellers. Thus, while in a complete network, competition to serve $B_1$ becomes less fierce as both sellers can sustain high prices selling to the weak buyer, in the asymmetric strong network, $B_1$ is able to take advantage of $S_2$ exclusively dealing with $B_2$, and to obtain from $S_1$ prices similar to the one in bilateral negotiations, which, in turn, are now significantly lower than $\lambda$.

### 5.3 Weak buyer

At some extent, such a preference for bargaining in a complete architecture is common to the weak buyer too. Clearly, it immediately turns out that the weak buyer is always strictly better off within a complete rather than in an asymmetric strong network.\textsuperscript{24}

\textsuperscript{23}Can we imagine the Italian Antitrust Authority trying to persuade the chairman of ENI that his company would make higher profits allowing a small competitor with a more limited portfolio of gas suppliers to freely negotiate with all ENI suppliers?

\textsuperscript{24}Also all the other intuitive results are confirmed for the weak buyer too. In particular, $B_2$ gets its worse equilibrium payoff in a $B_1$-short-side and its best in a $B_2$-short-side network. Again, it turns out that
From direct comparisons it also turns out that \( B_2 \) prefers to bargain in a complete network only when \( \lambda \) is high enough, while he is better off in an asymmetric weak architecture for lower levels of \( \lambda \). Infact, better connections can help \( B_2 \) to overcome significant disadvantages in the original trading capability of the weak buyer. However, a line of arguments which are the mirror image of the ones discussed for the strong buyer, implies that the protection of a more central node from the competitive pressure of \( B_1 \)’s outside option is no longer a sufficient trading guarantee when this weakness is less pronounced. Therefore, the weak buyer would prefer negotiating in a complete network exactly for levels of \( \lambda \) for which the strong buyer would not.

There is a tension between buyers’ interests. In fact, the strong buyer prefers to be embedded within a complete network when \( \lambda \) takes medium-low values, while within an asymmetric strong for high levels of \( \lambda \). On the contrary, \( B_2 \) prefers to negotiate within an asymmetric weak architecture when \( \lambda \) is medium-low and within a complete network when his reservation price is relatively high. The emergence of such a conflict of interests among buyers can be regarded as a fascinating prelude to the the investigation of the strategies of endogenous link formation by the traders. This goal is left for a companion paper.

5.4 Extensions and concluding remarks

We have analyzed the interaction between strategic negotiations and network structures in a bilateral oligopoly with identical sellers and heterogeneous buyers. We have provided a full characterization of all the subgame perfect Nash equilibria in pure and stationary strategies emerging in the negotiations stage within any fixed network architecture. We have then described the salient features of such equilibria and compare traders’ payoffs within and across networks.

We find that, depending on the inter-temporal discount factor and the dispersion of reservation values across buyers, negotiations may lead, even in a completely connected bargaining in an exclusive trade network delivers \( B_2 \) exactly the same equilibrium payoffs than negotiations in an asymmetric weak architecture. Such a positive externality from being better connected than in an exclusive partnership arises, again rather intuitively, also within a complete network, but only as \( \lambda \) is high enough.
buyers-sellers network, to multiple equilibria, coexistence of different prices, delays in trade and inefficient allocations. By comparing traders’ payoffs across networks, we then derive the endogenous bargaining power of each trader as a function of her position in the communication network. We show that, for intermediate levels of the buyers’ heterogeneity, the strongest competing purchaser may prefer to bargain within a market structure where communication and trading opportunities are easier and less constrained rather than in one with protected exclusive partnerships.

In a companion paper (Galizzi, 2010) we extend the present model to a two-stage game. In the first stage, firms on both sides of the market independently and simultaneously decide which traders on the opposite side they intend to be linked with. In the second stage, linked traders strategically negotiate within a fixed network. In particular, in the non-cooperative network formation game at the first stage, a link between two traders is formed only if both traders have decided to be linked together. In the second stage, given the network formed in the previous stage, traders in the bilateral duopoly strategically negotiate according to the bargaining protocol we have modelled in the present paper.

We consider the set of PSSPN equilibria discussed here and, moving backward, we solve the strategic network formation in the first stage. The main result is that, according to the equilibria of the bargaining stage, two networks can emerge as subgame perfect Nash equilibria of the overall game, either the complete or the asymmetric strong network.

Since multiplicity of equilibria is inherently related to the behavioural strategic interaction of traders in bilateral oligopolies, we validate the theoretical predictions of the model by means of an experimental analysis. An experimental test of the model, in fact, can not only overcome the difficulty of obtaining individual data on strategic behaviour in gas thin markets but, perhaps more importantly, allow for testing the theoretical results under the same controlled conditions as the theory itself. Our experimental results suggest that multiple equilibria in the strategic price and network formation are likely to

25To the best of our knowledge, except our work, the only experiment on bargaining in buyers-sellers networks is the one by Charness, Corominas-Bosch and Frechette (2005).

26Indeed, experimental analysis would allow us to test whether the model (quoting Hey, 1991) “...survives the transition from the world of the theory to the... real world – the world in which data is gathered”.

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emerge in bilateral oligopolies.

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References


A Appendix: Proofs

In the following, we report a sketch of the proofs for the two first Propositions only. The remaining proofs follow from analogous arguments and are therefore omitted. They are available on request from the author.

A.1 Proof of Proposition 1 (Exclusive trade network)

By an usual argument in bilateral bargaining, in equilibrium a trader offers her (his) exclusive partner just his (her) continuation payoff. Denote \( \delta V(S_i) \) and \( \delta W(B_i) \) the discounted values of the expected continuation payoffs by seller \( S_i \) and buyer \( B_i \), respectively, from entering a new negotiation stage. Imagine \( B_1 \) has been selected to make offers. In a PSSPN equilibrium, \( B_1 \) proposes \( S_1 \) a price \( p_{B_1}^* = \delta V(S_1) \), accepted by \( S_1 \). In fact, any price \( p_{B_1}' > \delta V(S_1) \) would still be accepted but would be strictly dominated. On the other hand, prices \( p_{B_1}' < \delta V(S_1) \) will be rejected by \( S_1 \), returning to \( B_1 \) his continuation value \( \delta W(B_1) \).

However, the proposal \( p_{B_1}^* = \delta V(S_1) \) gives \( B_1 \) a surplus \( 1 - \delta V(S_1) \) as good as the alternative payoff \( \delta W(B_1) \) from an unacceptable offer: in fact, \( \delta W(B_1) < W(B_1) = 1 - \delta V(S_1) \) always holds as the maximum surplus \( B_1 \) can obtain from negotiations in an exclusive trade network is what is left from his reservation price after paying to his exclusive partner \( S_1 \) her continuation payoff. Thus, proposal \( p_{B_1}^* = \delta V(S_1) \) is the optimal strategy by \( B_1 \) among all, acceptable and unacceptable, offers. Analogously, when selected, \( S_1 \) makes offer \( p_{S_1}^* = 1 - \delta W(B_1) \). In both cases, \( S_1 \) or \( B_1 \) trade at the agreed price and leave the market with the corresponding payoffs. The remaining traders \( S_2 \) and \( B_2 \), being linked each other, carry on further negotiations, which are equivalent to a standard bilateral bargaining with random order of proposers: \( B_2 \) and \( S_2 \) expect a surplus \( \frac{\delta \lambda}{T} \) each. The unique PSSPNE of the negotiations game within an exclusive-trade network following a selection of \( B_1 \) implies \( B_1 \) proposing a price \( p_{B_1}^* = \delta V(S_1) \), which is accepted by \( S_1 \), and \( B_2 \) and \( S_2 \) entering bilateral negotiations. The equilibrium payoffs are \( \Pi(B_1) = 1 - \delta V(S_1) \), \( \Pi(B_2) = \Pi(S_2) = \frac{\delta \lambda}{T} \) and \( \Pi(S_1) = \delta V(S_1) \). Analogously, the unique PSSPNE of the game following a selection of \( B_2 \) implies \( B_2 \) proposing \( S_2 \) a price \( p_{B_2}^* = \delta V(S_2) \), which is accepted by \( S_2 \), and \( B_1 \)
and $S_1$ entering bilateral negotiations. The equilibrium payoffs are $\Pi(B_1) = \Pi(S_1) = \frac{\delta}{2}$, $\Pi(B_2) = \lambda - \delta V(S_2)$ and $\Pi(S_2) = \delta V(S_2)$. Analogous results hold for random selections of sellers $S_1$ and $S_2$. By taking each above equilibrium payoff as weighted with $\frac{1}{4}$ probability, traders’ expected payoffs are $W(B_1) = \frac{1}{4}(1 - \delta V(S_1)) + \frac{1}{4}\delta W(B_1) + \frac{1}{4}\left(\frac{\delta}{2}\right)$, $W(B_2) = \frac{1}{2}\left(\frac{\delta}{2}\right) + \frac{1}{4}(\lambda - \delta V(S_2)) + \frac{1}{4}W(B_2)$, $V(S_1) = \frac{1}{4}\delta V(S_1) + \frac{1}{4}(1 - \delta W(B_1)) + \frac{1}{4}\left(\frac{\delta}{2}\right)$ and $V(S_2) = \frac{1}{2}\left(\frac{\delta}{2}\right) + \frac{\delta}{4}V(S_2) + \frac{1}{4}(\lambda - \delta W(B_2))$. Solved as a system, returns $W(B_1) = V(S_1) = \frac{1 + \delta}{4}$ and $W(B_2) = V(S_2) = \frac{(1 + \delta)\lambda}{4}$, that, in the limit case as $\delta \rightarrow 1$, approach the values $W(B_1) \rightarrow V(S_1) \rightarrow \frac{1}{2}$ and $W(B_2) \rightarrow V(S_2) \rightarrow \frac{\lambda}{2}$. We can summarize these results in Proposition 1. Similar arguments show that, within the strong-couple and the weak-couple networks, the traders expect payoffs $W(B_1) = V(S_1) = \frac{1}{2}(2 - \delta)$, $W(B_2) = V(S_2) = 0$, and to $W(B_1) = V(S_1) = 0$, $W(B_2) = V(S_2) = \frac{\lambda}{2(2 - \delta)}$, respectively.

### A.2 Proof of Proposition 2 (Supply-short-side network)

We describe each subgame of the negotiations game, starting after each random selection of the four traders.

#### A.2.1 $B_1$ proposes offers

In the supply-short-side network, whenever either buyer has been selected to make proposals, they can only make offers to the linked seller $S_2$. In particular, by an usual argument, in equilibrium $B_1$ proposes $S_2$ a price $p_{B_1}^* = \delta V(S_2)$, which is accepted by $S_2$. The equilibrium payoffs are $\Pi(B_1) = 1 - \delta V(S_2)$, $\Pi(B_2) = \Pi(S_1) = 0$ and $\Pi(S_2) = \delta V(S_2)$.

#### A.2.2 Seller $S_2$ proposes offers

With $\frac{1}{4}$ probability $S_2$ is selected to make offers to the connected buyers. By backward induction, we first describe, for any given proposed price $p_{S_2}$, the set of all possible pure strategies Nash Equilibria in the response game played by the two linked buyers. We then look for the optimal pricing strategy by $S_2$ in the offer phase. For a given proposal $p_{S_2}$,
the payoff matrix of the buyers’ response game, is as reported in Figure 4.

\[ \begin{array}{c|cc|c}
 & \text{Accept } p_{S2} & \text{Reject } p_{S2} \\
\hline
\text{Accept } p_{S2} & \frac{1}{2} (1 - p_{S2}) & \frac{1}{2} (\lambda - p_{S2}) & 1 - p_{S2} \\
\text{Reject } p_{S2} & 0 & \lambda - p_{S2} & \delta W(B_1) \\
\end{array} \]

Figure 3: Buyers’ response game in a supply-short-side network

In fact, two pure strategies are available to each buyer: either strategy \( \text{Accept } p_{S2} \) or strategy \( \text{Reject } p_{S2} \). Given that \( B_2 \) chooses \( \text{Reject } p_{S2} \), it is better for \( B_1 \) to \( \text{Accept } p_{S2} \), if and only if (iff) \( 1 - p_{S2} \geq \delta W(B_1) \), while, given that \( B_2 \) plays \( \text{Accept } p_{S2} \), it is better for \( B_1 \) to also accept it iff \( \frac{1}{2} (1 - p_{S2}) \geq 0 \), that is, iff \( p_{S2} \leq 1 \). On the other hand, given that \( B_1 \) accepts \( p_{S2} \), it is optimal for \( B_2 \) to also accept it iff \( \frac{1}{2} (\lambda - p_{S2}) \geq 0 \), that is, iff \( p_{S2} \leq \lambda \). Finally, given that \( B_1 \) Rejects \( p_{S2} \), it is optimal for \( B_2 \) to accept it iff \( \lambda - p_{S2} \geq \delta W(B_2) \).\(^\text{27}\)

The combination of \([B_1, B_2]\) pure strategies \([\text{Accept } p_{S2}, \text{Accept } p_{S2}]\) is a Nash equilibrium of the response game iff \( p_{S2} \leq 1 \)
\[ p_{S2} \leq \lambda \] , that is, whenever \( p_{S2} \leq \lambda \). On the other hand, pure strategies \([\text{Accept } p_{S2}, \text{Reject } p_{S2}]\) are a Nash equilibrium of the response game iff \( p_{S2} \leq 1 - \delta W(B_1) \), while \([\text{Reject } p_{S2}, \text{Accept } p_{S2}]\) are a pure-strategies Nash equilibrium iff \( p_{S2} > \lambda \)
\[ p_{S2} > 1 - \delta W(B_1) \] . Finally, \([\text{Reject } p_{S2}, \text{Reject } p_{S2}]\) are a pure-strategies Nash equilibrium iff \( p_{S2} > \lambda - \delta W(B_2) \), that is iff \( p_{S2} > \max \{1 - \delta W(B_1), \lambda - \delta W(B_2)\} \).

\(^\text{27}\)Hereafter, we use tie-breaking assumptions by which, if a trader is perfectly indifferent between accepting or rejecting an offer, he (she) accepts it; while, if a trader is indifferent between proposing acceptable or unacceptable offers, he (she) makes the acceptable one.
As the above discussed condition (1) \( \delta [W(B_1) - W(B_1)] \leq 1 - \lambda \) holds, the latter condition reduces to \( p_{S2} > 1 - \delta W(B_1) \). It can be easily reckoned that offer \( p_{S2} \) can never verify the set of restrictions \( \left\{ \begin{array}{l} p_{S2} > 1 \\ p_{S2} \leq \lambda - \delta W(B_2) \end{array} \right. \), as \( \lambda < 1 \) and \( B_2 \)'s continuation payoff is surely non negative, \( \delta W(B_2) \geq 0 \). Hence, there is no offer \( p_{S2} \) verifying \( \lambda - \delta W(B_2) \geq p_{S2} > 1 \). Therefore, only three possible combinations of strategies can represent Nash equilibria of the response game: either \([Accept p_{S2}, Accept p_{S2}]\) iff \( p_{S2} \leq \lambda \); or \([Accept p_{S2}, Reject p_{S2}]\) iff \( p_{S2} \leq 1 - \delta W(B_1) \); or, finally, \([Reject p_{S2}, Reject p_{S2}]\) iff \( p_{S2} > 1 - \delta W(B_1) \). Multiple Nash equilibria in the response game can arise whenever \( S_2 \) proposes an offer \( p_{S2} \) such that both \( p_{S2} \leq \lambda \) and \( p_{S2} > 1 - \delta W(B_1) \) hold. In such a case two alternative Nash equilibria coexist: one where both buyers accept \( p_{S2} \), the other where both buyers reject it. We will characterize all multiple equilibria in the response game and, for each of them, we will look for the PSSPN equilibria in the response game. The set of potential pure-strategies Nash equilibria can be narrower under specific levels of \( \lambda \) and \( 1 - \delta W(B_1) \). Consider either case I or II:

\[
\begin{align*}
\text{Case I: } & 1 - \delta W(B_1) > \lambda \\
\text{Case II: } & \lambda \geq 1 - \delta W(B_1)
\end{align*}
\]

Under case I, the buyers' response game show all three potential pure strategies Nash equilibria within some range: either \([Accept p_{S2}, Accept p_{S2}]\) iff \( p_{S2} \leq \lambda \), or \([Accept p_{S2}, Reject p_{S2}]\) iff \( p_{S2} \leq 1 - \delta W(B_1) \); or, finally, \([Reject p_{S2}, Reject p_{S2}]\) iff \( p_{S2} > 1 - \delta W(B_1) \). Not only the whole range of parameters is covered by some equilibrium, but the sets are also mutually exclusive, ruling out multiple equilibria. Whenever \( S_2 \) charges any price \( p_{S2} > 1 - \delta W(B_1) \) both buyers reject her offer, traders enter further negotiations, and \( S_2 \) gets \( \delta V(S_2) \). On the other hand, whenever \( S_2 \) proposes any price \( p_{S2} \leq 1 - \delta W(B_1) \), the offer would immediately be accepted either by \( B_1 \) only, if \( 1 - \delta W(B_1) \geq p_{S2} > \lambda \), or by both buyers, if \( p_{S2} \leq \lambda \), giving her a payoff of \( p_{S2} \). Among all such acceptable offers, \( p_{S2}^* = 1 - \delta W(B_1) \) is clearly a dominant strategy by \( S_2 \). Therefore, as long as \( \delta V(S_2) \leq 1 - \delta W(B_1) \), the best strategy for \( S_2 \) is to pro-
pose a price $p_{S_2}^* = 1 - \delta W(B_1)$, and, otherwise to make any highest, unacceptable, offer. However, condition $\delta W(B_1) < W(B_1) \leq 1 - \delta V(S_2)$ is always verified as the most $B_1$ can get from negotiations is what is left of his surplus once seller $S_2$ has been paid her continuation payoff. In fact, from condition (1), by negotiating with $B_2$, $S_2$ can only get lower payoffs as $\lambda - \delta W(B_2) \leq 1 - \delta W(B_1)$. Thus, for $\delta W(B_1) < 1 - \lambda$, $S_2$ offers a price $p_{S_2}^* = 1 - \delta W(B_1)$, which is accepted in equilibrium by $B_1$ only, while $B_2$ rejects it and leaves the market. Traders’ expected payoffs from case I equilibrium are $\Pi(B_1) = \delta W(B_1)$, $\Pi(B_2) = \Pi(S_1) = 0$ and $\Pi(S_2) = 1 - \delta W(B_1)$. It can be checked that $[\text{Reject } p_{S_2}^*, \text{Accept } p_{S_2}^*]$ is a pure strategies equilibrium of the response game. Given that $B_1$ accepts $p_{S_2}^* = 1 - \delta W(B_1)$, $B_2$ cannot profitably deviate by also accepting $p_{S_2}^*$ as it would give him a payoff $\frac{1}{2} (\lambda - 1 + \delta W(B_1))$, which is lower than the zero payoff from leaving the market, as $\delta W(B_1) < 1 - \lambda$ holds under case I. On the other hand, given that $B_2$ rejects, $B_1$ would get the same payoff $\delta W(B_1)$ if he rejects $p_{S_2}^*$ too. Finally, given the buyers’ behavior, seller $S_2$ cannot profitably deviate either: in fact, if she proposes any price $p_{S_2}' = p_{S_2}^* - \varepsilon$, still accepted at least by $B_1$, she would gain a lower surplus, while for any, unaccepted, $p_{S_2}'' = p_{S_2}^* + \varepsilon$, she would get her continuation value, which is lower since $\delta V(S_2) \leq 1 - \delta W(B_1)$.

**Case II:** $\lambda \geq 1 - \delta W(B_1)$ Under case II, the set of conditions \begin{align*} p_{S_2} &\leq 1 - \delta W(B_1) \\ p_{S_2} &> \lambda \end{align*}
identifies an empty range: in fact, $\delta W(B_1) \geq 1 - \lambda$ is incompatible with the conditions necessary to meet a $[\text{Accept } p_{S_2}, \text{Reject } p_{S_2}]$ PSSPN equilibrium in the response game. The whole range of parameters is covered by some equilibrium. However, in case II the conditions are no longer mutually exclusive. In fact, whenever $S_2$ proposes any offer $\tilde{p}_{S_2}$ such that $\lambda \geq \tilde{p}_{S_2} > 1 - \delta W(B_1)$, two alternative Nash equilibria co-exist in the response game: either both buyers accept $\tilde{p}_{S_2}$, or they both reject it. Consider the equilibrium (IIa) where both buyers accept any offer $\lambda \geq \tilde{p}_{S_2} > 1 - \delta W(B_1)$. A random tie-break determines which buyer trades with $S_2$ at $\tilde{p}_{S_2}$ and which leaves the market. The traders’ payoffs would be $\Pi(B_1) = \frac{1}{2} (1 - \tilde{p}_{S_2})$, $\Pi(B_2) = \frac{1}{2} (\lambda - \tilde{p}_{S_2})$, $\Pi(S_1) = 0$ and $\Pi(S_2) = \tilde{p}_{S_2}$. Among all
such acceptable offers, $p_{S_2}^* = \lambda$ is a dominant strategy for seller $S_2$. For any $p'_{S_2} > \lambda$, the response game shows a unique equilibrium where both buyers reject that offer, all traders enter a new round of negotiations, and $S_2$’s payoff is $\delta V (S_2)$. Thus, the optimal decision by $S_2$ is to propose an acceptable offer $p_{S_2}^* = \lambda$, as long as $\delta V (S_2) \leq \lambda$ and, otherwise, to make any unacceptable offer. As long as conditions $$\begin{cases} \delta W (B_1) \geq 1 - \lambda \\ \delta V (S_2) \leq \lambda \end{cases}$$ hold, there is a PSSPN equilibrium with accepted offers ($H_a$), characterized as follows. In the price offer phase, $S_2$ offers a price $p_{S_2}^* = \lambda$, which is accepted by both buyers. A random tie-break selects which buyer is trading with $S_2$, and which, instead, leaves the market. Traders’ expected payoffs from equilibrium $IIa$ are $\Pi (B_1) = \frac{\lambda}{2}$, $\Pi (B_2) = \Pi (S_1) = 0$ and $\Pi (S_2) = \lambda$. Consider the alternative case where, following an offer $\lambda \geq \tilde{p}_{S_2} > 1 - \delta W (B_1)$, the equilibrium ($IIr$) is such that both buyers reject $\tilde{p}_{S_2}$. Traders would enter a further round of negotiations and expect their continuation payoffs. Thus, when charging a price $\lambda \geq \tilde{p}_{S_2} > 1 - \delta W (B_1)$, $S_2$ expects a payoff $\delta V (S_2)$. Also for any $p'_{S_2} > \lambda$, both buyers reject the offer. Thus any price $p'_{S_2} > 1 - \delta W (B_1)$ returns $S_2$ the expected payoff $\delta V (S_2)$. On the other hand, any price $p'_{S_2} \leq 1 - \delta W (B_1)$ would be accepted by both buyers, and among all acceptable offers, $p_{S_2}^* = 1 - \delta W (B_1)$ is a dominant strategy. The best strategy for $S_2$ is to propose $p_{S_2}^* = 1 - \delta W (B_1)$, as long as $\delta V (S_2) \leq 1 - \delta W (B_1)$ and, otherwise to make any unacceptable offer. Again condition $\delta W (B_1) < W (B_1) \leq 1 - \delta V (S_2)$ is verified in the supply-short-side network as long as $\delta W (B_1) \geq 1 - \lambda$ holds. There exists a $IIr$ equilibrium characterized as follows. $S_2$ offers $p_{S_2}^* = 1 - \delta W (B_1)$, which is accepted by both buyers. A random tie-break selects which buyer is going to trade with $S_2$, and which, instead, leaves the market. Traders’ expected payoffs are $\Pi (B_1) = \frac{\delta W (B_1)}{2}$, $\Pi (B_2) = \frac{\delta W (B_1)}{2} + \frac{\lambda - 1}{2}$, $\Pi (S_1) = 0$ and $\Pi (S_2) = 1 - \delta W (B_1)$. It can be quickly checked that the above described strategies are pure strategies subgame perfect equilibria. Thus, under condition (1), three PSSPN equilibria can arise:

- If $\delta W (B_1) < 1 - \lambda$, there exists a PSSPN equilibrium where $S_2$ offers a price $p_{S_2}^* = 1 - \delta W (B_1)$ and only $B_1$ accepts it, while $B_2$ rejects it and leaves the market.
Traders’ expected payoffs from such PSSPN I-equilibrium are $\Pi (B_1) = \delta W(B_1)$, $\Pi (B_2) = \Pi (S_1) = 0$ and $\Pi (S_2) = 1 - \delta W(B_1)$.

- If the set of conditions holds $\delta W(B_1) \geq 1 - \lambda$, there exists a PSSPN equilibrium where $S_2$ offers a price $p_{S_2}^* = \lambda$ and both $B_1$ and $B_2$ accept $p_{S_2}^* = \lambda$ as well as any offer within the range $(1 - \delta W(B_1), \lambda]$. From the PSSPN IIa-equilibrium traders expect $\Pi (B_1) = \frac{1-\lambda}{2}$, $\Pi (B_2) = \Pi (S_1) = 0$ and $\Pi (S_2) = \lambda$.

- If $W(B_1) \geq 1 - \lambda$, there exists a PSSPN equilibrium where $S_2$ offers a price $p_{S_2}^* = 1 - \delta W(B_1)$ and both $B_1$ and $B_2$ accept $p_{S_2}^* = 1 - \delta W(B_1)$ while they both reject any offer within the range $(1 - \delta W(B_1), \lambda]$. From the PSSPN IIr-equilibrium traders expect $\Pi (B_1) = \frac{\delta W(B_1)}{2}$, $\Pi (B_2) = \frac{\delta W(B_1)}{2} + \frac{\lambda-1}{2}$, $\Pi (S_1) = 0$ and $\Pi (S_2) = 1 - \delta W(B_1)$.

### A.2.3 $B_2$ proposes offers

By analogous arguments, the optimal behaviour by $B_2$ is to propose $p_{B_2}^* = \delta V(S_2)$ if $\delta W(B_2) \leq \lambda - \delta V(S_2)$, and any $p_{B_2}' < \delta V(S_2)$ otherwise. If $\delta W(B_2) \leq \lambda - \delta V(S_2)$, the unique PSSPNE with acceptable offers of the game following a random selection of $B_2$ implies that $B_2$ proposes $S_2$ a price $p_{B_2}^* = \delta V(S_2)$, and $S_2$ accepts it. Traders expect $\Pi (B_1) = \Pi (S_1) = 0$, $\Pi (B_2) = \lambda - \delta V(S_2)$ and $\Pi (S_2) = \delta V(S_2)$. If $\delta W(B_2) > \lambda - \delta V(S_2)$, the unique PSSPNE with unacceptable offers implies that $B_2$ proposes $S_2$ any $p_{B_2}' < \delta V(S_2)$, and $S_2$ rejects it. Traders expect their discounted continuation values.

### A.2.4 Seller $S_1$ proposes offers

In the supply-short-side market, the isolated seller $S_1$ has the chance to offer some price at which, however, she would never be able to trade at. Therefore, whenever $S_1$ is selected to make offers, all traders expect their continuation values.
A.2.5 Description of the equilibria

We now combine each equilibrium outcome for any selection of the proposer and characterize the expected continuation values. Each payoffs is possible within a particular set of restrictions. Whenever an expression for the payoffs violates a condition, the corresponding combination of strategies can not be viewed as a candidate equilibrium. By repeating such a test, we eliminate all the combinations whose payoffs are not consistent with some conditions and, we characterize the PSSPN equilibria for the ones surviving the check.

For instance, consider a new round of the negotiations in which it is expected that, with \( \frac{1}{4} \) probability,

- \( B_1 \) offers \( S_2 \) a price \( p_{B_1}^* = \delta V(S_2) \), which is accepted by \( S_2 \), delivering the payoffs \( \Pi(B_1) = 1 - \delta V(S_2) \), \( \Pi(S_2) = \delta V(S_2) \), \( \Pi(B_2) = \Pi(S_1) = 0 \).

- as \( \delta W(B_2) \leq \lambda - \delta V(S_2) \) holds, \( B_2 \) offers a price \( p_{B_2}^* = \delta V(S_2) \) which is accepted by seller \( S_2 \), returning the payoffs \( \Pi(B_1) = \Pi(S_1) = 0 \), \( \Pi(B_2) = \lambda - \delta V(S_2) \) and \( \Pi(S_2) = \delta V(S_2) \).

- for any offer proposed by \( S_1 \), the expected payoffs for the traders are their continuation payoffs.

- as \( \delta W(B_1) \geq 1 - \lambda \) and \( \delta V(S_2) \leq \lambda \) are satisfied, \( S_2 \) offers a price \( p_{S_2}^* = \lambda \) which is accepted by both buyers, returning payoffs \( \Pi(B_1) = \frac{1-\lambda}{2} \), \( \Pi(B_2) = \Pi(S_1) = 0 \) and \( \Pi(S_2) = \lambda \).

Therefore, under conditions \( \delta W(B_1) \geq 1 - \lambda \), \( \delta V(S_2) \leq \lambda \) and \( \delta W(B_2) \leq \lambda - \delta V(S_2) \), we can compute the expressions for the expected continuation payoffs in a PSSPN equilibrium characterized by the above strategies for any possible random selection of the proposer. In fact, by taking each payoff as weighted with \( \frac{1}{4} \) probability, the expected continuation payoffs are \( W(B_1) = \frac{1}{4} (1 - \delta V(S_2)) + \frac{\delta}{4} W(B_1) + \frac{\lambda}{4} (\frac{1-\lambda}{2}) \), \( W(B_2) = \frac{1}{4} (\lambda - \delta V(S_2)) + \frac{\delta}{4} W(B_2) \), \( V(S_1) = 0 \) and \( V(S_2) = \frac{3}{4} \delta V(S_2) + \frac{1}{4} \lambda \). Solving the system returns \( W(B_1) = \frac{-9\delta + 3\lambda + 12 - 4\lambda}{2(3\delta^2 - 16\delta + 16)} \), \( W(B_2) = \frac{4\lambda (1-\delta)}{3\delta^2 - 16\delta + 16} \), \( V(S_1) = 0 \) and \( V(S_2) = \frac{\lambda}{4 - 3\delta} \), that, in the limit
case as $\delta \rightarrow 1$ approach the values $W(B_1) \rightarrow \frac{1-\lambda}{\delta}$, $W(B_2) \rightarrow V(S_1) \rightarrow 0$ and $V(S_2) \rightarrow \lambda$. We now check whether the expressions for the expected continuation payoffs are compatible with condition (1) and the restrictions $\delta W(B_1) \geq 1 - \lambda$, $\delta V(S_2) \leq \lambda$ and $\delta W(B_2) \leq \lambda - \delta V(S_2)$. We help our analysis by means of numerical simulations over the primitive parameters, namely, the intertemporal discount rate $\delta$ and the reservation price of the weak buyer $\lambda$, both contained by definition within a range $(0, 1)$. Simulations show that condition (1) and restriction $\delta V(S_2) \leq \lambda$ are verified for any value $\delta \in (0, 1)$ and that condition $\delta W(B_2) \leq \lambda - \delta V(S_2)$ is always satisfied. Moreover, condition $\delta W(B_1) \geq 1 - \lambda$ is satisfied as long as $\lambda \geq \bar{\lambda} = \frac{15\delta^2-44\delta+32}{7\delta^2-36\delta+32}$, that is when $\lambda$ is sufficiently high: just to give an idea, the condition is satisfied when $\delta = 0.85$, for values $\lambda \geq \bar{\lambda} = 0.842$. Therefore, for any value of the discount rate $\delta \in (0, 1)$ and for sufficiently high values of the weak buyer’s reservation price $\lambda$, the expected continuation payoffs in the above characterized equilibrium are consistent with the corresponding conditions. This, in turn, allows us to state Proposition 2 for what concerns equilibrium $SS2$. Analogous arguments drive the process of elimination of any combination of strategies inconsistent with the corresponding restrictions. Indeed, direct calculations and numerical simulations show that another equilibrium exists for intermediate values of $\lambda$, $\hat{\lambda} \leq \lambda < \bar{\lambda}$, and that an alternative equilibrium exists for sufficiently low values of $\lambda$, namely for the complementary case of $\lambda < \hat{\lambda}$, as stated in Proposition 2.
## Appendix: Tables of equilibrium payoffs

We report in the following tables the equilibrium payoffs of the traders within the different networks, and the values they approach in the limit case as $\delta \to 1$.

<table>
<thead>
<tr>
<th>Network/Payoffs</th>
<th>W(B1)</th>
<th>W(B2)</th>
<th>V(S1)</th>
<th>V(S2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Couple</td>
<td>$\frac{1}{2(2-\delta)}$</td>
<td>0</td>
<td>$\frac{1}{2(2-\delta)}$</td>
<td>0</td>
</tr>
<tr>
<td>Weak Couple</td>
<td>0</td>
<td>$\frac{\lambda}{2(2-\delta)}$</td>
<td>0</td>
<td>$\frac{\lambda}{2(2-\delta)}$</td>
</tr>
<tr>
<td>Supply Short Side</td>
<td>SS1</td>
<td>$\frac{4(1-\delta)}{5\delta^2-20\delta+16}$</td>
<td>$-3\delta^2-5\delta^4+4\delta^2+20\delta-16\delta$</td>
<td>$\frac{5\delta^3-40\delta^2+96\delta}{5\delta^2-20\delta+16}$</td>
</tr>
<tr>
<td></td>
<td>SS2</td>
<td>$\frac{-9\delta^2+12+12-4\lambda}{2(3\delta^2-16\delta+16)}$</td>
<td>$\frac{4 \lambda (1-\delta)}{3\delta^2-16\delta+16}$</td>
<td>0</td>
</tr>
<tr>
<td>Supply Short Side</td>
<td>SS3</td>
<td>$\frac{1}{2(2-\delta)}$</td>
<td>0</td>
<td>$\frac{1}{2(2-\delta)}$</td>
</tr>
<tr>
<td>B1-Short Side</td>
<td>B1S-A</td>
<td>$\frac{1}{4-3\delta}$</td>
<td>0</td>
<td>$\frac{1}{4-\delta} \left(1 - \frac{\delta}{4-3\delta}\right)$</td>
</tr>
<tr>
<td></td>
<td>B1S-R</td>
<td>$\frac{8-5\delta}{7\delta^2-36\delta+32}$</td>
<td>$\frac{8(1-\delta)}{7\delta^2-36\delta+32}$</td>
<td>$\frac{8(1-\delta)}{7\delta^2-36\delta+32}$</td>
</tr>
<tr>
<td>B2-Short Side</td>
<td>B2S-A</td>
<td>0</td>
<td>$\frac{\lambda}{4-3\delta} \left(1 - \frac{\delta}{4-3\delta}\right)$</td>
<td>$\frac{\lambda}{4-\delta} \left(1 - \frac{\delta}{4-3\delta}\right)$</td>
</tr>
<tr>
<td></td>
<td>B2S-R</td>
<td>$\frac{(8-5\delta)\delta}{7\delta^2-36\delta+32}$</td>
<td>$\frac{8\lambda(1-\delta)}{7\delta^2-36\delta+32}$</td>
<td>$\frac{8\lambda(1-\delta)}{7\delta^2-36\delta+32}$</td>
</tr>
</tbody>
</table>

Figure 4: Equilibrium payoffs within the strong-, weak-couple, supply-, and buyers-short-side networks.
Figure 5: Equilibrium payoffs within the asymmetric weak and strong networks.

Figure 6: Equilibrium payoffs within the complete network.
Figure 7: Limit payoffs within the supply-short-side, complete and buyers-short-side networks.

Figure 8: Limit payoffs within the asymmetric weak and strong networks.