

# Consent in a Multi-Issue Cooperative Game

## (extended abstract)

Gabrielle Gayer\* and Dotan Persitz†

This version: February 28, 2010

The standard cooperative game theory deals with a problem in which a group of individuals tries to allocate a resource among its members. The most popular solution concept in this literature is that of the “core”, which is the set of all imputations that are unanimously supported by the members of the group. In other words, there is no subgroup of individuals that could improve the payoff of its members on its own. One of the main deficiencies of this solution concept is that such a solution may not exist. Many other solution concepts were developed in order to overcome this problem.

We suggest an alternative approach to achieve consent in cooperative games. In this work we extend the concept of the core to multiple games that are solved together. As in the standard case, the resources of all games are allocated among the players. The players are informed about the total payment of all players in all games but are uninformed about the composition of the payments, i.e. about the payment each player receives in each game. An allocation  $x$ , is a solution of the

---

\*Department of Economics, The University of Haifa, Haifa, 31905, Israel. Telephone Number: (972)-4-8249588. Email: Gabi.Gayer@gmail.com.

†The Eitan Berglas School of Economics, Tel Aviv University, Tel Aviv, 69978, Israel. Telephone Number: (972)-3-6405816. Email: persitzd@post.tau.ac.il

multiple games if each player  $i$ , has a subjective perspective of the grand coalition's allocation in each games that is consistent with  $x$ . Moreover, no subgroup to which  $i$  belongs will be better off on its own according to  $i$ 's subjective perspective. We compare the set of solutions of the multiple games when they are solved separately to the set of solutions when they are solved jointly. For a single game both solutions are essentially the same, however for multiple games the former may be a strict subset of the latter. It turns out that our solution for the set of multiple games may exist even when some of the individual games have no solution.

For example, suppose three individuals are engaged in two issues, formalized as coalitional games with transferable utility. The issues can be represented by the following characteristic functions:

$$v_1(S) = \begin{cases} 2 & \text{if } |S| = 1 \\ 3 & \text{if } |S| = 2 \\ 4 & \text{if } |S| = 3 \end{cases} \quad ; \quad v_2(S) = \begin{cases} 2 & \text{if } |S| = 1 \\ 5 & \text{if } |S| = 2 \\ 8 & \text{if } |S| = 3 \end{cases}$$

The core of  $G(N, v_1)$  is empty since each player must receive at least 2, but the sum of payoffs can not exceed 4. The core of  $G(N, v_2)$  is:

$$\{x | x_1 = 2 + \alpha, x_2 = 2 + \beta, x_3 = 4 - \alpha - \beta, \alpha \in (0, 1), \beta \in (0, 1), \alpha + \beta \geq 1\}$$

Tying the two issues to each other enables the formation of the grand coalition in the joint problem. Suppose that each player's payoff is 4. These payoffs are sustainable given that each player believes that he gets 2 in each game, while the other players receive 1 in the first game and 3 in the second game. For each of them the subjective matrix is a perceived efficient matrix. Moreover, in each of the two games, every coalition  $S$  in which agent  $i$  participates achieves at least its worth.

In the paper we characterize the sets of multi-issue cooperative games for which a payoff vector for the grand coalition can be sustained. We focus on multi-issue games that include cooperative games with an empty core. Our results can be interpreted in the context of multilateral international negotiations when the mediator can manipulate the agents' beliefs regarding the proposed solution.