For Sylvie
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Preface

It is always difficult to know exactly where things really started and what I describe in this book is the result of a long journey. At various points I was influenced by ideas which took me off the path which I suppose one would call “mainstream economics” but I was slow to realise their importance. At the start I came to economics believing that it would help me to understand economic and social phenomena such as unemployment, inflation, inequality. However, Hugo Sonnenschein, who was my first adviser, told me that young people should do microeconomics since macroeconomics involved more wisdom than mathematics and wisdom only comes with age. Moreover, he said mathematics was indispensible if one was to become an economic theorist. John Chipman once explained that doing economic theory without mathematics was a bit like swimming the channel, an admirable feat, but hardly the easiest way of getting from England to France.

I fled Minnesota with its insistence on mathematics and looked for easier trails to follow at Princeton. However, Hugo had had an effect and I fell under the wing of Harold Kuhn, who became my thesis adviser and gave the best and clearest courses I have ever followed. He was the first to tell me that the basic mathematical structure used in theoretical economics was extremely simple. Everything is reduced to the maximisation of concave functions on convex sets. Then one looks at the first order conditions and, if one is a little more meticulous, one has a look at what happens when the constraints are binding. Things get dressed up in fancier clothing but the basic structure does not change. Werner Hildenbrand who became not only a co-author but a great friend taught me the importance of rigorous thinking and although we worked within the context of general equilibrium theory anyone who reads our texts can detect our dissatisfaction with the model and its underlying assumptions. Each of these three, Hugo Sonnenschein, Harold Kuhn, and Werner Hildenbrand, although they did pioneering work in the very fundamental foundations of general equilibrium theory had a healthy skepticism about the underlying assumptions of theoretical economics. Indeed, Hugo Sonnenschein’s work on aggregation was the basis for my disillusionment with general equilibrium. It was Harold Kuhn who persuaded me of the interest of game theory and its insistence on the importance of the direct interaction between individuals and Werner Hildenbrand who introduced me to Hans Foellmer, who became a friend and co-author and whose type of mathematical model underlies a great deal of the work described in this book. It was also Werner who made a radical shift in his methodological position and argued that we would do better to start with empirical facts rather than unverifiable assumptions based on introspection. He is a rare counter example to Max
Planck’s dictum about the incapacity of people to change their scientific stance and to adopt new ideas.

It was really in the middle of the ‘70s that I became aware of the fact that there was something amiss in the kingdom of general equilibrium. On the one hand there were the results of Sonnenschein, Mantel and Debreu, which to summarise, showed that no empirical evidence could falsify our assumptions about individuals. Then there was the result of Saari and Simon which showed that any process which would ensure that an economy would pass from an out of equilibrium state to an equilibrium would require an infinite amount of information. All of this meant that some other route might be interesting. The major problem was the passage from micro behaviour to the behaviour of the aggregate. The way round this was simply to assume that the aggregate behaved like an individual! Yet we knew that from the results that I have mentioned this was not justified by theory. This was exactly the way in which macroeconomics had passed from being a subject for the wise, to a “respectable” formal exercise based on “sound micro-foundations”. Open any modern macroeconomic text and you will find a microeconomic study of the simplest case, that of single “representative agent”. In no other discipline whether to the right, like physics, or to the left like sociology do you find the assumption that the behaviour of the whole is the same as the behaviour of a typical part.

With all this in mind I was intrigued by Hans Foellmer’s analysis of how the passage from micro to macro breaks down when people interact directly with each other. I remember a conversation in 1974 in an English garden on a rare sunny day with David Rand, a mathematician and colleague at the University of Warwick and we decided to look at demand behaviour as a system of spin glasses. We never did it and this was a big mistake on my part. But good ideas do not disappear when I became interested in the the recruiting behaviour of ants, Hans Foellmer showed me the way to analyse this with a very simple stochastic model. The switching behaviour of these ants led to the idea of switching between chartists and fundamentalists in financial markets which underlies the work that we later did together.

In this vein I tried in the ‘70s to apply some results from stochastic graphs to economic theory since I was convinced that the structure of the interactions between people was important. Much later I took this up again with Jernej Copic and Matt Jackson and we looked at how to identify clusters or communities in economics. I was astonished to find after 20 years how much resistance economists had to the idea of clusters and their importance in the economy. Yet there has been a growing acceptance of the idea that networks do play a significant role in the economy. In particular, the idea of an anonymous market in which people only interact through the price system seemed to be totally unrealistic. It was this that led me to look in detail at some empirical markets and in particular, the fish market in Marseille but also other fish and perishable goods markets as well as financial markets.

The chapter on fish markets is based on work with Mauro Gallegati, Gianfranco Giuloni, Wolfgang Haerdle, Dorothea Herreiner, Paul Pezanis, Annick Vignes, Nick Vriend, and Gerard Weisbuch. This topic has been a source of endless fun and wonderful expeditions to fish markets from Tokyo to Sydney, to Rungis, to Tromso, to Ancona and to Sete and Saumaty. I also profited from the meetings of the IIEFET, which allowed me to make contact with people who make a serious living from studying and
working on fish markets.

The chapter on financial markets borrows shamelessly from joint work with Hans Foellmer, Ulrich Horst, Roman Riccioti, Gilles Teyssiére, and Richard Topol. I am indebted to all of them. They accompanied me along various stretches of the financial markets path that I follow in that chapter, but none of them is responsible, if, at some point, I got lost! My interest in the foreign exchange market stems from the time that I spent at the Bank of England as a Houblon Norman Fellow. The Bank introduced me to a number of the major trading rooms in London. I was made particularly welcome at was then Chemical Bank’s trading room and was particularly flattered by a remark from one of the traders. I asked him why his boss was prepared to let me ask questions which slowed things up when millions of pounds were at stake. He answered, “we give you better treatment than the other economists who come here for one reason. When they come here, they inevitably tell us what we should be doing whilst you are trying to find out what we actually do!” I must also say that I am comforted by the fact that senior people at the Bank such as Andy Haldane take the idea of the economy as a complex system and the importance of networks very seriously, particularly in their analysis of the recent crisis.

The chapter on contributions to public goods is based on work done with Walid Hichri who did his doctorate with me and patiently ran numerous experiments with fortunately very similar results as we found out that people cannot simply be classified as more or less generous or altruistic.

One of the people whom I most admire and who could, in many ways, be thought of as the intellectual father of this book, who was one of the pioneers of the analysis of the relation between micro and macro behaviour is Tom Schelling and I have had the privilege of discussing some of these ideas with him. A few days before he won the Nobel prize, I was at the Institute for Advanced Study in Princeton and the graduate students at the university held a poll to name the next winners of the prize. I told my wife that I was going to the economics department to vote for Tom Schelling because I thought that he really merited it, but that I was sure that he would not get it. There was a huge rainstorm and I decided not to get my bike out and did not vote. When the winner was announced I sent an e-mail to Tom recounting this story and said how happy I was to have been wrong about him not getting it. He replied in typically laconic fashion, saying “not as glad as I was!”.

The chapter on segregation is directly inspired by his famous model. Nick Vriend a student and friend pushed the analysis further and then thanks to Dejan Vinkovic an astrophysicist whom I met at the Institute for Advanced Study at Princeton we developed a physical analogue of Tom Schelling’s model. This allowed us to explain the very different sorts of segregated clusters that can form as a result of individual preferences for the race and the income of their neighbours. This also meant that I ran into all the objections to this sort of analysis on the basis that people are not like particles. Having been interested in ants and where I ran into the same criticism I have become hardened to this sort of attack which I find wrong-headed.

In the early 90s a number of us were interested in pursuing ideas about direct interaction between heterogeneous agents in economics and Mauro Gallegati had the wonderful idea of starting a series of workshops on the subject, (WEHIA). This series still continues and after several years in Ancona has moved around from Italy, to Hol-
land, Germany, the U.S. and China. The workshops gave rise to an association ESHIA of which I am happy to be the current president. A journal, JEIC, (yet another) has sprung up, and has added to the other outlets such as the JEDC and JEBO which have been sympathetic to these ideas.

Another framework for the discussions around these ideas was the research programme called “Complex Markets” financed by the European Commission. Mark Salmon doggedly tried to organise the unorganisable and I benefited enormously from discussing these ideas with him and with Michele Marchesi, Cars Hommes, Mikhail Anufriev, Thomas Lux and the other members of the group.

Another idea that underlies much of what is said in this book stemming from the dissatisfaction with homo oeconomicus, concerns what precisely we mean by the identity of an economic agent and with Ulrich Horst and Miriam Teschl we have tried to clarify the nature of identity as it evolves as a result of experience and interaction with others. In particular the influence of the groups people belong to and their impact on those groups is at the heart of the sort of problems generated in analysing the evolution of an economic system. Amartya Sen, was always a source of wisdom and erudition on this subject and helped me to think a little more clearly about it.

Apart from those whom I have already mentioned, I could not even begin to list all of the people who have helped me develop the ideas here but here are the names of some of those who contributed to the way that I think, (possibly wrongly and surely superficially), about economics. All of them are friends and some have become co-authors. None of them is responsible for any errors or misperceptions. I apologise immediately to any that I have left out and to any that I include who do not feel comfortable at being in the list.


I also owe a huge debt to Nobi Hanaki, my colleague and friend who patiently put this manuscript into presentable form.

Finally my biggest debt is to my wife Sylvie Thoron for her support, encouragement, and constructive criticism and most importantly for having put up with me all of this time.
Chapter 1

Introduction

“A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it”


“There is something fascinating about science. One gets such wholesale returns of conjecture out of such a trifling investment of fact”

Mark Twain, Life on the Mississippi (1883)

“We have in our discipline been led up the wrong path by the invisible hand of the demon, and because it takes both time and money to make an engine we are producing on a large scale “aeroplanes” which have no engine”.


1.1 Introduction

At the time of writing the world was being shaken by an upheaval in the financial sector comparable to that of 1929. These events in world financial markets have, to say the least, given economists pause for reflection. The explanations given for the collapse of the structure are clear and convincing. Individual banks extended credit to those wishing to buy homes with little regard for the capacity of the borrowers to pay. If the
unhappy borrower did not fulfill his obligations the bank recovered the home, the price of which was rising. The loans in question were distributed among banks worldwide, through instruments which packaged loans of various quality together. This, we were told, was a good thing because it diversified the risk. However, with a weakening of the U.S. economy the number of defaulters grew and, worse, prices in the housing market no longer rose. At this point, banks started to examine their positions and to evaluate the losses and potential losses due to the “subprime” loans contained in the instruments they were holding. Many major banks found that their positions were more than delicate and began to seek ways of redressing them. However, the crucial problem was that banks did not know which of their counterparts were in trouble and thus stopped lending to other banks. The freezing of the interbank market brought the whole system to a halt since banks are constantly in need of being able to finance various transactions and habitually borrow from each other to do so. The solution which may or may not eliminate or reduce the problem was, at the time of writing, to inject enormous amounts of money into the system, to rescue AIG, a huge insurance company whose credit-default swaps underpinned the credit market and to essentially guarantee the bad debt. In addition, the two largest mortgage banks in the U.S were effectively nationalised. Several banks in Europe were rescued from bankruptcy and to all intents and purposes nationalised. The crisis had global consequences and an important impact on the real economy. Despite the concerted efforts of the major central banks and governments, it is far from clear how long the consequences will last.

All of this is a story of contagion, of interdependence, interaction, networks and trust. Yet these notions do not figure prominently in economic models. A first line of defence offered by economists to justify this, is that we are talking about financial markets here and that these are intrinsically different from the rest of the economy, even if the two interact. But is this really the case? Whether we are talking about models of financial markets or of the real economy our models are based on the same fundamental building blocks. The most important of these is the idea that individuals act in isolation and the only interaction between them is through the price system. All that we have to do, to deduce the behaviour of the economy at the aggregate, or macro, level is to add up the behaviour of the individuals who make it up. In effect, the behaviour of the aggregate can be assimilated to that of an individual.

Economists are not alone in this. Both politicians and commentators use explanations such as “the market was afraid of the oncoming recession” to justify a fall in prices, or that “the newly published growth forecast made the market more optimistic”, as if the market viewed the world with one mind. Yet, the idea of explaining the collective panics or collective “exuberance”, to use Alan Greenspan’s famous phrase, that we periodically observe, as reflecting the identical, or average behaviour of individuals who neither contact nor observe those around them, seems curious. The recent near-collapse of the world’s banking system does not seem to correspond to the collective result of individual banks optimising in isolation and unconsciously coordinating on a disastrous solution. What is involved is a great deal of local interaction, of transmission of information, views and expectations from one actor to another. Large systems with micro characteristics of this sort are studied in physics, biology and also sociology. There, it is recognised that the system may switch rapidly from one phase to another and that this will be dependent on its internal organisation and not on some exogenous
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The uncomfortable thing about the sort of models that I am referring to, is that there is no necessary proximate cause for a sudden shift in the aggregate state, no culprits to blame and no easy remedy to prevent similar occurrences in the future. This is not the case with economic models and macroeconomic models in particular. The relations between the variables are essentially fixed and the system functions at equilibrium just as in a mechanical system. The only thing that may perturb the evolution of the economy is an external shock from which the economy adjusts, by assumption, to a new equilibrium. Out of equilibrium dynamics are not a central issue in economics and Gerard Debreu said explicitly that their analysis was too difficult and that was why he had never ventured in that direction. So, the most interesting aspects of economics if the economy is viewed as a complex interactive and adaptive system, are absent in macroeconomic models based on the General Equilibrium view.

In other words the vision of the world reflected in modern macroeconomic models leaves out aspects of the economy which seem to be central to understanding how it functions and evolves. Indeed, the problem that intrigues many people when they first come to economics is that of explaining how the myriad of disparate individual economic activities come to be coordinated. A modern economy is composed of millions of agents who interact directly and indirectly with each other. Each of them knows a great deal about the activities they are engaged in and a lot about the people whom they interact on a regular basis. They interact intensively and directly with some individuals and less often and more indirectly with others. They have a great deal of very local information but, know much less about the behaviour of the whole economy, other than through some summary statistics. Yet, despite the fact that most of the individuals in the system are not aware of each other’s existence, collectively their activities are remarkably coordinated. How is it that all these individuals each of them with specific information and abilities organise themselves, for most of the time, in a consistent and relatively predictable way? What is more, most of the individuals involved have considerable confidence in the system to handle external perturbations though what I will argue in this book is that endogenous changes are much more important than exogenous shocks. Again, in general, whether the change is exogenous or endogenous the economic system seems to be capable of making local repairs and organises itself to do so, I have almost no knowledge of the specific way in which the coffee trade is organised in order to guarantee that coffee will find its way into my cup. Nevertheless, should coffee no longer be available I am convinced that somebody would set about making it available. This is not because that individual necessarily has special talents but rather that the organisational structure is there and he just has to insert himself into it. Thus, collectively the economy organises itself into a framework which is independent of any particular individual but which coordinates the activities of all the individuals.

But, what will be one of the major themes of this book, is that while the economic system, and in particular, the financial system may give the impression that it is functioning well, from time to time it may slide into a totally different state. The argument of this book will be that such movements are intrinsic and not due to some exogenous shock. In the introduction to a popular book, entitled “Emergence” the author starts

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1He made this observation in an interview with E Roy Weintraub, See Weintraub().
with a graphic description of the movement of slime finding the path of least resistance without any of its components having any intention to do so. The movement of the slime may be almost imperceptible for a time and then suddenly accelerate as the slime slides down a steeper slope. The analogy with the economy is far from perfect since the movement and changes in the system do come from the, at least locally, purposeful behaviour of the individuals who make up the system. However, none of them can really be held responsible, for radical changes in the aggregate economy.

Before proceeding, I should make one thing quite clear. I am not suggesting that what we observe in the real world is something that looks essentially like what one would expect if one believed in a fully fledged General Equilibrium model populated by rational optimisers. Indeed, even though the structure of the economy may be quite robust, what I will argue is that the system will evolve and may occasionally go through very large changes which would not be consistent with a static equilibrium view. Furthermore, I would not suggest, for example, that what we are observing are efficient situations in any sense. A great deal is being achieved in terms of coordination but there is almost always room for improvement in every direction, as Schumpeter firmly believed. Let me come back to the recent “credit crisis”, which illustrates this well. From a situation where many dimensions of the world economy seemed to have achieved some degree of stability, we were suddenly precipitated into what has been described as the “worst crisis since 1929”. Did this happen as the result of some major shock to the economy? Not at all. What had happened is that norms had developed and become established. In adopting these norms the individuals are probably unconscious of their aggregate consequences. In the case of the financial crisis, the rules of the game were gently modified in the banking sector. It became acceptable to lend to people who had little chance of being able to repay their loans, it became acceptable to use more and more leveraged positions, and it became standard practice to hive off dubious loans in the form of derivatives with the argument that the risk was being “diversified”. However, this slow and almost unconscious shift was at the root of the crisis.

Yet, as more and more risky loans were issued, in a world where house prices were booming nobody saw this as a real problem. The practices in question became standard procedure and this, at the very micro level, made them acceptable. Furthermore, through the dispersion of these risks through derivatives throughout the global banking sector there was seen to be no systemic threat. The individuals or banks making the decisions were not aware that their increasingly interdependent positions were generating a threat to the stability of the whole system. In fact the individual actors all felt themselves to be well protected having, in effect, insured their positions with others. There was no central mind to perceive this. The system was, indeed, as Hayek argued, organising itself but this self-organisation, contrary to a standard and largely ideological view was not stabilising. Indeed, as I have said, it needed only a small downturn in the property market for banks to start becoming concerned about who was holding the bad risks. As soon as this happened banks became wary of lending to each other and the interbank credit market dried up. To repeat, the system froze without any specific event causing it and without the actors in the system having foreseen the collective result of their individual actions.

The situation was saved, at least temporarily, by massive injections of liquidity but not without the effective failure of some banks, and with major consequences for
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the real economy. But, what is important is that these were purely pragmatic measures, often orthogonal to both what models would have suggested and to ideological convictions. The only explanation given by those taking the decisions is that “in exceptional circumstances we need exceptional measures”. But it is difficult to put much confidence in measures which are taken neither on theoretical grounds nor on any well-defined policy view. What it seems to suggest is that those taking the decisions have little understanding of how the system really functions and evolves and that the models which they or those who advise them, use, are simply too detached from reality.

How does this evolution compare with that of the sort of view of the economy that I wish to propose? Firstly, the economy and the financial sector had organised itself into a highly interdependent system. Paradoxically the extensive interlocking of the components and the heavy trading of derivatives actually concealed information rather than revealing it. Thus, the system self organised its own self-destruction leading to a radical change in the aggregate situation. But once again, this is a story of interaction and interdependence and the breakdown of relations of trust which had emerged and not one of an external shock to a stable market.

So, the question is not to find an alternative explanation as to how the economy arrives at an equilibrium in the classic sense, it is rather, what sort of framework has it developed to achieve all of the coordination that we do observe and how does that self organisation sometimes lead to major phase changes? There are therefore two levels on which to argue. On the one hand we would like to explain how all the agents in the economy come to coordinate their daily activities in a relatively stable fashion. On the other hand we have to explain how a system functioning in this way, once again with no central authority to control it can suddenly evolve into a crisis?

1.1.1 Markets and coordination

A first answer would be that a large part of the explanation of coordination of the first type, is that markets do the work. Markets, their organisation and the relationships that develop within them have long held a fascination for historians, anthropologists and sociologists. Economists have, despite the work of North and others, tended to take some specific model of market organisation as given and then to examine the aggregate behaviour of the market. The intricacies of particular forms of market organisation are not thought of as being relevant to the aggregate outcome. Frequently the individuals are thought of as acting independently of each other and linked only through the price system. In this case, those interested in aggregate outcomes make a short cut. They argue, at least implicitly, that we can add up the activities of individuals and that the result will preserve the characteristics of the individuals. Thus, in macroeconomics, the economy or the market is generally assumed to behave like an individual.

In fact it is clear that the situation is much more complicated than this. The organisation through which activities are coordinated is complicated and, for example, it is reflected in the networks that link people together. People participate in many different networks and this has an effect on their economic behaviour. The choices of an individual will be governed by the place which he occupies at work, by the way in which his friends and neighbours behave, by his religion and the associated network and so forth. Firms are themselves networks as are many other economic entities. The
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network structure which develops over time in a society, or an economy, plays a crucial role in determining individual and hence, collective economic outcomes. Moreover, like a biological organism the closer one looks at the structure the more complicated its organisation appears. Yet the natural question is as I have suggested, how is it that from the highly complex networks of interaction between agents, reasonably coherent aggregate behaviour emerges? An alternative to the standard approach would be to suggest something more radical and to argue that it is the type of organisation rather than the individual behaviour that is central to achieving coordination. Once in place, the organisational structure, itself, coordinates individual activities and makes them consistent. Over time this structure emerges as a result of the interaction between individuals and gradually plays a more and more important role, as rules, checks and balances are incorporated into the system.

This view of the economy is much closer to that of social insects than to the traditional view of how economies function. Economists do not like the idea of comparing the economy to an ants’ nest or to a bee-hive. Their principle preoccupation is with knowing whether the state in which an economy finds itself is efficient or not. They are no doubt convinced that the way in which social insects organise themselves is, in some sense, efficient, but they are not convinced that the efficiency is achieved in the same way as it is in an economy or market. Thus, if you ask most economists what is the basic question that concerns them, they will probably answer that it is to understand what the equilibrium of the economy or market that interests them is like, and whether it entails an efficient use of resources. If one frames the problem in this way, the essential idea is to define an idea of what consists an equilibrium and what its properties are. The term “equilibrium” is slightly misleading because there is at least an implicit suggestion that it is something like a “rest point”. In other words one could think of the complex system I have described as settling to such a state. The modern definition of equilibrium in economics is much less than this, it is simply a description of an allocation of resources to the individual consumers and firms, from which nobody, given the constraints imposed by the system, would have any interest in deviating. So, the idea is a purely static one. Economists study the properties of such states and the most important results in welfare economics turn around the idea that equilibria are, in a certain sense, efficient.

Yet there is a prior problem that is posed by the behaviour of social insects and by economic agents, and it is that which I raised at the outset. It is that of explaining how the individual economic activities, in a modern economy, come to be coordinated at all. Economic agents constantly interact with each other in different ways and for different purposes and somehow out of these individual interactions a certain coherence at the aggregate level develops. There is no special reason to believe that the economy will, of its own accord, settle to an equilibrium, but there is no denying that there is a great deal of coordination. Indeed, it is probably the case that system never settles to anything like a steady state, but moves restlessly forward, rearranging itself, but with discernible patterns. This seems a much more persuasive view of what the economy looks like rather than one which is essentially in an equilibrium and occasionally gets knocked off course by some exogenous shock. Disappointingly economics has rather little to say about this sort of idea. Yet there are echoes of this in both Schumpeter and Hayek. Schumpeter’s evocative term, “creative destruction” speaks for itself. Hayek’s vision
of information as being dispersed throughout the economy, with each individual having his own limited amount, and with no place or mind in which all this was gathered, but with the system organising itself as individuals interact is also in the spirit of the sort of model I have in mind.

My main argument throughout this book will be that it is the interaction between individuals that is at the heart of the explanation of many macroeconomic phenomena and that we will never move towards an understanding of such phenomena without considering this interaction as a central feature of the economy and not as just something which impedes the functioning of what would otherwise be a perfect system. At almost any point in the book, a serious economist could reasonably object, by arguing that economic theory has already taken such difficulties into account. Yet, what I wish to argue is that we do so by modifying and completing the original model and that what we should be doing is much more fundamental, we should be building models in which interaction and interdependence are the central motor of the economy. The simple fact that economic agents do, in fact, communicate with each other and learn from each other that they also infer information from the actions of others and, most importantly, in most markets that they trade with each other, and not anonymously, leads to phenomena which cannot be explained by treating individuals as isolated maximisers.

There is, of course, an approach to economics, the game theoretic one, which takes specific account of the direct interaction between agents. However this is at the opposite extreme. In that framework, every player takes account of what every other player does and moreover knows that the others do so. For example, one can think of a market as a game and model a situation in which every agent is consciously and strategically interacting with every other agent. Furthermore every individual believes that the others are capable of reasoning as well and fully as himself. This involves the famous common knowledge problem, “he knows that I know that he knows that I know”. This leads to basic logical difficulties, since an infinite regress is involved. Though the academic debate on this subject took place in the late sixties it was already anticipated by Charles Schulz in 1965 where he illustrates the difficulties involved in this sort of situation, (see figure 1.1).

In addition this approach produces two further difficulties. Firstly, it is difficult to characterise the equilibrium behaviour, what should the two do with the football? Secondly, such a vision does not allow that whom one interacts with may be a matter of choice. Thus, far from interacting with all the other actors, economic agents choose those with whom they trade, and to treat a market, for example, as an anonymous game, ignores the specific, intricate trading and relational networks that develop and influence market outcomes. Agents may well interact strategically but there are few markets where they do so with all the other agents. The sort of intricate reasoning involved is perhaps better suited to specific situations in which individual interact strategically for some particular allocation problem. Rather than attribute this sort of reasoning capacity to people in general, which might be appropriate in some very limited circumstances, it seems more realistic to model people in a rather more modest way. They are not purposeful, in the sense that they fully optimise, and take account of the functioning of the economy but they follow rules which they use to make the best choice within the

\[\text{For an interesting discussion of this see Binmore (1990)}\]
Figure 1.1: PEANUTS
1.2. RATIONALITY

But, in a sense, this seems to be what the competitive economic model does. People only react to anonymous market signals and take no account of the behaviour of others nor of how that behaviour impinges on them. Individuals are assumed to conform to our hypotheses of rationality but make their rational decisions in isolation. This is certainly a simple, if not simplistic view of how people behave. But, much worse, there is the downside, nothing is said about how the coordination is achieved and, in particular, who decides upon and sends the market signals. It is worth looking fairly carefully at the components of the standard competitive model to see what it is that does not correspond to the picture of an economy that I want to convey. There are several ingredients to be examined. In particular, there are, the rationality of the individuals, their isolation and their coordination via the market mechanism.

1.2 Rationality

Let me start with the first: rationality. The obvious question for a newcomer to economics, but one which he rapidly forgets, is, do the axioms concerning the rationality of individuals correspond to some common sense notion of rationality? Recall that rationality is defined as a certain number of restrictions on peoples’ preferences. By the time we got to the mid 1950s there was little doubt as to the “correct” approach to this. On what preferences are, Hirofumi Uzawa (1971), says simply,

“Preference relations, in terms of which the rationality of human behaviour is postulated, are precisely defined as irreflexive, transitive, monotone, convex and continuous relations over the set of all conceivable commodity bundles. A demand function associates with prices and incomes those commodity bundles that the consumer chooses subject to budgetary restraints”.

Yet if we examine these hypotheses one by one we might well question their plausibility. An assumption such as that of continuity of preferences corresponds to no natural notion of rationality, unless one has a very broad interpretation of what it means to be rational. We can make it sound plausible by saying something like the following: if bundle of goods x is strictly preferred to bundle y of goods, then any bundle sufficiently close to x will be strictly preferred to any bundle sufficiently close to y. Changing the amounts of the various goods in the two bundles by very small amounts indeed should not change one’s preferences over them. Yet, this all turns on another standard assumption, that goods are infinitely divisible, which, as we well know, they are not.

Why do we do all this? Because, with this assumption we will be able to show that individual demand is a continuous function of prices. When we add up all our continuous individual demands we will obtain continuous aggregate demand and we will be able to prove the existence of an equilibrium. Yet all of this seems somehow backwards. Long ago, Cournot remarked that even if goods were indivisible and individual demands were discontinuous it might well be, that at the aggregate level, things
would smooth out and we would observe essentially continuous aggregate demand. Furthermore, he argued that as the result of the aggregation process, we might, for example, well observe what seems to be a continuous monotonically declining market demand curve, at the aggregate level, while this was far from true at the individual level. In other words, buyers of fish together wish to purchase less fish as the prices of the latter increase, and this in quite a smooth way. However, this is not necessarily because all buyers are reacting smoothly and even in the right direction to the changes in prices. Thus, as we move up to the aggregate level, the indivisible nature of goods becomes unimportant and the jumps in individual demands as prices change become insignificant. The situation becomes even more complicated at the individual level when individuals interact and trade with other individuals, but, as we will see, in actual markets this may not interfere with aggregate behaviour.

The other assumptions as to rationality are just as hard to justify. Transitivity, which seems eminently reasonable as a postulate, cannot be tested in reality, since individuals are never faced with exactly the same alternatives twice. Of course, one can ask individuals what they would prefer among certain alternatives and, indeed, if one does so, it is easy to lead people into intransitivity. Interestingly, when one does this, a typical reaction is to apologise and to wish to change some of the stated preferences. Thus, individuals somehow believe that they should have transitive preferences even if their choices show otherwise.

Other aspects of our assumptions have been questioned and scrutinised. What is the meaning of having preferences over future bundles of goods. How do I know what my preferences will be when I arrive at a future point in time? In particular, if my experiences influence my tastes how can I know what I will turn out to prefer. There was an advertisement for Guinness which said, “I don’t like Guinness. That’s why I have never tried it”. This seems absurd to most people but is perfectly consistent with an economist’s view of preferences. Since my preferences are well defined I do, in fact, know whether I like Guinness or not. Therefore there is no reason for me to try it, if I happen not to like it.

Even if we accept a lot of the underlying assumed structure of preferences we can run into trouble. Think of someone who has at each period a utility function defined over current bundles of goods and when viewing the future, he either knows what his utility function in future periods will be or believes that it will be the same as it is now. He then discounts future utility. If he discounts every period at the same rate then he will, at least, be consistent in time. When he arrives at a later period he will not find any conflict between his preferences for goods, seen from that point in time, and the choices that he had made earlier for that time. However if he attaches somewhat greater weight to immediate consumption, paradoxes and inconsistencies can arise. The widely discussed hyperbolic discounting problem is an example of this (see Frederick et al. ())). Again, this sort of consideration is far from new and it is only the rather recent idea that the static axiomatic approach is the “scientific” one that has made us lose sight of the problems that it entails. Just to illustrate the temporal choice problem it is worth citing David Hume’s (1739-40/1978) discussion. He says:

“In reflecting on my action, which am to perform a twelve-month hence, I always resolve to prefer the greater good, whether at that time it will
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be more contiguous or remote; nor does any difference in that particular make a difference in my present intentions or resolutions. . . But on my nearer approach, those circumstances, which I at first over-look’d. begin to appear, and have an influence on my conduct and affections. A new inclination to the present good springs up, and makes it difficult for me to adhere inflexibly to my first purpose and resolution”

David Hume, A Treatise of Human Nature, p. 536

Hume is doing no more than underlining an elementary fact, that circumstances and relations change, as do people themselves, therefore to imagine that people are unchanging or that they are perfectly capable of anticipating the changes that will occur is simply unrealistic, and indeed, I would argue, unnecessary.

Many of the difficulties that arise when we look carefully at the underlying assumptions that we make on individual preferences have been highlighted by the introduction of considerations from psychology into our analysis and this has been reinforced by the evidence from experimental economics. These two strands have led to the development of behavioural economics in which many of the standard assumptions on economic behaviour are questioned. Here we are in an odd situation, many economists reject, out of hand, the sort of model I am proposing because it is not based on “sound micro-foundations”. By this is meant that if the aggregate behaviour is not derived from that of people who, at the individual level, obey the standard axioms of rationality, then such a model is not scientific. Yet there is a steadily accumulating body of evidence that people, even in carefully set up experimental conditions, do not behave as they are supposed to do in theory. Heaven alone knows what they do when they are let loose in the real world with all its distractions. So while the lack of full rationality is used as the reason to reject models which assume less than that, the empirical evidence is that people do not, in fact, obey the axioms in question.

This is not the place to review all the criticisms of the standard analysis of economic behaviour, and I could go further, as some have done, and ask if the notion of preferences as a well defined order over present and future consumption makes any sense at all. Yet, without this basic structure, we are in difficulty since our science is built on welfare foundations and if we let this notion of rationality go, it is difficult to make statements about what is “welfare improving” or what is Pareto efficient. Without the welfare underpinnings the basic theorems of Welfare Economics lose their sense, and, in turn, the basic justification for the market mechanism, that it leads to efficient outcomes, disappears.

The important point here is not that individual welfare is irrelevant or inexistent, but rather that the formal structure that we have imposed on individual preferences is too restrictive. Who would quarrel with the idea that individuals know when they feel better off at some point in time, making one choice rather than another? Yet the idea that their choice, when viewed over time, satisfy the sort of intemporal consistency that we would like to impose seems highly implausible. People change, their experience matters, and the structure that, we, economists impose on the choices that we believe people make, do not take this into account. John Stuart Mill, in another context, that man is heavily influenced by what he has lived through and he, therefore, would not
have accepted the idea that economic agents could be characterised by their fixed and immutable preferences. He said,

“...the vulgar error of imputing every difference which he finds among human beings to an original difference of nature. As well it might be said that, that of two trees, sprung from the same stock, one cannot be taller than the other but from greater vigour in its original seedling. Is nothing to be attributed to soil, nothing to climate, nothing to difference of exposure - has no storm swept over one and not the other, no lightning scathed it, no beast browsed on it, no insects preyed on it no passing stranger stript off its leaves or its bark?”

John Stuart Mill, The Negro Question, 1850

This said, it seems reasonable to assume that people are inclined to move towards preferable situations in some more limited sense and not to perversely choose outcomes which make them feel worse off. But, one can think of many ways in which this could be expressed and one does not need to impose the formal structure on preferences that we have become used to. People may use simple rules of thumb and may learn what it is that makes them feel better off, they may have thresholds which when attained, push them to react.

This will play a role in what follows, since I will argue that if we allow for interaction and the emergence of economic organisation we need to impose much less structure on individual behaviour. Thus, rather simple individuals can, collectively, achieve quite sophisticated outcomes without any of them having a full knowledge of what is happening and indeed without respecting the canons of rationality, in the standard sense. Individual ants have a very limited understanding of the organisation in which they exist and are unaware of the presence of many of their fellow inhabitants but collectively they provide for the needs of the colony and its reproduction.

At this point, some well brought up economist might interject that if it is the organisation of the economy that interests us, this description of an ant-hill, is not far from a description of the competitive economy, a la Arrow-Debreu. Individuals are isolated, they make their optimising decisions in isolation and all that they respond to are the market signals. What is more and, this is insufficiently recognised, the system requires very little information to function at equilibrium. Yet, there is a major difference, since in the Arrow-Debreu world all the individuals are reacting to some common signals which have no counterpart in the ant world, nor in some parts of the economic world. Indeed, as I have said, Hayek argued long ago that the information in the economy remains dispersed and is never brought together into signals available to everyone.

“The problem of a rational economic order is determined precisely by the fact that the knowledge of the circumstances of which we make use never exists in concentrated or integrated form, but solely as the dispersed bits of incomplete and frequently contradictory knowledge which all the separate individuals possess. The problem is thus in no way solved if one can show that all of the facts, if they were known in a single mind (as
1.3. EQUILIBRIUM

we hypothetically assume them to be given to the observing economist), would uniquely determine the solution; instead we must show how a solution is produced by the interactions of people each of whom possesses only partial knowledge.”

Friedrich von Hayek (1945)

Here we are at the heart of the problem, for the economist is interested in the equilibria of the system, and is not concerned with how the interactions between the individuals determine the state of the economy and, in particular, whether they would produce an equilibrium.

1.3 Equilibrium

Yet, I cannot argue that we need a different vision of an economy without giving an account of the basic elements of the existing model. Having observed that the underlying rationality assumptions seem, at best suspect as a foundation for real behaviour, let me turn to the problem of equilibrium. Suppose, however, that we make a leap of faith, and imagine that individuals do satisfy the rationality axioms and furthermore that the organisation and transmission of information is somehow achieved. Indeed, suppose, as in the standard model, that there is a single price for each good and that it is known to everyone. Individuals simply need to know these prices and this, coupled with their income, generates the constraints that, together with their preferences, yield their demands and, of course, their excess demands for goods. The standard argument is now simple. What is needed is a vector of prices that will make these excess demands consistent, in the sense that in aggregate there is zero excess demand for all commodities. Thus all that the market mechanism has to do is to transmit the equilibrium price vector corresponding to the aggregate excess demands submitted by the individual economic agents. The information required to make this system function at equilibrium is extremely limited. In fact, a well known result of Jordan (1989) shows that the market mechanism not only is parsimonious in terms of the information that it uses, but, moreover, it is the only mechanism to use so little information to achieve an efficient outcome in the sense of Pareto.

Should this remarkable observation not be enough to make us accept the standard General Equilibrium model and to leave it at that? Although we may quarrel with the assumptions about individual behaviour, contrary to what is often said, we do not impose any knowledge of how the economy works on the individuals. They just have to choose their demands, or supplies, if we include production, and then transmit them to the central price making mechanism. True, for all of this to work out, the preferences of the individuals have to satisfy our exacting requirements, otherwise we could not be sure that there is a system of prices that would clear all the markets. Nevertheless, one might argue, that, despite the problems that I have already raised, preferences are unobservable, and therefore, it should not be too difficult to accept the standard assumptions.
Two objections can be raised to this. Firstly, how legitimate is it to accept a set of unverifiable assumptions, based on the introspection of economists, just because they yield satisfying properties of the equilibrium of an economy? Secondly, and much more important, nothing is said about how the economy actually reaches the equilibrium. If ever it were there, then we might take some comfort from Jordan’s result. Yet, it will not have escaped the reader’s attention that I have not said what the central price setting mechanism consists of, nor how it chooses the equilibrium prices. Many tales can be told to justify the process of achieving competitive prices but these often assume, if implicitly, things that are not in the model, individuals who set and change prices when faced with excess demand or excess supply for example. These stories lead us away from the austere structure of the basic model and, as we will see, would require much more information than is specified by the Jordan result. Indeed, we can be quite precise in showing why the parsimony of the information used by the market mechanism is illusory. To see this it suffices to recall again, that so little information is needed for the economy to function at equilibrium.

However, if one is to claim any realism for the market mechanism one has to know not only how informationally demanding the mechanism is at equilibrium but also how much information it requires to get there and whether it will get there. This is the problem of stability.

1.4 Stability

As Michio Morishima (1964,) pointed out,

“If economists successfully devise a correct general equilibrium model, even if it can be proved to possess an equilibrium solution, should it lack the institutional backing to realise an equilibrium solution, then the equilibrium solution will amount to no more than a utopian state of affairs which bear no relation whatsoever to the real economy”

Actually, I would go further and suggest that the very equilibrium notion that we use is not relevant as a description of the real economy. I want to argue for a system in which the individuals are continuously changing and adapting their choices as a function of changing circumstances and their contacts with others and that the system is also, in consequence continually changing. The once and for all decisions that would generate an equilibrium from which the economy would not move do not correspond to any reality that we might observe.

If we are to evaluate the usefulness of the notion of market equilibria, we have then to look at the problem of stability. Such equilibria are surely only of interest, as Morishima said, if they can be attained through a reasonable adjustment process. In other words, should markets find themselves out of equilibrium, it should be possible to explain how prices would adjust so that equilibrium would be restored. A process, which is due to Walras, and which has a very plausible economic interpretation is the so-called “tatonnement” process. This simply specifies that the prices of those goods

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3This is the subject of a longstanding debate in economics. John Stuart Mill was a firm believer in the value of introspection while Jevons did not find it legitimate for example.
for which demand exceeds supply, that is, where there is excess demand, should rise and those for which there is excess supply should fall. Even those with no formal training in economics would probably find such an idea intuitive. Most people would find it plausible that if there is too much of a good on the market its price will fall. How exactly this happens is, of course, left unsaid. Still this “tatonnement” type of adjustment seems reasonable.

Yet, as we know from the well-known results of Sonnenschein (1972), Mantel (1974) and Debreu (1974), even with the typical strict restrictions on preferences, the equilibria of economies are not necessarily stable under this adjustment process\(^4\). This is unfortunate since the tatonnement process requires little more information than the Walrasian mechanism at equilibrium. Yet the lack of stability is of great importance. If equilibria are not stable under some plausible adjustment process, then their intrinsic interest becomes very limited indeed. If we cannot be sure that an economy will arrive at an equilibrium why should we devote so much time to studying these states? As Hildenbrand (1994) says,

“When I read in the seventies publications of Sonnenschein, Mantel and Debreu on the structure of the excess demand function of an exchange economy, I was deeply consternated (sic). Up to that time I had the naive illusion that the microeconomic foundation of the general equilibrium model, which I admired so much, does not only allow us to prove that the model and the concept of equilibrium are logically consistent, but also allows us to show that the equilibrium is well determined. This illusion, or should I say rather this hope, was destroyed, once and for all, at least for the traditional model of exchange economies. I was tempted to repress this insight and continue to find satisfaction in proving existence of equilibrium for more general models under still weaker assumptions. However, I did not succeed in repressing the newly gained insight because I believe that a theory of economic equilibrium is incomplete if the equilibrium is not well determined.”


One suggestion would be that the problem lies with the adjustment process rather than with the standard or General Equilibrium model. If a more general adjustment rule were to be specified, perhaps the equilibria of the economy could be shown to be stable. Yet, what became immediately clear after the innovative work of Smale (1976) was that stability could only be achieved at the price of an enormous increase in the amount of information.

Smale’s Global Newton Method is an extension of the standard tatonnement process for finding the equilibria of an aggregate excess demand function. However, as already mentioned, it uses a great deal of information. Without entering into technical details, let me just mention that what is needed is a knowledge of all the partial derivatives of the aggregate excess demand function and this greatly increases the size of

\(^4\)In fact Herb Scarf, (Scarf (1959)) had already given an example of an economy with a unique but unstable equilibrium but many had nurtured the hope that some appropriate restrictions on preferences might eliminate the problem.
the messages necessary to make the mechanism function. Although the process leads to equilibria from a large set of starting prices it still does not guarantee convergence from any arbitrary starting point. An additional problem is with the economic content of the process. While the original tatonnement process has a very natural interpretation, despite valiant efforts by some economists, this is not the case for the Newton Method.

What is worse, is that the problem of the informational content of the Newton Method cannot be avoided, if one hopes to find a process which will converge globally, that is from any prices to an equilibrium. This is what Saari and Simon (1978) showed. Furthermore, all the alternative adjustment processes that have been constructed to date have no natural economic interpretation. There have been many efforts to construct globally and universally stable price adjustment processes since and in a certain sense Kamiya (1990), Flaschel (1991) and Herings (1996) succeeded. Yet if one looks closely at these results there is always some feature which is open to objection.

All of this seems, to me at least, to suggest that there is no hope of finding an economically interpretable adjustment process which will converge to an equilibrium in the standard sense from any price vector independent of the economy. In fact Saari and Simon’s result might be thought of as saying that such a process would require an infinite amount of information.

The unfortunate conclusion of all this is that the informational requirements of an adjustment process that would work for all economies seem simply to be too extreme. Should one then be satisfied with adjustment processes that are specific to the particular economy in question? This is hardly reassuring for those who argue for the plausibility of the equilibrium notion. This would be a far cry from the standard argument that the competitive market mechanism has universally desirable properties. Alternatively one could argue that, in real markets, it is something like the tatonnement process that is at work and that economies are simply not, in general, stable in this sense. However, if one accepts this position then one has to focus on the disequilibrium dynamics of economies. In this case much more has to be analyzed if one is to understand the transmission of information as the economy evolves over time.

To understand how the economy reaches a particular state, I would suggest, requires an understanding of the organisation of the economy. There is considerable discussion of this in experimental economics. While recognising that individuals do not conform to the standard assumptions on rational behaviour subjects seem often to coordinate and to settle down to a particular collective state. What is more, many experimental economists would argue that the final result of all this interaction is often very close to that that would have been predicted by standard economics.

Yet standard economic theory paints a very different picture of the functioning of an economy. Organisation is assured by the market which coordinates individual activities by transmitting signals to those individuals, informing them of the constraints with which they are faced. Each individual has preferences according to which he makes the best choice possible given the constraints he faces. If the result is not consistent the market will adjust its signals and thereby change the constraints of the individuals until their choices are consistent. The organisation is given and nothing is said about where it comes from. The question that economists pose is the following. Which “states of the economy” are efficient and how are these related to the “equilibria” of the market system? Again, an equilibrium means here that individuals make the choices that they
1.5. A SIMPLER WORLD

want, given the market signals and that these choices are mutually consistent. The standard result and one which lies at the heart of all recommendations in favour of the market system, is that market equilibria are efficient. Thus, the basic paradigm in economic theory is one in which individuals take decisions in isolation, using only the information received through some general market signals, such as prices, to make their decisions. The standard model does not deny that agents interact but, as Samuelson said, they only do so through the price system. Indeed, a way of characterising the efficient markets hypothesis, so pervasive in the financial markets literature, is to say that all information private and public is aggregated in the price system. Thus no agent has any interest in searching for information other than from prices. Direct interaction is not an integral part of market behaviour according to this view.

In the standard model, all agents have the same information. The information available is global and not local. Although we know from Jordan’s (1979) well known result that astonishingly little information is needed to achieve efficiency, the description of the way in which the mechanism functions is hardly realistic. Agents are isolated from each other and if there are asymmetries in their information there is frequently no discussion as to their origins.

1.5 A simpler world

Consider another vision of the world in which individuals function in a limited neighbourhood and most of their information comes from those with whom they interact. Furthermore, their reasoning capacities are limited and they adapt rather than optimise. Is it not possible that in such a world the collective outcome has certain desirable properties? What I have just described corresponds very well to the situation in an ants’ nest or a bees’ hive. This is very different from a world in which individuals are intelligent and calculating and are not limited in their understanding of the economy.

Such a vision of the economy is anathema to those who are convinced that humans, unlike ants, anticipate, reason and decide consciously as to what they want to do. Although this is true to a certain extent, it is also true that the choices made by any economic entity are heavily constrained by the place they occupy in the economic structure. However, if we accept this view we are immediately faced with a dilemma. An individual’s behaviour and contribution to economic activity depends on the role he fills and not just on some intrinsic characteristics. This means that it is of no use looking at some “representative individual” in order to understand what will happen at the aggregate level. You would not imagine looking at the behaviour of a representative ant if you wanted to predict the evolution of the activity of the nest. In this view aggregate activity is not a blown-up version of individual behaviour. The passage from micro to macro is more complex than a simple adding up of independent individuals. Macro-economic behaviour surely reflects the underlying micro-economic behaviour, but as two leading neurologists point out, the passage from the individual to the aggregate level is not trivial. As they say

“Major advances in science often consist in discovering how macroscale phenomena reduce to their microscale constituents. These latter are often
counterintuitive conceptually, invisible observationally, and troublesome experimentally. Knowledge of the molecular and cellular levels is essential, but on its own it is not enough, rich and thorough though it be. Complex effects, such as representing visual motion, are the outcome of the dynamics of neural networks. This means that while network properties are dependent on the properties of the neurons in the network, they are nevertheless not identical to cellular properties, nor to simple combinations of cellular properties. Interaction of neurons in networks is required for complex effects, but it is dynamical, not a simple wind-up doll affair.”

Churchland and Sejnowski (1995)

Similarly in economics, if we are interested in macro-economic relations concerning the reaction to changes in various aggregate variables we should not start at the level of the isolated rational individual.

Once again this will not be welcome to economists who wish to found macro-economics on “sound micro-foundations”. What do we mean by this? We mean that each individual solves a rather complicated optimisation problem faced with well defined constraints and that the result translates directly into the aggregate. Of course, we do not mean “solve” in a conscious calculating way. Each individual is endowed with preferences which satisfy certain standard widely accepted properties corresponding to rationality. He then simply chooses the best alternative, according to those preferences, among the set of alternatives available to him. Over a century we have become very good at characterising the solution to this sort of maximisation problem. The “marginal revolution” could be cynically described as the introduction of the constrained maximisation of concave functions on convex sets into economics. Since we know how to write down the first order conditions for this sort of problem it was easy to give a rather natural interpretation to them. “An individual will contribute labour until the marginal utility he gains is just equal to the marginal disutility of his effort”, might be such a statement. Such reasoning is not confined to economics. I heard an entomologist explaining the behaviour of bees by saying that the bee will continue extracting pollen from a flower until the marginal effort necessary to get an extra unit of pollen is just equal to the effort required to fly to another flower and obtain a new unit of pollen there. Similar explanations are widespread in the “optimal foraging” literature, (see Stephens and Krebs (1986) for example).

The early efforts of economists to rationalise behaviour culminated in the laying down of formal axioms governing what constitutes rationality. But where did these axioms come from? Were they the result of intensive examination of human behaviour? Surely not. Yet the idea that rationality should be defined by axioms placed on preference orderings became accepted as the “scientific” approach to describing economic behaviour. Moreover, a great deal of effort and mathematical skill has been spent on weakening the conditions imposed on preference relations. It was argued that weakening the assumptions in this way would make them more acceptable. Little was said about the basic underlying assumption that individuals are indeed characterized by such preference orders. Taking this as given, the goal was to weaken the basic axioms as much as possible. Today, it is recognized that none of the standard axioms, even in
their weakened form, are derived from observation of choice behavior but rather are the result of pure introspection by economists. Yet this recognition has nothing to do with recent developments. Werner Hildenbrand (1994) cites quotations from economists such as Lionel Robbins (1935), Tjalling Koopmans (1957), and John Hicks (1956), all of whom were well aware of this. Worse, later in his life, Vilfredo Pareto concluded that individuals make quite arbitrary choices and spend the rest of their time justifying them! He actually said:

“Whoever wants to make a scientific study of the social facts has to take account of reality and not of abstract principles and the like... In general, men act in an nonlogical way, but they make believe that they are acting logically”

Thus, skepticism about the bases of the utility theory on which the theory of demand is based is long-standing among highly reputable economists. It is thus well understood that these axioms are the result of introspection by economists and worse, some of us suspect, are there for mathematical convenience rather than as a valid description of what constitutes rationality. If this is so why should we be so attached to our model of rationality and why should we wish to base economic analysis on it? Part of the explanation is inertia. We have become familiar with the use of this model and know how to manipulate it correctly. Another part is due to a lack of alternatives. What constitutes good theory if it is not to be based directly on the optimising individual? Yet people like myself argue that by adhering rigidly to this approach we have restricted our capacity to explain aggregate phenomena.

Not only are the foundations of rational behaviour, as it is described in economic theory, weak, but we cannot pass from the individual level to the aggregate level without contemplating the impact of the interaction between individuals. The theme of this book will be that the very fact that individuals interact with each other that causes aggregate behaviour to be different from that of individuals.

One reaction to this is to say that, if it is aggregate behaviour that interests us, we should only be concerned with looking at the relationship between aggregate variables and not be concerned with the, possibly complicated, interrelated micro-behaviour that generates those relationships. Whilst any economist knows that individual economic agents constantly interact with each other in different ways and for different purposes, it could be argued that to analyse this is unnecessary for the explanation of macroeconomic phenomena.

For example, it is clear, as I have observed, in the neurosciences, that it is neither necessary nor desirable, to revert to the study of the behaviour of neurons to explain thought processes. Yet the practice of analysing macro-relationships without considering their micro-foundations is now, in economics, almost universally considered as unscientific. Indeed, and here one has to be careful about what means by explain, economists are not alone in claiming that macro phenomena should be explained by an analysis of the underlying micro-behaviour of a system and this is clearly shown by

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5I have discussed this in more detail in Kirman (200?) where I argue that we have followed a long and difficult path in developing demand theory, for example. But unfortunately it is not at all obvious that we took the right road and we have climbed a peak which does not give us a very helpful view of the landscape.
the quote at the beginning of this book. But the objection to the current use of microfoundations is of a different order. It is not the idea that one may be interested in what happens at the micro-level that is in doubt, it is the way the micro level is linked to the macro level that is dubious. The line of reasoning is simple. If we start with well behaved individuals we will obtain well behaved aggregates. Well behaved individuals have nicely structured behaviour derived from their optimising behaviour. Simply adding them up will preserve those properties at the aggregate level.

However, this whole approach has been undermined by theory. We have known for a long while that some of the structure of individual behaviour is lost when they are aggregated. To take a simple example, individuals, under standard assumptions satisfy the Weak Axiom of Revealed Preference, (WARP). This simply says that if an individual, in some circumstances, chooses alternative A when alternative B was available to him, it is never the case that he chooses B when A is available. This property is fundamental to consistent demand behaviour but we know that even if each of two individuals satisfies WARP their average choices may not. Thus even this basic property does not hold in the aggregate. It is not reasonable to attribute the characteristics of individual behaviour to aggregates even if the individuals in question are well behaved. Proceeding from consistent individual behaviour to consistent aggregate behaviour is not legitimate. We may observe well-behaved individuals and badly behaved aggregates. This is one direction, but the reverse is more intriguing. Can we observe good aggregate behaviour which is generated by less than well behaved individuals? The answer is yes and, as I have remarked, experimental economists have clearly found a lot of evidence pointing in this direction. How then should we explain this? My answer and the theme of this book is that it is the way in which individuals interact and the way in which that interaction is organised that co-ordinates activities. This is not a real answer for the theorist for we now have to develop theories as to what we may expect as behaviour from a system of interacting agents. What precisely this theory should consist of is not clear.

One avenue is that suggested by Werner Hildenbrand (1994). As you may imagine, many economists find his position too radical. He argues that we should simply abandon preferences, and the individual demands derived from them, as our primitives. It is worth looking at how he proceeds. He suggest identifying certain, empirically observable, characteristics of collective behaviour, in his case the distribution of choices, from which we can rigorously infer some regularity of aggregate behaviour. What he shows is that if we look at the distribution of individuals’ consumption choices and if that distribution “spreads out” enough as income increases, then the economy will satisfy, at the aggregate level, the “generalised law of demand”. The latter simply says that the vector of the aggregate consumptions of each good moves in the opposite direction to the vector of prices. This is a generalisation of the idea of downward sloping curves for each commodity. We have no micro theory to explain why consumption choices should spread out in this way and any such theory would have to take into account the interaction between agents. Nonetheless, Hildenbrand gives us an empirically testable fact from which he infers formally the good behaviour of aggregate demand. This reverses the normal line of reasoning. He starts from the position: “if this empirical relation holds I can prove that the following “law” must hold”. This puts the burden of proof back where it should be. We simply have to check the validity of the empirical relation.
1.6 HETEROGENEITY

The rational optimising individual is not necessary for his argument.

1.6 Heterogeneity

A crucial feature of Hildenbrand’s argument is that individuals are heterogeneous and, indeed, it is this heterogeneity that gives structure to the demand behaviour at the aggregate market level. A well trained economist will observe that in the General Equilibrium Model for example we allow for the heterogeneity of individuals so there is nothing new in this. However, in that context, heterogeneity is given and is simply captured in the different preferences of different individuals. Yet, we should go much further, since we know that in a “self-organising” situation, individuals may adopt different tasks and that this may be advantageous from the point of view of society. Thus heterogeneity may actually be an emergent feature of an economy.

The first clear description of this was given by Adam Smith with his pin factory and his explanation of the simple idea that, by giving to each individual a simple and specialised task, the productivity of the whole could be improved. Indeed, if animals or humans specialise on specific tasks then they require much less cognitive capacity and thus overall productivity is increased since the tasks can be done faster. This “division of labour” is a remarkable and fundamental phenomenon which can be found in a wide number of circumstances particularly in social insects, where different tasks are taken on by different members of the hive or nest. The nature of increasing returns or “non-convexities of the production set”, is one that has been widely discussed in economics ever since Adam Smith

Whilst the precise modelling of this phenomenon has been very limited, its nature is intuitive. Specialisation means that one can no longer describe overall activity as the result of one typical agent but the phenomenon is so prevalent that we can surely not afford to ignore it. A fascinating example is given by honeybees who specialise in their tasks. Not only do we find heterogeneity of roles but these roles change. The kind of work performed by the worker depends largely upon her age. The first three weeks of her adult life, during which she is referred to as a house bee, are devoted to activities within the hive, while the remainder are devoted to field work, so that she is called a field bee, (see Adjare (1990)). Thus there is important heterogeneity even within individuals; The tasks for house bees are varied but temperature control is one of the important duties. When the temperature is low, bees cluster to generate heat for themselves, but when it is high, some of them fan their wings to circulate air throughout the hive. The general hive temperature required is between 33 ° and 36 °C, while the brood chamber requires a constant heat of 35 °C. Honey has to be cured in order to ripen, and this also requires the help of circulating air. According to Crane (1999), 12 fanning bees positioned across a hive entrance 25 cm wide can produce an air flow amounting

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6Jean-Michel Grandmont (1992) made a major contribution in this direction by arguing that if preferences are sufficiently dispersed in the standard general equilibrium model, then the economy would behave like an individual satisfying the standard conditions and the aggregation problem would be solved. His contribution has given rise to an extensive discussion as to what precisely his idea of dispersion entails, and despite the intuitive appeal of his argument, it does not seem to solve the problem and thus redeem the standard framework.
CHAPTER 1. INTRODUCTION

to 50-60 litres per minute. This fanning can go on day and night during the honeyflow season.

What is the lesson here for us? The typical economist’s response to this phenomenon would be to consider a representative bee and then study how its behaviour responds to the ambient temperature. This would be a smooth function of temperature, wing beats going up with the temperature. Yet this is not what happens at all. Bees have different threshold temperatures and they are either on, beating at 11,400 beats per minute, which produces their distinctive buzz, or off. As the temperature rises more bees join in. Thus collectively with very simple 1,0 rules the bees produce a smooth response. This sort of coordination, with each agent doing something simple, can only be explained by having a distribution of temperature thresholds across bees. Aggregation of individuals with specific local and differentiated simple rules of behavior produces smooth and sophisticated aggregate behavior. When there are non-convexities and specialisation is advantageous, individuals can organise themselves so that it can emerge. The same is true in markets where information is dispersed and agents are not in contact with all the others.

Each agent can learn to do something specific and a situation can be obtained in which different individuals are behaving differently. Furthermore the cognitive requirements of the tasks that they specialise in may be much less than if they had to undertake a whole range of activities. This division of labour is such a longstanding and prevalent feature of our society that the names that people are given often reflect professions. What people do becomes an intrinsic part of their identity, and conditions the choices that they make. This was already observed by Epictetus when he said,

“The individual reference looks to the occupation and will of each person. The lyre player is to act like a lyre player, the carpenter as a carpenter, the philosopher as a philosopher, the orator as an orator”

Epictetus Discourses 3.23.4-5.

The profession, or specialised activity, one undertakes conditions one’s tastes, and one’s place, in society. Thus this voluntary diversification is, to some extent, self-perpetuating. Yet, as a result of this diversification collectively the system may act in a more sophisticated way than its components. This is the hallmark of a complex system.

Another interesting example of coordination is provided by bees pollinating. For plants it is preferable from a fitness point of view not always to be pollinated by the same bees. This avoids too much inbreeding or within plant pollen transfer. Thus, variance in nectar yield could be an effective way to avoid repeat visits by bees and to enhance fitness. The problem is that this variance could be the result of the pollinator’s activity. There is a delicate coordination problem. Bees exhibit risk averse behaviour in the sense that the lower the nectar yield the further afield they forage. Plants can profit from this by discouraging repeat visits through nectar yield variation. How would this work? The idea is that by having a high variance, “loyalty” would be discouraged. On the other hand it may not be the plants themselves that vary, and it could be that yield or production is affected by harvesting and that it is this alone that leads to the variance. A study by Keasar et al. (2007) on the relation between visits by bees to
rosemary plants, suggests that there is a feed-back, the foraging activity of pollinators increases the plant-generated variability in nectar yields and such an increase, in turn, reduces further visits by pollinators. Thus the plant variability is at the origin of the dispersion of bees between plants. As the authors point out, however, at very low densities of pollinators loyalty may be positive since any fertilisation is better than no fertilisation at all. When pollinators are scarce, it will be advantageous to recruit bees and to try to keep them. But the main message is that there is some advantage to be gained at current levels of pollination in having heterogeneous nectar yields. Once again, an aggregate collective behaviour emerges from the unintentional coordination of heterogeneous individuals.

Coordination in achieving a collective task and how it is achieved is, as I will continue to insist, a central problem for economics. How this is managed by individuals who do not have the collective interest in mind, is a fascinating question, and the fact that it can be done by insects, suggests that we do not necessarily have to attribute extraordinary reasoning powers to economic agents.

1.7 A simple example of coordination

A very simple example of this sort of problem was given by Brian Arthur (1989) and is usually referred to as the “El Farol Bar” problem or somewhat perversely by the many physicists who have analysed the problem as the “minority game”. He considers a simple example in which 100 customers must decide whether to visit the local bar or not. They prefer to be at the bar if there are strictly fewer than 60 others there and prefer to be at home otherwise. Their problem, then, for each individual is merely to forecast the number who will attend on the night in question and then to take the appropriate decision. The obvious equilibrium is one in which there are 60 people at the bar. This is an example of the well-known Nash equilibrium in which nobody has an incentive to change his decision given the choices of the others. However, Arthur’s question is, how would the customers arrive at such a situation?

If one uses full-blown game theoretic rationality, each client contemplating the action of the other will be led into the infinite regress which I have already described. Arthur suggested as an alternative that agents might try to solve the problem and simply use forecasting rules to work out how many people will be at the bar and then decide accordingly. He simulated exactly this and showed that using very simple forecasting rules and concentrating only on the known history of attendancethe system will settle very close to equilibrium.

He proceeded as follows. Each artificial customer was assigned a number of forecasting rules chosen at random from a larger set of such rules. These rules are of the form, “attendance will be the same as last time” or “attendance will be the average of the last four periods” for example. Each agent then uses that rule which is currently performing best and takes the appropriate action either going to the bar or staying at home. In this way, a new attendance observation is generated. Perhaps surprisingly, the system rapidly settles down to attendances which are close to 60. Thus, with individuals using simple myopic rules, ignoring the feedback from individual choices to aggregate outcomes, that is, reacting to a “field variable”, as Aoki (1996) would
put it, the system finds it way close to equilibrium. From an aggregate point of view this seems very satisfactory, particularly given the complicated feedback from the actions to the variable being predicted. From a welfare angle the situation is not so good since every time attendance is above 60 there is a majority of individuals who are unhappy. Moreover, there is no reason to believe that all the individuals learn to forecast correctly. For example imagine that 60 of the individuals systematically forecast an attendance of one individual and always go to the bar whilst the 40 others all forecast 100 and never go. All the individuals will be wrong all the time but the system will be in equilibrium. This should not happen in Arthur’s model, since agents use the quality of their forecast as their criterion but it illustrates the problem that a collectively rational situation need not be generated by individual rationality. Indeed, the normal outcome in the sort of situation described will be one in which most agents predict well and a few will make systematic mistakes.

The simple message of this example is that individuals operating in a rather simple and myopic way without contemplating the impact of their interaction can nonetheless produce a collectively reasonable outcome.

Later in the book, I will deal in some detail with a problem of this order, that of how people coordinate on how much to contribute to a public good. We will see that, in the experiments that I describe, the agents coordinate on a collective outcome but individually behave in different ways. As I have already said, however, the benchmark for a coordinating mechanism is the market and the examples I have just discussed do not involve markets.

1.8 Markets

So, let me come back now to one of the central themes of this book, the way in which markets function and how they coordinate the activity of the participants. In the anonymous market of pure theory, people do not trade with each other, nor is the way in which traders meet and undertake their business specified. Yet in reality, some markets are organised on a pairwise trading basis, some as an auction, some have a central clearing system. In some, prices are posted whilst in others all prices are negotiated. This, of course, suggests that different structures arise for varying goods in different circumstances. Thus, when faced with a specific example of market organisation, the question would become why does this particular market form emerge in this particular context? A convincing argument is that markets organise themselves over time and that the norms and customs that characterise a market emerge as this happens. If this is the case, it would not be unreasonable to suppose that the outcome of a market would depend on the way in which the market is organised. Yet there is little room for such considerations in standard market models.

Nevertheless, the idea that market organisation has an important impact on economic outcomes is, of course, far from new. Historians have spent a great deal of time discussing the emergence of economic organisation and Moses Finley (1973) suggests

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7There have, of course, been some attempts to incorporate these considerations into market models and one of the rare examples in which this was fully developed is the book by Frank Fisher (), though his market remains an abstract one.
that there was no market in the modern sense, in which interaction is anonymous and through prices, in the classical world. Yet, Peter Temin has contested this view. What is clear is that markets in various forms are one of the most longstanding features of economic life. As Braudel observed,

“Ce très vieux type d’échange se pratiquait déjà à Pompei, à Ostie ou a Timgad la Romaine, et des siècles, des millénaires plus tôt: la Grèce ancienne a eu ses marchés; des marchés existent dans la Chine classique comme dans l’Egypte pharaonique, dans la Babylone où l’échange était si précoce… En Ethiopie, les marchés par leurs origines se perdent dans le temps”

Ferdinand Braudel, “Les Jeux de l’Échange”

Although my main argument here is that markets and their organisation are, in large part, emergent phenomena, it is also true that the state has played a major part in their development and in their limitation. The trading of certain commodities, in some countries, has been regarded as the business of government and, this is still true today. This tradition dates back a long way. In Imperial China for example, the state controlled to a great extent some “key commodities” including salt during the Ming and Qing dynasties, and wine and iron and steel during the Han dynasty

Much later, medieval markets in England were characterised by personal and hierarchical relationships. Later again, as McClean and Padgett (1997) point out, markets in Renaissance Florence were strongly influenced by networks of family relationships and, as this was modified, market outcomes changed. Thompson (1971) describes how markets in 18th century England displayed a pattern of relationships and implicit rules governing those relationships. This concern with the influence of market arrangements did not, as is sometimes suggested disappear with the “marginal revolution”. As Walker (1996) clearly explains Walras was preoccupied with the way in which exchanges and communication in markets are organised. Nevertheless, as Walker also points out Walras’ interest in real markets such as the Paris Bourse was superficial and his description shows that he was not really aware of how the Bourse actually functioned.

Nevertheless, the standard paradigm, which is one in which individuals receive prices from some unspecified source such as an auctioneer, is normally referred to as “Walrasian”. Again, as Walker explains, Walras never specified such an auctioneer and indeed each of his models, which were not however, based on empirical examples, specifies carefully how individuals meet and change prices. He was preoccupied with explaining much more than the solution of a simultaneous equation system, (see Walker (2005)). Marshall (1920) is also at pains to point out the importance of the way in which markets are organised for economic outcomes. But, he also, in his famous discussion of the corn market, remains at a hypothetical level and discusses what the actors on the market might do and how this would bring about an equilibrium. He did not study the empirical evidence from any particular market. This is important, since what we observe is that theoreticians even when they pretended to be discussing how markets function remained at an abstract level and distanced themselves further from the empirical evidence. Thus market organisation became something which was assumed rather than observed. Of course, there was an important line of thought which
treated the market as an evolving phenomena and this was associated with the Austrian school and particularly with Hayek and Schumpeter. Hayek (1989) suggested that organisation or “order” is an emergent phenomenon and concluded that the market in modern terms was but one form of spontaneous order. He did, in fact, suggest that this form of organisation was superior to others thereby making explicitly the point that the efficiency of outcomes is intimately linked to the way in which the economy is organised. This is perhaps the weakest part of his analysis since he finds it difficult to justify his assertion that market organisation is optimal in some sense.

The major contribution to economic thought made by North (1990) was to insist that market institutions are indeed, important. To the anthropologist, such an assertion seems trivial, (see for example Geertz’s (1978) remarkable work on the functioning of the Moroccan bazaar.) But for the theoretical economist it is much less so. North is at pains to explain that closer analysis of institutions is essential to economic history whilst economic theory tends to focus on timeless and frictionless transactions. He suggests therefore that institutional analysis is an appropriate way of using some of the lessons from neoclassical economic theory in economic history but does not go on to suggest that theory itself might need to be substantially modified in the light of the institutional considerations that he raises. His emphasis is on the constraints and incentives provided by institutional organisation.

What I want to argue, in this book, is that, if we are to take markets seriously as places, whether virtual or geographical, where individuals interact directly and influence each others’ actions, then economic theory itself has to be modified. Rather than simply look at the constraints imposed by institutions and then apply standard theory within those constraints it would be better to develop a theory which encompasses the emergence of those constraints. Markets are complicated phenomena, even for the same product, they are far from uniform in their structure. Their rules change and they are more or less restricted by government regulation. But, given their importance in the economy, it seems unreasonable to leave their organisation and evolution to one side.

With this in mind, I will come back to the subject of markets in two chapters each of which will use a particular type of market as an example. The first will use fish markets and the other, financial markets. These two examples are extreme, in the sense that the product sold on one are totally perishable. There are essentially no inventories whilst many financial products have, at least in principle, an infinite life. In the financial markets chapter, I will explain how the mutual interactions between individuals can lead them to change their forecasts, opinions and choices. This can lead to “herd behaviour”, financial bubbles and characteristics of aggregate financial time series which are difficult to explain with standard models based purely on the analysis of individual behaviour. In the chapter which takes fish markets as an example, I will try to show how important it is to understand the structure of relationships in markets if one is to be able to make sense of the behaviour of a market at the aggregate level. This brings me to an important point and one which, in fact would merit a book on its own and that is the role of networks in economics.

Networks

Indeed there are two excellent books which have recently appeared on this subject, (see Jackson (2008) and Goyal (2007)).
1.9. MACROECONOMICS AND THE NEED FOR NEW MODELS

In much of the economic literature very little play is made of the idea that the structure of the interaction between individuals is important, but, as Lucas said,

"Applications of economic theory to market or group behaviour require assumptions about the mode of interaction among agents as well as about individual behaviour"


In sociology a great deal of emphasis is put on the networks in which individuals are “embedded”, (see e.g. Granovetter (1985) and White (1981)). One of the arguments made is that markets cannot be insulated from social structure because different social relationships will result in informational asymmetries, for example, which will provide some parties with benefits and leave others at a disadvantage. Such asymmetries are widely discussed in economics but little attention is paid to their origin.

Perhaps more important, is the fact that individuals’ choices are heavily influenced by those they are in contact with. Individual preferences, expectations and aspirations are all conditioned by those of our “neighbours”, that is, those with whom we interact. It has long been recognised that such interactions can influence the nature of aggregate outcomes\(^9\), but what is also important is to analyse the structure of the interactions. A perfectly competitive market might be thought of as a star with individuals linked to a central price setter, such as an auctioneer. A game theoretical model in which the nodes are the individuals would have every node linked to every other one, since all the players consider the actions and reasoning of all the others. This would be a complete graph. The graph corresponding to real economic situations is somewhere in between, with densely linked local clusters linked by longer links to other clusters.\(^{10}\) I will come back to the organisation of interaction and the importance of networks in the second chapter. For the moment, just let me assert that it is not only interaction but the organisation of that interaction that is important and, what is more, that organisation will change as individuals build and destroy links with others.

1.9 Macroeconomics and the Need for New Models

The basic idea of this book is that the economy is a constantly evolving system, which not only evolves but, in so doing, may go through sudden changes which are endogenously produced. In the chapter on financial markets, I will return to this and suggest that market prices are heavily influenced by the ways in which market participants influence each other. This recalls the work of Minsky (1986) who argued that these markets would have long periods of tranquility punctuated by periods of collapse and high volatility. He argued that this was an intrinsic feature of these markets and was due to the use and abuse of the institutional rules that emerge. More generally, one might ask why macroeconomic models seem to do such a poor job in reproducing or


\(^{10}\)An example of this is the sort of graph described by Duncan Watts (1999) as a “small world” graph.
fitting macroeconomic data. What I will argue is that this is because modern macro-
models have stripped away much of the detail of economies as if the organisation and
evolution of institutions, networks and even social and religious factors were outside
the purview of economics. But all of these factors do have an important impact on the
aggregate economy. As a leading econometrician, David Hendry says,

“The marked gap that exists between macro-economic theory models and
applied econometric findings arises because much observed data variabil-
ity in macroeconomics is due to factors that are absent from economic
theories. Various sources of non-stationarity impinge on macroeconomic
data, deriving from technical progress, new legislation, institutional change,
Financial innovation and political factors including conflicts, inducing both
evolution and structural breaks which change the distributional properties
of the data.”

_Hendry (2005)_

The idea that institutions and many social features could play an essential role in
the evolution of the economy is far from new though the drive towards a purer more
mathematical framework has tended to push such considerations to one side. An earlier
example of such preoccupations is provided by Schumpeter (1926) when he says:

“economic theory usually contains statements about 'social institutions’,
such as ‘property’, ‘inheritance’ and ‘the family’, and that these institu-
tions are ‘partly economic’ and ‘partly non economic in nature’. Social
institutions therefore cannot be analysed with conventional economic the-
ory; pure economic theory is only applicable to topics such as value, price,
and money. Something else is needed – a ‘theory of economic institutions,
basically within economic theory’. And this something else is economic
sociology” (Schumpeter 1926)

But this idea that the evolution of economic institutions could be successfully in-
cluded in economic theory is far from becoming widely accepted. In fact it is not clear
how far Schumpeter was prepared to go in that direction and an alternative interpreta-
tion is that, as Shionoya puts it nicely:

“economic sociology is therefore conceived by Schumpeter as a “bridge
between history and theory” or as a “compromise between the generality
meant by theory and the individuality meant by history” (Shionoya 1997: 200).

All of this suggests that, given the incapacity of modern macroeconomic models to
even envisage the sort of major crisis we have been experiencing, that not only do we
need new theory but that we need to invoke the influence of the social and institutional
framework in which the economy is embedded. But the two things are not separate,
many economists invoke the failure of institutional arrangements as an explanation for
the crisis. The idea being, at least implicitly, that if the institutions can be appropriately
rethought or regulated the models which have become standard will be adequate. But
this is a fundamental mistake, if we do not integrate the coevolution of norms and
institutions into our models and do not take account of historical lessons we will not
progress towards a better understanding of the economy. It would, of course, not be
fair to say that such ideas have not been present, there has always been a part of the
economics profession that has argued along these lines. Pareto, when he was working
on sociology, many members of the Austrian School, and Fred Hirsch (1978), in his
“Social Limits to Growth”, all made this sort of argument. However, this had little
impact on the majority of the profession, but with the onset of the recent crisis social
norms, and the importance of the influence of others has come back to the forefront. In
particular, as soon as we talk about social influence we cannot avoid looking at human
psychology which is absent from standard economics and is condescendingly classified
as being in the realm of “behavioural economics”. But this is at the root of the sort of
contagious phenomena that we observe. This point of view is forcefully argued by

To return, for a moment, to the lessons to be learned from history, it is odd that we
choose in periods of relative calm to ignore history but in periods of sudden change
or high volatility to rush back to historical events. There exists a substantial literature
on crises, (see e.g. Bagehot (1873), Minsky (1986), Kindleberger (1989) and Lei-
jonhufvud (2000) and the many analyses of the Great Depression), but these are all
regarded as specific cases in which the international economy somehow deviated from
its equilibrium path.

In Colander et al. (2008 and 2009) we have argued that we have to develop macro-
economic models which not only take full account of, but are centered around the
interaction between individuals and in which contagion, phase transitions and out of
equilibrium dynamics are analysed. Furthermore, it is not sufficient for economists to
claim that they have no responsibility for the evolution of the economy. Modern macro-
economic models are widely used by central banks and governmental authorities and
the authors of such models must have some responsibility for the use to which they are
put.

Building and developing new models based on the sort of considerations I have
mentioned into account is a daunting task At least for the moment no complete paradigm
seems to be emerging which could in any way rival the existing general paradigm at
least for completeness or elegance. Yet, neither of these qualities mean that the existing
paradigm is very useful for explaining economic phenomena. It is this, of course, that
makes the task of developing alternatives worthwhile. Indeed, the road seems clear
even if difficult to follow. What we have to do is to build models in which individuals,
even if myopic and with limited and local information take their decisions in function
of each other. The theoretical idealist might envisage a full-blown game theoretic ap-
proach. However, this imputes a level of analytical reasoning which, apart from its
logical problems, does not seem appropriate for the analysis of whole markets. An
alternative is to give the agents elementary rules of behaviour and to study the outcome
of the interaction of such agents. In rather simple cases we may be able to find ana-
lytical results and these will give us a guide as to what to expect if we then simulate
more general versions of our models. There are various levels at which we can practise
this, so-called, agent based modelling. The most optimistic and least constrained is
the “artificial life” approach which puts the minimum of structure and constraints on the agents in the model and then sees what evolves. This was the approach adopted in economics by Leigh Tesfatsion (1997) and also the one used to build the Santa Fe stock market. Another approach is to give some basic rationality to the agents but to make it much less restrictive than in standard economic theory. Thus, agents may learn from experience, but will seek to improve according to some criterion.

In the rest of this book, I will give examples of such approaches and in all of them we will see that systems of simple interacting agents with limited local knowledge can generate aggregate phenomena which could not be foreseen by looking at the individuals in isolation. All of these will, I hope, help to emphasise the basic messages of this book. Aggregates do not behave like individuals. The individual actors in the economy have a very restricted view of their environment. Their rationality does not correspond to that which the standard economic axioms impose. The system organises itself but, in such a way that its evolution is difficult to predict. Sudden major changes in the state of the system can occur without any external shock.

I hope that the various chapters will convince the reader that we have to rethink our basic view of the economy and put interaction and collective behaviour in the centre of the picture and that we should also change and simplify our depiction of the economic agents who make up the system. Indeed it is the view of the economy as a complex interactive system that is the common thread that runs through the book. To define what is meant by this I can do no better than to cite Herb Simon,

“Roughly by a complex system I mean one made up of a large number of parts that interact in a nonsimple way. In such systems, the whole is more than the sum of the parts, not in an ultimate metaphysical sense, but in the important pragmatic sense that, given the properties of the parts and the laws of their interaction, it is not a trivial matter to infer the properties of the whole. In the face of complexity, an in-principle reductionist may be at the same time a pragmatic holist.”

*Herbert Simon (1962, p. 267)*

Taking this view is a challenge, it means putting on one side many of the convenient tools to which we have become accustomed and trying to build a framework which will accommodate more of the economic phenomena that we actually observe. Nevertheless, it seems to be a challenge that we not only might like to accept but one which is necessary for us to accept if we are to advance the basic agenda of economics.
Chapter 2

The Structure of Interaction

2.1 Introduction

Up to this point I have insisted on the fact that direct interaction between agents plays a crucial role in determining aggregate outcomes. However, to justify this assertion, as I said in the first chapter, I have to say more about how this interaction is organised and, in particular, who interacts with whom. To analyse this, we need to know about the network of links between the individuals whether consumers, firms or other collective entities. Networks and network analysis play a central role in many disciplines and for a long time their role in economics was ambiguous. For many economists the study of networks was limited to the analysis of the functioning of physical networks such as the railway, the telephone system or the internet for example. Yet, more recently it has been recognised that networks, are, in fact are much more fundamental and pervasive than this and this is well illustrated by Goyal’s (2007) and Jackson’s (2008) recent books on economic and social networks. Almost any serious consideration of economic organisation leads to the conclusion that network structures both within and between organisations are important. Let us go back for a moment to the crisis of confidence in the world economy. Here we see the role of networks. Bad risks from the American mortgage market, had been bundled with good risks into derivatives and these had diffused through the international banking system. Up to that point the financial system, thought of as a network of banks, had become larger and more connected and it was argued that the resultant diversification of risk was a stabilising influence. Yet, there was little analysis of the effect of the increasing connectivity of the network on the probability of an epidemic of negative impacts. The problem is that as risk is diversified into different instruments those who buy it lose track of the underlying asset. Thus, while the risk is diversified the information is not. When this happens, an epidemic of mistrust can develop as each bank in the network is wary of lending to another who may have inherited the risks that turned out to be bad. Worse, banks find themselves not only with assets which may turn out to be “toxic” but the market may revise its valuation of these assets. Thus, the fact that various banks have been obliged to reassess their losses as a result of the subprime episode and its consequences was not
only due to their discovering the true nature of their assets but also to their revaluation downwards by the market. The resultant losses of the banks enhance the epidemic of mistrust. It is possible that it is simply the increased connectivity of the network that has favoured the development of such an epidemic. But, in fact, the problem is more subtle than this. Propagation of information or of shocks may be more likely and the effect may be bigger in networks that are much less than fully connected.

This problem was already discussed by Allen and Gale (2000). Using a network structure involving four banks, they showed that the spread of contagion depends crucially on the pattern of interconnectedness between banks. When the network is completely connected, with all banks having exposures to each other such that the amount of interbank deposits held by any bank is evenly spread over all other banks, the impact of a shock is easily absorbed. Every bank suffers a small loss and there is no contagion. By contrast, when the connectivity of the network is lower, with banks only having exposures to a few counterparties, the system is more fragile. The initial impact of a shock is concentrated amongst neighbouring banks. Once these succumb, the premature liquidation of long-term assets and the associated loss of value bring previously unaffected banks into the front line of contagion. Thus, the structure of the network heightens rather than damps the effect of a shock. Indeed, there is evidence that even in large, apparently anonymous markets, participants trade or interact with a rather small group of other traders. Thus the fact that the participants are clustered into limited groups may cause the propagation of a shock which was not particularly large at the outset.

In general what we want to know, is whether it is true that a network which emerges from a particular evolution of trading relationships which are mutually advantageous can become fragile without those who participate in it realising what is going on.

Let me quote Haldane (2009) of the Bank of England, when talking about the development of the banking network before the global financial crisis. He gives details of that evolution and then goes on to say,

“This evolution in the topology of the network meant that sharp discontinuities in the financial system were an accident waiting to happen. The present crisis is the materialisation of that accident.”

_Haldane (2009) p. 4._

It is only too clear, ex post, why this should be of interest to economists. But it is also true that, while economists have now recognised the importance of networks, they have for long been regarded as peripheral to economic analysis. As should have become clear by now, I would suggest that they together with the interactions that they mediate, are actually central to our understanding of how an economy functions. At the risk of being repetitive, let me say again that we have to acknowledge that the direct interaction between agents and the way in which that interaction is organised has fundamental consequences for aggregate economic outcomes. When agents are directly linked to each other and influence each other, the relationship between the behaviour of individuals and the behaviour of aggregate variables will be different than in the anonymous market situation in which all agents are linked to each other only through the price
system. What we observe at the aggregate level will not mimic what we observe at the individual level, nor will it correspond to the behaviour of some “representative individual”. Moreover that rationality which we attribute to economic individuals in order to justify and analyse the behaviour of aggregates may have to be modified. Thus what I will argue, in this chapter, is that the structure of the relationships between individuals, firms or groups is of profound importance if we are to understand aggregate or macro economic behaviour. We should, indeed, be interested in the passage from micro to macro economic behaviour, but this cannot be understood without taking into account the way in which individuals’ decisions and actions are influenced by the networks of connections that link them to other agents. Furthermore, one will not, in general, be able to represent the behaviour of the aggregate as the behaviour of some average or representative individual. Just as neurologists would not think of explaining behaviour by studying the changes in a representative neuron nor should economists try to explain aggregate phenomena in this way.

This does not mean that one should not be interested in what happens at the micro level, but rather, that the passage to the aggregate level is mediated by the network structure in which individuals find themselves. Neurologists will continue to examine what happens at the molecular level but would not argue that there is some simple passage from that level to the aggregate activity of the brain which does not involve the network of interactions between neurons.

Of course, as economists, unlike neurologists, we do not usually descend as far as the level of the neurons of economic agents, but, as interest in so-called “neuroeconomics” has developed it has been argued that economic behaviour is very much determined by the network of neurons that is activated in a certain situation and that as the situation changes another network may become active. Thus even at this level it is the network structure of the neurons that is important (see Oullier et al (2008). To return to another analogy, we would not expect to be able to explain how much food is stored by a colony of ants by looking at the behaviour of individual ants in isolation. The organisation of the ants plays an essential role. This example raises an important point. Far from complicating things, taking direct account of interaction and the networks which organise it, actually makes life simpler for the economic theorist. This is because the reasoning and calculating capacities we need to attribute to economic agents may be substantially less than it is in standard models. A central theme of this book is that individuals operating with simple rules in a limited context may, together, generate rather sophisticated behaviour on the aggregate level. In other words, aggregation itself may be what is structuring market or group behaviour.

When interaction and the networks through which it operates are well defined, this will be important in determining aggregate economic phenomena. This, as I have explained, has the great advantage of allowing us to start from more plausible and less comprehensively rational models of individuals than those that we normally use. The first step is then to understand how networks influence aggregate outcomes. For this we can either take networks to be fixed and deterministic, or fixed and stochastic. The next step is to understand how these networks form and if, and why, they persist. Here, we can consider the evolution to be mechanistic according to some criterion of fitness or one can think of the links in the network as being consciously and strategically chosen by the individuals who constitute the nodes (see recent surveys by Jackson (2007 and...
To many social scientists the interest of the problems examined here must seem evident and it is worth taking a quick look at the relationship between economic and sociological analysis, since the latter tends to take it for granted that network structures are at the heart of the explanation of social phenomena whereas the former has, until recently, attached less importance to them.

2.2 Economics and Sociology.

The idea that networks of relations at various levels have an important effect on economic activity is familiar in sociology. At a general level, as Robert Putnam has said, one can think of “social capital” as “features of social organisation, such as networks, norms and social trust that facilitate coordination and cooperation”. He views networks as an important means of ensuring that groups function well together. In sociology, a great deal of emphasis is also put on the networks in which individuals are “embedded”, (see e.g. Granovetter (1985 and 1997) and White (1982 and 2002)). Lazega (2001), for example, puts a lot of weight on the way in which individuals in an organisation are constrained by their relations with those who have come to be regarded as “powerful” figures. Thus, it is suggested that most economic mechanisms are a mixture of formal and informal rules or “conventions”. The latter are thought of as emerging as a result of the network structure of the society in which the particular economic mechanism and the individuals who operate within it are situated.

Consider the approach adopted by Baker (1984) in his study of a major securities exchange and by Abolafia (1996) in his analysis of the Chicago Commodities Exchange. Baker shows that the volatility of options prices is dependent on the type of network structure and, in particular, the size of the subset within which the agents in a market operate. Abolafia explains how informal rules emerge, are tested, and are consequently modified. The changes in rules after what are considered as abuses are testimony to this. The evolution of rules against “insider trading” and the reaction of the markets to the silver trading episodes of the 1980s are good examples. But what is the role of networks here? An important feature is that the way in which markets are organised into networks allows differential access to information. As I observed in the first chapter, whilst models with asymmetric information are widespread in economics little attention is paid to the origins of that asymmetry. Yet the fact that individuals operate within a limited network provides just such an argument. Indeed sociologists defend the idea that markets and market outcomes cannot be insulated from social structure because different social relationships will result in informational asymmetries, for example, which will provide some parties with benefits and leave others at a disadvantage.

As economists try to integrate network structures into their considerations they should perhaps be more inclined to take seriously the work of sociologists who have typically regarded networks of various sorts in society as being of great importance in determining how society behaves. However sociologists have, in general, explicitly rejected what has come to be regarded as the “ultra rational” behaviour attributed by economists to the individuals in society. Thus from their point of view the derivation by
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Economists of the characteristics of aggregate situations from the rational behaviour of isolated individuals is at odds with the network approach. Sociologists regard collective behaviour as the result of the interaction between individuals situated in relation to each other in social networks, and indeed some think of the network as more important than the individuals that make it up. (see Luhmann (1984) for example). Economists and in particular macroeconomists, on the other hand, with rare exceptions, have assimilated collective behaviour to that of an individual. Two things, then, seem to separate the approach of the economists from that of other social scientists. Firstly, there is the lack of any organisational structure in standard economic models and secondly there is the assumption of complete rationality.

Of course if we accept a more sociological view of the economy, we are faced with a standard but, nonetheless important, difficulty. Which part of the system can be considered as exogenous and which can be thought as endogenous. Social ties also have their “raisons d’être”. However, at some point, one is obliged to limit the environment within which one works. But if one limits the scope too much then models become so “partial” that they lose any general relevance. Indeed, as I have mentioned, it was his dissatisfaction with the scope of economics that motivated Pareto’s work on sociology. Ideally, one would like to have an explanation as to how social networks emerge, that is how the whole system organises itself. But even the problem of how economic links emerge is difficult and only recently has it been the subject of systematic analytic treatment.

Nevertheless, there has been a burgeoning interest in economics in the question of “social interactions” (see Blume and Durlauf (2004), Brock and Durlauf (2001a and 2001b), Glaeser et al (2003) and Gui and Sugden (2005)). If one accepts that people’s preferences reflect their identity and also that their identity is affected by their place in the social network, then the ordinary analysis of choice has to be modified. One has to add to the features that condition the choices of an individual, the characteristics of the individuals with whom he is linked. This has led to a discussion in the economic literature, (see Akerlof and Kranton (2000)). This changes the nature of the problem since there is a simultaneity problem if individual choices are influenced by the choices of their neighbours in the social network. However, the usual approach is to consider that the characteristics that influence individuals through the network are given and are not changed by interaction. A more ambitious approach is to allow for individuals choosing their neighbours and hence the influences that they will be subjected to, (see Horst et al. (2007)).

2.3 The economic consequences of networks

Note that the basic theme of this book provides justifications for emphasising the importance of networks in economics. Firstly I claim that interaction other than through the market mechanism is important. This may be true even when there is no specific network structure. Indeed there are situations in which interaction is general in that no agent is linked to a particular collection of other agents. Even interaction of this sort, if it is not mediated by a market in which agents are isolated and anonymous can change aggregate outcomes. Let me go back, for example, to the most extreme version
of interaction, that studied by game theory. Every player takes account of what every other player does and moreover knows that the others do so. The network of links between individuals is complete and, what is more, it is being fully and consciously used. This leads to two problems. Firstly the idea that everyone is taking into account the actions of others means that only limited examples can be fully analysed. Furthermore the assumption that every player is taking account of each others’ actions and that he knows that the others are doing so and that they know that he is doing so and so forth leads to basic logical problems, as I mentioned in the first chapter (see, for example Binmore (1990)).

There are thus two extreme approaches in economics. On the one hand there is the standard model where individuals are essentially independent, act in isolation and their activities are coordinated by market signals. On the other, there is the full game theoretic model in which individuals are completely interdependent but would have to be endowed with prodigious powers of reasoning. Models in which agents are linked through an incomplete network of contacts lie between the two.

There are many economic situations in which there are links between some, but not necessarily all, agents. These are frequently referred to as models of local interaction. In order to discuss local interaction we must define what we mean by local and this, in turn, means that we must impose some structure on the space of agents. This will determine the distances between agents, which in turn will determine who is a neighbour of whom. The structure might depend on geographical distance as in locational models (see Gabszewicz and Thisseur (1986) for example), closeness in characteristics (see Gilles and Ruys (1990)) or the potential gains from bilateral trade. Thus, countries may form free trade agreements and detach themselves from previous trading partners, (see Crawford and Fiorentino (2005)). In other contexts, individuals in developing countries will tend to insure risk and health through other individuals who are members of their family for example, (see Fafchamps and Gubert (2007)).

In this chapter, I shall consider the interaction structure as being modelled by a graph where the agents are the nodes and two nodes are linked by an edge if the corresponding agents interact. In particular, I will restrict attention to the class of undirected graphs, which says that if \(a\) interacts with \(b\) in some way then \(b\) interacts with \(a\) in the same way. Within such a framework, it will generally be the case that individuals interact directly with those who are their neighbours, that is those who are near to them. But, again, for this to be meaningful it is necessary to define a notion of “nearness”. For example, in a graph, one might ask how many links are there on the shortest path from individual \(a\) to individual \(b\), thus defining a “distance” between \(a\) and \(b\). In this context, we might, for example, assume that each agent is influenced only by a limited (finite) number of other agents who are within a certain distance of him. Such individuals are usually referred to as the agent’s “neighbours”. At this point, once the graph is given and the distance chosen, the “neighbourhood structure” is defined a pri-

\[1\]This, of course, excludes a whole class of interest where the link is activated by one of the individuals and then in order, for example, to undertake a transaction. In many economic examples we have to handle the transmission of information from a central source and this is clearly best handled by considering the links as directed. Evstigneev and Takar (1995) have modelled some economic equilibrium problems using directed graphs but, as they indicate, the task is made difficult by the fact that some of the mathematical tools which make the undirected case tractable are not available for directed graphs.
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ori. However, it is important to bear in mind that a more ambitious aim of the study of economic networks is to make this structure endogenous. Nevertheless, before moving to the problem of the formation of links, it is worth analysing the consequences of introducing the idea of local relationships among individuals.

Models with a network structure of interaction allow us to analyse an important problem which is often alluded to in economics but not often analysed. As agents’ interactions are limited to a set of neighbours, changes will not affect all agents simultaneously but rather diffuse across the economy. This is clearly relevant when discussing the contagion of one bank or hedge fund by another in financial markets. The way in which such diffusion will occur, and the speed with which it happens, will depend crucially on the nature of the neighbourhood structure. It is the connectivity of the graph of relations that will be essential here. In many cases, a specific structure is imposed and thus the connectivity of the graph will be determined exogenously. For example the most typical example of a graph structure used in economics is that in which agents are thought of as being placed on a lattice and interacting with the individuals nearest to them. But as will become clear, many alternative structures of links can be considered, (see Myerson (1977)).

In the deterministic case it is clear that, once the particular network of communication is established, analysis of many problems becomes straightforward. Suppose that the graph of relationships is given and the “distance” between two agents is defined as the number of links in the shortest path between the agents. The set of individual \(a\)’s neighbours, \(N(a)\), consists of all agents within some distance \(k\) of \(a\). Suppose we are interested in the speed with which a signal is propagated through the population. As I have mentioned, this speed depends on the connectivity of the graph. But for the moment I have not said what this means, although the intuitive idea is clear. A convenient, and frequently used, measure of connectivity is the “diameter” of the graph, i.e. the maximum of the distances over all the pairs of agents. In the usual lattice type of model, the diameter of the graph becomes very large as the number of agents increases and conventionally when there is no path of links from one agent to another the diameter is taken to be infinite but it is clear that one might consider two graphs with an infinite diameter but where one is obviously “more connected” than the other. In the implicit graph associated with a non-cooperative game we are at the other extreme, however, and the diameter is one, that is every agent is in contact with every other one regardless of how many agents there are. Thus we can visualise three types of model already.

The classic Walrasian model in economics is one in which individuals are linked to some central figure often referred to as the Walrasian auctioneer who sends price signals and to whom they communicate their supplies and demands. This gives us a star shaped graph as in Figure 2.1.

In many of the standard studies of interaction in economics a simple lattice is chosen and the neighbours of an individual are often considered to be the four agents directly linked with that individual. Thus in Figure 2.2 the blue individual has the red

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[2]The lattice is usually taken to be one or two dimensional, (see Durlauf (1990), Benabou (1996), Blume (1993) and Ellison (1993)).

[3]Actually, as Walker () has been at pains to point out, the auctioneer does not figure in Walras’ work.
individuals as neighbours.

The neighbours might also be the eight adjoining neighbours in the case in which the individuals are located on a chequer board as in the well known Schelling (1971), segregation model. In the case of game theory all players in an $n$ person game are linked to each other as in Figure 2.3.

Gilles and Ruys (1990), working within the deterministic approach developed by Myerson (1977) and Kalai, Postlewaite and Roberts (1978), adopt a different approach and use the topological structure of the underlying characteristics space to define the notion of distance. Thus, agents nearer in characteristics communicate more directly. Translating this into a network structure means attributing links to those people who are closest in terms of some common characteristic. To an economist interested in how trading relationships are established, this seems a little perverse. In general, the possibility of mutually profitable trade increases with increasing difference in characteristics whether these be differences in tastes, endowments or abilities. Identical agents have no trading possibilities. However, there are other economic problems, the formation of unions and cartels, or other coalitions, for example, for which this approach may be suitable (see Demange and Wooders (2005)). The important point is that the relational structure is linked to, but not identical with, the topological characteristics structure. Once again the main feature of such models is that the aggregate outcomes that emerge may depend crucially on the underlying communication structure.

In many cases in economics links are not used with certainty but with some probability. Either one can think of a fixed set of links but which are only used randomly, for example I am in principle linked to millions of stores in the world by internet, but I use very few of these links and even those I do use, I may not use all the time. I may simply take the Yellow Pages and choose a few at random. Alternatively, one can think of the
Figure 2.2: Figure 2.2

Figure 2.3: Figure 2.3
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links themselves as being random and this may be the case for many social interactions which happen as the result of random meetings. In either case, once the interaction is stochastic, the problem becomes more complicated. We are not so much interested in the interaction itself but with the consequences of that interaction. To examine this and to see what aggregate patterns will emerge one has still to specify the nature of the communication graph linking agents. As I have mentioned one might think of a lattice structure. In general, however, one can consider any network and define the “neighbourhoods” accordingly. However the determination of how individuals depend on others is now random. Let me now have a look at each of the two ways of viewing such stochastic interaction.

2.4 **Given fixed network: stochastic interaction**

A first step is to consider the graph of communication as given but to assume that the dependence of the state of any individual on his neighbours is stochastic. Thus, once the graph is specified, neighbourhoods are defined and one assumes that the agent \( a \) is influenced by the individuals in his neighbourhood, \( N(a) \) in a probabilistic sense, that is, the probability of the agent \( a \) being in a state \( s \) is conditioned by the states of his neighbours. To see what is going on I have to use slightly more formal terms but the reader may wish to skip over this.

Consider each agent \( a \) in the set \( A \) of all agents as having one of a finite possible number of characteristics in a set \( S \). Then the set of all possible states of the economy is given by \( S^A \), where a state of the economy \( \omega \) associates each agent \( a \) with particular characteristics \( s \) in \( S \), i.e.,

\[
\omega : A \rightarrow S
\]

and \( \omega(a) \) denotes the characteristics of agent \( a \) in state \( s \).

Let \( \eta \) specify the environment of agent \( a \), i.e., the characteristics of all the agents other than \( a \). Now the probability law of an individual \( a \) is given by \( \pi_a(s/\eta) \) that is, his probability of having characteristics \( s \) given the characteristics of the other agents. Note that in our framework the only relevant part of \( \eta \) for \( a \) will be the states of the neighbours of \( a \). In this way we can write down the probability of each individual of finding himself in state \( s \) given the states of his neighbours. The first problem is to write down an aggregate probability law \( \mu \) or “global phase” that is consistent with the \( \pi_a \), or, put alternatively, to construct an aggregate law \( \mu \) from individual laws, i.e.,

\[
\mu(\omega(a)) = \pi_a(s/\eta)
\]

By making the assumption that all agents have a positive probability of being in every state and by assuming that there is a finite number of agents and of possible states for the individual, one can use a standard theorem, (see Spitzer (1971)), which guarantees that there is a unique “global phase” or a unique distribution of the states of the economy, that is consistent with the underlying micro characteristics. In other words it is enough to know the individual probability laws to know what the probability is that the economy will be in a particular state.\(^4\) But two things are at work here.

\(^4\)The use of this sort of approach is analysed in detail in Durlauf (1997) and Blume (1993).
Firstly, we would like to know what happens as the number of agents becomes large. In this case we might hope that all the random interaction between individuals might wash out and we could safely deal with the mean values and treat the system as if it were deterministic. Secondly we might not like the idea that all agents have a positive probability of being in any state. To take the latter first, Beth Allen (1982) gave a very simple example involving the adoption of technologies. She shows that even in the one technology case to her positive probability condition is necessary, Let $1$ indicate the state of having adopted the technology and $0$ represent not having adopted it. Let there be two agents, i.e.

$$A = \{1, 2\}$$

and consider the following local characteristics

$$\pi_a(1|0) = \pi_a(0|1) = 0 \text{ for } a = 1, 2.$$  

That is there is zero probability that an agent will adopt a technology different from that adopted by the other one.

Then $\mu$ such that

$$\mu(1, 1) = q \text{ and } \mu(0, 0) = 1 - q$$

is consistent with $\pi$ for $q \in [0, 1]$ and there are thus an infinite number of global phases. In other words, although the individual probabilities are well defined we have no idea as to the probabilities of the two possible outcomes in which both of the actors adopt the same technology.

So we see the role of a rather restrictive assumption in linking the probabilities at the individual level to those at the aggregate level. We needed this to ensure that the global outcome was not indeterminate. The question we can now ask is what happens as the economy becomes large? Can we still assign probabilities to each state of the whole system in a way which will be consistent with the individual probabilities? Suppose that we can do this. Furthermore suppose that all the individuals are identical in that their local probabilities are the same. Thus the whole system is characterised by the symmetry of the interactions between the actors. Föllmer (1974) explains that two disturbing things can happen. There may be more than one aggregate probability law consistent with the individual specifications as in the example we have just seen, and worse, some of these may not treat identical agents symmetrically. The latter case in which the conditional aggregate probabilities may vary from agent to agent, is called a symmetry breakdown.

Föllmer’s contribution shows that, even with a completely symmetric local structure, non-homogeneities may arise at the aggregate level. Think of an economy with a large number of agents each of whom draws his characteristics, for example his resources and endowments, from the same distribution and each of whom depends on his neighbours in the same way. One might hope that some sort of “law of large numbers” would apply and that with enough agents the prices that constitute an equilibrium would be independent of the particular draws of the individual. However, in the case in question, even in the limit it may not be possible to specify what the equilibrium
prices of such an economy will be. If interaction is strong enough, no heterogeneity is required at the individual level to produce these difficulties. Put another way, if agents are sufficiently dependent on their neighbours, aggregation does not remove uncertainty. Strong local random interaction amongst agents who are a priori identical may prevent the existence of a determinate equilibrium. This is very different from models with randomness at the individual level, but where the uncertainty is independent across individuals. We are accustomed to the idea that the noisy behaviour of individuals will be wiped out in the aggregate but as soon as this independence is removed things become much more complicated. When, as in the current economic crisis, individuals start to take risks and “insure” these risks with other individuals, the result can be radical at the aggregate level if the risks are not independent. This does not depend on differences between the individuals in question. Thus even with a highly symmetric underlying micro-structure, aggregate behaviour is unpredictable with sufficient interaction agents who are, a priori identical, may not be treated identically. The aggregate patterns which emerge simply do not reflect the symmetry of the microstructure. The very fact that the probabilistic interaction is mediated through a network structure and that the states of individuals are dependent on their neighbours is enough to change the relation between individual and aggregate outcomes. Moreover it is not the particular structure of the network that is important, even the simplest lattice structure can generate these problems.

2.5 The dynamics of interaction

In general, we are not really interested in the static analysis of what happens when people influence each other but would like to know how the states of the economy evolve over time. Föllmer’s random field approach which I have just discussed has been extended to consider dynamic stochastic processes. Indeed, much of the recent work on local interaction and interdependence has examined the evolution of behaviour of interacting agents. The specification of the local interaction plays an important role in characterising the nature of aggregate behaviour that will emerge over time. Of course, the literature involves models of rather simplistic situations, but these have the merit of making it clear what is going on. Good examples of this are the articles by Blume (1993) and Durlauf (1997) which both illustrate the use of the model adopted by Föllmer. Blume looks at a situation in which individuals play a “game” against their neighbours. Here the structure of links is fixed and the results of the interaction between players depends on their choice of strategy. Each player chooses a strategy and once this is done the payoffs materialise. Given the results the players modify their strategies over time. In a deterministic setting people would just choose a new strategy at each point in time, for example that which has performed best in the past. Here, rather than choosing a particular strategy, it is the probabilities of playing different strategies that are modified according to the difference in performance between a player’s own strategies and those played by the opponents. This reflects an old idea that, on the one hand, people want to benefit from past experience, and on the other, they want to explore new strategies or actions. In fact, the deterministic choice situation can be thought of as the extreme case of the stochastic one. Suppose that the parameter
governing the revision is very large or in other words, if the players attach a great deal of importance to their previous experience, then this corresponds to choosing the “best response”. Put alternatively, the player will take the action which has performed best up to that point, with probability one. For smaller values of the revision parameter however, the population may fluctuate between different states and may have a non-degenerate limit distribution. In other words, the population may never settle down to a steady state. The sensitivity of the adjustment of players’ strategies to those of their neighbours and the structure of the network determine the nature of the aggregate outcomes.

From this we begin to see how randomly interacting agents can generate movements between the states of a population or economy at the aggregate level. But one thing on which I want to insist is the difference between global and local interaction. This is very nicely illustrated by an example given by Ellison (1993). Again the structure of the situation is very simple and involves individuals playing a game, but it shows the basic idea. He compares a model with global interaction in which all players play against each other, with one in which players are situated on a circle and play against one of their neighbours. Here again the graph structure is very specific and is not itself stochastic. However, there are mutations in individual behaviour which means that from time to time players randomly make mistakes.

In this example the pairs of players play a simple coordination game. In the simplest such game there are two strategies, if the players both choose the same strategy a for example they each earn 1 and if they both choose b then they both earn 2. If they do not coordinate they both earn nothing. Thus there are two equilibria, (b, b) and (a, a), the first of which dominates the other in the sense that, in the “good” equilibrium, pay-offs are higher than in the “bad” equilibrium. Now, think of a situation in which players have to choose one strategy and then to use it with all the players they play with. In this case they have to decide how many of the players are playing a and how many b. In the example I just gave, a player has an interest in playing a if more than one third of the population is playing that strategy otherwise he is better off playing the other one.

Given the random mutations in peoples’ choices, in both the global and the local interaction cases the system fluctuates between coordinating on the “good” and “bad” Nash equilibrium. However the switch to the “good” equilibrium is achieved much more easily in the local case. It is clear why this is the case. In the coordination game I just described there is a threshold of the number of agents playing one strategy and once this threshold is passed the player is better off choosing it. If an individual is situated on a circle and he therefore has just two neighbours, then it may be enough for one mutation to occur to make him switch strategies. Such a mutation is, of course, much more likely than in a situation in which an individual has many neighbours and the simultaneous mutation of a significant number of them becomes highly improbable. This simple example provides an important lesson however, the change of a system from one state to another, may actually be more likely when agents are connected in a limited and local way than in a situation in which all agents are, at least potentially, in contact. To go back to what we learned from Föllmer it is the very fact that individuals are influenced by those around them that may cause problems. In a sense, we lose the notion of independence which lies behind so much of the reasoning behind models with large numbers of agents.
2.6 Random Graphs and Networks.

An alternative approach is to consider the graph structure itself as random. It is, of course, a methodological question, as to whether this is fundamentally different from the situation with a fixed network which is used randomly by the individuals. However, in the case where the network is random the notion of a neighbour is less well defined. But, and I will come back to this shortly, it is possible to link the probabilities that agents will interact to some underlying notion of the distance between them.

Now economists will typically be interested in the equilibria of a system. So suppose that once the communication network is established, a corresponding equilibrium notion is defined. Then one can study the characteristics of the equilibrium that arises from the particular structure one has chosen, since the agents will be constrained in their choices by the network. However, when the interaction is stochastic, the network, or the links used, will be a particular realisation of a random drawing and, in consequence, the outcome will, itself, be random. In the basic Markov random field model which I have already discussed, interaction is probabilistic but between neighbours in the lattice or some more general graph, and the graph structure is given exogenously. Haller (1990) for example, links the deterministic Gilles and Ruys approach to the stochastic one by basing the probability that communicational links exist, directly on the topological structure of the attribute space. This is, of course, just a complicated way of saying that people who are similar will have a high probability of being linked whilst those who are different will not. However, the stochastic graph approach allows for more complicated neighbourhood structures than this, since it allows agents, who are not “near” in terms of some underlying characteristics still to have a positive probability of communicating. In fact, the famous “small worlds” graphs introduced by Duncan Watts (2000) are characterised by having very long links together with clusters of closely linked individuals.

So, in the stochastic graph approach, the basic idea is to consider a set of agents $A$ and to attach probabilities to the links between them. Let $p_{ab}$ denote the probability that individual $a$ “communicates” with agent $b$. In graph terms, this is the probability that a link exists between nodes $a$ and $b$. The graph is, as before, taken to be undirected, i.e., the existence of the arc $ab$ implies the existence of $ba$ and thus one way communication is ruled out.

In the case where there is a finite number of agents in the set $A$ this is easy to envisage. The resulting stochastic graph can be denoted $\Gamma_g(p_{ab})$.

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5This was introduced by Kirman (1983) and developed by Kirman, Oddou and Weber (1986), Durlauf (1989) and Ioannides (1990).

6This, of course, excludes a whole class of interesting economic models in which the direction of communication is important. Much of production involves transactions which are necessarily in one direction from inputs to outputs, for example. This can be taken care of by specifying whether the link is active and then specifying the nature of the transaction. More difficult to handle is, for example, the transmission of information from a central source which is clearly best handled by considering the links as directed. Evstigneev (1994) has modelled some economic equilibrium problems using directed graphs but, as he indicates, the task is made difficult by the fact that some of the mathematical tools, which make the undirected case tractable, are not available for directed graphs.
2.7 CONNECTIVITY

The easiest way to envisage this is to think of the matrix in which each entry is the probability that \( a \) is linked with \( b \) and here I will only consider the case in which the matrix is symmetric, i.e. where \( p_{ab} = p_{ba} \).

If there is no obvious underlying topological structure which is thought of as affecting the probability that agents communicate, then, one could consider \( p_{ab} = p \) that is the probability of interaction is the same regardless of “who” or “where” the individuals are. Thus global interaction is a special case of this model.

2.7 Connectivity

It is often the case that our idea about how connected the world is, differs from the real situation. When we meet someone and find, to our surprise, that we have a common friend we have a tendency to consider that this was, a priori, highly unlikely. It is, of course, more likely than it would have been had the other person been literally picked at random but an interesting question is just how unlikely was it? I will try to explain, in what follows, why even in the purely random case the probability of having a common friend is much higher than one might have expected.

Indeed, the graph representing the links through which interaction takes place may become very highly connected as the number of agents increases, provided that the probability that any two individuals are connected does not go to zero too fast. Before explaining this it is important to know what one means by connectivity. A fairly standard measure of the connectivity of a graph is its diameter. This is defined quite simply. Consider all pairs of agents and for each pair count the number of links on the shortest path between them this is referred to as the distance between the two individuals or nodes. Now take all pairs and look at the distance between them. There will be a pair or pairs with the longest distance. This distance is referred to as the diameter of the graph. Now the idea is to see what happens to the diameter as the number of individuals increases. To understand what is meant by this, consider a result of Bollobas which was used by Kirman et al., (1986) to prove their main result. He shows that if the probability that any two agents know each other in a graph with \( n \) nodes, \( p_{ab}^n \), is greater than \( \frac{1}{\sqrt{n}} \) then as \( n \) becomes large it becomes certain that the diameter of the graph, \( D (\Gamma^n (p_{ab}^n)) \) will be 2. More formally,

\[
\lim_{n \to \infty} \text{Prob} (D (\Gamma^n (p_{ab}^n)) \geq 2) = 1
\]

In other words it is sure that any two individuals will have a “common friend” if the graph is large enough. Thus, as was observed in Kirman et al. (1986), one should say on encountering someone with whom one has a common friend, “it’s a large world” and not make the more conventional observation.

This somewhat surprising result suggests that, as sociologists have long observed empirically, relational networks are likely to be much more connected than one might imagine. This is important in economic models, since the degree of connectedness determines how fast information or a “technological shock” diffuses and how quickly an epidemic of opinion or behaviour will occur. Thus the aggregate consequences of
a local change will be very much linked to the structure and degree of connectivity of
the network through which information flows.

It is important to note that the result just evoked depends crucially on the fact that
the actors in the network are linked with uniform probability or, slightly more generally,
that the pair of agents with the lowest probability of being linked should still be above
the lower bound mentioned. This “small world” problem is rather different to that
studied by Watts (1999) but he looked at networks in which the links are changing
stochastically and I will come back to this. The dynamic evolution of the state of the
individuals linked in a graph like structure is particularly interesting since the stable
configurations of states, if there are any, will depend on the graph in question and some
of the results from other disciplines (see Weisbuch (1990)) can be evoked in the context
of economic models.

In this context it is interesting to examine what happens when, although agents
modify their behaviour in the light of their own and their neighbours’ experience, the
consequences of their behaviour may affect other agents further afield. Weisbuch et
al. (1994) for example show how agents may chose polluting or non polluting devices
from local experience but their choice may result in pollution which diffuses widely.
The consequence of this may be a rather sharp division into areas in which all the
agents have adopted one type of device while in another area the alternative device will
be used.

In financial markets the role of connectivity is far from clear, on the one hand it
may improve the diversification of risk, but this may be illusory when the risks are
correlated, and on the other hand, it may increase the rapidity of contagion. As Gai and
Kapadia (2008) say,

“While greater connectivity may reduce the probability of contagion, it
could also increase its spread should problems occur. Adverse aggregate
shocks and liquidity risk also amplify the likelihood and extent of con-
tagion. Our results suggest that financial systems may exhibit a robust-
yet-fragile tendency. They also highlight how a priori indistinguishable
shocks can have vastly different consequences financial market partici-
pants and policymakers would be unwise to draw too much comfort from
the resilience of financial systems to past shocks.”

In particular they emphasise the knock-on effects that can occur, as they say,

“… conditional on the failure of one institution triggering contagious de-
faults, a higher number of financial linkages also increases the potential
for contagion to spread more widely. In particular, greater connectivity
increases the chances that institutions which survive the effects of the ini-
tial default will be exposed to more than one defaulting counterparty after
the first round of contagion, thus making them vulnerable to a second-
round default. The impact of any crisis that does occur could, therefore,
be larger.”

Again, we see the problem with complex systems, simple arguments as to the im-
pact of the structure of networks are likely to be misleading and it is important to
understand the interdependence of these features. Connectivity is only one part of the puzzle albeit an important one.

2.8 Emerging networks: The choice of links

So far I have considered economic activity as being organised with a network structure which is given exogenously. That is, either the underlying set of links was given or the probability that they form was exogenous. However one of the most interesting challenges in examining the functioning of an economy is to include the evolution of the network structures themselves. If one wants to proceed to a theory of endogenous network formation, a first step might be to find out which organisations of individuals are "stable" or "equilibria". Once one finds such states then one would define a dynamic process of network evolution and see whether such situations would constitute "rest-points" of such a process. Thus, such rest points would be arrangements, or networks, which would not be subject to endogenous pressures to change them. This, as it stands, is not a well formulated concept. More of the rules under which agents operate have to be spelled out. The dynamics of the system will be defined by the way in which links are formed. One choice in economics have been to consider links as being consciously chosen by individuals. To define an equilibrium, one might require that no pair of linked individuals would want to break a link in such a situation, and furthermore, that no pair of currently unlinked agent would both prefer to be linked. An alternative has been to allow the links to be created as a consequence of the advantage that individuals have accrued from using them. An equilibrium would then be a situation in which further experience would not lead agents to change their partners. To understand the problems involved, consider first situations in which agents consciously choose their partners.

2.9 The strategic formation of networks.

A substantial literature has developed in which economic agents deliberately and strategically choose their partners. Thus the strategy of a player will be to choose those players to whom he wishes to be connected. Of course since links are undirected there has to be a rule which determines what happens when the choices are inconsistent. Once such a rule is given one can then see which choices of strategies by individuals constitute an equilibrium in the sense that I have mentioned, which is that the graph resulting from the choices of the individuals would be immune to change. By this, I mean that if agents have the possibility to add or remove links, which graphs would remain unchanged? This is a question which has been studied by Jackson and Wolinsky (1996) and others. They consider models in which links are valuable to players since they connect them directly and indirectly to others. However they may also be costly to form or maintain. They assume that the greater the distance one player is from another, the lower the corresponding utility that their being linked gives. They then examine the characteristics of stable graphs and this provides a reference point against which to set the rest-points, if any, of a model of evolving networks. Here the notion of equi-
CHAPTER 2. THE STRUCTURE OF INTERACTION

librium is that of a Nash equilibrium and what one is looking for is a graph structure where no individual would like to change his choice of partners given the choices of the other players. Jackson has contributed a number of papers in this direction as has Goyal (for surveys see Jackson (2008) and Goyal (2007). Their aim is to characterise the structures of equilibrium graphs.

To be more specific, consider an example given by Bala and Goyal (2000). The idea is that individuals can profit from the information that others obtain from those to whom they are linked, by creating such links themselves. There is a cost to creating a link but one gets the benefit of the knowledge flowing through the network to which one is connected, subject to some delay or decay. The architecture of equilibrium networks if knowledge is widely dispersed is striking. The networks will be of a star form with one agent at the centre and the others linked directly to her. This is, of course precisely the same form as that of the competitive market mentioned earlier. Two characteristics of such graphs are important. They display, by definition, a high degree of centrality and they are very highly connected. The diameter, or longest distance between any two participants is two. In this example only those who create links pay for them and so nobody pays for more than one link and everyone is closely linked with everyone else. A difficulty with such structures and one that has been noted in the context of transportation networks is that they are very vulnerable to the breakdown of any link. Each individual is dependent for all his information on one link to one player.

A second example is due to Goyal and Joshi (2002) and studies an oligopoly model in which firms can collude with others. Firms engage a certain amount of resources in the pairwise arrangements and in return have lower marginal costs. The firms then compete by setting quantities. It turns out that if marginal costs diminish by a constant amount for each link formed then, given the type of competition, there are increasing returns to link formation. The resultant architecture of the graph is interesting. It is formed of a dominant group or clique and a number of isolated players. This configuration is familiar from the work on coalition formation, (see Bloch (1995), Thoron (2000) and Demange and Wooders (2005) for example). It is also characteristic of certain stochastic graphs. What is intriguing is that we can see an asymmetric structure form even though all the players were symmetric to start with. Suppose that we return to Goyal and Joshi’s example and now allow firms to make side payments to other firms. In this case the star form re-emerges since one player at the centre can benefit from the increasing returns and can make compensating payments to the others. The structure of the network that emerges depends crucially on the rules under which the agents operate.

Jackson and Watts (2000) have also contributed to this literature by examining the equilibrium structures of graphs formed strategically. A number of problems remain in this area since one often assumes that players choose their links according to some pre-established protocol and this protocol is ad hoc. In other words players choose their links sequentially but the order in which they do so is specified by the modeller. Furthermore, the pay-offs to the players depend on the structure of the graph but the way in which this dependence is determined is often not specified. All that one knows is that each graph structure attributes a payment to each player. In other words each player knows what his pay-off will be if he knows the structure of the graph linking all the players. It seems a little difficult to believe that this would be the case, particularly if
there are many players. However, if the individuals do not have this sort of information they will not be able to choose which links to form or to break. What is obviously important here is how the pay-offs for the individuals depend on the graph. If only local links affect pay-offs, for example, the choices of partners becomes easier, but this sort of structure is not always discussed in the literature. One last observation is in order. If agents were really intelligent they would reflect on the response of others to their modifications of links. So, if I make or break a link to another agent how will this affect his pay-off and will he then be motivated to change his own links. Typically in economic models they are myopic and do not reflect on this sort of problem. Although, in a certain sense, graphs emerge in this context are static in the sense that the pay-offs are known and once formed they are fixed. Furthermore, no learning is involved, all the players know everything from the outset and the graphs that are stable are determined by that knowledge. Perhaps one should think of this sort of process as a sort of equilibrium selection rather than as a genuinely dynamic process.

2.10 Emerging Random Graphs

It seems to me however, that the most interesting challenge in this area, is to study the evolution of the communications graph itself. Such a process may not be due to choices by the agents. Work by Watts (1999), for example, has shown how structure may emerge as links are replaced by other links. What he examines is a situation in which agents have a fixed number of links with others and, for example they are all situated on a circle and are just linked with their immediate neighbours. Then he draws one of the existing links at random and replaces it with a new link. This may be to any agent, in particular, it could be to one to whom the distance was great in the original graph. Adding such links drastically increases the connectivity of the graph. This procedure is repeated and what emerges is a typically clustered structure in which closely linked individuals in a small group are linked to other groups through one or two links. In a pure random graph distances are short as we have seen but there is almost no clustering. However, in the sort of “small world” graphs studied by Watts (1999) the situation is different, distances are still short but there is a great deal of clustering. Here the few long links between different groups keep the distance down but most of the interaction is within small groups. This sort of clustering is observed in many empirical situations in economics.

Potts (2001) has argued that the small world structure is consistent with the idea that knowledge is concentrated within entities such as firms but is exchanged with other firms through the long links provided by markets. What is lacking for the economist in these models is any explanation of why links are being drawn and replaced. If an economic agent is to be linked to another one would like to be able to give some reason for that happening. It should also be mentioned that the degree distribution of such graphs does not correspond to many empirical examples.

As I have already said, an obvious way in which to proceed is to specify models in which the links between agents are reinforced over time by the gain derived from those links. An interesting counter example to this is the work of Michael Chwe (1994) who considered agents who are “far-sighted” and contemplate the reactions to their actions.
links. Thus longstanding economic relationships would be derived endogenously from the agents' experience. Instead of thinking of learning only at the individual level, the economy as a whole could be considered as learning and the graph representing the communication in the economy as evolving.\footnote{For a discussion of the formal problem of evolving networks see Weisbuch (1990).}

Nick Vriend (1995), presents a first step to simulating a model in which either the links themselves or the probability that they will be used over time evolve. He considers a market in which buyers learn where to shop and firms learn, from experience, how much to stock. Thus, the network here consists of the links between a set of buyers and a set of sellers. The interesting question is how will this network evolve over time? In this model firms sell indivisible units of a homogeneous good, the price of which is fixed and agents demand at most one unit. This is the most rudimentary model possible because the only criterion for success that the buyers have is whether they are served, and sellers are only concerned with providing the correct amount to satisfy demand. Nevertheless it is particularly interesting to note the development and persistence of a non degenerate size distribution of firms even though all firms are identical to start with. Furthermore some buyers always return to the same store, whilst others continue to search. There is empirical evidence for this sort of division of activity both on product markets and in financial markets. Thus, in Vriend's model, relationships between traders do evolve over time and a number of stable bilateral arrangements emerge. Vriend adopts what has come to be called the "artificial life" approach, that is, his agents are initially totally ignorant and merely update the probability of performing actions given the rewards that those actions generate.\footnote{Tesfatsion (1995) gives a full account of the artificial life approach to economics and its merits and drawbacks.} To be more precise, he uses a "classifier system". In this agents have conditional rules, consisting of an "if... then" statement, and then modify the choice of rules in the light of experience.\footnote{This so-called "classifier" approach was introduced by Holland (1992).}

In an extension to Vriend's original model, Kirman and Vriend (2001) wished to see whether making the behaviour of the buyers and sellers more elaborate would change the sort of network that emerges. They consider individuals who make more than one encounter in a trading day. Sellers now set prices they charge to each of their customers and allow the latter to choose whether to accept or refuse these prices. Here the number of rules to choose from is vastly greater than in the original model and this poses considerable problems if only for computational reasons. Nevertheless, individual buyers in the model soon learn which prices to accept and which to reject. Furthermore, sellers start to discriminate between buyers and charge their loyal customers different prices than those set to "searchers". Interestingly, some sellers set high prices to buyers and give them priority when there is insufficient stock to serve all the customers. Others do the opposite, giving low prices to loyal customers but serving searchers first at higher prices. Although the former strategy yields higher profits, individuals who adopt the low price strategy for their loyal customers get "locked in" and are unable to learn their way to the alternative strategy. Thus a "dominated" type of behaviour coexists with a superior one. The outcome of the process through which the market organises itself would have been impossible to predict by looking at the individuals in isolation and is clearly linked to the structure of the graph of relations which emerges. I will give a
In another example of a model in which links evolve, Tesfatsion (1997) considers individuals who have the possibility of accepting trading opportunities with other traders\textsuperscript{11}. Trading corresponds to playing a repeated prisoner’s dilemma game. However, players can choose whether to play against other players or not on the basis of expected payoffs. These payoffs are based on previous experience with that particular partner, thus the capacity to identify those with whom one interacts is crucial. Positive probability is assigned to being matched with “acceptable” players, while the links between those couples of which one member finds the other unacceptable are assigned probability zero. The criterion for accepting another player is whether the expected payoff from playing against that individual is above some threshold level. The possibility of rejecting partners can lead, as Tesfatsion shows, to the emergence and persistence of groups with different pay-offs. This seems quite reasonable since there is no reason for people continuing to interact with people who consistently play in an unacceptable way. This recalls another contribution by Hauk (\textsuperscript{12}) who referred to “wallflowers”, people at a dance who cannot find partners. She wanted to know what would happen if people could be ostracised if they played hostile strategies and showed that this could improve cooperation.

Tesfatsion’s results are, like most of the work in this area, obtained from simulations and are thus open to the standard criticisms. What one really wants to obtain are analytic results, providing a benchmark against which to measure the results of such simulations. The basic problem and the approaches to analysing it can all be posed within a simple framework. Recall that what we are interested in, is the evolution of the graph representing the interaction between the different individuals in the economy. The problem is to move on from the previous specification of a random graph to one where that graph itself changes over time. In the basic model links exist, or are used, with certain probabilities, but then the question that immediately arises is how will this change over time? Thus the underlying idea is to look at how the probability that certain links between agents will be used evolves. What should become clear is that, the dynamics of the process we are interested in can be seen as the evolution of probability distributions over time. To see why this is so we have to see how a random graph can be seen as a probability distribution, and then to show how the distribution evolves.

One way of viewing a graph with \( n \) nodes is by its incidence matrix. The latter is nothing other than an \( n \times n \) matrix with elements \( a_{ij} \) which are equal to one if the link between \( i \) and \( j \) exists and to zero otherwise. With this in mind we can model an evolving graph for a fixed population of size \( n \). One way of doing this is to consider the set of all possible graphs, that is the \( 2^{n^2} \times n \times n \) incidence matrices and to define a probability distribution over them. Thus a random directed graph is nothing other than a point in the unit simplex \( S \) in \( R^k \) where \( k = 2^{n^2} \) with the appropriate reduction in dimension for an undirected graph since the matrix is then symmetric\textsuperscript{12}. The evolution of the random graph is then described by a mapping from \( S \) into \( S \) and the dynamics will then be determined by the particular form of learning used to update the probabilities.

\textsuperscript{11}This is a development of the type of model developed by Stanley et al. (1994).

\textsuperscript{12}This is an alternative to the way of specifying random graphs mentioned previously, which was to draw each of the links \( a, b \) with a probability \( p_{a,b} \).
attached to the links. This all seems rather technical but is nothing other than a formal
description of the transition from one random graph to another. The law governing the
transition will, of course, be determined by the particular application being analysed.

To see what is going on, observe that a vertex of the simplex corresponds to a deter-
mministic network whilst the barycentre corresponds to the uniform probability model
which in turn is the same as the situation in which all agents are linked to each other
with probability $\frac{1}{2}$. As I have said, the question of how networks evolve can be seen
in this general framework. Different models yield different ways of modifying the
probability distribution corresponding to the communication network of the economy.
The dynamics engendered by the different specifications may yield very different re-
results. In some cases there may be convergence to a deterministic graph, in others, there
may be convergence, not to a vertex, but to some other particular point in the simplex.
In this case the network will not be deterministic but the probability distribution over
the links in the graph will converge over time. It may well also be the case that the
dynamics never converge and that no limit distribution exists. Careful specification
of the updating mechanism is needed to determine how the resulting network evolves.
Thus, depending on the particular specification one chooses one should be able to ob-
serve the evolution of trading groups and partnerships in markets and the development
of groups playing certain strategies amongst themselves in models based on repeated
non-cooperative game models for example. It should, of course, be clear that the evo-
lution of purely deterministic graphs is a special case of the model just discussed.

An obvious extension, already mentioned, of this analytic framework is to consider
agents as having several different types of functional links. They might be linked with
fellow workers in a firm, with trading partners, or with members of a household for
example. We are all embedded in many networks through our family, our work, our
recreation and many other activities. However the analysis of this sort of multi-layered
graph seems to be rather intractable and in economic analysis we have had to content
ourselves with the simplest cases.

2.11 An example: The emergence of buyer-seller net-
works

In the chapter on fish markets we will see how the approach just suggested translates
into more concrete form. There I will consider a model of a simple market similar to
that already mentioned of Kirman and Vriend (2001) in which Weisbuch et al. (2000)
analyse a wholesale market in which buyers update their probability of visiting sellers
on the basis of the profit that they obtained in the past from those sellers. What is shown
is that a clear pattern of relations emerges and that the buyers on the market are divided
into two groups, those who are loyal to one seller and those who “shop around”.

Why, in this example, is it important to understand the nature of the trading re-
lationships that emerge? The answer is that the aggregate efficiency of the market
depends on them. A market where agents are predominantly searchers is less efficient,
in a certain sense, than one with many loyal traders. When searching is preeminent,
sellers’ supply will often not be equal to the demand they face. Some buyers will be
unsatisfied and some sellers will be left with stock on their hands. This is particularly important in markets for perishable goods.

All of this recalls an earlier model of Whittle (1986) where there are two sorts of individuals, farmers and traders. Under certain conditions markets may emerge with well structured relationships between buyers and sellers where previously there were only itinerant traders.

The idea that networks have played an important part in markets is one which is far from recent. As an example of a contribution which analyses the evolution of historical networks in an explicit way is that by McLean and Padgett (1995). This is related to Weisbuch et al. (2000) and to an early study by Cohen (1966). McLean and Padgett study data on transactions in Renaissance Florence\footnote{Matt Jackson (2008) also examines this example in some detail.} for as Dobbin (2004) says,

> “Evidence that social networks shaped Florentine economic practices comes in several forms. In one set of analyses, business partnerships and marriage partnerships showed striking parallels, reflecting the move from a traditional and locally based system to a modern and cosmopolitan system. In the wool industry, partnerships among elites tended to be based on shared location, whereas elite bankers deliberately partnered with bankers from other locales. These differences are reflected in marriage patterns: a local pattern of marriages prevailed among wool-making families and a more cosmopolitan pattern of marriages among banking families.” Dobbin (2004) p.14.

In particular, Mclean and Padgett (1995) look at which markets were disordered and which were ordered, in the sense that order corresponds to an essentially deterministic graph in which each buyer is linked with one seller and disorder corresponds to each potential link having the same probability. As the theory developed by Weisbuch et al. (2000) predicts, the two markets, those for banking and wool which were the most active and generated the greatest profits were ordered whereas the others were disordered. However, the interpretation by Mclean and Padgett is worth examining. In their vision, disordered markets correspond to perfectly competitive ones since, if every buyer has the same probability of going to any seller then the authors claim that the same price will prevail everywhere. In this case, individuals have no incentive to privilege any particular seller.

Yet, there is another feature of the market which has to be taken into account. This is the inefficiency of the purely random market. As I have mentioned, if agents search at random, there will always be sellers in excess supply or demand, and there will also be unsatisfied buyers. Therefore, even in the case of durable goods, there will either be unnecessary stocks of goods or rationing. Buyers in ordered markets have learned to take into account the possible lack of goods, and their loyalty leads to a more efficient matching of demand and supply. Thus, even with a uniform price, there is pressure for loyalty to emerge. To describe the random market as reflecting competitive pricing does not seem to be justified. In fact, one might have the opposite view. Switching between suppliers must involve the risk of not being satisfied. Therefore buyers will only “shop around” if there is known to be a non degenerate price distribution, otherwise they have
no reason to switch sellers. Mclean and Padgett’s argument could only be justified if they produced data to show that the “law of one price” held in disordered markets. Yet, as they point out, no data are available on the prices at which the transactions were made in the Florentine markets.

2.12 Preferential Graphs

So far I have said little about the structure of economic graphs yet we would like to know what sort of characteristics we might expect to see in various sorts of economic situations. One of the features of graphs that has received attention in the economics literature, is the degree distribution. If individuals are nodes, then the degree of an agent is how many other individuals he is linked to. The degree distribution is then the frequency distribution of degrees across nodes. Take, as a benchmark, the standard random graph where links are drawn with a certain probability and then one can see what the degree distribution it generates should look like. However, it is well established that many social and economic networks do not have a degree distribution consistent with one generated by such a graph. In particular, in many applications it seems that the distribution often has too much weight in the tails, that is more extreme degrees are observed too frequently. This “fat tails” property has been observed in many contexts and it will return in other contexts in this book. A distribution that has this property is one that is due to Pareto and is one which is “scale free” and is often known as a “power law”. Denote by $d$ the degree and by $f(d)$ the relative frequency of the nodes with degree $d$, then one can write the power law as:

$$f(d) = ad^{-b}$$

for given parameters $a$ and $b$. One way to derive such a distribution is to assume that a new links are added to existing nodes with probabilities proportional to the degree of those nodes. This is the basis for Zipf’s Law for the size of cities, where new inhabitants are added to cities in proportion to the size of those cities. Barabasi and Albert (2001) developed a model of this type and referred to it as one of “preferential link attachment”. Buried in many models is an assumption of this type. For example, in some models, more new capital will accrue to firms who already have high levels of capital. This will generate power laws in the distribution of capital across firms. Yet in many empirical cases networks seem to have degree distributions somewhere between those corresponding to the random graph and the “scale free” case. In particular, the distribution and structure of clusters associated with graphs with a power law degree distribution seems inconsistent, in many cases, with empirical evidence and as a result hybrid models have been developed which seem to fit better with the empirical facts, (see Jackson and Rogers (2007)).

2.13 Another example: Free trade agreements

An obvious example of structured networks is that of free trade agreements involving several countries. One of the problems with economic models using graphs, as I have
mentioned, is that the payments from the graph structure are not made specific so the relationship between the structure and the pay-offs is not clear. So, although we are told that for each graph of relationships what the pay-offs to all the individuals are, there is little indication as to where those pay-offs come from. However, there are some attempts to justify the relationship between the pay-offs and the graph structure in economic terms. Indeed this is the justification for arguing that the graph governing the interaction between individuals is important in the economy. An example of the sort of thing I have in mind is provided by Goyal and Joshi (2006) who build a model of free trade agreements which gives a specific form to the pay-offs and is worth spelling out in some detail here.

Suppose that there are \( n \) countries and in each there is a single firm that can sell either at home or abroad. As in the classic customs union literature suppose that initially tariffs are prohibitive but if countries join an agreement all tariffs between them are removed. Once countries have chosen their partners, that is those with whom they have links the network \( g \) is determined. Denote by \( N_i (g) \) the countries that are in an agreement with country \( i \) then only firms from these countries can compete in \( i \)’s market. Denote the output of firm \( j \) sold in country \( i \) by \( Q_{ij} \). The total sold in that country is then:

\[
Q_i = \sum_{j \in N_i (g)} Q^i_j + Q^i_i
\]

Each country has the same linear demand curve given by: \( P_i = \alpha - Q_i \) where \( \alpha > 0 \) and they each have the same constant marginal cost of production \( \gamma \) with \( \alpha > \gamma \). It is clear that the number of firms selling in country \( i \) is then \( \# N_i (g) + 1 \). For each firm \( j \) active in that market the output sold there will be:

\[
Q^i_j = \frac{\alpha - \gamma}{\# N_i (g) + 2}
\]

With this we can now write the gain for country \( i \) by:

\[
\Pi_i (g) = \frac{1}{2} \left[ \frac{(\alpha - \gamma) (\# N_i (g) + 1)}{\# N_i (g) + 2} \right]^2 + \sum_{j \in N_i (g) \cup \{i\}} \left[ \frac{\alpha - \gamma}{\# N_i (g) + 2} \right]^2
\]

This is, of course, a very simplistic expression which is just the sum of consumer surplus and the domestic firm’s profit.

Now, given this simple structure, Goyal and Joshi (2006) examine what graphs will be stable. In other words which free trade agreements will be viable. What they show is, that if the gains are of this form, then the only pairwise stable networks are either the complete network or a network with two components, one isolated country and all the others in a completely connected network. Furthermore, the only efficient network, in the sense that there is no improvement possible for any country which does not imply a loss for some other country, is the complete one. This is a rather clear result and reveals the three effects at work. Firstly the domestic firm loses when the agreement is signed on its domestic market, but secondly it gains profit from its access to foreign markets. Thirdly prices fall so domestic consumers gain. Whether or not a country will remain isolated depends on the trade-off between these factors and the size of the agreement.
An interesting empirical question is the nature of existing free trade agreements and in certain cases there are cliques such as NAFTA, though contrary to the assumption in the model, external tariffs are not prohibitive. In examining empirical examples, as Furusawa and Konishi (2005) and Zisimos (2007) have done, one has to bear in mind that many of the assumptions of the simple model just presented are violated.

There are many other examples of how graph structures can have an effect on economic outcomes and a number of these are discussed in Rauch and Casella (2001) and in Podolny and Rauch (2007). In particular, there are examples in which agents generate positive and negative externalities for each other and the activities of individuals will be highly influenced by those around them. Examples range from technological spillovers\footnote{The sort of model discussed by Arthur (1989) can be easily extended to examine the diffusion of technologies through a graph structure.} to pollution, and from imitative behaviour to the contagion effects that can arise in financial markets\footnote{The models of financial markets in which agents are led to change forecasts such as those of Lux and Marchesi (1999) and Brock and Hommes (1997) can be interpreted as networks in which individuals are linked to “gurus” who provide them with forecasts and who change their links as a function of the success of the rules, see Follmer et al (2005).}. In particular, in many situations there will be communities of agents who interact more intensively with each other than with those outside their own group. This may be because of some underlying common characteristic or because the individuals find it beneficial to do so. One interesting problem is to try to identify clusters in empirical networks. To take a simple approach adopted by Copic et al. (2009), think of agents within a community interacting with each other with a certain probability $p_{in}$ and with those in other communities with probability $p_{out}$ and suppose that these probabilities are the same across all communities. Now given the empirical evidence on actual interactions between agents try to find the probabilities and the partition of the individuals into groups that was most likely to have generated the observed data. This maximum likelihood approach enables one to rank all the possible community structures in terms of the probability that they generated the observations. In the paper cited we applied the method to citations between academic economic journals and managed to sort the journals into communities. This is a problem of very limited interest to most readers, except for academic economists who have long had the suspicion that there are clubs of economists who cite each other and publish in particular journals. However, it does not take much imagination to think of more important economic applications.

**2.13.1 The identification problem**

In all of the preceding discussion it was taken for granted that the network structure of economic activity does have an influence on the economic outcomes. In the theoretical models that have been developed, this is what is being assumed and many references to observed collective behaviour such as that of fads or fashions or imitative are based on the premise that the influence of networks is important in determining such phenomena. However, as Manski (1995) has pointed out the situation is complicated when it comes to testing such propositions empirically. Manski specifies three different explanations for correlated behaviour in groups or networks:
2.14 CONCLUSION

“contagious effects, wherein the propensity of an individual to behave in some way varies with the behaviour of the group, exogenous (contextual) effects, wherein the propensity of an individual to behave in some way varies with the exogenous characteristics of the group, correlated effects wherein individuals in the group have similar individual characteristics or face similar institutional environments”

What we are assuming in many of our economic models is that agents are influenced by their neighbours either by their choices, their expectations or by some other form of direct interaction. How can we distinguish what appears to be a clear case of contagion from a situation in which individuals choose independently but share some characteristics which makes them choose similarly? To give a rather facetious example suppose that we observe three individuals who know each other each go out and get drunk on New Year’s eve. A natural inference would be that they had discussed the matter and had mutually influenced each other. Suppose, however that on careful investigation we find that they are all Scottish and that people of Scottish origin habitually get drunk on New Year’s eve. In this case there is no contagion, but rather a simple shared characteristic. The question then is, how can we distinguish between these two possibilities?

The answer is, as pointed out by Brock and Durlauf (2001b), that it is very difficult and that we are in danger of overemphasising network effects. There is therefore a need to be prudent before going down the road of the sociologist Luhmann who implies that individuals do not matter and that activity can be interpreted through the network alone.

If one wants to consider the impact of networks in reality, perhaps what is most lacking in economics is good empirical evidence for the importance of network effects rather than anecdotes. There are isolated examples of such work such as that by Burt (1992), Glaeser et al. (2003) and Granovetter (1985) but this lies at the frontier with sociology and there is relatively little work to examine the results of local interaction mediated through networks. Much of this type of work is surveyed by Brock and Durlauf (2005) who use it to give an account of the importance of the identification problem raised by Manski. However, the real problem as I pointed out at the outset is with accepting that networks are central to the functioning of economies. Until this is recognised we will continue to search for explanatory factors other than the effects of interaction in order to be able to come back to our models with isolated individuals. In each case we can probably find such explanations and this will prevent us from moving in the right direction.

2.14 Conclusion

What I have tried to do in this chapter, is to give an account of how the direct interaction between individuals or firms or banks who are linked through a network structure can have an important impact on aggregate economic outcomes. The formation of expectations, the trades that are made, people’s preferences and the way in which people imitate the behaviour of others all produce collective outcomes which can be very different from those that would occur if the individual actors were isolated from each
other and only interacted through anonymous markets.

Throughout this discussion it is the relationship between individuals rather than some sort of mechanical or technical network that is under discussion. However, it is also important to understand that institutional arrangements which may hinder or catalyse interactions may have a large effect. But, the central idea was to focus on the impact of the direct and often interaction between individual decision makers and how they are influenced by the network in which they are situated. The idea is to move in the direction of sociologists who have always argued that understanding the networks in which economic activity is “embedded” is fundamental to understanding collective behaviour. The notion that economists can safely ignore the way in which individuals interact and the structure that mediates that interaction has had its day. Yet, to move beyond this simple statement and determine the mechanisms through which the network structure exerts its influence on aggregate economic outcomes is a task which is far from being accomplished. Furthermore, the study of how networks actually emerge, in reality, is the most challenging.

For the time being there are two rather separate strands in the literature, that which is based on the strategic choice of links and that in which the use of links is reinforced by the experience of the users of those links. Whether these two approaches will yield similar results remains to be seen. In some sense this is an evolutionary problem such as that faced by evolutionary game theory. To what extent do the networks that evolve over time represent some sort of equilibrium? The empirical answer depends very much on the horizons involved and the speed at which the results of interaction diffuse. Nevertheless, it seems to me clear that network analysis is becoming more and more central to economic thinking as indeed it should be.

Indeed, understanding the structure of the networks that make up the economy is not just an intellectual exercise, it is important for very practical reasons and policy makers are coming to appreciate this. I will leave the last word on this subject to Haldane of the Bank of England,

“Deregulation swept away banking segregation and, with it, decomposability of the financial network. The upshot was a predictable lack of network robustness. That is one reason why Glass-Steagall is now back on the international policy agenda. It may be the wrong or too narrow an answer. But it asks the right question: can network structure be altered to improve network robustness? Answering that question is a mighty task for the current generation of policymakers. Using network resilience as a metric for success would help ensure it was a productive one.” Haldane (2009).
Chapter 3

Fish Markets: An Example of the Emergence of Aggregate Coordination

“It is a peculiar fact that the literature on economics…contains so little discussion of the central institution that underlies neoclassical economics—the market.” (North, 1977, p.710)

3.1 Introduction

So far I have argued that the direct interaction between individuals is central to economic activity and that to understand that interaction and its consequences you have to analyse the networks that govern it. Let me now turn to those institutions which, in economics in general, are anonymous and skeletal but which, in reality are just the opposite. Of course, I mean markets. Indeed if you wanted to find an example of how some sort of order emerges from the complicated interaction between many people with differing interests and characteristics you would not have to look further than the markets that exist for the millions of goods traded every day. Markets fascinate people, they often seem to be chaotic without any obvious organisation. Commodities markets contain hundreds of agents shouting and signalling to their counterparts. Although the stock exchange has physically disappeared in some countries such as France it has been replaced by an electronic order book, linked to thousands of actors scattered around the globe trading when they wish, and receiving and inferring information from many different sources. Auction markets for cattle are run by auctioneers who speak so fast and incomprehensibly that only those in the know have any idea as to what is going on. Art auctions are apparently staid affairs but are attended by people who often represent the rich and make their offers known by minimal signs to the auctioneer. Tension is often high despite the impeccable behaviour of the participants. Fish markets, the subject of this chapter, are nearer the other end of the spectrum in terms of discretion, and visible
physical activity. Because of their atmosphere and their situation, either on the coast, or in the very heart of cities, they have a very particular appeal. This is perhaps because of the smell, the fact that they take place at unearthly hours, or the idea that the harvest of the sea is supplied by brave men pitting their strength and wits against the elements. These markets have been there since the dawn of history and endless accounts have been written describing them.

In this chapter, I will take a look at this very special type of market, and I will use fish markets to illustrate the basic theme of this book. Why fish markets? One of the things that is important here is firstly that they present a particular interest for economists. This is partly for historical reasons and partly because the fact that fish is perishable means that there is no physical link between markets on successive days. This means that if one wants to analyse them, complicated considerations of inventories are eliminated. But, more important from my point of view is that these are places where many people meet together, transact affairs in different ways and in different institutional settings. Some markets like those in Tokyo, Sydney or Ancona in Italy are organised on an auction basis, some, such as Marseille are organised on the basis of pairwise trading. In Iceland, some auctions are English, ascending price, some are Dutch, descending price. On the same day on each of these markets, different buyers pay different prices for the same fish, prices change over the day, neither buyers nor sellers know how much fish is available at the opening of the market, though they may have some indications. Some come often to the market, some visit it infrequently. On markets with pairwise trading, some buyers purchase their fish systematically from the same seller, some shop around for the best price. Loyal customers pay more than bargain hunters on average which is far from being true of all markets. Yet out of all of this emerges some sort of global coordination. On those days when more fish is available prices are, on average, lower than on those days when fish is scarce. Although the price for the same type of fish sold by the same seller to different buyers, may vary, the distribution of prices across the market, changes remarkably little over time.

My basic aim here is to provide some evidence for one of the rather simple claims that I make in this book, which is that the aggregate behaviour of a market may well present clear regularities which could be thought of as representing some sort of collective rationality. However as will become clear, there will be evidence for another basic claim which is that this aggregate regularity should not be considered as corresponding to individual rationality. Indeed, I want to reinforce my argument that the process of aggregation itself, may lead to regular behaviour on the aggregate level and I will give empirical evidence for this. What I hope will become clear, is that the evidence from the fish markets I examine, reinforces another of my arguments. This is that the relationship between the behaviour of the individual participants and the market as a whole is mediated by the way in which the market is organised. The way in which the market allocates resources depends on the type of market institution.

Using data for two particular markets, the wholesale fish markets in Marseille and Ancona, I will argue that, a fish market may behave at an aggregate level in the way that economists might expect, but that this is an artefact of the aggregation of a complicated interactive system and not the reflection of conventional individual behaviour. At the risk of being repetitive, the point here is that “nice” aggregate behaviour may appear in a situation in which individuals are clearly not acting in isolation and where the market
3.2 Fish Markets: Background

Fish markets have a long history both in terms of the description of how they function and in terms of their economic analysis. From Roman times on, they have been extensively described and analysed. A detailed account of the functioning of the surprisingly sophisticated main fish market in ancient Rome is given by De Ruyt (1983). The first market “bubble” is probably that for red mullet, a Mediterranean fish which became highly prized at the time of the Romans. Cicero, Horace, Juvenal, Martial, Pliny, Seneca and Suetonius all discuss in detail the price of this fish which they considered to be unreasonable and based on a fad. The price of large specimens of the fish rose to extraordinary levels during the Roman empire and at one point three specimens fetched 30,000 sesterces. Even allowing for the problems of converting to modern prices, (the consensus conversion gives $300) this was out of proportion to other consumption goods. As a result the emperor Tiberius was moved to impose a sumptuary
tax on the fish market. The bubble burst and Macrobius noted later that prices had become “reasonable” again. Fish markets were an important feature of Mediterranean life and since this was one of the first areas in which markets developed it is not surprising that there are many accounts of their functioning from the time of classical Greece till the present.

I have already said that fish markets have a particular interest for economists. This is because they exhibit two features one of which I have mentioned, which make them a natural subject for economic analysis. Firstly, fish is a perishable good and, as a result, stocks cannot be carried over from one day to another. This makes the formal economic analysis of the market simpler. Secondly the organisation of such markets varies from location to location with little obvious reason. In Iceland, for example there are 32 auctions, 18 of these are English, i.e. rising price and 14 are Dutch, i.e. descending price. At Lorient in France fish is sold through a combination of pairwise trading and auction, whilst at Sete it is sold by Dutch auction and at nearby Marseille by pairwise trading. The fish market in Sydney is conducted as two simultaneous Dutch auctions and at Ancona in Italy there are three simultaneous Dutch auctions and the same buyers bid on all of them. The comparison of different outcomes under different forms of organisation is an obvious research topic for economists. However, though I believe that this is a good justification for economists’ frequent references to fish markets, this aspect is one which has not received much attention to date. The last part of this chapter is a step in that direction.

3.3 Fish Markets: Previous Economic Analysis

In addition to having been the subject of many historical accounts, fish markets have a long tradition in the economic literature. They were the subject of a debate between John Stuart Mill (1869) and William Thornton (1870). The question at issue was the nature of the prices charged for the same type of fish during auctions. How could different prices for the same type of fish be observed? The two points of view were, firstly that there were either several possible equilibria, or no equilibrium prices at all, whilst the opposing point of view was that, what were observed, were out of equilibrium or disequilibrium prices. This debate was reopened by Negishi (1986). What emerged as the central issue of this debate, and one which is a recurring issue in economics, is the nature of demand and how we should interpret the notion of equilibrium, and whether such a notion is appropriate for a market such as that for fish.

Fish is essentially perishable, and sellers buy their stocks before the market opens. This is why markets such as Marseille, where trading is pairwise and no prices are posted, can be considered as ones in which, in each period, stocks are fixed and prices

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1 His article was followed by a discussion by Ekelund and Thommesen (1989) to which he replied in Negishi (1990).

2 In fact this is not quite true since some species can be sold again the next day. However, buyers are perfectly aware of which fish have already been placed on the market therefore there are no inventories of fresh fish. It was this feature that led Pareto and Marshall, or to be more precise, the followers of Marshall, to use it as an example.

3 Hicks (1989) pointed out that Marshall himself, actually talked about the corn market whereas his successors used the fish market to avoid the problems posed by the carrying over of inventories.
are used as a strategic variable by both buyers and sellers. The latter is not true on all fish markets, and, of course depends on the sort of market organisation which is being considered. In auction markets such as Sete the stocks are, in fact, fixed, they are determined by the quantities waiting on the boats lined up at the quay behind the auction hall, but since the catch has not been unloaded, the total is not known to the bidders in the auction, who start bidding as soon as the catch from the first boat is landed. In Ancona, where the catch is unloaded the evening before from the fishing boats the bidders have a general idea as to the amount of fish available but no detailed information. However, on these markets, the prices can hardly be considered as strategic variables since they are set through the auctioneer. But, on a market such as Marseille, which is the first market I will consider here, the prices are, as I have said, set by pairwise negotiation. Since no prices are posted in Marseille and total stocks are not common knowledge at the beginning of the day information is far from perfect. As I have said, this is also true of the maket in Sete, since the boats offload their catches as the auction progresses and the only guide to total quantity is how many vessels are left at any point in time. It is also interesting to note that the boats play a sort of game over the order in which they will land their catch. They do this by jockeying for their places in the line. In Ancona, on the other hand, the fish is already unloaded and waiting to be placed on the belts in the auction hall, and hence the buyers have a better overall idea of how much fish is available at the outset.

Most authors who have studied fish markets in the past have not made much reference to the particular organisation of the market but have simply analysed it in terms of a standard competitive market and have estimated a demand system for the fish market. Examples of this are Gorman (1959), in a well known unpublished paper, and Barten and Bettendorf (1989). Yet, I claim that the material organisation of the market is paramount in determining certain features of the allocation process or more prosaically the “division of the spoils”. Before making the arguments to justify this, let me first describe some of the details of the first market for which we had data, that in Marseille.4

3.4 The Marseille Fish Market (Saumaty)

At the time that the data was collected, the wholesale fish market for Marseille, situated at Saumaty on the coast at the Northern edge of Marseille, was open every day of the year from 2 a.m. to 6a.m.5 Over 500 buyers6 and 45 sellers came together, although they were not all present every day and they transacted more than 130 types of fish. Prices were not posted. All transactions were pairwise. There was little negotiation and prices can reasonably be regarded as take it or leave it prices given by the seller. The data set consists of the details of every individual transaction made over a period of three years. The data was systematically collected and recorded by the Chambre

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4Annick Vignes started collecting this data by hand, a monumental task, but then found that all the data was centrally recorded and after some negotiation she got access to it.
5The market now, unfortunately for Annick Vignes and myself, since I went there from time to time and she visited it regularly at the times mentioned, is now open during the day rather than at night.
6In fact some 1400 buyers appear in the records but many of these were hardly present at all.
de Commerce de Marseille which managed the market at that time. The following information is provided for each transaction:

1. The name of the buyer
2. The name of the seller
3. The type of fish
4. The weight of the lot
5. The price per kilo at which it was sold
6. The order of the transaction in the daily sales of the seller.

The data runs from the 1st of January 1988 to the 30th of June 1991. The total number of transactions for which we have data is 237,162.

Although no auction mechanism has been set up, when the market was being reorganised, provision was made for the establishment of an auction. Neither buyers nor sellers were favourable to this development and so the market has remained organised as previously. This is curious for an economist as one might have found the following simple argument convincing. Suppose that the current process allocates the goods in an efficient way, then the total gain generated by the market is maximal and the problem is how to divide it between the two groups, the buyers and the sellers. However the new process would presumably change this distribution. As we are talking about a zero sum game, one of the parties must win and the other lose. Why, then, was neither in favour of the change? If the current mechanism does not achieve efficiency then the auction, if efficient, would generate greater profits and both parties could gain.

There are two possible reasons that one could invoke for rejecting the change to an auction. Firstly, the buyers and sellers may simply have thought that an auction would be less efficient. Secondly and more plausible, they may just have been afraid of operating in a system with which they were not familiar. This seems the most obvious explanation for the sort of inertia that is often observed in institutional arrangements. Finally, the role of the actors and the actors themselves might change with the new mechanism. The sellers currently offer 90% of their fish from sources other than the Mediterranean sea. This fish is bought at auction from other ports. Would these sellers be able to buy like this and then to put the fish up for auction a second time or would shortcuts develop between the major buyers and the other markets?

Be all this as it may, the current arrangement in Marseille gathers together a large number of buyers and sellers who negotiate with each other over a stock of fish which is already determined at the opening of the market. Given the number of agents involved in the same place at the same time on a regular basis one might be led to expect that, after a while, prices would converge, in the sense that the same type of fish would be sold at essentially the same price on a given day. This, as I will show, is far from being the case. Furthermore, one might expect to see evidence of the standard assertion that, as the day progresses, prices diminish. This is again not true.
3.5 Market properties and Individual Behaviour

As I have remarked, markets are complicated affairs, organised in different ways and containing an intricate network of interacting relationships and fish markets are no exception. It is therefore not at all clear that the behaviour one will observe in the aggregate in such markets will correspond to some enlarged version of the behaviour of the individual in the classical competitive environment. Yet, in economics, aggregate behaviour is often tested to see if it meets restrictions that can be derived directly from individual maximising behaviour. Thus it is common practice to treat data arising from aggregate purchases of some commodity over time, as if these were the expression of the competitive demand of some representative individual. This approach involves a number of implicit assumptions, in particular that the underlying micro-data observed can be thought of as corresponding to individual Walrasian demand, and furthermore that aggregation considerations do not invalidate the use of restrictions derived from individual behaviour.

My basic argument here is that this is the wrong way to look at things. What the participants in markets do, depends on who they are in contact with. In the sort of market I am describing here, the individuals know each other and meet regularly. They have learned which partners to trade with and what sort of fish they will be able to buy and sell and at what prices they will trade with each of their partners. They have arrived at this situation by trying different prices in the past and by noting the reactions of their partners. Gradually a trading network emerges and the quantities that are traded and the prices at which transactions are made, evolve. This is a very different vision than that of a centralised market in which individuals have no contact with each other and react only to central price signals.

In this section, I will explain, following Hardle and Kirman (1995), that empirically, for the particular market that we study, in spite of the complexity of its organisation, certain standard properties of aggregate behaviour can be shown to hold. In particular we studied the property which might be thought of as “downward sloping demand curves” for individual fish. We found that such a property holds at the aggregate level although it does not hold at the individual level. Thus, in a certain sense, this behavioural “regularity” is more apparent at the aggregate than at the individual level.

At this point some well trained theorists would object and argue that it is obvious, from my description that this is not a perfectly competitive market and that it should be modelled, not as a competitive one, but rather as a full-blown game and that, in this case, one might also be able to derive the monotonically declining price quantity relation. In other words, if we think of individuals who anticipate that their own behaviour will have an influence on the market, and who anticipate the reactions of the other participants, then we might find that this leads to a downward sloping relation between price and quantity sold. However there is no theoretical reason to expect any such simple aggregate relation. In neither the competitive nor the strategic case is there any simple passage from individual to aggregate behaviour. Such a conclusion is directly in the line of work by Becker (1962)\footnote{This idea has been developed by Gode and Sunder (1993).} who showed that downward sloping demand
curves at the market level could be derived from random individual choice behaviour subject only to a budget constraint. Whereas he summarised his result as saying that “households may be irrational and yet markets quite rational”, a better summary of the results here would be that “complicated interactions between individuals may lead to simple aggregate properties.”

I hope that it will become clear that it is inappropriate to think of purchases on the sort of markets considered here, as corresponding to Walrasian demand, and that it is not appropriate to think of prices, at any particular point in time, as equating aggregate supply and demand although the result of the process is that very little fish is left over at the end of the market.

3.6 The impact of market organisation

What is important here, is that it is the very organisation of the market for the product in question that prevents it from being competitive in the standard sense. Although it has often been argued that, in markets of this sort, the presence of a sufficient number of buyers and sellers will be sufficient to drive the price to a competitive one, this may well not be the case and casual empiricism suggests that considerable price dispersion may not only occur but may persist. In this case the notion of a single “market price” loses its significance. This problem arises in the other types of market arrangements used for the sale of fish. If different lots of the same type of fish are auctioned off successively, for example, the average price will not necessarily correspond to the price which would have solved the Walrasian problem for that market. Yet, with rare exceptions such as Laffont and Vuong (1993,1995) the standard approach in the empirical literature has been to treat even auction data as if it were generated by competitive behaviour. The problem here is that techniques for the econometric analysis of data arising from differently organized markets such as auctions, for example, have been little developed and there is always a temptation to return to standard and sophisticated techniques, even if these should not really be applied to the type of market in question. This explains the flight back to standard analysis in papers on this subject.

Two justifications are commonly used to suggest that the competitive outcome is a reasonable prediction more or less independent of the type of organisation. One is that used by those who perform experiments with “double auctions.” They argue that, despite the fact that in the markets that they examine individuals set prices and propose quantities, the result of this process is the same as that would have been obtained through the competitive mechanism. This argument reinforces my basic point. The fact that we observe an aggregate result that conforms to the predictions of a particular model is not enough to justify the conclusion that individuals are, actually behaving as they are assumed to do in that model.

A different justification is offered by Barten and Bettendorf (1989) who are well aware of the basic difficulty. They suggest that the aggregate behavior in the fish market

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8It should be noted here that Wilson (1977) and Milgrom (1981) have argued that in this framework too the price will converge to the “true value” when enough objects are auctioned among enough buyers. Thus here again economists could argue that prices aggregate all the information that might be circulating in a market.
3.6. THE IMPACT OF MARKET ORGANISATION

can be reduced to that of a Walrasian mechanism by looking at an inverse demand system. They reason as follows:

“Price taking producers and price taking consumers are linked by traders who select a price which they expect clears the market. In practice, this means that at the auction the wholesale traders offer prices for the fixed quantities which, after being augmented with a suitable margin, are suitably low to induce consumers to buy the available quantities. The traders set the prices as a function of the quantities. The causality goes from quantity to price.”

Although the authors are only making explicit what is commonly done and work in an auction context, it is clear that one should prove that, even if the auction price is well defined, it is indeed related to prices charged to consumers through a simple mark-up. Necessarily, if different wholesalers pay different prices for the same product and the mark-up principle does apply, then a distribution of prices will be observed on the retail market. However, it would be the case that for one seller all of his customers would pay the same price for the same good, so he would not be seen to discriminate between different buyers.

This brings me back to an important point. The Marseille market does not function as an auction, and individual traders strike bargains amongst themselves. They are well aware of each others’ identities, and, as a result, different prices can be, and are, charged to different purchasers for the same product. Thus discrimination is a factor in generating the distribution of prices which as I have already mentioned is an important feature of the market. There are significant variations in the average prices paid by different buyers (see Kirman and McCarthy [1990]) and a similar phenomenon has been observed by Graddy (1995) for the Fulton fish market in New York. One of the interesting results that has stemmed from more recent work on this market (see Graddy and Hall (2009)) is that although price discrimination is widely practiced, at least in theory, there is little gain for sellers from behaving in this way. This reinforces another argument of this book that the behaviour of individuals in relation to each other may not correspond to what one would expect from a pure optimiser. In any event price discrimination exists and, as a result, there is considerable price dispersion. This means that reducing prices to averages may well lose a significant feature of the data. Furthermore, it means that the average price cannot be regarded as a reasonable sufficient statistic for the way in which the market produces prices and that other properties of the price distribution must be taken into account. This reduces the plausibility of the argument advanced by Barten and Bettendorf.

As I have mentioned, an alternative approach would be to suggest that the fish market can be modeled as a situation of strategic interaction and then to define the appropriate game theoretic equilibrium price notion. If there is such an equilibrium, given the heterogeneity of the actors on the market, the fact that they meet pairwise, and that there is imperfect information, such an equilibrium will surely correspond to a non-degenerate price distribution (see Diamond [1987], Butters [1977], and Sorensen [2000]) rather than to a single “competitive” price for each type of fish.

A first simple test, therefore, as to whether this market can be thought of as being a good approximation of a competitive one, is to see whether there is, indeed, significant
price dispersion. If this is the case, some sort of model in which the interaction between agents is considered and there are many, would be more appropriate. Such evidence, at least in the Marseille market, is overwhelming as is pointed out in Hardle and Kirman (1995) and Kirman and Vignes (1991). Successive prices charged by one seller, for the same type of fish, to different buyers may differ by as much as 30% for example. Figure 3.1 illustrates the distribution for the prices for two types of fish over a week.

![Figure 3.1: the distribution for the prices for two types of fish over a week.](image)

This raises a second question. Given that there is significant dispersion of prices, does this situation correspond to some sort of equilibrium? Almost all of the models in the economic literature, which predicate a distribution of prices, (see Hopkins (2008) for an excellent survey), either maintain that the distribution of prices is fixed from
the outset or argue that there is an *equilibrium distribution* of prices. This is also what one might expect from a simple game theoretic analysis. Yet, observing the sort of difference in mean prices and variance from day to day, one would be hard put to argue that the distribution is constant over time. Even standardising to the same mean does not help. There are two possibilities, either the way in which this market is organized simply precludes any sort of intertemporal stability, or alternatively, perhaps the distribution taken over a longer period is stable.

Suppose that the latter were the case. Then the distribution of prices taken over a suitably long period might remain unchanged from one such period to the next. In this case, an equilibrium, or steady state distribution might be an appropriate concept. It is important to understand what is meant here by distribution. What we examined when analysing this problem in Hardle and Kirman (1995) was the total number of kilos transacted in each price interval. The alternative would be to count the number of transactions at each price level. Of course, the two would be equivalent if each kilo was the object of a separate transaction. In fact this is precisely how many authors avoid having to make this distinction since, in the literature on price dispersion, individuals are typically assumed to demand one unit of an invisible good (see Rothschild [1973] and Diamond [1987] for example). Thus the distribution of prices is given by

\[ h(p_j) = \frac{\sum \text{quantities sold at prices in } j\text{-th interval}}{\text{Total quantities sold}} \] (3.1)

Given the fluctuations in daily activity and the varying presence of different traders it seems unlikely that there would be any constancy from day to day for example, and this is already clear from looking at Figure 3.1.

However, as I have already remarked, if the market is, in a certain sense repeating itself, we might hope to find some sort of regularity for longer periods. We could, then, think of the market as moving from one day’s state to the next in a way which reflects the stochastic reactions of all the individuals to the current situation. In this case we could think of a very different equilibrium idea, that of the limit distribution of this process. So what one would look at would be the “time averages” of the fish prices over time and see if, in the long run, these settle down. In this case, if we plot the distribution over a sufficient period, it should not change if we do the same for a later period. This seems to be a very sensible notion of equilibrium but one which is far from the standard ones. It is worth remarking that I have not written down a theoretical model and shown that it does indeed have a limit distribution. Thus, the problem of the stability of the distribution of prices over time remains an empirical question. In Hardle and Kirman (1995) we tested for the constancy of the price distribution of each of four fish, trout, whiting, sardines and cod from month to month over three successive months. A month might seem a rather short period but the results for whiting were striking as can be seen in figure 3.2 (the results for the other three fish were very similar).

On the horizontal axis are the transaction prices and the figure is a non parametric smoothing of the histogram defined in Eq. (3.1). Although the visual evidence is convincing, we also tested formally for the intertemporal stability of the price distributions for these fish.

\[^9\text{We do exactly this in Foellmer et al. (2005) but for financial markets.}\]
Figure 3.2: need caption
3.7. PRICE QUANTITY RELATIONS

We proceeded by fitting the distributions for each of the months and then seeing by how much the distance of the fit varied from that of each of the other months. We then tested whether the differences between the fitted distributions were significant or not, (full details of the tests for the stability of the distribution are given in Hardle and Kirman (1995)). The important thing is that we could not reject the hypothesis that the distributions were constant over time. That is, when we considered the following hypotheses:

\[
H_0 : f_i = f_j, \quad i \neq j \\
H_1 : f_i \neq f_j, \quad i \neq j
\]

(3.2)

for each of four fish over the three months in question, in none of the cases could we reject.

There are, of course, a number of technical problems which I will not enter into here. Just as an example, although the evidence from the fitted densities seems to be clear, for the statistical tests for stability to be valid, the observations should be independently identically distributed. This cannot be strictly true, since certain buyers systematically pay higher prices for example. Although these buyers are probably of particular types, restaurants etc., they are only identified by code. There is, therefore, no prior information on which to condition and to treat them as being different.\(^\text{10}\)

Despite these caveats, there is very persuasive evidence that the distributions of the prices of each fish are remarkably stable over time. This in turn leads me to suggest that the market has organised itself so that a rather stable structure has emerged, even though a great deal of rather complicated interaction may be going on at the individual level. This is, of course, one of the main messages of this book, out of interactions between different subsets of individuals well defined aggregate behaviour can emerge.

3.7 Price quantity relations

The previous discussion shows that although the market has a certain sort of aggregate stability it cannot be characterised as competitive. However, one might then ask whether it satisfies any of the standard aggregate properties that one would have expected from a competitive market. If such properties can be established then, given the complicated nature of this particular market they may well, like the stability of the price distribution, be “emergent” rather than the simple sum of well behaved autonomous individuals.

What sort of property do I have in mind? A classical property is that of a monotonically declining price quantity relation for the transactions of each of the fish chosen. Note that I did not use the term “demand curve” and I will come back to this in a moment. In Hardle and Kirman (1995), we used non-parametric methods to fit two different aggregate price quantity relations and found that these relations, in contrast to those at the individual level, did indeed exhibit monotonicity. Economists, faced with the evidence at the aggregate level, might be tempted to say that what we are looking

\(^\text{10}\)In treating our observations as drawn from the same population in this way we were following Theil [1971] for example, who in his “convergence” approach thought of N consumers as independent elements of an infinite consumer population and the parameters of their utility functions as identically distributed.
CHAPTER 3. FISH MARKETS: AN EXAMPLE OF THE EMERGENCE OF AGGREGATE COORDINATION

This would fit very well with the idea that what changes from day to day is the supply of fish. In older markets or more local ones, the variations in supply were due to weather and variations in the amount of different fish in the areas being fished. The use of radar has led to the location of fish being easier to track but the weather remains a random variable. Once again, the Marseille market is much more complicated than that. Fish comes largely from other sources and these are so widespread that the weather is not a real factor, and furthermore the amounts on sale in Marseille depend on the prices in other ports both at home and abroad. This means that we cannot think of a fixed demand faced with an exogenously varying supply. As I suggested earlier, the obvious question is as to whether what we are looking at does, in fact, correspond to what one normally defines as demand.

At this point, I should put my cards on the table and say that all that the evidence shows is that the market organises itself in such a way that when less fish is available prices are, on average, higher. Yet there is a long tradition which has examined the sort of evidence that I have presented here and which has typically claimed to be estimating demand relationships.

3.8 A digression on “demand”

Many economists, I think rightly, would say, “what you are looking at is not “demand” in any standard sense”, while others would argue that I am doing nothing other than estimating aggregate demand. So it is worth spending a little time on just what it is that we are observing.

Until Marshall, there was considerable discussion as to the correct definition of demand for a single commodity. However, in the more formal literature there was convergence on the rather abstract Walrasian notion that individual demand simply represents the quantity of goods that an individual would purchase at given prices which he was unable to influence. The subsequent theoretical literature concentrated largely on the extension of the analysis to interdependent markets and the problem of demand systems rather than single demand equations while still maintaining the abstract Walrasian approach. Until recently, the idea that demand should be treated in this way has not really been challenged either in the economic nor in the econometric literature. In moving in this direction, the profession lost interest in the idea that market structure might be of importance in determining the outcome of the market. By this, I do not mean that there was no interest in whether individuals have market power, in the sense that they could influence prices, rather that the interest in the institutions through which transactions are made and prices are set, became marginal. The essential thing was that the individuals accept a “market price” as something over which they have no control.

Over the twentieth century the theoretical literature converged on the precise definition of demand that is to be found in the Arrow-Debreu model. Econometricians used this as their justification for concentrating on more sophisticated techniques for the estimation and identification of demand systems. The agreed definition, that of

11 Of course, if there is any substitutability between different species of fish one needs some sort of generalised law of demand such as that specified by Hildenbrand (1983), and I will come back to this later.
competitive demand, concerning the quantities of goods an individual would buy at
given prices, were he only constrained by his income, was retained. In a pioneering
contribution by Working (1927) the conceptual nature of demand and supply are not
questioned. The only real problem for him was that of which of the two was fluct-
uating over time. Thus, as far as he was concerned, the major problem was that of
"identification".

However for many markets, and this is the object of the exercise here, the con-
ceptual competitive framework is not satisfactory. For example, in our particular case,
the wholesale fish market in Marseille, all transactions are bilateral and no prices are
posted. When we look at the relation between the prices charged and the quantities
purchased on this sort of market, a number of questions which were very present in the
early debate as to the appropriate notion of "demand", recur.

To understand the basic issue, it is worth trying to answer a basic question. What are
the implicit assumptions underlying the usual empirical analysis based on Walrasian
demand theory and are they are appropriate here?

The first problem that arises is whether the purchaser of a good is, in fact, the final
consumer? If this is not the case then one would have to show that properties of indi-
vidual demand carry over to properties of quantities purchased by an intermediary at
different prices. Think of how individual demand is derived. The individual choose that
bundle of goods that maximise his utility and with all the classic assumptions this leads
to certain properties of demand. Yet, the retailer wants to obtain the maximum profit
when faced with a collection of such individuals. Why should the demand properties
of the individual buyers carry over to the demand of the intermediary or wholesaler?
If one considers the simple case of a purchaser who is a retailer and has a monopoly
locally of the product that he buys and resells, then it is easy to construct examples
in which this will not be the case. This question was raised by Working (1927) and
mentioned again in the classical studies by Schultz (1938), who although using indi-
vidual properties of demand made his estimations using data for farm prices and not
shop prices. More recently, in the specific study of the Belgian fish market already
mentioned, Barten and Bettendorf (1989) refer to this question.

The second problem that arises, even if one accepts that the wholesalers have the
same sort of demand as the final consumers, is whether the market does function "com-
petitively". Then there is the question already discussed by Working, the question of
identification, which has given rise to an enormous econometric literature, in this case,
separating out supply changes from demand changes. In a truly Walrasian, or Arrow
Debreu world such a distinction could, of course, not be made, since all transactions
over time are the result of supply and demand decisions taken in some initial period.
However this problem is usually circumvented in the empirical literature by making
an implicit assumption of stationarity and separability, i.e. that the market is somehow
repeated over time, and that decisions are taken in the same way at each point in time.
This should, of course, be tested but, if the assumption is justified, it does mean that
one can talk meaningfully of successive observations of the same market. The evidence
that I have given of the stability of price distributions over time does comfort the idea
that there is some sort of intertemporal stability of this market, but does not suggest that
market outcomes are similar from one day to the next. So, in this case the appropriate
theory is that referred to as temporary general equilibrium theory.
The idea here is that each day, the participants in the market form their expectations of future prices. They then form their demands and supplies and the market subsequently clears. Next period expectations are modified, in part, as a function of today’s prices and the market reopens and is again cleared. The problem with this is that, if all the agents have full knowledge of future prices the problem can be solved and indeed, markets would not need to reopen. However if there is uncertainty then, expectations have to be taken into account. The classic way around this from Muth on is to make the heroic assumption that expectations are “rational”, that is that all the market participants know the correct distribution of future prices. (I will come back to this in the chapter on financial markets).

There are two ways of justifying this, neither very satisfactory. Firstly, one can imagine that all the individuals are so clever that they can work the whole problem out and find the equilibrium distribution of prices. Secondly, and perhaps a little more reasonably, we can say that there is a distribution of prices such that, if all the individuals concerned believe that prices are drawn from it, and have the corresponding expectations, then this will be an equilibrium. We then say nothing about why individuals would have such a distribution in mind nor how they would learn to believe this. If expectations are not fully rational, and we have no reason to believe that they should be, then a number of basic difficulties arise. For example, short run demand loses many of the properties of its Walrasian counterpart. It does not satisfy homogeneity or the Weak Axiom of Revealed Preference for example (see, e.g., Grandmont [1983]). Trying to fit a demand system to an economy or market functioning in this way and basing it on the usual theoretical restrictions makes little sense therefore.

One might, however argue that there is little effect of prices today on prices tomorrow in the fish market. In other words we might accept the idea that changes in the prices of fish do not result in a large amount of intertemporal substitution, that is fishmongers are relatively myopic, then thinking of a sequence of equilibria in a market which repeats itself is more acceptable. Once again this explains why, when considering particular markets, fish has been so widely used as an example, (e.g. by Marshall, Pareto, Hicks) since with no stocks, successive markets can be thought of as more or less independent. In our case, when fitting our price quantity relations we are implicitly treating price changes as resulting from random shocks to the supply of fish although the amount available is, at least in part, a result of strategic choice.

We are avoiding a major difficulty here, since part of the vision that I want to convey portrays individuals in the market as learning. Thus one reason why prices in one period may affect those in another is that individuals are learning which prices to set or to accept. There are two observations to make about this. Firstly, since our market opens with a lot of uncertainty about the amount of fish available, then individual buyers will indeed learn during the day. They might also be learning how to behave in general and thus might react differently to the same situation from one period to the next. In a sense, we are assuming that the more general learning has taken place, although we have also studied this problem. This is a bit like the evolutionary argument that the current form of an organism is best suited to its current environment and that there are no significant changes in the morphology in the short or medium run. This is reasonable in a physical and slowly changing environment, but in a market where the environment is made up of other individuals who are also learning there is no ob-
vious reason to believe that the prices will settle to some stationary distribution and that agents will cease to learn. A nice illustration of this is given in figure 3.3 where the scientist believes that he has trained the rat to put food in the hopper when the rat pushes the button marked “food”, but where the rat is persuaded that he has trained the scientist to do so.

Figure 3.3: Who is learning?

But there is an even more important problem here which is that of aggregation. If we fit a demand system in the usual way we are assuming that market behaviour corresponds to that of an individual. More precisely, we are arguing that individuals respect the standard conditions on their characteristics, maximise in the usual way and that what we observe at the aggregate level, which is just the sum of the individual demands, reflects the properties of the individuals. In other words, if the aggregate
does behave as an individual should, then one claims that that is how individuals are actually behaving.

But what if the individuals are much less disciplined and that the aggregate behaviour is the result of the aggregation process rather than just reflecting the individuals? Think of Gary Becker’s (1962) analysis, which I mentioned earlier, and in which he showed that individuals choosing at random from their budget sets could collectively produce well behaved demand. Think also of Gode and Sunder’s (1993), “zero intelligence” agents who collectively achieve a competitive equilibrium. Is this what is going on here? The first thing to do to see if this is appropriate, is to look at individual behaviour. Unfortunately, or, perhaps fortunately from my point of view, examination of individual data reveals none of the properties that one would expect from standard individual demand. To see this look at figures 3.4a and 3.4b where quantities of fish of a particular type, sole, purchased by two different buyers are plotted against the price at which those quantities were purchased.

It would take an extremely optimistic econometrician to argue that the relationship between prices and quantities in these cases are best fitted by monotonically declining relations. Thus, even if such properties are found at the aggregate level, they cannot be attributed to individual behavior.

This is one side of the problem of aggregation. The other is that even if individuals did happen to satisfy certain properties it is by no means necessary that these properties carry over to the aggregate level (see e.g. Sonnenschein [1972] Mantel [1974] and Debreu [1974]). The two taken together mean that there is no direct connection between micro and macro behavior which is, after all, the main message of this book. This basic difficulty in the testing of aggregate models has been emphasised by a number of authors, (see Kirman [1992], Summers [1991] and Lewbel [1989]) when discussing representative individual macro models but as Lewbel observes, this has not, and is

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12The examples shown here are not at all exceptional we examined hundreds of such individual relations and for almost none of them was there any significant evidence of a monotone declining relationship.
unlikely to, stop the profession from testing individually derived hypotheses at the aggregate level. Hence although some empirical properties of the aggregate relationships between prices charged and quantities purchased can be established I would suggest that these should be viewed as the result of interaction and aggregation and not of standard maximizing individual behavior.

This should not stop us however, from trying to see whether or not the data do satisfy certain properties which would result in a standard model with isolated individual maximisers. This is of interest in itself. Now suppose that the market does exhibit such features and one claims as I do, that they do not correspond to classical individual maximizing behavior. Then it is incumbent on me to try to explain how the market organizes itself so that this comes about. A particular feature that one does observe in our case, is that over the day markets do, more or less, clear in the sense that the surplus left unsold never exceeds 4%. Furthermore, since sellers become aware, from the reactions of buyers to their offers, of the amount available on the market, and vice versa, it would not be unreasonable to expect average prices to be lower on those days where the quantity is higher. However, the situation is not simple. For example, some buyers transact early, before such information becomes available, and others only make one transaction for a given fish on a given day. Thus to deduce such a property formally would require very strong and unrealistic assumptions.

But first things first. To begin with, I have to establish that a property such as that of a monotone and negative relation between quantities purchased and prices is verified by the market data. This requires an empirical examination of the behavior of the market. The proposition to test when considering the four fish taken as examples, is whether the quantities purchased at each price \( D(p) \) for those fish displays the monotonicity property, i.e. for \( p \neq p' \) and \( p > 0, p' > 0, p \in R^+ \).

\[
(D(p) - D(p')) \cdot (p - p') \leq 0
\]

In particular, such a property, if \( D(p) \) is interpreted as a standard demand system, is described as the “Law of Demand” by Hildenbrand [1983] following Hicks. It implies that each partial “own demand curve” for the fish should be downward sloping. Since there is no a priori reason to impose any sort of functional form on the system the simplest approach is to make a non-parametric fit of the price quantity relations to establish a weaker property, that for each individual fish they are negatively sloped.

Such an approach is open to the criticism that it does not take into account substitution effects between fish. There are three responses to this. Firstly, many buyers such as restaurant owners have a pre-determined vector of fish quantities which they do not vary much in response to relative price changes. Secondly there are other buyers who only buy one type of fish and therefore do not substitute. Lastly some of the exogenous factors influencing the amount of fish available, such as weather are common to many fish thus limiting the amount of substitution possible. For all of these reasons each of the four fish is analysed separately.

Undertaking an analysis of the “demand” for each fish, amounts to eliciting some of the basic characteristics of the data. Basically, the idea is to take the data for a given

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13 Of course to take observed quantities purchased as representing a marginal curve is not correct since the ceteris paribus condition is violated. However this makes the resultant monotonicity more, rather than less, convincing.
fish and aggregate it by taking the quantity of that fish sold on a particular day and the weighted average price for that day. There is a problem of separation of strategies here. There are not only variations in the supply of fish due to weather etc. but more fish is landed on active market days by choice. The variations over the week are due in part to obvious institutional factors (fish-shops are closed on Sundays) but also to more indirect ones. As Robbins (1935) observed before his discussion of the market for herring in England:

“The influence of the Reformation made no change in the forces of gravity. But it certainly must have changed the demand for fish on Fridays”.

The resulting data are fitted by non-parametric smoothing methods, (for a full account see Härdle (1990)). Non parametric methods are more demanding than parametric approaches, since they enable one to pick up any lack of monotonicity of the fitted curve over some particular price range. Despite this, in all four cases the fitted curves are indeed monotone decreasing over a large part of their range and an example is given in Figure 3.5.

Simple inspection of a graph is, of course, not sufficient but formal evidence is given in Hardle and Kirman (1995), that the monotonicity property of the price quantity relation is robust. Furthermore aggregation across all fish produces even more striking results as can be seen from Figure 3.6.

The important thing to re-emphasise here is that the “nice” monotonicity property of the aggregate price quantity curves does not reflect and is not derived from the corresponding characteristics of individual behaviour. Nor indeed, given the previous discussion, should we expect it to be.

### 3.9 A Simple Model

At this point one might argue that for complicated markets so much is going on at the micro level that it is not worth trying to build a formal model of such a situation. One should be content to have found regularity at the aggregate level. This seems to me to be throwing in the sponge a little too quickly. It is surely worth at least reflecting on the nature of the model involved. What sort of formal model would be appropriate for fish markets? Let me then sketch a simple model of the sort of market typified by Marseille and restrict myself, for simplicity, to the case of one type of fish. What follows may seem rather arid but constitutes a formal description of a very simple situation. Since my conclusion is that this is not a very useful exercise the reader may wish to skip the details.

Consider the market for one perishable product with $m$ sellers and $n$ buyers. The market evolves in a fixed number $T$ of rounds. Each seller $i$ has strategies which at each round $t$ specify a vector $X_{it} \in \mathbb{R}^N_+$ of the prices which he will charge to each of the buyers. A strategy for each buyer $j$ specifies at each round $t$ a demand function

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14 An alternative approach which yields similar results, (see Hardle and Kirman (1995), is to take the total quantities of fish sold at each individual price over the whole period.

15 In Kirman and Vignes [1991] we considered a continuum of buyers and sellers but this was to facilitate the solution of the technical problem of establishing the continuity of strategies.
Figure 3.5: Need caption
Figure 3.6: Figure 3.6
q_{jt}(p) : R_+ \rightarrow R_+$. In both cases the choice of the prices set and the demand functions will depend on two things: firstly, the strategies of the other players and, secondly, on who has met whom in the market. The model is then completed by specifying a matching process which, in keeping with the literature, will be assumed to be random. Thus a matching at time $t$, a realisation of the random variable, will be a mapping $g$ from the integers $J = \{1, \ldots, n\}$ to the integers $l = \{1, \ldots, m\}$. A probability distribution must then be specified over the outcomes of the matching process for every time $t$. One might think, as an example, of each buyer as choosing a seller with uniform probability $\frac{1}{m}$, independently at each time $t$. However many other matching processes could be considered including those in which some particular buyers and sellers are always matched together. A best strategy for a buyer $j$, then, will consist for each realisation of the matching process, for the associated price vectors of each seller $i$ and demand functions of the other buyers $h \neq j$ of a demand function for each period $t$. Similarly for a seller it will consist of specifying the best price vectors for each matching and each period.

The market described by this simple model can be envisaged as follows:

**Period 0**: Initial stocks become available.


**Period 2**: Given their information about what happened in period 1, sellers respecify prices, buyers respecify demands. Matching occurs. Exchanges follow.

**Period T**: Last specifications by sellers and buyers, last matching and exchanges.

As it stands, I have done no more than give a framework which enables me to define the concept of an equilibrium. To characterise precisely the nature of an equilibrium requires that the strategies chosen in response to the strategies of others be derived from maximising behaviour. For example a buyer might maximise his expected utility at each round $t$ given the known strategies of the other players and the matching up until $t$. One important point to emphasise is that any strategy must be such that if the information set up to time $\tau$ is the same in two realisations, the next component of the strategy at time $\tau + 1$ should be the same. Thus it is important to specify what is known at each time. If, for instance, as I have already mentioned, the individuals know only their own initial stocks and only observe their own transaction outcomes, they will be much more limited than if they observe everything that has occurred. Furthermore, it may well be the case that individuals actually choose to condition strategies only on a limited part of the information they have available. Even if these problems are surmounted. Technical difficulties in proving the existence of an equilibrium remain and these are illustrated by Kormendi [1979] and Benabou [1988].

### 3.10 Does this help?

What I wish to suggest is that, even if we make the appropriate assumptions to obtain an equilibrium, we have made little progress, since what really interests us are the
characteristics of the outcomes in the market. As it stands, even if the existence of equilibrium can be proved, it has little to say in terms of observable and testable behaviour, either at the aggregate or individual level. For example, whether or not we give a complete specification of the maximising behaviour of the individuals, the sort of model I have sketched could not rule out extensive price dispersion (particularly since price discrimination is possible as each seller knows the buyers’ characteristics). Furthermore there will be no necessary tendency for prices to decline during the day as is commonly supposed and, as I have mentioned, there is no a priori reason to assume that individual buyers will, or will not, search.

If we accept this outline of the formal structure as corresponding to the sort of process involved in the market, it is not surprising that the outcomes do not necessarily satisfy standard competitive properties at the individual or aggregate level, since observed transactions are the results of the interaction between buyers’ and sellers’ strategies. What we are looking at is a game and not a very simple one at that. The observed purchases reflect the demand of agents, given what they have observed, and given the strategies of the other players. If there is an equilibrium it should be the case that the market clears at the end of the day. The difference from day to day on the market is due to the amount of fish available. This is the result in part of exogenous factors such as weather which does not play much of a role on the Marseille fish market now, but is also due to the choice of sellers when anticipating demand changes. This latter factor should be incorporated into a complete model but this makes things even more complicated since, in that case, the market days are no longer independent.

All of this suggests, to me at least, that there is not a great deal to be gained by attempting a full-blown optimising approach to modeling the sort of market that we are dealing with here, not because we cannot prove the existence of an equilibrium but because we cannot obtain any simple testable characteristics of that equilibrium. Perhaps the most convincing argument of all is to pass some time on this market and to observe the participants and how they behave. The habits and relationships that people have developed over time seem to correspond much more to things learnt by the force of experience rather than to conscious calculation. Whether, as in evolutionary game theory, the result of this learning is that they behave as they would have done had they been able to make all the necessary calculations is an open question. What seems plausible is that people use simple rules and that they have modified them over time. The result is that the market evolves towards a well defined structure which has many interesting features but which could not be deduced from looking at any typical or representative participant. Simple individual behaviour generates complicated but recognisable aggregate features. This observation leads me to suggest a different sort of model.

3.11 Models with less rationality

If we accept then, that the equilibrium in a fully optimising game-theoretic context is difficult to characterise, and that it is a poor description of reality, we have to turn to a different type of model. Assume that agents use simple rules to make their choices. If this is the case then the notion of “equilibrium” outcome has to be redefined since it
will depend on the rules chosen. It may be the case however, that this sort of approach based on more rudimentary behaviour may be more effective in reproducing the sort of phenomena we observe on real fish markets. In particular there are a number of phenomena which are of interest in their own right and which can be examined in this way. For example we need to be able to generate not just a non-degenerate distribution of prices but also the sort of trading relationships that emerge.

To illustrate this point just let me remark that the sort of network of trading relationships that we observe in markets like Marseille have rather specific patterns. These must, at least in part, be responsible for the sort of price dispersion that is observed. In addition, although these networks may play an important role in determining market outcomes, nothing is said in the sort of game theoretical model discussed above, about their impact or the evolution. They could be included in that sort of model in two ways. Firstly, the network could be taken as given and this might be thought of as restricting the information available to the individuals. Secondly, and much more ambitiously, they could be formed strategically, but both of these approaches would merely make the analysis less tractable. What we need is a way of looking at the emergence of these networks which does not make unreasonable demands on the calculating capacity of the individuals involved. Before doing this it is perhaps worth asking what we actually observe.

So what is the nature of the trading relationships on the Marseille fish market? There is a remarkable dichotomy. On the one hand, there are those buyers who regularly buy from the same seller and are extremely loyal, and on the other hand, there are people who shift between sellers all of the time. Yet, there are few buyers who are basically loyal but occasionally shop around. This, of itself, seems to be a feature that one should try to explain. If one tries to go back to a full game theoretic model this becomes extremely complicated because one has to develop now a dynamic game in which the experience of playing with each seller is taken into account. Alternatively, one has to think of a situation in which people have strategies which are so complicated that they can take into account all the possible prices they might face from each of the different sellers at each point in time.

What I would like to suggest here, is the idea of developing a much simpler theoretical model in which people simply learn from their previous experience and they in consequence change their probability of visiting different sellers as a result of their experience. What I will argue is that models of this sort which attribute very little computational ability or general reasoning capacity to individuals may be capable of generating specific features of real markets. This sort of "bounded rationality" approach has received a lot of attention but is often dismissed for its lack of rigour. In fact the analysis of the evolution of the "state" of the market in the model can be perfectly rigorous given the specific choice of rules for the agents.

This is not widely accepted in economics and one might wonder why. The reason seems to me simple, choosing rules of thumb for agents is regarded as ad hoc. However, we have come to accept that the restrictions that we impose on the preferences of individuals, unlike other behavioural rules, are not ad hoc. Therefore, if we replace those assumptions, which by their very nature cannot be empirically tested, by other rules, we are subject to the criticism that we lose the rigour of "proper micro foundations". Let me simply suggest that maximisation of a well defined preference order
is not necessarily a reasonable assumption when both the order to be maximised and the set of alternatives are highly complicated, and that something is to be gained from simplifying our account of individuals’ behaviour in complicated situations. One response to this is that in the process of using simple rules individuals converge to those rules which work best and therefore act just as if they were maximising in the standard way. This is Lucas’ (1988) position. There are two objections to this. First, we have to show that the learning process converges, and if it does that it corresponds to the maximisation in the original problem. Second, learning processes usually involve learning about something which is not changing. But here, the learning is influenced by the behaviour of other individuals who are also learning. It is by no means clear that we will have convergence in such a situation. The answer to this is that we are interested in modelling the results of interactions between individuals following simple rules, not just as a way of justifying a theoretical equilibrium but rather as a vehicle for understanding empirical reality.

As I have mentioned, the relationships between individuals form an important part of the structure of a market and when we look at the situation on the Marseille market we see a number of features of these relationships. The purpose of the next section is to try to build a framework to understand these.

3.12 Trading relationships within the market.

To do this, consider a model which we developed as a simplified version of the Marseille fish market (see Weisbuch et al. (2000)). There we consider a situation in which buyers do not anticipate the value of choosing sellers but rather develop relationships with sellers on the basis of their previous experience.

To be more precise, there are $n$ buyers indexed by $i$ and $m$ sellers indexed by $j$. The buyers update their probability of visiting sellers on the basis of the profit that they obtained in the past from them. If we denote by $J_{ij}(t)$ the cumulated profit, up to period $t$, that buyer $i$ has obtained from trading with seller $j$ where

$$J_{ij}(t) = \Pi_{ij} + (1 - \gamma) J_{ij}(t-1)$$

and where $\Pi_{ij}$ is the profit that the buyer $i$ makes if he visits seller $j$ and the latter still has fish available. (I assume for the time being that the profit does not vary over time).

Then the probability $p_{ij}(t)$ that $i$ will visit $j$ in that period is given by,

$$p_{ij}(t) = \frac{e^{\beta J_{ij}(t)}}{\sum_k e^{\beta J_{ik}(t)}}$$

where $\beta$ is a reinforcement parameter which describes how sensitive the individual is to past profits. This non-linear updating rule will be familiar from many different disciplines and is also widely used in statistical physics and I will use it again in this book. It is known as the “logit” rule or, in game theory as the “quantal response” rule.  

\footnote{I have developed this sort of argument at length in Kirman (2006) where I suggest that we have gone down the wrong route in modelling demand.}
3.13. A LITTLE FORMAL ANALYSIS

The rule is based on two simple principles. Agents make probabilistic choices between actions. Actions that have generated better outcomes in the past are more likely to be used in the future. Such a process has long been adopted and modelled by psychologists (see e.g., Bush and Mosteller, (1955)). It is a special form of reinforcement learning. It has also been widely used in evolutionary and experimental game theory (see Roth and Erev (1995)) and a more elaborate model has been constructed by Camerer and Ho (1999). The particular form chosen here is used extensively. It is found in the model developed by Blume (1993), for example, to analyse the evolution of the use of strategies in games. This approach has the great advantage that it requires no specific attribution of rationality to the agents other than that they are more likely to do what has proved to be successful in the past.

3.13 A little formal analysis

To simplify matters at the outset we will start with a continuous approximation of our model which is actually in discrete time. Furthermore, we will replace the random variables by their expected values. This is referred to as the “mean field” approach. In this way it is easy to see that the change in cumulated profit for the buyer is given by,

$$\frac{dJ_{ij}}{dt} = -\gamma J_{ij} + E(\Pi_{ij})$$

(3.5)

Using the learning rule that I have given we know the probability for agent $i$ to visit seller $j$ and can therefore calculate the expected gain from that visit. Recall that there are two things involved here, firstly the probability that the seller $j$ still has fish available when buyer $i$ arrives, and secondly the probability that the latter chooses seller $j$. So the expectation is given by,

$$E(\Pi_{ij}) = Pr(q_j > 0) \Pi_{ij} \frac{\exp(\beta J_{ij})}{\sum_k \exp(\beta J_{ik})}$$

(3.6)

Now consider an even simpler case where the seller is sure to have fish, in which case we have,

$$Pr(q_j > 0) = 1$$

(3.7)

Now simplify even further and look at the case where there are just two sellers and furthermore each time a buyer visits one of the sellers he receives a fixed profit of $\Pi$ and find the equilibrium level for the cumulated profit for a buyer from seller 1 and this will of course be when

$$\frac{dJ_1}{dt} = 0$$

(3.8)

Substituting this gives,

$$\gamma J_1 = \Pi \frac{\exp(\beta J_1)}{\exp(\beta J_1) + \exp(\beta J_2)}$$

(3.9)

Now take the difference between the profits from the two sellers and we have,

$$\Delta = J_1 - J_2$$

(3.10)
If we now substitute we have the following expression,

$$\Delta = \frac{(\exp(\beta \Delta - 1))\Pi}{(\exp(\beta \Delta + 1))\gamma}$$  \hspace{1cm} (3.11)

We now have simply to solve this equation for $\Delta$ and this gives two cases. First, consider

$$\beta < \beta_c = \frac{2\gamma}{\Pi}$$  \hspace{1cm} (3.12)

In this case, when the importance attached to previous experience is below the critical value $\beta_c$ we have,

$$\Delta = 0, J_1 = J_2 = \frac{\Pi}{2\gamma}$$  \hspace{1cm} (3.13)

There is a single solution and the cumulated profits from both sellers and hence the probabilities of visiting them are the same.

However, when $\beta > \beta_c$ then there are three solutions and $\Delta = 0$ is unstable and there is a rapid transition at $\beta = \beta_c$. By which we mean that as soon as $\beta$ passes above the critical value the probabilities of visiting each seller become rapidly very different. All of this is illustrated in Figure 3.7.

To repeat, what we can see is that as the value $\beta$ that is, the importance that the buyer attaches to previous experience increases, there are two stable solutions for the probability (the relation between $J$ and the probability is given by the equilibrium condition), that he has of visiting seller 1. When he has that probability he has the complementary probability of visiting seller 2. Thus he will spend most of his time with one seller and much less with the other. As $\beta$ increases towards 1 the buyer becomes completely attached to one or the other seller. So in this very simple case we see that loyalty is a property of the equilibrium when $\beta$ is high enough.

However, remember how we arrived at this solution. We built a simple approximation by using the “mean field” approach and then with a number of extreme assumptions derived the solution. The mean field approach involves making two approximations. We considered that the process was a continuous one and furthermore that we could replace the random variables by their expected values. In addition we considered the case with just two sellers each of whom gave equal profits and always had fish to sell.

The next step was to see if this simple analysis held up in more complicated situations. To do this we took, as an example, the case in which there are three sellers in the market. Now what we did was to simulate the actual stochastic process and not to rely on an approximation. As in the analytical model, at any point in time, the buyer chooses with certain probabilities between the three sellers. Thus he will be represented by a point in the three simplex or triangle as illustrated in Figure 3.8 below. Each buyer is represented by such a point in the simplex and the nature of the relationships will be illustrated by a cloud of points. A buyer who shops around in a purely random way, that is who is equally likely to visit each of the three sellers, will be represented as a point in the centre of the triangle. If, on the other hand, he visits one of the sellers with probability one then he can be shown as a point at one of the apexes of the triangle.

Thus, at any one point in time, the market is described by a cloud of points in the triangle and the question is how will this cloud evolve? If buyers all become loyal
Figure 3.7: The transition from random buying to loyalty as a function of $\beta$, (source Weisbuch et al, (2000)).
to particular sellers then the result will be be that all the points, corresponding to the buyers will be at the apexes of the triangle as in figure 3.8b. This might be thought of as a situation in which the market is “ordered”. The network of buyers and sellers becomes deterministic. On the other hand, if buyers learn to search randomly amongst the sellers, then the result will be a cluster of points at the centre of the triangle, as in figure 3.8a. What we saw in the analytical model taken from Weisbuch et al. (2000), is that which of these situations will develop, depends crucially on three parameters $\beta$, the reinforcement factor which represents the importance attached to previous experience, $\gamma$ the discount rate and $\Pi$, the profit per transaction. The stronger the reinforcement, the slower the individual forgets and the higher the profit obtained from sellers, the more likely is it that loyalty will emerge.

It might be asked whether or not the features of the actual market in Marseille do reflect the sort of behaviour predicted by this, admittedly primitive model. What the model suggests is that the transition from disorder to order, or more prosaically from shopping around to loyalty as $\beta$ changes, is very sharp. The change will depend on a critical value $\beta_{ci}$ of $\beta$ which will be different for each buyer $i$ and will depend on the frequency of his visits to the market and his profit. It is easy to see why higher profits obtained will make a buyer stick to the seller that gave him those profits but the importance of the frequency of visits needs a little explanation. If a buyer comes infrequently to the market then his information from previous visits is less pertinent than which the regular visitor got from his last visits. He will therefore discount previous experience more than his loyal counterpart. This will lead him to reinforce his tendency to shop around.

Before leaving the theoretical model, one observation is in order. The patterns illustrated in figure 3.8a were obtained with buyers all of whom had the same $\beta_{ci}$ and the same is true for figure 3.8b where the value of $\beta_{ci}$ is, of course, lower. But what happens in the simulations if the group is mixed as it is in reality? In the loyal buyers
situation, the sellers learn to sell the correct amount because they have fixed and regular customers. In the random shopper situation, sellers cannot predict accurately the amount that they will sell. Therefore there is more waste, some buyers are unsatisfied and some sellers are left with fish on their hands. Now, will it not be the case that the presence of random shoppers will interfere with the profits of the loyal and thus weaken their loyalty? Sellers will sometimes have insufficient stocks because they have been visited by random shoppers and this could have a negative effect on those who were normally loyal to those sellers. We simulated the situation with equal proportions of loyal or low $\beta_{ci}$ and high $\beta_{ci}$ buyers. Interestingly, the presence of the random shoppers did not prevent the development of loyalty as is shown in figure 3.9. Those whose low $\beta_{ci}$ led them to be loyal remain loyal and those whose high $\beta_{ci}$ led them to shop around continue to do so.

What then should we expect to observe on the real market if what our model suggests is right? As I have said, the important conclusion is that the division between loyal buyers and random shoppers should be quite sharp and one should not expect to find individuals who shop around to some extent but are somewhat more loyal to some sellers than to others. This is precisely what is observed on the Marseille fish market. The behaviour of buyers is highly bimodal. Consider the case of cod as an example. In Figure 3.10 the histogram of the number of buyers visiting different numbers of sellers is shown. There is a concentration of buyers who only visit one seller and then a distribution of individuals who visit several sellers with a median of 4 per month.

The extent of the loyalty of customers for certain types of fish can be observed from the fact that, for example, 48% of all buyers bought more than 95% of their cod from one seller, the seller of course, not being the same for all of these buyers. 33% of buyers bought more than 95% of their sole and 24% bought more than 95% of their whiting from one seller. In both the whiting and sole markets more than half of the buyers buy more than 80% from one seller. Furthermore, as the theory predicts, those sellers with the highest sales, and those who come to the market most frequently are those who are most loyal.

What is interesting here is that there almost no values of $\beta$ in the interval for which buyers are partly loyal. Thus the critical value of $\beta$ moves behaviour changes very rapidly. Such a rapid change is referred to as a “phase transition” in physics. As I have explained, in the Weisbuch et al. (2000) model we derive this sort of “phase transition” in a particularly simple model of stochastic choice. We used the “mean field” approach. The latter is open to the objection that random variables are replaced by their means and, in consequence, the process derived is only an approximation. This allows us to obtain an analytical solution but at the price of replacing the stochastic model by a deterministic one. The alternative is to consider the full stochastic process but this was not tractable. To study the stochastic model itself and to envisage several trading opportunities during the day we were forced to resort to simulations to see whether the theoretical results from the simple deterministic approximation were retained in more complex situations\(^\text{17}\). We saw one example of this but do the results carry over to more complicated situations? As it happens they do, and quite elaborate versions

\(^{17}\)A detailed discussion of this sort of problem is given by Aoki (1996), who thinks of the buyers in the markets as being partitioned between the sellers and each buyer as having a probability of transiting from one seller to another. He looks at the limit distributions of such a process.
Figure 3.9: The probabilities of buyers visiting the three sellers when buyers have high or low values of $\beta$
3.14. AN EVEN SIMPLER MODELLING APPROACH

As is clear from the previous sections the problem of modelling even such simple examples as fish markets is that if one tries to incorporate some of the realistic features of the microscopic interaction between the actors, the model rapidly becomes analytically intractable. One answer to this is that provided by so called “multi agent” modelling (see for example, Arthur et al. (1997) and Epstein (2007)). In this approach, agents are endowed with simple rules which govern their reaction to their economic circumstances. As time goes on, they place more weight on those rules which turn out to be more profitable.

In one sense the previous model is a special case of this since the rule that the agents use can be thought of as determining the choice of seller in a particular way depending on the success of transactions with that seller in the past. However, once the rules are extended to cover a variety of choices the behaviour of the model quickly becomes too complicated to model formally. In the previous approach we started with simple behaviour in simple circumstances and then found a formal solution. We then simulated more elaborate versions of the model to see whether the results still held.

Suppose that we wish to use the same sort of approach but want to analyse several features of the market at the same time. This may well make it too difficult to provide...
even a simple formal model. This suggests, by invoking Occam’s razor, simulating a
model with as simple an updating procedure as possible. In such a model one hopes to
find, as emergent features, some of the salient aspects of the real empirical markets that
interest us. In Kirman and Vriend (2000) we developed a simple model which produces
three of the features of the Marseille fish market. These are firstly, the division between
loyalty and shopping behaviour on the part of buyers that we have already mentioned.
Secondly, there is the price dispersion even for the same species. Lastly, we might hope
that sellers learn to handle their clients in a way which corresponds to what actually
happens in reality. In the theoretical model developed in Weisbuch et al. (2000) the
problem of handling several periods in the day was already sufficient to oblige us to
resort to simulations. Adding now the extra complication of pricing behaviour and
the handling of clients means that there is little hope of producing a comprehensive
theoretical model which will reproduce all the characteristics of the real market so we
built a simple agent based model in which agents interact with each other and learn in
so doing.

In the simple simulated model we developed in Kirman and Vriend (2001), ten
initially identical sellers and one hundred initially identical buyers met in the market
hall for five thousand days for a morning and an afternoon session. They traded single
individual units of a perishable commodity. Here we make two simplifications. The
morning and afternoon sessions correspond to the idea that there are several opportu-
nities to trade during the day. Taking two periods allows us to take account of the idea
that the possibility of trading later in the day has an influence on the prices that buyers
will accept and sellers will propose early in the day. It would, of course, be more re-
alistic to consider more trading opportunities in the day. The single unit assumption is
frequently used but can be criticised on the grounds that when buyers require different
amounts this may influence what they pay. Leaving these caveats on one side for the
moment I can now describe the model.

On each day the sequence of events is the following.

In the morning before the market opens the sellers purchase their supply outside
the market for a given price that was identical for all sellers and constant through time.
Thus, like the small open countries in trade theory, we assume that the participants
on the Marseille have no influence on what happens in the outside world. The market
opens and the buyers enter the market hall. Each buyer requires one unit of fish per
day. All buyers simultaneously choose the queue of a seller. The sellers then handle
these queues during the morning session. Once the sellers have supplied all the buyers
who are willing to purchase from them the morning session ends. All those buyers who
are still unsatisfied choose the queue of a seller in the afternoon. Of course, the only
sellers present in the afternoon are those who did not sell all their stock in the morning.
Sellers now sell to those buyers who are willing to purchase from them and the end of
the afternoon session is then reached. All unsold stocks perish. Those buyers who did
purchase fish, resell that fish outside the market, at a given price that is identical for
all buyers, and constant through time. Each buyer can visit at most one seller in the
morning and one seller in the afternoon.

What are the decisions with which the actors are faced? Buyers have to choose
a seller for the morning session. They then have to decide which prices to accept or
reject during the morning session. If necessary, they also have to decide on a seller
for the afternoon. Lastly, they must decide which prices to accept or reject during the afternoon session. Sellers have also four decisions to make. They must decide what quantity to supply. They must decide how to handle the queues with which they are faced. They must decide which prices to set during the morning session and which prices to set during the afternoon session.

In the model described each individual agent uses a Classifier System for each decision and this means that each agent has four such systems “in his head”. A simple stylised classifier system is presented in Figure 3.11

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<thead>
<tr>
<th>condition</th>
<th>action</th>
<th>strength</th>
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<td>if ....</td>
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Figure 3.11: A classifier system , (source Kirman and Vriend (2001))

Each classifier system consists of a set of rules. Each rule has a condition “if......” and an action “then......” and in addition each rule is assigned a certain strength. The classifier system decides which of the rules will be the active rule at a given point in time. It checks the conditional part of the rule and decides amongst all of those rules for whom the condition is satisfied which to choose. This is done by a simple auction procedure. Each rule makes a “bid” to be the current rule and this bid = current strength + ε, where ε is white noise, a normal random variable with mean 0 and fixed variance. The rule with the highest “bid” in this auction becomes the active rule. The white noise means that there was always some experimenting going on and there was always some probability that a rule, however bad, will be chosen. At time $t$ the classifier system updates the strength $s_t$ of a rule that has been active and has generated a reward at time $t - 1$ as follows:

$$s_t = s_{t-1} - c \cdot s_{t-1} + c \cdot \text{reward}_{t-1}, \text{ where } 0 < c < 1.$$ 

Hence, as long as the reward generated by the rule on day $t - 1$ is greater than its strength at $t - 1$ the strength will increase. The strength of each rule converges to the weighted average of the rewards generated by that rule. What the reward is, will depend on the rule in question. Supposing that in our market example the rule for the buyer is of the form, “if the price proposed by the seller for one unit of fish in the morning is 11 euros then accept”. The reward for using this rule would then be the profit that is generated by using it. In this case the reward would be the price at which the unit of fish is sold on the retail market minus the price paid (11 euros). When the model is started the strengths of all rules are equal.

What the agents in this model are doing is learning by an even simpler version of reinforcement learning than that encountered previously. Details of the particular rules
in the simulation model of the Marseille fish market can be found in Kirman and Vriend (2001).

Although such an approach seems to be innocent of theoretical pre-suppositions it should be noted that the very choice of the rules amongst which the agent chooses has an impact on the outcomes. Ideally, one would like to start with agents who are totally ignorant. However, this would imply that they would somehow generate a set of rules with which they would experiment. This pushes the analysis back many stages to a very fundamental level. What is done here is in line with standard practice which is to provide the agents with a set of rules and simply note that this, to some extent, conditions the outcomes of the process. As an example, consider the fact that we would like agents to learn how to handle the queues with which they are faced. In an ideal world we would like the agents to realise that their handling of the queues is important and then we would like them to work out for themselves how to handle them. As it is, by giving different rules explaining how to handle queues the modeler is already biasing the behaviour of the seller by suggesting to him what it is that is important in generating his profit. However, what is not biased is the choice amongst the rules presented. Thus, the rule chosen will be the best available for handling queues amongst those presented, given the agent’s experience, but he might well himself have focused on some other aspect of the market. With these reservations it is still worth examining the results of the simulations and to see to what extent they reflect reality.

One might ask whether some of the features of the market could not be modelled in a more theoretical way. For example, in the last session it seems as if buyers and sellers are faced with a version of the ultimatum game (see e.g., Guth and Tietz (1990)). Since, in the model sellers propose a price and buyers either accept or refuse it would seem that the sensible price to propose is just slightly less than the price at which the fish can be sold on the outside market and this indeed is the sub-game perfect outcome of the ultimatum game. However, it has long been noted that this is not what one observes in experimental outcomes nor in reality. One obvious reason for this is that we are observing a repeated game in the market. Thus, a refusal today has implications for behaviour tomorrow even if agents are not aware of this. Buyers learn to accept or reject prices on the basis of the profitability of doing so, whilst sellers learn in a similar way which prices to ask. What is crucial here as noted by Gale et al. (1995) and Roth and Erev (1995) is that the relative speed of learning on each side of the market will govern which outcomes occur. The importance of this will become clear as soon as we look at the results of the simulations.

Let us first look at the prices asked and accepted in the morning as shown in Figure 3.12.

There is first of all a considerable period during which learning takes place and then prices settle to 10 which is one greater than the price at which fish is bought by sellers outside the market and one greater than the perfectly competitive price. What is interesting is that during the learning period, which is what governs the final outcome, two things are going on. Sellers learn to ask prices close to the ultimatum price which is 14 euros, one less than the price at which fish can be sold on the outside market. However, buyers do not learn as quickly to accept such prices. Where does this difference come from? The answer is that initially some buyers will accept high prices having not learned to do otherwise. This will encourage sellers to charge such prices.
Figure 3.12: Prices asked and accepted in the morning session, (source Kirman and Vriend (2001)).
However buyers will start to find out that they can obtain higher profits by refusing high prices and accepting lower ones. There are always some such prices to be found. As sellers learn that buyers are not accepting their prices, they start to decrease the prices asked and simultaneously, as buyers observe that prices being asked are descending, they start to decrease their acceptance levels. Once again, sellers learning “leads” that of buyers and as a result the prices converge. In the afternoon, there are also two separate learning processes going on and once again convergence occurs but to a higher price (11euros) than in the morning. The evolution of afternoon prices can be seen in Figure 3.13.

![Figure 3.13: Prices asked and accepted in the afternoon session, (source Kirman and Vriend (2001)).](image)

This might seem extraordinary since if buyers become aware that prices in the afternoon are higher than prices in the morning they should presumably always buy in the morning. This is not correct. The reason is simple, those people who reappear in the afternoon have been selected. To see this, consider a situation in which the distribution of prices asked is the same in the morning as in the afternoon. Suppose now that those buyers that encounter prices in the upper tail of the distribution reject in the morning.
the result of this would be that the average price paid in the morning will be lower than
the average price paid in the afternoon. The whole point here is that it is not the average
price that is rejected whereas what is shown in the figures is the average at any point in
time. In Figure 3.14 the price distribution over the last 2,500 days is shown and it can
be seen that it does not become degenerate and concentrated on one price.

Figure 3.14: The distribution of prices , (source Kirman and Vriend (2001))

Thus, a phenomenon which is observed on the real market in Marseille, as we saw
earlier, emerges in our artificial fish market.

A second feature of the real market is that which has been discussed earlier, that of
“loyalty”. In the previous model we simply established the pattern of loyalty but did
dnot suggest any macroeconomic consequences of that feature. To pursue this question
we need to have some measure of loyalty and then to examine its impact. To do this
we construct an index of loyalty which has a value equal to one if the buyer is perfectly
loyal to one seller and has a value equal to $1/n$ where $n$ is the number of sellers when
the buyer systematically visits each seller in turn, that is, when he has the most extreme
“shopping around” behaviour. More specifically, the loyalty index is given by:

$$L_{ij}(t) = \sum_{x=1}^{t} \frac{r_{ij}(t-x)}{(1+\alpha)^{t-x}}$$

This is an indicator of how often buyer $i$ visits seller $j$. It is a global statistic covering the whole period but there is a discount factor represented by $\alpha$. The parameter $r_{ij}(t)$ is a counter which increases with each visit of $i$ to $j$. Here we took $\alpha = 0.25$ and $r_{ij}(t) = 0.25$ if buyer $i$ visits seller $j$ at time $t$ and $= 0$ otherwise.

In the spirit of this approach, nothing was built into the rules of the sellers to make them privilege loyal buyers. We wanted to see whether this sort of behaviour would emerge.

The sort of rules they had were of the form:

"If loyalty = a certain value then choose a certain probability of serving that client”,
"If loyalty = a certain value then charge $p$”

What probability will be chosen depends on whether the seller learns to favour loyal customers or not. Which $p$ is charged depends on how successful that choice turns out to be.

The time series of average loyalty is shown in Figure 3.15.

What happens is that 90% of the buyers actually get a higher payoff by being loyal as can be seen in Figure 3.16. What this means is that when basically loyal customers shop around, as they do stochastically from time to time, the profit realised is lower on average than when they buy from their regular supplier.

Furthermore, nine out of ten of the sellers get a higher profit when dealing with loyal buyers as shown in Figure 3.17. In other words the profit, on average from a loyal customer is higher than from a random shopper. Here the difference in revenue represents the fraction of the average revenue from loyal customers above or below the average profit realised from transactions with casual buyers.

This is a reflection of the fact that what is happening here is not a zero sum game. Only when a transaction takes place do buyers and sellers realise a profit. Thus, payoffs will be highly conditioned on acceptance and rejection, and on prices asked. The question then becomes how do loyal buyers tend to be handled by sellers? In all but one of the cases, sellers learn to give priority in service to loyal buyers but to charge them higher prices than random shoppers. Buyers learn that when they become loyal their profit is higher since they are more likely to be served even though they pay higher prices. Thus, loyalty is profitable both to buyers and sellers.

What about the one seller who did not find loyal customers more profitable than shoppers? This seller learned to charge low prices to loyal customers but to give them low priority in the queue. One might ask why he did not learn to adopt the more profitable strategy learned by the other sellers. The answer here is that with the sort of local learning that is going on a move towards better service and higher prices for loyal customers can never develop. To make such a move would imply increasing prices and improving service. However, buyers will immediately observe the higher prices and will not necessarily immediately observe better service in terms of priority
Figure 3.15: The evolution of loyalty, (source Kirman and Vriend (2001))
Figure 3.16: The pay-off advantage for buyers of loyalty, (source Kirman and Vriend (2001))
Figure 3.17: The pay-off advantage for sellers of loyalty, (source Kirman and Vriend (2001)).
in the queue. This will lead them to reduce their probability of visiting this seller. As this seller observes that his customers are drifting away he will go back to his former behaviour and will therefore never learn his way to the more profitable strategy. However, it is interesting that, in the model this seller still makes profits so he does not disappear. Thus there is at least one explanation for the dispersion of profits that one observes on the market. No figures are available to document this but there is a consensus on the market that some sellers make considerably more profit than others and the argument of our model would be that they have simply reinforced on more profitable rules.

This very simple rudimentary artificial fish market model manages then to reproduce some of the features of the real market. For example, it is interesting to note that on average in the Marseille fish market loyal buyers pay higher prices than shoppers. Those buyers who buy more than 50% of their fish per year from one seller pay, on average 5% more than the other buyers even though almost all the large buyers are loyal. Thus here we have a clear organisational feature which has emerged and which has had a very specific consequence for the market price distribution. Such a model has the advantage that it can always be extended to examine other aspects of the real market whereas to attempt to construct a theoretical model which incorporates all of these is a more than ambitious task.

3.15 Another type of market: The Ancona Fish Market.

I have dwelt at length on the emergence of certain aggregate properties on the Marseille market. However, the organisation there is very special. It is based on pairwise trading with no posted prices. Many, if not most, fish markets are organised on an auction basis and, as I have observed earlier this was proposed, but refused, in Marseille. Nevertheless, it is well worth asking the question as to how much the aggregate behaviour of the market depends on the type of organisation. As a comparison we have analysed the fish market in Ancona, on the Adriatic coast of Italy, which is is organised as three simultaneous Dutch, (descending price) auctions, (see Gallegati et al. (2009)). We have detailed data for all the transactions made on this market and this provides us with an opportunity to compare the data from this market with that in Marseille and see if similar stylized facts emerge. Secondly we also have the possibility to see if our data exhibits the features found in auction data or predicted by auction theory. Let me start by giving a brief description of the Ancona fish market known as MERITAN.

3.15.1 Description of MERITAN

The MERITAN (“MERcato ITtico ANcona” Italian for Fish market of Ancona) is open 4 days a week (Tu.-Fr.; 3.30-7.30). It consists of 3 simultaneous Dutch auctions with about 15 transactions in total per minute. The total value of the fish sold amounts to 25 millions euros per year. Each type of fish is arranged in cases of about 5-7 Kilograms. Each morning the vessels are randomly assigned to one of the three conveyor
belts and the assistants in the market begin putting the cases on it. When the selected seller has one of his cases put on the belt, the price display is set (the auctioneer decides the initial price) and starts going down while the case moves toward the end of the belt. Buyers watch the three displays and can bid on one or more of them, the first person to push the button at the price that has been reached wins the auction for that case. There are about 170 buyers. 20 of them are wholesalers while 150 are retailers (intermediaries, outdoor market sellers, fish shops); in any case the buyers are not the final consumers. There are 70 sellers.

The data we used are relative to one of the three conveyor belts since the others were not being electronically recorded at the time. They cover the period from the 19th September 2002 to the 28th of May 2003. The database represents 53555 transactions for a total weight of 360115 kg. During this period 70 sellers and 149 buyers exchanged fish of 110 transaction classes on this specific conveyor belt (data for the whole market are more comprehensive). Note that what we call a transaction class is different from a species (see the second column of the table below for examples of transaction classes).

The data are collected daily on the market computer. For each case traded the data provides:

- The day, month and time (hour and minute) of the transaction.
- The weight.
- The price per kilo and total.
- The identification number of the seller (vessel).
- The identification number of the buyer.
- The transaction class (T.C. hereafter) and its identification number.

The following table gives the number of transactions and the total weight for the main TCs, buyers and sellers.

To complete the description of the market we examine buyers and sellers size distributions. We define the size as the total weight of the fish bought or sold by the agents over the period. As shown in figures 3.18 and 3.19, there is no dominant size among the sellers while the buyers are clustered on the small size.

The distribution of buyers presents a notable peak while that of sellers is rather flat. On the other hand it is evident that there are a few very large buyers. Here, as in Marseille, we are dealing with a heterogeneous group of buyers and sellers but this time they interact through a central auction mechanism. The buyers observe who purchased which lot and can therefore, in principle, act strategically. However, since there is a transaction every 4 seconds, it would seem more plausible that the buyers use rules of thumb rather than detailed and complicated calculations. Again we cannot rule out the possibility that the two have come to coincide as a result of learning over a long period. Some anecdotal evidence that buyers are not able to calculate their optimal strategy is provided by the auctioneer at Sete in the South of France, a fish market

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18 The graphs are obtained non parametrically using kernel smoothing techniques with a Gaussian kernel, where h is the bandwidth.
### TRANSACTION CLASSES

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<th>ID</th>
<th>Name</th>
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<th>kg.</th>
<th>ID</th>
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<td>1623</td>
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<td>7096.78</td>
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Table 3.1
Figure 3.18: The distribution of the total amounts bought on the Ancona Market (source Gallegati et al. (2007)).
Figure 3.19: The distribution of the total amounts sold on the Ancona Market. (source Gallegati et al. (2007))
which also operates on a Dutch auction basis. I observed that sometimes he started the auction at a price which was immediately taken and that this might indicate that the lot would have been taken at a higher price. He admitted that this was true but said that he did this to speed things up and to stop the buyers thinking strategically. In this way he claimed that he was increasing his total revenue. He gave, as an example, bridge and pointed out that if one played relatively slowly one had time to reflect on which cards had passed while this becomes impossible if one is forced to play quickly.

3.16 The empirical evidence

We took a closer look at two questions which, we believe might help us to understand whether the market mechanism does have an impact on market behaviour.

First, we analysed the way in which prices are formed and the dynamics of this process.

Second we investigated the effect of the auction mechanism. The two main questions here are:

- Does the auction destroy buyer-seller relationships?
- Despite the auction mechanisms are there buyers (sellers) who systematically obtain lower (higher) prices than others?

3.16.1 Price-quantity relation

First of all we want to check the existence of a negative relationship between price and quantity as we did for the Marseille market. To make this kind of test we cannot use the data from each transaction. Remember, that the fish are sorted into cases with more or less equal weights, this means that a plot of quantity (weight) and price for each transaction would yield an inelastic relation. To avoid this kind of problem we plot average daily variables for price and quantity.

Figures 3.20a and 3.20b show the negative relationship between price and quantity for two different TCs. The negative slope can be observed as in the Marseille market if we pool all the TCs (see figure 3.21, where the solid line represents the Nadaraya-Watson nonparametric regression with Gaussian kernel). Here, as for the Marseille fish market we use non-parametric regressions which makes the task of establishing a negative slope for the price quantity fit more difficult but, when found, more convincing.

We find a similar result to that in Marseille, that aggregate behaviour does not always have a counterpart in the microeconomic data. Indeed plotting the date for a single buyer often yields a rather different picture: the price quantity relation becomes indeterminate at low quantity levels. We show this phenomenon in figure 3.22 where we select the days on which one of the largest buyers bought less than 200 Kg. of fish. In other cases examining the data for medium buyers one can observe a reduction of the price volatility for increasing quantities, but the price has no trend as seen in the
Figure 3.20: Price-quantity relation for two T.C., each dot correspond to the observation for a day.

Figure 3.21: Price-quantity relation for all T.Cs, each dot correspond to the observation for a day.
right panel of the same figure. So any aggregate characteristic is not the reflection of individual behavior.

Figure 3.22: Price-quantity relation for buyer 78, each dot correspond to the observation for a day.

On the other hand, if we put the data for all buyers on the same graph (we get 9184 observation) and perform some linear or non parametric regression, we get a nice downward sloping relation. It is interesting to note that the nonparametric evidence for the negative relation increases if we select the data points corresponding to low quantities. This is remarkable because the individual data appear to be absolutely random for low quantities.

How does this random individual behaviour translate into a well behaved aggregate relation? Once again we come back to the basic theme of this paper, the market aggregates complicated individual behaviour and it is the aggregation that produces the regularity at the aggregate level. Think of the way in which we normally obtain the aggregate demand. We do so by summing up the corresponding individual functions, and we assume that the individual curves are "well behaved". But this assumption is justified by all the standard unverifiable restrictions placed on preferences. The evidence suggests that the individuals do not behave at all as if they obey these restrictions. There are many ways of justifying this lack of consistency if one wishes at any price to recover standard demand properties. One should condition the demand choices by the day of the week, by the total quantity expected on the market, etc. But why should we persist in this venture? Think of an early approach proposed by Cournot and afterwards developed by a number of other economists. Cournot argued that individuals essentially have a reservation price for a unit of a good. They will buy the unit if the
price is below their reservation price otherwise they do not make a purchase. If we aggregate over many individuals with different reservation prices we will obtain a well behaved downward sloping demand curve. In other words the characteristic form of the aggregate demand curve can be obtained just using the heterogeneity of very simple buyers. Again, recall Becker’s result that if individuals choose randomly on their budget constraint this will generate downward sloping demand at the aggregate level. Thus individuals who are like Gode and Sunder’s (1993) “zero intelligence” traders still generate well behaved demand.

But do we actually see the desired relation at the aggregate level as we did in the case of the Marseille market? In figure 3.23 we select the datapoints corresponding to a weight of more than 450 kg. (9144 observations). This is simply because the density of the points decreases with the daily quantity, so if we include all the points we get a bad fit with our non-parametric method, given the actual bandwith. For each price we sum the total quantity of goods transacted and we do indeed get a downward sloping curve.

![Figure 3.23: Ols and Nadaraya-Watson (with h = 30) fit of the aggregate price-quantity relationship.](image)

Although we are dealing with a market organised by auction rather than through pairwise trading we see the same contrast between more or less random individual demand and well behaved aggregate demand.
3.17 Price dynamics

The next question that we asked and one which has a long history in economics, is how do the prices evolve over the day? To analyse the price dynamics during the day we developed two types of graph. We first ranked the daily transactions by the time of day in which they occured and then we performed averages for the transaction with the same rank. As shown in figures 3.24a and 3.24b the average price goes down as the rank of the transactions increases. A strange feature appears which we also observed for certain species of fish on the Marseille market: For a large number of T.C.s the average price starts increasing for the last transactions. Without knowing the exact information available to bidders it is difficult to explain this. However, if certain buyers need certain quantities of particular fish and suspect that the market is coming to an end then this could make them increase their bids.

(a) TC13  
(b) TC54

Figure 3.24: Average price for each rank of transaction of two TCs (the first dot on the left, for instance, is obtained collecting the price of the first transaction for each day and computing the average).

The second type of plot for the analysis of price dynamics is shown in figure 3.25. Here we plot the data corresponding to the day with the largest number of transactions for the specific T.C. (the horizontal axis records the time of the transaction so a value of 4.5 means that the transaction took place at half past 4 in the morning).

It is evident in general that the price volatility decreases as the auction proceeds. The early prices exhibit a turbulence that disappears over the period of the auction. This is probably due to two things. Firstly at the outset there are buyers that have to buy very
Figure 3.25: Price dynamics with in a day
3.18 Loyalty

A very interesting question is to ascertain whether the auction mechanism destroys buyer-seller relationships. As I have mentioned, in Weisbuch et al. (2000) we found strong evidence for loyalty of certain buyers to sellers in the Marseille fish market. But there the market is characterised by bilateral bargaining and the buyer could, in principle, make the rounds and collect every seller’s price to choose his best action. However many do not do so and as we saw loyal buyers actually pay more than random shoppers. Now the question is what does loyalty mean in an auction market. Remember that the name of the vessel which caught the fish is posted as the fish arrives on the band. So, it is possible that some buyers come to appreciate the fish from particular boats. Since the buyers are not the consumers they presumably realise greater profits from this fish. It is therefore possible that we see loyalty emerge. This does not mean that there is no uniform basis for judging the quality of fish. Supposing one vessel has the “best” fish. Then the prices for the fish from that vessel will be bid up and this may exclude a number of buyers. Thus we may see buyers becoming loyal to different buyers and paying different prices as a consequence. In fact, buyers in Ancona learn to become loyal as in Marseille but the pattern is somewhat different. What sort of measure should we use to calculate the extent of loyalty? A typical measure is the Gini index which indicates how spread out among different sellers are the purchases of a particular buyer. We calculate the Gini index for each buyer. In figures 3.26a and 3.26b we show the Lorenz curve for the two extreme cases (the least concentrated in the left panel and the most concentrated in the right one).

To have a global picture of the market we made a smoothed (Gaussian kernel) frequency distribution of the Gini index among buyers. A significant share of buyers have a Gini index equal to 0.4 and almost all have their index between 0.35 and 0.55. As I have said the buyer-seller relationship is different from that in the Marseille fish market. There, as I have described, (see Weisbuch et al. (2000)) we found basically two type of agents: those who were totally loyal (these would have a Gini coefficient of 1) and those that did not care whom they buy from. (these would have a Gini coefficient close to 0). The auction mechanism washes out this distinction since the
distribution in figure 3.27 is single peaked. However, we do see some preference on the part of different buyers for the vessels whose fish they buy. Different buyers are loyal to different vessels even though this loyalty is far from total. Can we say more about loyalty? An obvious question is as to whether loyalty depends on the buyer’s size. Figure 3.27 shows that the amount of loyalty increases with the size of the buyers up to a given value. Beyond this level the concentration index decreases or stays stable.

This is probably because very large buyers are forced to neglect the source of the fish if they are to get all the fish they need. Before looking at what prices different buyers and sellers pay, which is linked to the loyalty problem, one remark is in order. Vessels are of different sizes therefore if there was one very large vessel and many small ones, for example. It could be the case that many buyers buy a disproportionate amount of their fish from that one vessel. Thus loyalty would be a reflection of the size of the vessels. It could also be that different vessels manage to obtain very different prices which could sort the buyers into classes. With this in mind we can look at the prices that buyers pay and sellers obtain.

### 3.19 Buyers and Sellers Price performance

The first question here is: are price performances related to the amount of fish transacted? Basically it seems that the amount of fish bought or sold has almost no influence on the prices associated with the buyers and sellers. This conclusion is robust for all
Figure 3.27: Loyalty
buyers. However we did observe, for sellers, that the vessels with larger catches never sold at an average price lower than 7 euros, while some of the smaller vessels sold at lower average price on some days. So, to find out if there are some buyers that systematically pay higher (lower) prices than others or some vessels that get higher (lower) prices than others we did a slightly more sophisticated exercise.

For each subject (buyer or seller) we calculated the monthly average price and rank the subject by his price (prices are taken in increasing order). For each month, we thus associate the number one to the subject with the lowest price, two to the second lowest price and so on. Because the data cover 9 months, a subject that was present on the market in every month has a vector of 9 numbers, denoting the ranks, associated with him. If individual $x$ has the vector $[12, 5, \ldots]$ this means that there were eleven other people with an average price lower than his average price in September, 4 people with a lower average price in October and so on. To evaluate the performance of the subject we established a threshold (for instance 10) and counted the number of times the rank of the subject was less or equal to the threshold. What we found was that there are several buyers on the market who perform systematically better than others, and there are also those who regularly rank low. The same is true for the sellers.

What we see here is once again the diversity of behaviour at the individual level and some rather coherent aggregate structure. The market can be considered as an algorithm which transforms the varying individual demands into prices and organises itself so that all the fish is sold. As one might hope higher average prices are obtained on days when supply is shorter but this is not a reflection of classical individual demands. There are individuals who pay, on average, higher prices than others but sometimes these individuals beat their colleagues and obtain fish at low prices. In the same way it is not true that the prices of fish from those vessels that realise higher average prices always dominate those of their lower price competitors. There are many explanations for this and, for example, a buyer who normally pays high price may obtain almost all of his requirement early in the day and be prepared to wait and to try to complete his quota at a lower price.

### 3.20 An agent-based model

How might we model all of this in a way which captures the constraints that the buyers face but is not too complicated? In Gallegati et al (2009) we developed a very simple model which attempts to do this.

What we did is to build an agent based model using some of the observed features of MERITAN. I shall only sketch the outline of the model here but the full details can be found in Gallegati et al. (2009). To start with we assumed, for example, that sellers have no possibility of taking decisions so their behaviour is not modelled (their cases

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19Of course the quality of fish is also important for our analysis. Unfortunately there is no way to have this information from the dataset. It is claimed by those running the market that the variation in quality is very limited, since the fish are all caught in the same area and it is all landed the day after the catch. The lack of variation in quality is also true for the Marseille market but, rather because the classification scheme for the different types of fish is extremely precise. Given, in addition, that fish must be sold on the same day there is little room for quality variation.
are delivered before the market opens and are placed on the belt in a random order).\footnote{In reality this is not quite true since the sellers can determine a reserve price below which the fish will not be sold. However, this is done before the auction starts and does not change as the auction progresses.}

What we wanted to do then is to model the buyers’ behavior. Here our question is: how simple can we make a buyer’s behaviour and still reproduce certain regularities we observe in the data? The strategy is to endow each buyer with the simplest rule we can. This is in contrast with more standard practice which is to either define a certain type of optimising behaviour or to use a learning mechanism. Of course, this is, in a sense, an engineering rather than theoretical approach. It is at the other extreme from the game theoretical model outlined earlier for the Marseille market and does not attribute to the agents the learning capacity that they had in the ACE model described before.

What we did was to take into account the variables that influence the buyers’ decisions. One important factor is that the buyers have a rough idea of the total quantity of fish available when the auction starts. They can infer this information in several ways. First of all the opening hour of the auction is an important indicator: if the fish is abundant the vessels stay longer at sea. Second, the auctioneer gathers a lot of information before opening the auction. This information is transmitted to the buyers by the prices at which he starts the sales of each case at the beginning.

On the Ancona market the buyers are not consumers and they are faced with a final demand function which of course affects the bid and some knowledge of its form is required. It is not a gross simplification, we were told by the market organisers, to assume that each buyer has a fixed quantity that he wants to buy on any given day. We assumed that buyer’s average bid price is an increasing function of the quantity, remaining at time $t$ before he reaches his goal (where $q_{it}$ is the quantity he already bought), and a decreasing function of the total quantity available at time $t$. We then added a random element to the bid and were able to generate data that had many of the characteristics of the empirical data. The details of the model are given in Gallegati et al. (2009).

While it is still possible, at least in theory, to behave intelligently under the Dutch auction mechanism on the MERITAN, the agent based model we built shows that behavior does not have to be very complicated to generate certain observed facts. On the contrary, very simple behavioral rules coupled with agents heterogeneity are able to reproduce the latter. Once again it is more the interaction of the different agents rather than extremely sophisticated strategic behaviour that produces the aggregate phenomena on the market.

### 3.21 Conclusion

The message of this chapter is rather simple. Markets are an important feature of all economies. Each market is characterised by an organisation and structure which will have an impact on the outcomes observed. Fish markets, as markets for particularly perishable goods, are especially suitable for economic analysis and have frequently been used in the past as an example. However, even in this case standard models do not seem to be well adopted to shedding light on the nature of the economic outcomes that one might expect. Curiously enough the examples which I have examined here, those of the fish markets in Marseille and Ancona, do exhibit rather a lot of regularity at the
aggregate level. Nevertheless this is not due to individuals behaving, in isolation in a regular way as in the standard competitive model. The complicated organisation of this sort of model breaks any simple link between individual and aggregate behaviour. A number of the special features of this market such as the special trading relationships that have developed are difficult to account for in the standard framework. I first suggested a simple theoretical approach which does capture the formation of such trading relations. Furthermore, it seems that an even simpler approach referred to as “multi-agent” simulation based on very simple rules for learning from past experience is rather successful in reproducing some of the features of the real fish markets chosen as examples. In particular these models generate price discrimination which is linked to the structure of the relations between the market participants. Furthermore the comparison between the markets in Marseille and Ancona highlights the link between organisational features and aggregate outcomes.
Chapter 4

Financial Markets: Bubbles, Herds and Crashes

“I can calculate the motion of heavenly bodies, but not the madness of people”

Isaac Newton

4.1 Introduction

From what must seem to be a rather esoteric example, fish markets, I will now turn to financial markets. The recent events that I mentioned briefly in the first chapter and that have shaken the world economy, have led economists to take a long look at their role in the economy. These markets fascinate people, in part because of the spectacular sums involved and, in part, because their performance has direct consequences for many individuals. This could not have been clearer than in the autumn of 2008. Financial markets suddenly shifted into an extremely volatile phase for which nobody was directly responsible. As I have said, for economists, in most contexts, markets achieve the coordination of individual choices but how precisely they do so is mostly left unexplained. In the discussion of fish markets, I hope it became clear that markets play different roles in the coordination of the actions of many individuals who interact in various ways. Furthermore the outcome depends on the particular structure of the market. Yet, economists discuss at length such topics as “the efficiency of market outcomes” and of “market allocations” whilst paying almost no attention to the way in which markets themselves function in reality.

Financial markets are, in a sense, at the opposite end of the spectrum from markets for perishable goods like fish. Fish markets are of particular interest to economists, precisely because almost none of today’s goods are carried over to the future. Markets on different days are linked because the participants learn from their experience but individuals are not taking positions in the market based on their expectations of tomorrow’s price for today's fish. In financial markets, just the opposite is the case. What
drives these markets are the expectations of the participants of the future value of assets and most of these assets do not have a finite life. Forecasts of the future and how they are made are central to understanding the functioning of financial markets.

For the outsider, markets, and in particular, financial markets are cauldrons in which agents risk millions of euros or dollars daily. The atmosphere is highly charged and as Coates and Herbert (2008) show, the way in which traders react and confront each other is strongly linked to their testosterone level. Lo et al. (2005) measured the physical manifestations of stress in traders whilst they were actually trading. Those with the most emotional reactions, on average, perform worse than their “colder” colleagues. These actors who are driving the formation of prices and their intense interaction are light years away from the abstract and somewhat sterile view of markets that characterizes much of pure economic theory and which bears little relation to the vision of the ordinary man of these institutions.

More importantly the reductionist view of theory is far from describing the vision of those who actually participate in markets or who are responsible for regulating them. Such markets are characterised by a whole relational and conventional structure which has evolved over time. One might well argue that it is this tissue of conventions that actually regulates the markets and prevents the individuals within them from behaving in a completely extreme fashion. In fact, as Aboulafia (1997) when discussing financial markets observes,

“Markets are socially constructed institutions in which the behavior of traders is suspended in a web of customs, norms, and structures of control.”

Even the use of the term “trader” implies an activity that is little described in most economic models. Here I will attempt to provide an approach to analysing financial market which, at least, allows one to reproduce some of the stylised facts of such markets. I shall remain at a rather modest level and argue that we can develop models which incorporate the direct interaction between agents in a market and that, attributing some rather simple features to the individuals, modifies market outcomes considerably. In the sort of models I have in mind the individuals are not blind in the evolutionary sense, they try to make choices according to some criterion, but do not optimise in the full sense, nor do they have the information necessary to do so. The sort of example presented here might be thought of as being among the bricks from which we might hope to build a more complete and realistic framework within which to analyse the functioning of markets.

4.2 The “standard” approach

Before proceeding it is worth examining the basic premise of this chapter. It is easy to find assertions to the effect that standard theory does not deal in a satisfactory way with the functioning of markets. Indeed, a well known economist remarked recently that as soon as he read the phrase, “standard economic theory cannot handle this problem”, he would move on to another paper. Usually the people making the assertion seemed not to be aware of the real content or scope of modern economic theory. To avoid this
4.2. THE “STANDARD” APPROACH

pitfall, let me first look at what is meant by the “standard” theory in this context and in particular the standard theory of financial markets, which are the examples I shall use.

Consider the simplest description of the “competitive mechanism” which is at the basis of almost all theoretical models. Individuals accept prices as given, choose or “demand” the best bundle of goods available at those prices subject to their budget constraint. An equilibrium allocation is one where the quantities demanded by the individuals add up to those available in the economy and correspond to those chosen by them at some prices \( p^* \) which are then referred to as equilibrium prices.

Before continuing, two critical remarks are in order. Competitive equilibrium is generally thought of as an individualistic, decentralised solution concept. However, for this to be strictly true, one would have to specify how prices are actually set. Besides this, there is a basic objection to the competitive solution, in that it only makes sense if the passive price taking behaviour can reasonably be justified. The typical response of the economist is that this assumption makes sense if there is a large number of agents and that none of them individually, therefore, could have any influence on prices and hence on the outcome of the allocation process. In fact, if one wishes to be rigorous, there must be an infinite number of agents if each of them is to have strictly no influence. Thus if we finesse the problem of who sets prices, the competitive or Walrasian solution could be thought of as acceptable for very large economies.

However, as a picture of how markets actually work it is seriously deficient. There is no communication, interaction nor even trade between individuals. Yet, few would argue with the idea that this is still the vision that permeates economic theory. Suppose now that we make a leap of faith and we leave these considerations to one side and accept the idea of a market with isolated, price taking participants. We are still left, in financial market models, with further assumptions which need to be examined. The first of these is the “efficient market” hypothesis.

4.2.1 The efficient market hypothesis

Once we enter the world of financial markets we are in a world in which time, uncertainty and predictions about the future play a role. Variables are linked to each other, as indeed they are in the General Equilibrium model in the sense that the values of some variables will directly determine the values of others. Since there is uncertainty, prices are stochastic variables. Furthermore, in this view, individuals take decisions as if they knew the underlying stochastic process governing the evolution of prices. This is the “rational expectations” hypothesis. In the majority of models of financial markets, a closely related assumption, the “efficient markets” hypothesis, is imposed. This asserts that the prices of financial assets are determined by fundamental variables, the evolution of which is exogenous to the model. For example the value of a stock is determined by the discounted sum, the present value, of future dividends. Furthermore this hypothesis says that, at any point in time, asset prices already contain all known information. They are, therefore, as informative as is possible. The only way that this could be compatible with the arrival of new information, for example about the underlying fundamental variables, which could have an impact on the prices would be if that information were completely unpredictable. Indeed, if it were predictable then someone could make that prediction and gain money from it. To understand what is
meant by unpredictable one could think of a series of random shocks arriving, each of which cannot be predicted. This means that they must have zero mean. This is what is called the random walk hypothesis and dates back to the work of Bachelier (1900) on the Paris Bourse. It has received much attention since it implies that, in the long run it is not possible to make above-average returns in the stock market by trading, except through persistent luck or by obtaining and trading on the basis of information which is not available to others and which is known as inside information.

There are three common forms in which the efficient market hypothesis is commonly stated varying from weak form efficiency, through semi-strong form efficiency to strong form efficiency, each of which, in principle, has testable implications. Weak form efficiency, for example, says that no profit can be made from the information contained in previous prices. This would imply that no amount of time series analysis can generate a successful investment strategy. Furthermore one can test the structure of the series of increments in prices to see if they exhibit the stochastic structure implied by the hypothesis.

At the other extreme, the strong form efficiency hypothesis asserts that the prices of assets reflect all information and no one can earn excess returns. This, of course ignores the possibility of insider trading. In fact, analysis of the US stock market has shown that people do trade on inside information. However, it was also found that other individuals watched the trades made by those with inside information and made similar trades thereby reducing any profits that the inside traders could make. Were this process instantaneous there would be no problem of insider trading. The fact that it is not, is one of the reasons for being dissatisfied with the standard model of financial markets.

It is worth noting that the strong hypothesis does not mean that some people will not make more profit than others at some point in time but does suggest that such differences should disappear over time.

At this point, it is worth reflecting on what models, incorporating the efficient markets hypothesis would predict for financial market prices and then to see what sort of deviations from the predicted behaviour occur in actual markets. A first observation is that the standard view of the evolution of the prices of financial assets is, as I have observed, that they behave as a random walk or more precisely as geometric Brownian motion. This is what is at the basis of the Black-Scholes model. In such a case prices $S_t$ at time $t$ would be characterised by the following equation:

$$dS_t = \mu dt + \sigma dW_t$$ (4.1)

In other words there is a drift over time plus some random variable. Such an equation has the advantage that it is useful for pricing derivatives but, as we will see, it misses some essential features of empirical financial time series that we would like to capture. Consider the following example illustrated in Figure 4.1.\textsuperscript{1} The time series, which is that of the German Dax stock market, index from May 1994 to May 1999 seems to behave very much as predicted by Equation (4.1).

But now look at what happened in the subsequent periods as shown in Figure 2. There was an abrupt shift in direction and the market moved swiftly into a downturn.

\textsuperscript{1}This illustration was given by Ulrich Horst.
Figure 4.1: The German Dax index from 1994-2000
However, if we now complete the series by the observations from 1999 onwards the picture changes and we now have to explain the sudden turning point as seen clearly in Figure 4.2.

We need an explanation for the sudden turn-around of the market. One idea is to suggest that there was some major exogenous shock and this is what one would have to do to maintain the efficient market hypothesis. Typically, such a sudden shock could be related to some major item of news, but there was no news of significance in the case in question. Indeed, Bouchaud (1) has shown that, in the great majority of major turning points in the leading stock market indices there was no significant item of news. An alternative approach and the one that I will follow here, is to try to build models, which will produce such turning points as endogenous phenomena.

Those who retain the efficient markets hypothesis try to reconcile theory with observations by arguing that, what appear to be deviations from it, are due to imperfections such as costly transactions, slow diffusion of information and the non-negligible power of some market actors. Thus the basic model is still as specified but there are real world imperfections. This is a common explanation of economic data which are inconsistent with the theory.

In this chapter it is the opposite vision that prevails, that is, the sort of events that one observes on financial markets are a fundamental feature of the process that generates the data. This is because what one is observing is a market in which information is constantly being transmitted between individuals, in which information is inferred from the actions of others and where the trades are organised on a sequential basis. It is this that is the cause of bubbles and cycles and not some exogenous shocks. There is not some sort of equilibrium process which is periodically perturbed, the path of prices reflects an endogenous process, and the fluctuations are an integral part of that process.

What is most interesting is that Bachelier’s work which is at the basis of the efficient markets hypothesis went unnoticed for many years and then was taken up with such enthusiasm by economists. Yet, from the outset there were those who saw that the whole structure was unsatisfactory for exactly the reasons that I have outlined. At the time that Bachelier wrote his thesis, Henri Poincaré the great French mathematician wrote the thesis report. He clearly stated that one should not take this seriously as a way of modelling financial markets, as individuals do not act independently but are constantly influenced by others and will always be prone to herd behaviour. To cite him precisely,

“Quand des hommes sont rapprochés, ils ne se décident plus au hasard et indépendamment les uns des autres; ils réagissent les uns sur les autres. Des causes multiples entrent en action, et elles troublent les hommes, les entraînent à droite et à gauche, mais il y a une chose qu’elles ne peuvent détruire, ce sont leurs habitudes de moutons de Panurge. Et c’est cela qui se conserve.”

Henri Poincaré La Valeur de la Science 1908

This can be translated as

“When men are close to each other, they no longer decide randomly and independently of each other, they each react to the others. Multiple causes come into play
Figure 4.2: The German Dax index from 1994-2004
which trouble them and pull them from side to side, but there is one thing that these
influences cannot destroy and that is their tendency to behave like Panurge’s sheep.
And it is that which is preserved”. 2

But Poincaré’s warning 3 went unheeded and, for a long period, the efficient markets
hypothesis ruled the roost until the day when Alan Greenspan ruefully admitted before
Congress that the

“the whole intellectual edifice collapsed in the summer of last year”


In the light of all this it is worth looking again at what might be the best way to
analyse financial price movements.

4.2.2 The notion of equilibrium

To see what I have in mind, return to the example of the stock market index, and we
are presented with two alternatives. Either, before the major turning point, prices were
on an equilibrium path and then some significant exogenous news or shock happened
which knocked the series off that path, or there was a “bubble” and prices had become
detached from the fundamentals. Those who adhere strictly to the standard model do
not accept the existence of bubbles and would argue that some real events must be
involved in any large change in trend. Indeed, it can be argued that the relationship
between fundamentals and asset prices over time is highly “non linear” and that small
changes in today’s values may lead to large changes in the future, thus significantly
changing current asset prices.

It is difficult, however, to believe that there could be a sudden change in the funda-
mentals which would lead agents to simultaneously agree within half a day that average
returns over the major U.S. stocks in the future had gone down by over 20%. Yet this
is what would have to be argued for the October 1987 episode on the New York Stock
Exchange and, indeed Merton Miller (1991) suggests that substantial changes in the
future can result from very small changes in the present and that such an explanation is
not inconsistent with the crash. Yet, this seems to be suggesting a complicated response
to a question for which more plausible and simpler answers can be found.

The same difficulties arise in explaining sudden and substantial changes in ex-
change rates, do they really simply reflect modifications in expectations about future
fundamentals? Not so long ago the euro dollar exchange rate was steady for a long pe-
riod around 0.8, since then it moved abruptly above parity and recently it has settled for
a period around 1.5. What information about the real economies in Europe and in the
U.S provoked such a radical change? Suppose that there was a tight but, maybe com-
plicated, relation between the fundamentals and the rate which is constantly present. In

2For a complete and entertaining account of this period and the origins and development of the efficient
markets hypothesis, see Fox (2009).

3It is also worth remarking that Poincaré had a correspondence with Walras, in which he chided the latter
for his assumptions of the “infinite egoism” and “infinite farsightedness” of economic agents. The former he
could accept, at a pinch, but the latter seemed, at best implausible. At the very outset of economics’ journey
down the path to Arrow-Debreu, Poincaré was already pointing out the difficulties, but unfortunately for
economics his observations fell on deaf ears.
this case, how does one reconcile the two ideas frequently expressed by traders that on the one hand, “fundamentals matter in the long run” but, on the other hand, they do not drive exchange rates in the short run? A simpler explanation would be that bubbles occur and periodically collapse but that the timing of the crash is unpredictable. This means that traders with short horizons, even if they are convinced that, at some point, prices will return to the level consistent with fundamentals will continue to follow the current trend. This will reinforce the bubble and is far from being irrational. This is in contradiction with those who argue that bubbles are simply due to the irrationality of traders and even further from the view of others like Garber (2000) who argue that the major historical bubbles can be attributed to fundamentals. Thus, in his view, there are no bubbles in the sense of prolonged departures from fundamentals. For him, presumably, the shift in the euro dollar exchange rate merely reflects some profound change in the relative fundamentals of the two areas. But what would be puzzling if one was inclined to believe this is the suddenness of the change.

Rejecting bubbles because they do not sit well with our theory does not seem a persuasive idea. Indeed, there is a long history concerning bubbles and there is a substantial body of evidence for their existence, (for a nice historical example of contagion effects, see Kelly and O’Grada (2000))

Another major quarrel with the efficient markets hypothesis and the close relation between asset prices and the fundamentals was caused by the so-called “excess volatility” puzzle. A number of studies have revealed that the volatility of stock prices was significantly higher than that of the dividends on which the prices were supposed to be based in theory. There are at least a priori grounds for arguing that both the bubbles and the excess volatility problem merit closer examination.

4.2.3 Bubbles and excess volatility

As I have observed, one of the major difficulties with standard models of asset markets, and particularly of foreign exchange markets, is that what is difficult to explain is not so much the appearance of “bubbles” which grow but the fact that they eventually burst. If one leaves to one side the idea that they simply do not exist, it is worth looking at the theoretical literature on the subject. There is, for example, a substantial literature on “rational bubbles”; see, e.g., Evans (1991). These contributions point out that bubbles need not be inconsistent with rational expectations. However, such models suffer, in general, from the lack of an underlying market framework and also from the fact that such bubbles will be permanently explosive.

An alternative explanation for the failure of prices to track fundamentals is that they come to be correlated with some external events which, while not being relevant, become so, simply because the market participants believe them to be fundamental. A number of economists have shown how prices can be correlated with initially irrelevant exogenous variables, “sunspots” (see for example (Cass and Shell (1983)). What is more Woodford (1990) has shown that individuals can rationally learn to believe that sunspots play the role of fundamentals and, as a consequence these beliefs will be fulfilled. Typically, the analysis in such examples wishes to show that prices will settle

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to some equilibrium, albeit a “wrong” one. This raises again some of the fundamental questions that I have already mentioned. Is there an equilibrium price, if so is it constant, and if not can we think of some other more appropriate notion of equilibrium? How is it related to fundamentals?

The excess volatility puzzle refers to the fact that the prices of stocks seem to exhibit much greater volatility than that of the fundamentals which are supposed to underlie them. In foreign exchange markets there is a tendency to think of the fundamentals as the arrival of news. Yet as De Grauwe and Grimaldi (2001) observe, the volatility of the exchange rate by far exceeds the volatility of the underlying economic variables which were the basis for the news in question. Flood and Rose (1995) found that while the movement from fixed to flexible exchange rates led to a dramatic increase in the volatility of the exchange rate no such increase could be detected in the volatility of the underlying economic variables. This contradicted the ‘news’ models that predicted that the volatility of the exchange rate can only increase when the variability of the underlying fundamental variables increases.

### 4.2.4 The predictability of asset prices

Some further observations about the theory of financial markets are in order. As I have remarked, a standard argument about the evolution of prices of a financial asset is that if it were predictable then agents could make money. Thus, the only evolution possible is one of a series of random shocks, hence the notion that stock market prices must evolve as a random walk, an idea which, as I have mentioned, dates back to Bachelier. This argument is, of course, open to an obvious objection. If there is predictable structure present in price series it can only be exploited if agents are able to or learn to, perceive it. Thus as Brock et al. (1992) have shown, technical trading rules do have some predictive value in stock markets. Of course, once this is understood, their predictive power should disappear.

One thing is clear from these remarks. Prediction is based on observed prices and this prediction determines people’s actions. These actions, in turn, determine prices. As we have seen, this means that prices can become correlated to some external phenomenon such as “sunspots” because agents believe them to be so linked. Alternatively the system may exhibit endogenous fluctuations (see Guesnerie and Wood (1992)). Again, prices here incorporate the predictions of individuals as well as engendering these predictions. Any change in the information of agents which leads them to change their predictions will, in turn, naturally change prices.

A second observation is that the structure of financial markets and the way in which information is passed may, of itself, influence the evolution of prices. To take as a simple example the foreign exchange market, traders who have very short horizons, deal on the basis of orders from individuals or institutions with longer horizons. The traders essentially have to “close their positions” at the end of the day. Their trades will be influenced by this requirement and will in turn influence the bids and asks they make. This will modify the evolution of prices over time and may, in turn, influence the transactions longer term agents wish to make. Such a process may not necessarily be either stationary or stable. Indeed the very organization of the market may influence the way in which prices evolve and may explain why prices do not remain firmly linked to
4.3 Informational Cascades and Herd Behaviour

This brings me to another point. If agents act sequentially they may well infer information from the acts of others. This may lead to “herd behaviour” and a literature has now developed explaining how such behaviour may lead to price bubbles in financial markets (see Banerjee (1992), Sharfstein and Stein (1990), Bikchandani et al. (1992), Welch (1992), Hirschleifer (1993), Kirman (1993, 1994) and for a comprehensive account, Chamley (2004)). Thus as trading develops, individuals may influence other individuals’ expectations and fluctuations may occur as a result, or individuals watching others may abandon their own information in favour of that conveyed by the actions of others and an “informational cascade” may result. Figure 4.3 illustrates a famous experiment carried out on Times Square and where some students by gazing up at the sky induced a whole crowd to do so. The idea here is that which underlies “herd behaviour” in financial markets, if people are doing something they must have a reason for doing it.

Models of “herd behaviour” suggest that there is some externality generated by the choices made by members of the population. For example in Banerjee’s (1992) model individuals sequentially choose options from those indexed on a line segment. One of these options is profitable, the others are not. Each player receives a signal
with probability $\alpha$ and this signal is correct with probability $\beta$. People choose their options sequentially. Thus observing the choices of previous individuals may reveal information about the signals that they have had. Banerjee looks for what is called a Bayesian Nash equilibrium and finds that the equilibrium outcome will be, from a welfare point of view, inefficient. However, if the population is large enough, that the probability that none of the $N$ players will choose the “profitable” option is bounded away from zero. But, and this is important from our point of view, the equilibrium to which choices tend is highly variable across different plays of the same game. Indeed, the initial choosers have a major influence on the final outcome.

### 4.3.1 The restaurant example

A very simple example explains the origin of the Banerjee problem. There are two restaurants $A$ and $B$ and there is a public signal, such as the Michelin guide which is 60% reliable, that is which has a 60% probability of being correct. Suppose that the signal suggest that $A$ is better. Suppose that individuals also receive a private signal which has a 90% probability of being correct. Moreover, of 100 potential clients 90 receive a signal that $B$ is better and 10 that $A$ is better. Thus the aggregate information suggests that $B$ is highly likely to be better. Now if all this information were somehow to be published all individuals would choose $B$, ignoring any crowding problems. However what happens if individuals choose sequentially? If one of the players who received the private signal $A$ chooses first he will choose $A$ since both the signals available to him indicate that $A$ is better. Suppose that individuals also receive a private signal which has a 90% probability of being correct. Moreover, of 100 potential clients 90 receive a signal that $B$ is better and 10 that $A$ is better. Thus the aggregate information suggests that $B$ is highly likely to be better. Now if all this information were somehow to be published all individuals would choose $B$, ignoring any crowding problems. However what happens if individuals choose sequentially? If one of the players who received the private signal $A$ chooses first he will choose $A$ since both the signals available to him indicate that $A$ is better. The second player observes signal $B$. Since signals are assumed to be of equal quality, and he knows from the first player’s behaviour that he received signal $A$, the two signals cancel out and he is only left with the public signal and he therefore chooses $A$. The third player is thus left in a situation where he can infer nothing about the information received by the second player from that player’s choice. He is therefore in the same position as that player was and chooses $A$. By the same reasoning the unsatisfactory result occurs that all clients end up at the almost certainly worse restaurant. Paradoxically both welfare loss and instability would be reduced by preventing some people from using other than their private information. Thus reducing interaction would be better for all involved. The Banerjee market phenomenon is strongly related to the feedback between players and what is of particular interest is the crucial role played by the sequential structure of the moves.

Bikhchandani et al. (1992) emphasise the fact that after a sufficient time the cumulated actions of other actors contain so much information that an individual will have an incentive to ignore his own information and a “cascade” will start. A facetious illustration of this is provided in figure 4.4.

Although this cartoon seems just to illustrate the madness of traders, it captures the point I have been making. The individual who mistakenly thinks that he has heard “sell” is not really behaving irrationally. The signal was wrong or wrongly interpreted but given what he perceived the signal to be is he not justified in using it? In many types of analysis one does not allow for the information in a signal to be poorly transmitted. But, providing that the information value of an individual’s signal is bounded, Smith and Sorensen (2000) show that there is a positive probability that a cascade which leads to a socially inefficient outcome will occur. In other words, once one allows for
4.3. INFORMATIONAL CASCADES AND HERD BEHAVIOUR

Figure 4.4: An Information cascade
the possibility of wrong information a bad cascade can occur, just as in the cartoon.

As I have mentioned, dropping the sequential nature of the decision-taking can eliminate the problem but in many financial markets, for example, the sequential structure of decisions plays an important role. The prices observed on financial markets are the prices at which each transaction is made and transactions are clearly made sequentially in reality. Sequential trades will not necessarily lead to herding, however, and Moscarini and Ottaviani (1998) argue that if the underlying state of the world is changing over time then conditions can be given in which no cascade at all or only temporary cascades can occur. Smith and Sorensen (2000) furthermore show that if people are of different types, each type can herd on a specific opinion. Thus heterogeneity, another feature frequently absent from standard models, can reinforce the herding phenomenon and different types can converge on different beliefs.

What is particularly useful about this literature is that it makes explicit how the efficient markets hypothesis breaks down. As the situation progresses, people cease to look at their own information. Since the way in which private information gets incorporated into public knowledge, is because someone acts on the information, it is clear that when people cease to act on their own information it is lost. This is exactly what happens in the restaurant example and shows how it is possible for information not to get incorporated into the public domain.

### 4.3.2 Another explanation for herding

In the previous example people were extremely rational but this did not prevent the system moving to a bad position. There are also simpler case in which individuals reacting rather mechanically to the people that they encounter can generate outcomes that do not coincide with any usual notion of equilibrium. In Kirman (1993) for example, I discussed the evolution of two opinions over time in a population. The basic idea was stimulated by the observed behaviour of ants who, when faced with two apparently equally productive food sources concentrate largely on one for a period of time and then for another period, focus their attention on the other. This is due to the recruiting process, which is such that, the more ants are being fed, the stronger the trail to the source and the higher the probability of an ant, leaving the nest, of going to that source. Using a simple stochastic model developed by Hans F"ollmer and which led to later joint work which I will describe at the end of this chapter, it is shown that provided there is a minimal amount of “noise” in the system the proportion of ants feeding at each source will stay close to 1 or to 0 for a long time and then switch to the other extreme.

The feedback involved can either be thought of as a stronger trail, or, if recruiting is of the tandem type, of a higher probability of meeting a successful forager from the food source that is currently most frequented. The appropriate equilibrium notion here is then not some fixed proportion but rather a limit distribution of the underlying stochastic process. Thus, thinking of the state of the system as $k/N$, the proportion of ants at the first source, we can write $f(k/N)$ for the limit distribution and this should be viewed as the proportion of time that the system spends in any state $k/N$. What
Hans Föllmer showed was that if one lets $N$ become large, and approximates $f$ by a continuous distribution $f(x)$ where $x$ takes on values between 0 and 1, then this distribution will be a symmetric beta distribution, i.e. of the form

$$f(x) = x^\alpha (1 - x)^{\alpha - 1}$$

(4.2)

and with appropriate assumptions on the parameters of the original model the distribution will be concentrated in the tails. Thus the process will indeed spend time at each limit and occasionally switch between the two. In other words, the ants in the original situation were concentrated on one food source and then switched to the other but there was very little time where the ants were divided evenly between the two sources.

In a symmetric situation with a priori symmetric individuals, aggregate behaviour displays violent stochastic swings.

This sort of model can be applied to a market for a financial asset as is done in Kirman (1991) and the abruptness of the change there is amplified by including into agents’ observations a signal concerning which opinion is held by the majority. Here observing the behaviour of the majority provides additional information over and above that obtained by direct encounters. One has, of course, to model the market in such a way that agents do actually gain by acting with the majority.

This model is related to that developed by Topol (1991) for example and he refers to “mimetic contagion”. Ellison and Fudenberg (1993) also develop a model in which not only how well a certain choice has done but also its “popularity” is considered. In financial markets agents are faced with two problems, first to forecast future prices, and second, on the basis of that, to choose the best investment to make. How they forecast prices can also be thought of as a choice as to which forecasting rule to use.

### 4.3.3 Agents’ forecasts

From what I have just said it is clear that investment and speculation depend, on the forecasts that people make as to future prices. The obvious question is then, how do agents make their forecasts? In keeping with everything I have said up to now it seems reasonable to assume that agents have different possible forecasting rules and must somehow choose between them. This is very different from the rational expectations approach in economics, which assumes that agents all have the correct vision of the stochastic process which governs prices in the future. In the latter case we ignore any possibility that agents might learn over time about how prices evolve and simply assume, to close the model, that all of them understand the “true process” and forecast appropriately. There is no place in that equilibrium view for learning about the price process.

The rational expectations approach seems to be just a convenient construct for making equilibrium consistent but does not tell us anything about how agents actually form their forecasts and modify them. Furthermore, whilst it is true that rational expectations are consistent, it is not at all obvious that learning will lead to them. To be a little clearer, if we take the position that agents have to learn how to form their expectations, then as they learn, they will modify the latter. This, in turn will modify their

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5Private communication.
demand and this will then change prices. Now, using the updated forecasting rule they will make a new forecast given the prices that have been realised and so forth. Why, as everybody learns in this way, the process should converge and why, in particular, it should converge to rational expectations, is not clear.

If we think that agents do learn about the environment in which they function, then we should try to model the learning process and incorporate it directly into our model of a financial market. In the examples that I will use I suggest a rather simple approach. The basic idea, as I have said, is that agents have a number of forecasting rules amongst which they must choose and the way in which they choose amongst them has to be specified.

Let me look at two problems posed by this restriction, in turn. What we need to know is how many forecasting rules there are and what form they take. Provided that they are well defined, we can have any finite number of such rules. It is often the case that authors choose two rules as an example but there is no need for this.

What form the rules should take poses an important question. Should they be based on past prices alone or should they be based on the values of certain economic variables which are considered to be “fundamental” to the economy. In the first case, future prices are forecast by extrapolating from past prices and, however sophisticated, such rules are referred to as “chartist”. On the other hand, rules which use the values of other economic variables are frequently referred to as “fundamentalist”.

In the chartist case we have to specify the sort of extrapolation that the agent does. It could be a very simple moving average or some rather sophisticated time series construct. In either case we have to specify how many observations from the past the agent uses. That is, how long a memory he has. This choice is not innocent and the stability of the price process can be seriously affected by the presence of agents forecasting on the basis of very limited memory.

For the fundamentalist forecasters, there is one thing that is important and that is the choice of the variables which are considered as being fundamental in determining the price of the asset in question. Typically, the fundamentals are considered as evolving exogenously. In fact, what the fundamentals are and how related they are to the price of the asset in question is a question that remains largely unanswered. What is worse, there is no reason to believe that agents have a view of the relationships linking economic variables which is fixed and if agents are persuaded of the relevance of some new variable it may actually become relevant. As I already mentioned, Michael Woodford (1990) showed how people could learn rationally to believe that some exogenous phenomenon has an effect on prices. In his case the exogenous variable was referred to as “sunspots” which, by their nature, have no intrinsic impact on the prices of assets. Yet, as people come to believe that prices are affected by sunspots they modify their demand for assets and this, in turn, causes them to be correlated with the sun spots. Thus, what are initially irrational beliefs, become rational. This illustrates the very basic idea that what is fundamental depends to a large extent on what is perceived to be fundamental.

Once the set of rules, whether “fundamentalist”, extrapolatory, or some combination of the two, is established, we have to specify how agents decide amongst the rules.

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6These terms were first coined in the foreign exchange literature by Frankel and Froot (1986)
Then having done that, we can determine their demands and this will, depending on the equilibrium concept we are using, determine the equilibrium price or the series of market prices. If we are looking at a sequence of equilibrium prices then the mechanism is simple. Forecasts determine demands, demands determine prices, these prices together with previous prices determine new forecasts, which in turn, generate new demands and hence new prices and so forth.

To come back to my main message, the choice of forecasting rule is based on the success of such rules in the past, but that success is determined by how many people follow the rules. The interaction between the individuals in what amounts to “herding” is thus driving the evolution of prices. It is this that prevents the market from tending towards some sort of steady state. In a recent article by Brazier et al. (2008), macroeconomic stability is studied from this point of view. The argument here being the following. The fact that one view as to how monetary policy works comes to predominate and leads to a long period of financial stability, in no way guarantees that such stability will persist as people switch to another view. Thus the comforting view that we have finally learned how to keep the economy on a healthy, financially stable trajectory is easily belied if the individuals in the economy coordinate on some other view of how the economy works. This is closely related to an argument made by Brock and Hommes (1997). Thus even indirect interaction between the individuals in a market or economy, particularly as regards their beliefs, can lead to major aggregate movements which are difficult to explain in the standard model.

To get a more specific idea of how all this works consider the following simple example:

4.3.4 An example

This example is a simplified version of that used in Kirman et al (2004) and is closely related to the model developed in Kirman and Teyssiere (2004). There is a foreign exchange market on which are traded two currencies domestic $d$ and foreign $f$ whose values are linked by the exchange rate $s$. $s$ is the price of one unit of foreign currency in units of domestic currency, so that, from the point of view of a domestic investor a devaluation of her currency means an increase in the value of $s$. There are $n$ investors, who measure their wealth in units of $d$. They have the same risk aversion and the same utility functions but they may hold one of two views as to the evolution of the exchange rate. Thus the driving force behind the dynamics in this model will be the variations in the proportions of those following each of the forecasting rules.

The next thing to choose is the set of forecasting rules. Just to simplify matters consider two types of rule, fundamentalist and chartist but, in fact, any finite number of rules would fit into the same framework.

Now, think about the investor who wishes to forecast the value of $s_{t+1}$. If she is a fundamentalist she believes there is an equilibrium value $\bar{s}_t$ to which the exchange rate will revert. Suppose that these beliefs have the following form:

$$E^f(s_{t+1}|I_t) = \bar{s}_t + \sum_{j=0}^{M_f} \nu_j (s_{t+j+1} - \bar{s}_{t+j}) \text{ with } \sum_{j=0}^{M_f} \nu_j = 1$$

(4.3)
If she is a chartist her forecast as to the future value of the exchange rate will be an extrapolation of its past values. Give these extrapolations the following form:

\[ E^c (s_{t+1} | I_t) = \sum_{j=0}^{M_c} h_j s_{t-j} \quad \text{with} \quad \sum_{j=0}^{M_c} h_j = 1 \]  

(4.4)

Now define the following variables at time \( t \):

- \( \rho \) is the dividend in foreign currency paid on one unit of foreign currency;
- \( s_t \) is the exchange rate
- \( f_i^t \) is the demand by individual \( i \) for foreign currency
- \( d_i^t \) is the demand by individual \( i \) for domestic currency
- \( r \) is the interest rate on domestic assets

The individual’s wealth at time \( t \) is determined by her investments in foreign and domestic assets and what she earned on each of them respectively. That is,

\[ W_i^t = (1 + r) d_i^{t-1} + s_t (1 + \rho) f_i^{t-1} \]  

(4.5)

At each point in time the individual’s demands for foreign and domestic assets must satisfy the budget constraint:

\[ W_i^t = d_i^t + s_t f_i^t. \]  

(4.6)

The gain for an individual in period \( t \) is given by:

\[ g_i^t = W_i^t - (1 - \omega) W_i^{t-1} \]  

(4.7)

where \( \omega \) is the discount factor and the cumulative gain is given by:

\[ G_i^t = W_i^t - (1 - \omega)^{t-1} W_i^1 \]  

(4.8)

where \( W_i^1 \) is individual \( i \)'s wealth at the beginning of period one (before she chooses the rule to be used).

However, the gains at each period are determined by the demands for domestic and foreign currency in the previous period and these are in turn determined by the forecast that the individual made as to the exchange rate in the next period. The latter depends on whether she was following the fundamentalist or chartist rule.

The next step is to specify how the forecasting rules are chosen. The particular assumption made here is that the choice of rule is probabilistic and the probabilities of choosing a rule depend on an individual behavioural learning process. This idea will be familiar after the analysis of loyalty on the Marseille fish market. As we saw earlier, the choice can depend on a number of different things. For example, as in the case of the ants, the agents can change their forecasts depending on whom they have met. Here, as an illustration, the choice will depend on the success of the rules in the past in terms of the profit that was obtained when using them. In fact, the probabilities could depend on other measures of success, such as the accuracy of the forecasting rules in the past.
but in any particular model we have to decide on what governs the choice of rule. We give a rather general framework in Foellmer et al. (2005). Note that, if the choice of rules is to depend on the profitability of the rules in the past, we need to keep a total of the gains obtained by individual $i$ up to period $t$ by following the fundamentalist rule and similarly for the chartist rule. The formal way to do this is as follows: Define a random variable $\theta_i^t$ which will take on two values $F$ and $C$, that is:

$$\begin{align*}
\theta_i^t &= F \quad \text{with probability } p_i^t(F) \\
\theta_i^t &= C \quad \text{with probability } 1 - p_i^t(F) = p_i^t(C)
\end{align*}$$

Now we can define an indicator function for the random variable and this is simply:

$$\begin{align*}
I_t(F) &= 1 \quad \text{if } \theta_i^t = F \quad \text{and} \quad 0 \quad \text{if } \theta_i^t = C \\
I_t(C) &= 1 \quad \text{if } \theta_i^t = C \quad \text{and} \quad 0 \quad \text{if } \theta_i^t = F
\end{align*}$$

Given this we can now add up the gains from using each forecasting rule and this leads to the gains for an individual when she was using each forecasting rule being:

$$\begin{align*}
G_i^t(F) &= \sum_{r=1}^{t-1} I_r(F) (G_i^r - G_i^{r-1}) \\
G_i^t(C) &= \sum_{r=1}^{t-1} I_r(C) (G_i^r - G_i^{r-1})
\end{align*}$$

with $G_i^0(F) = G_i^0(C) = 0$.

### 4.3.5 The probability of choosing a rule

As I have said, here we will assume that the trader chooses her rule on the basis of previous success with that rule. More specifically take the logit rule used earlier and transpose it to this example.

Thus, assume that the probability of following the fundamentalist rule is given by,

$$p_i^f = \frac{e^{\beta G_i^f}}{e^{\beta G_i^f} + e^{\beta G_i^c}}$$

which can conveniently be rewritten as with the obvious notation

$$p_i^f = \frac{1}{1 + e^{\beta \Delta G_i^{f+c+1}}}$$

where $\beta$ is a constant which reflects the importance the individual attaches to previous experience and obviously, since we only have two rules, the probability of becoming a chartist is given by:

$$p_i^c = 1 - p_i^f$$

The particular rule chosen here for the probability for the transition from one rule to another will be familiar from the chapter on fish markets and a number of economic applications (see Anderson et al. (1990), Brock (1993), Brock and Durlauf (1996). Its use has been justified by the fact that it has been widely used in the psychological literature on reinforcement learning, and in game theory, where it is referred to as
the "quantal response rule". One way of deriving it is to consider the problem of the trade-off between obtaining information about sources of profit and exploiting those which have proved profitable in the past. This "exploration" versus "exploitation" arbitrage can be analysed by maximising a linear combination of the gain to be had from trying new alternatives and the expected gain given the experience in the past with the different rules,(see Brock (1993) and Weisbuch et al. (1997)). In our case the market participants can be thought of as favouring the rule that has been most successful in the past but still having a positive probability of trying other rules.

Given that the choice of forecasting rule is stochastic, the investor’s first piece of information is the rule \(\theta_i^t\) she has drawn. What is the remaining information \(I_i^t\) of a domestic investor \(i\) at time \(t\)? She has her observations of the past values of her demands for foreign and domestic assets in the previous periods, the vector of observed exchange rates up to period \(t-1\) and the cumulated gains that she has realised from using each of the two forecasting rules, fundamentalist and chartist. Thus we have:

\[
I_i^t = \{d_{i,t-1}, f_{i,t-1}, S_{t-1} = (s_1, \ldots, s_{t-1}), G_{t-1}^i(F), G_{t-1}^i(C)\}
\]

The total information available to the individual \(i\) once her forecasting rule has been determined is given by \((I_i^t, \theta_i^t)\). Note, however, that we do not assume that the agent was able to calculate or find out, in some way, what she would have earned had she used the alternative rule at any point in time. Thus learning here is personal rather than social. This is far from a minor observation since if individuals only have their own experience to go on they will have different histories and therefore different probabilities of using the various forecasting rules. If they all know what they would have obtained had they used other rules then everyone would attribute the same probability to each rule. Thus the heterogeneity of behaviour would be eliminated.

The choice of rule determines the demand of an agent once we know her objective function. Given the specific choice of the \(i\)th individuals’ objective function we can determine her demand for foreign assets and, given her budget constraint this will also determine her demand for domestic assets.

### 4.3.6 Demand for foreign currency

Since we are dealing with an example and so as not to alienate other economists, I shall choose a utility function from which the demand function is derived and, to avoid any analytic difficulties, I shall choose the very simple Mean Variance utility function which exhibits constant absolute risk aversion, (CARA) This function has, as is well known, some undesirable features, but from an analytic point of view it has a number of advantages. In particular, its maximisation does not depend on the agent’s wealth in the current period. For any investor \(i\) the expected utility function \(E\left(\tilde{W}_{t+1}^i | I_i^t, \theta_i^t\right)\) is defined by:

\[
E\left(U\left(\tilde{W}_{t+1}^i | I_i^t, \theta_i^t\right)\right) = E\left(\tilde{W}_{t+1}^i | I_i^t, \theta_i^t\right) - \mu V\left(\tilde{W}_{t+1}^i | I_i^t, \theta_i^t\right)
\]
where \( \mu \) is the measure of risk aversion. Recall that the agent’s wealth at the next period \( t + 1 \) is given by:

\[
\tilde{W}_{t+1} = (1 + \rho_{t+1}) \tilde{s}_{t+1} f_t^i + (W_t^i - s_t f_t^i) (1+r) = (1+r)W_t^i + f_t^i ((1 + \rho_{t+1}) \tilde{s}_{t+1} - (1 + r)s_t)
\]

(4.17)

So the investor has the following problem

\[
\max_{f_t^i} \mathbb{E} \left( U \left( \tilde{W}_{t+1}^i \big| I_t, \theta_t^i \right) \right) = \mathbb{E} \left( \tilde{W}_{t+1}^i \big| I_t, \theta_t^i \right) - \mu \text{Var} \left( \tilde{W}_{t+1}^i \big| I_t, \theta_t^i \right)
\]

(4.18)

The expectations of the investor depend on whether she is following the fundamentalist or chartist rule i.e on \( \theta_t^i \) and whichever rule she follows her expectations will be conditioned on the current exchange rate \( s_t \). Hence using first order conditions we can write the demand for agent \( i \) as:

\[
f_t^i \equiv f_t^i (s_t, \theta_t^i) = \frac{(1 + \rho) \mathbb{E} \left( \tilde{s}_{t+1} \big| I_t, \theta_t^i \right) - (1 + r)s_t}{2\mu(1 + \rho)^2 \text{Var} \left( \tilde{s}_{t+1} \big| I_t, \theta_t^i \right)}
\]

(4.19)

Since the expectations in (4.3) and (4.4) are conditioned on which rule the agent is following and recalling the definition of the indicator function for the random variable given in (4.10) we can write:

\[
f_t^i (s_t, \theta_t^i) = f_t^i (s_t, F) I_t(F) + f_t^i (s_t, C) I_t(C)
\]

(4.20)

Finally we shall suppose that the demand for the individual \( i \) when she is using the rule \( F \) or \( C \) is given by replacing the expectation in (4.16) by the expectations for each rule which we will define as before, that is:

\[
\mathbb{E}^F (s_{t+1} \big| I_t) = \tilde{s}_t + \sum_{j=0}^{M_f} \nu_j (s_{t-j+1} - \tilde{s}_{t-j}) \text{ with } \sum_{j=0}^{M_f} \nu_j = 1
\]

(4.21)

for the fundamentalist and:

\[
\mathbb{E}^C (s_{t+1} \big| I_t) = \sum_{j=0}^{M_c} b_j s_{t-j} \text{ with } \sum_{j=0}^{M_c} b_j = 1
\]

(4.22)

for the chartist. Of course one could write down a demand function from the outset, but would immediately be accused of being ad hoc. I leave it to the reader to decide if that would be more ad hoc than the utility function I have chosen.

### 4.4 The concept of equilibrium

Now that we have defined the agents’ demands all that remains is to see how prices are determined. Within most economic models this involves the choice of an equilibrium notion. Here again, there are various possibilities. A first idea would be to consider that the market clears at each period, and hence that we are looking at a series of
temporary equilibrium prices. This is an important restriction. At each point agents form their demand and an exchange rate is found such that these in aggregate are equal to the total available supply. Once the equilibrium exchange rate for this period is established, agents observe it, modify their expectations and hence their demand. Then the process is repeated. In this case today’s demands are set equal to today’s supply but markets will reopen in subsequent periods. This implies two things. Firstly that agents do not have a “correct” view of the future at each point in time and secondly, that we have a series of equilibria but no overall equilibrium. At each point in time there is a price but these prices do not necessarily converge to a steady state, though much of the early work on this theme did focus on the characterisation of such steady states.

It is worth noting a number of the assumptions made here. First, I have made the assumption that agents are only interested by their wealth in the next period. This is an important simplification, agents could be less myopic and this would mean that their longer term expectations were important. However we would then come back to the rationality problem, in the sense that agents’ predictions would not necessarily be consistent with the actual outcomes. To resolve this dilemma, as we have seen, some economists would insist that the expectations of agents should be consistent with the stochastic price process and that this should be a requirement for equilibrium. Such so-called “rational expectations” equilibria, have the advantage of intertemporal consistency, but do not explain how agents arrive at the appropriate expectations. As I have observed, the “sunspots” literature shows that, if one tries to introduce a reasonable learning process for individuals, the economy or market can learn to have consistent expectations but which have no connection with the fundamental variables of the economy, and, what is more, their beliefs will be self-fulfilling. Furthermore, insisting on rational expectations in the context of our example would change the dynamics completely and would mean that we would have to concentrate on the steady states which were the focus of attention for a considerable period of time, (for an excellent discussion of this problem see Grandmont (1983)).

Since our interest is in models which generate characteristics of empirical price series and these involve considerable fluctuations, this is not a very productive line to follow. In particular what is of interest for us is to see how prices evolve as agents modify their forecasts in the light of the prices that they observe.

4.4.1 The feed-back from equilibrium prices to forecasts and hence demand

An important feature of financial market models is the feedback from prices to demand. If, as in our example we take the temporary equilibrium approach the sequence is quite simple. Excess demands in one period determine the market price in that period but this price is then added to the price history which will condition agents’ forecasts. However, as soon as an agent modifies his forecast he modifies his demand for the financial asset in question. This in turn will mean that the equilibrium price in that period will be changed. Given that we are not looking for some sort of convergence we can look at the equilibria in our example and how they evolve over time.
4.5 Equilibria in the example

Remaining then with our myopic framework, we need to determine our sequence of temporary equilibria. First we have to determine the aggregate demand for foreign currency of the investors which is given by:

\[
\Phi_t \equiv \sum_i f_i^t(s_t, \theta_i^t) = \sum_i f_i^t(F) I_i^t(F) + \sum_i f_i^t(C) I_i^t(C) \tag{4.23}
\]

or if we define the number of investors who use the fundamental rule at time \(t\) as \(N_t(F)\) and the total number of investors as \(N\) and \(k_t = \frac{N_t(F)}{N}\) then we can rewrite aggregate demand as:

\[
\Phi_t = N [f_t(F)k_t + f_t(C)(1 - k_t)] \tag{4.24}
\]

The equilibrium exchange rate \(s^*_t\) is then given by:

\[
\Phi_t - \frac{X_t}{s^*_t} = 0 \tag{4.25}
\]

where \(X_t\) is the random liquidity supply of foreign currency coming from underlying trade for example. We could, incidentally, change our model so that we looked directly at excess demand and we could dispense with the exogenous supply of foreign exchange. However, in our example we obtain the following expression for the equilibrium exchange rate:

\[
s^*_t = \frac{1}{X_t} \mu (1 + \rho)^2 \sigma_s (1 + r) \left[ e^a_{t+1}(F) y_t + e^a_{t+1}(C) (1 - y_t) \right] - (1 + \rho) \sigma_s^2 \tag{4.26}
\]

where \(s^a_{t+1}(F)\) and \(s^a_{t+1}(C)\) represent the expectations under each of the two rules, fundamentalist and chartist as to the next exchange rate \(s_{t+1}\) that is:

\[
s^a_{t+1}(F) = E(\tilde{s}_{t+1}|I_t, F) \quad \text{and} \quad s^a_{t+1}(C) = E(\tilde{s}_{t+1}|I_t, C) \tag{4.27}
\]

and where \(X_t\) is the exogenous supply of foreign exchange already mentioned and which we will take, in the simulations, to be random noise.

When the model we have just described is simulated, it replicates many empirical stylised facts. There is long memory that is the correlation between different period prices declines more slowly than would be suggested by the standard model, and there are periods of bubbles and high volatility. The switching of regimes from ones dominated by chartists to ones dominated by fundamentalists is likely to be what provides a large part of the explanation for the existence of the long memory so often detected in financial time series. However, as suggested by Kirman and Teyssiere (2002), this is really “spurious long memory”. Bubbles occur and always explode, as can be seen from Figure 4.5. In fact, the bubbles correspond to periods where the chartists take over and the price path becomes detached from the fundamentals.

We can clearly see that the prices, or exchange rate, in our case, where agents choose their forecasting rules probabilistically, there is no tendency towards a particular equilibrium price. More generally, depending on the source of randomness in the
Figure 4.5: The evolution of the simulated asset prices and the fundamentals.
model, which might be in terms of the agents’ incomes, or in terms of some exogenous demand for the asset, prices might follow a mean reverting random walk, geometric Brownian motion or some other such process. We cannot hope to find some steady state to which the process will systematically return, each time it is perturbed.

### 4.5.1 An extension of this approach

In our example and in a great deal of the literature, the focus is on what happens in a market for a single asset. However, a realistic model of the foreign exchange market has to allow for the fact that there are different types of traders those from each of the countries involved, and this is what is done in Kirman et al. (2005). For each country, the domestic currency is a foreign currency for traders from other countries. Therefore one should model the behaviour of the agents in all the countries, those for whom investment in the currency in question represents a “safe” investment whilst the others will have to bear the exchange risk. In particular, it is interesting to note that this enlargement of the model allows one to forego the, rather artificial, outside supply of foreign exchange.

The results obtained show that a rather basic model, in which traders of different national origins trade in the same currency using simple rules to form their expectations, can generate complicated dynamics at the aggregate level. What drives the dynamics of the exchange rates are the dynamics of the proportions of domestic or foreign investors who chose the fundamentalist strategy to determine their expectations. The equilibrium exchange rate’s time path varies according to the values of the crucial parameters of the exploration/exploitation process used by the domestic or the foreign investors when forming the probabilities of choosing the different forecast rules. Most interesting is the fact that, in the example of the model which is simulated and which is limited to two countries, fundamentalist traders in one or the other country can determine the evolution of the exchange rate. In the simulations of this model what happens is that the proportions of the fundamentalists go to the extremes in the two countries but are not necessarily synchronised. If the fundamentals are not completely correlated in the two countries, as is the case in this simulation, then when fundamentalists dominate in both countries neither has correct expectations. Thus, in contrast to the previous example, part of the dynamics is due to the relative movements of the fundamentals in the two countries.

However, although the particular time path followed will depend on the specific values of the parameters it is also worth noting that the general structural characteristics of the process do not depend on this choice.

### 4.6 Relations with the literature

What I hope has become clear, with the example and its extension, is how agents may change their expectations as they learn and, given the self reinforcing aspect of their expectations, how prices will move with the expectations. What the basic example shows is that it may be the case that, as a result of the self-reinforcing character of the process, agents will cease for some periods to believe in any fundamentals and
that they will become “chartists” or trend chasers. In this case markets detach themselves from any fundamentals for a period but then return, (for a good overview of theoretical aspects and empirical evidence for this see Hunter, Kaufman, and Pomerleano (2003)). Such a vision fits well with the burgeoning econometrics literature on “switching regimes” initiated by Hamilton (1989) and provides an economic framework for this phenomenon. As agents switch from one belief to another there will be periods where there is considerable heterogeneity of expectations, consistent with empirical observations; see Chionis and MacDonald (2002). Yet, over time, agents cannot be said to be behaving irrationally.

What I suggest then, is that changes in the distribution of expectations as a result of the mutual influences of agents, play a key role in explaining the evolution of prices. Of course, in the standard “representative agent” model there is no place for any such interaction and resultant heterogeneity of expectations and many authors have suggested that this is the reason for the poor predictive power of such models. This argument has been made in particular for the foreign exchange market and evidence for this is given by Meese and Rogoff (1983), Frankel and Rose (1995) and Cheung, Chinn, and Pascual (2005). Note again, in the sort of model proposed here, heterogeneity is largely a temporary phenomenon, and for much of the time people have similar expectations.

Once we accept the idea that expectations can be heterogeneous, even if temporarily, it seems that we have to move away from the standard “rational expectations” framework. One idea is simply to introduce agents who systematically have “wrong” expectations but who may survive nevertheless. Such models were pioneered by De Long et al. (1989, 1990) who introduced the notion of “noise traders”. Such a solution to the problem is not very appealing and, if one takes account of the idea that agents may learn, it is difficult to accept that certain actors will persist in their error.

Another alternative is to introduce dispersed information into the model and this recalls Hayek’s ideas about the fact that each individual has his own information and even if one agent were to be able to centralise all the information it is not clear that he could guide the market or economy to an equilibrium. One approach suggested by Townsend (1983) is to have symmetrically dispersed information and to analyse the consequences of “higher order expectations”, expectations about others’ expectations (an idea that goes back at least to Keynes and the “beauty queen” problem. Townsend’s contribution shows that a small amount of non-fundamental trade may generate considerable volatility since traders perceive movements in asset prices as conveying information about future values of fundamentals; see Allen, Morris, and Shin (2003). Again, despite the more sophisticated reasoning attributed to agents, a certain degree of irrational behaviour is needed to generate the results. As the market evolves however, the prices return to their fundamentals. Yet, this return should not be, and is not, in the model I have presented, permanent. If agents learn about the true fundamentals and the prices then adhere to those fundamentals as in Bacchetta and van Wincoop (2003), bubble-like departures would cease to occur. In the model I have presented here, such bubbles will continue to develop and to burst. This gives a more realistic meaning to the widely held view, that “in the long run fundamentals matter”. Prices will inevitably return to those implied by the fundamentals but will just as inevitably leave them again. This is, of course consistent with the fact, that on average, in the short run, fundamentals are poor predictors of prices.
4.7. AN ANALYTIC APPROACH.

Heterogeneous expectations, at least for certain periods, are also necessary to explain other phenomena. Without them, one could not explain as Bacchetta and van Wincoop (2003) point out, the enormous volume of trade on financial markets. Spot trading on foreign exchange markets in 2001 was approximately $1.2 trillion per day, for example. If all agents have the same expectations it is difficult to understand who is trading with whom!

A number of other models have been constructed in which agents change their views as time passes and in some of these the changes can be self reinforcing. This will generate heterogeneous expectations both across the traders and over time. A particularly relevant contribution is that of Brock and Hommes (1997) in which agents can use one of two predictors. One of them is costly but when all agents use it the process is stable. The other is cheaper but when used by many individuals induces unstable behaviour of the price process. Thus their model has periods of stability interspersed with bubble like behaviour. They present analytical results to characterise this possibly chaotic behaviour of their system. There are many other models in which agents have different forecasting rules, and these are often summarized, as in our example, as “chartist” and “fundamentalist” views. Many such models, when simulated, generate time paths of prices which switch from one expectations regime to another and which generate “realistic” time series, but, in general, no analytical characterization of their properties is provided.

4.7 An analytic approach.

The reflections above bring me back to an earlier puzzle. Is there any appropriate notion of equilibrium in such markets? If there is, it cannot be thought of as a deterministic state around which there are fluctuations. Nor can it be thought of as a single price path towards which the market price will converge after some time. The situation is not without hope however. What we suggest, in Foellmer et al. (2005) as a possible answer, is a probabilistic notion of equilibrium where the distribution of asset prices does not change through time. In the long run we know what the probability of a certain price will be but the system will always exhibit changing prices. The distribution of prices can be considered as an equilibrium distribution. In fact, if as in the example, agents choose their forecasts in a stochastic way, then with certain restrictions, it can be shown that the price process is ergodic and has an unique limit distribution. This means that time averages converge to their expected values under the unique stationary distribution. This gives an alternative notion of equilibrium. In particular, in the presence of chartists, the limit distribution exhibits fat tails and long memory. This is illustrated in Figure 4.6 where what is shown is the distribution of prices with and without “chartists” in a simulation of the model. It is the presence of the extrapolators that fattens the tails of the price distribution.

It is also the presence of chartists that generates bubbles and this is illustrated in Figure 4.7.

What we show is that, despite the fact that the balance between the numbers of agents holding different expectations is constantly changing we can nevertheless characterise the structure of this process in the long term. In order to be able to do this we
The Distribution of Stock Prices

Figure 4.6: Simulated stationary distribution of asset prices with and without chartists: the presence of chartists generates “fat tails”
4.7. AN ANALYTIC APPROACH.

Figure 4.7: A bubble and the corresponding fraction of chartists.
do not have to limit the proportion of chartists. This might seem to be necessary, since when they take over the process could explode. In fact, it is only necessary to limit the probability that individuals become chartists.

Thus we do not have to rely purely on simulations to understand the nature of the process and we do not simply have to argue in theoretical models, as some have done that, in such situations, “anything can happen”.

What is going on here is that, despite the rather complicated structure of this sort of model, under appropriate assumptions, some structure of the price process does emerge. Despite the switching of opinions at the individual level and the mutual influence of the agents, there is a well-defined market process. In other words, we can characterise this process and talk about certain of its long term properties. This does not mean that the system will “settle down” to a particular price, unless it is shocked from the outside, and it does not mean that we can identify specific “equilibrium prices”. We can make statements as to how much time exchange rates will spend in a certain interval in the long run, but we cannot make predictions as to when the process, will, for example switch back to “fundamentals” after a bubble. This seems to make very clear the explanation for the, apparently schizophrenic, attitude of traders towards fundamentals mentioned earlier. “In the short run they can be ignored whilst in the long run they matter”.

4.8 Conclusion

The purpose of this chapter has been to give an indication of an approach to modelling financial markets which in keeping with the spirit of this book, allows for relatively simple behavioural rules and which, in simple examples, produces price series with plausible characteristics. However, much remains to be done. From the outset I have emphasised the role of markets and the importance of the way in which they function. Yet in the examples I have discussed in this chapter I concentrated on one specific aspect of markets. This was the way in which the interaction between agents influences how they forecast prices. The equilibrium notion remained a simple market-clearing process, at each point in time, with no real justification.

An alternative and more appealing approach would be to try to model the process of price formation in a more realistic fashion. On many financial markets prices are recorded for each transaction and some system such as an “order book” is used to match bids and offers. In this case one could model the supplies and demands of individuals as resulting from current and expected prices and could use the type of algorithm actually used on markets such as the Paris stock exchange to match the bids and offers.

It would be interesting to see if, in this more realistic, framework, price series still exhibit the same characteristics, long memory, temporary bubbles, and excess volatility. While in a simple framework we may hope to replicate some of the theoretical results in Foellmer et al. (2005), it is likely that we will have to pursue the simulation of agent based models to get a clearer idea of the limitations and applicability of the theoretical results. Nevertheless, this seems to be a more promising and realistic route than that followed by models which envisage a market as a mechanism which
experiences occasional deviations from a steady state.
Chapter 5

Public Goods: A coordination problem

5.1 Introduction

From the outset, the theme of this book has been that in complex systems, the behaviour of the aggregate is not the same as that of the component parts. In analysing the behaviour of a system of interacting particles in physics or a biological system the behaviour of the organism is not similar to that of the individual particles in the one case or cells in the other. The thing to be emphasised is that there is the phenomenon of “self organisation”. Somehow the components organise themselves in such a way that their collective behaviour is coherent. As I have said there are several possibilities. One of these is that what we observe at the aggregate level corresponds to what we are led to expect by our theory. This theory is normally based on the rational behaviour of maximising individuals. The temptation is then to conclude that because the aggregate is well behaved, the individuals must, indeed, correspond to those in our theoretical models. This is, as I have already emphasised, a non-sequitur. We saw this when examining the buyers on the Marseille fish market. At the aggregate level, the demand curves for fish have the nice monotone declining property basic theory would lead us to expect, yet examination of the demand of the individual buyers reveals no such property. In this case, just because the aggregate behaves as it would if all the individuals satisfied the assumptions made about them, it is not necessarily the case that the individuals actually behave in that way. The aggregate behaviour is the result of the aggregation and not of the fact that the individuals are “well behaved”.

Yet, to come back to my basic argument, in economics we frequently build models based on hypotheses about individuals’ behaviour and then test these on aggregate behaviour. We jump the aggregation problem by assumption. Then, if we cannot reject our individual based model at the aggregate level, we conclude that it is a valid model of how people are behaving. At the risk of being repetitive let me insist that one of the purposes of this book is to suggest that this is a misleading approach.

Where is this problem most apparent? Suppose we have an empirical or experi-
mental system which does not behave as our model, based on the assumption that the whole acts like a rational individual, predict. In that case we can reasonably reject the model. However, as Summers (1994) pointed out, what we may be rejecting is not the model of individual behaviour but the assumption that the whole acts like an individual. Either the individuals are not behaving as claimed or when they are aggregated their collective behaviour no longer corresponds to that of the individuals, taken separately, or on average. In either case there is a problem with the model either at the individual level or with the aggregation that it assumes. This was the sort of situation in the analysis of financial markets.

But what happens if the system being observed does behave as predicted? Here, the task is much more delicate. We now have to examine the behaviour of the individuals themselves to see if they behave as they are assumed to. I have discussed markets at some length and two things have become clear. Firstly, there is a lot of structure in the interaction between individuals on markets and this has an impact on the outcome. Secondly, in some markets, and in financial markets in particular, this interaction will lead to aggregate outcomes which seem to be in contradiction with standard theory. What I will do now is to look at one of the many other economic situations in which individuals coordinate their activity and where the relationship between individual and aggregate behaviour is much more complicated than it might seem at first sight. In particular, I will examine an example where it would seem that we could give a convincing explanation for aggregate behaviour based on an analysis of how individuals behave. Indeed, the great majority of the contributions on this subject, that of contributing to public goods, has been devoted to doing just this.

When dealing with markets the benchmark theoretical assumption has been that the participants behave, as is frequently assumed in economic theory, as isolated price takers. Their behaviour obviously influences the aggregate outcome but they act as if they were not conscious that this is what is going on. To be more specific market actors may react to market signals without worrying about the fact that those market signals depend on their behaviour and that of the other individuals in the market. But, as I have emphasised, in many situations, this is not a realistic approach.

But suppose now that individuals do try to anticipate the behaviour of others or, at least, are aware of the influence of that behaviour. How will they act? This question has, of course, received a great deal of attention in the economic literature and is the territory of game theory. The difficulty with the pure game theoretical approach, from the complex systems point of view, is that individuals are interacting but are not following rather simple local rules. They are engaged in far-sighted sophisticated reasoning. People, in this view, act strategically, that is, they take account of the impact of their actions and the influence of others. Moreover, they know that the others are behaving similarly. This brings us straight to the “common knowledge” problem which goes back at least to Hobbes and was discussed by Hume (1740) who was perhaps the first to make explicit reference to the role of mutual knowledge in coordination. In his account of convention in A Treatise of Human Nature, Hume argued that a necessary condition for coordinated activity was that agents all know what behavior to expect from one another. But, he does not argue that by so doing they will reach a good outcome. He uses reasoning which will be familiar to all of those who have studied game theory. He gives the example of two farmers who expecting abundant crops must decide whether
“Your corn is ripe today, mine will be so tomorrow. “Tis profitable for us both that I should labour with you today, and that you should aid me tomorrow. I have no kindness for you and know you have as little for me. I will not, therefore, take any pains upon your account; and should I labour with you upon my own account, in expectation of a return, I know I should be disappointed, and that I should in vain depend upon your gratitude. Here, then I leave you to labour alone. You treat me in the same manner. The seasons change and both of us lose our harvest for want of mutual confidence and security” David Hume, A Treatise of Human Nature, pp. 520-521.

Thus he anticipated the famous “Prisoners’ Dilemma” problem. We are now at the opposite end of the spectrum from the isolated individual in the market. Here the actors are consciously taking account of the rationality of the others and the logic we attribute to them involves implicitly an infinite regress, “he knows that I know that he knows that I know….” This problem was raised again much later by Schelling (1960), and was first clearly enunciated in a more formal way by Lewis (1969) and expounded upon by Aumann (1976). Not only is there a problem with the infinite regress, but there are even logical problems with the way in which we assume that people are thinking.

The usual theoretical quest is to find an equilibrium for such strategic situations. How do we find equilibrium strategies? A standard argument runs as follows. “If he plays strategy $s$ then I will respond with strategy $t$ and knowing this he would prefer to do something else. Therefore strategy $s$ will not be played. But, there is an old problem here. If he is the rational player that I am assuming then he will never play strategy $s$. So the counterfactual assumption, that he plays $s$ cannot be made. This sort of conundrum shows the sort of logical problems one runs into if one attributes such “eductive” reasoning to individuals. This sort of difficulty is discussed by Binmore (1990) for example. So here we are faced with a completely different problem than that we found in the basic market theory. There, individuals took no account of the effect of interaction. Here, the behaviour that we would need to attribute to individuals, if we assume that they think things through completely, is not only very sophisticated but also suffers from logical inconsistencies. Yet one might well ask whether this is a reasonable way of thinking about how people behave in reality. In the market case the objection to the standard theoretical approach is that individuals in reality do not consider themselves in many situations as anonymous and isolated but do interact with each other, even if only locally.

In the game theoretic case we assume that individuals interact consciously with all the others and are well aware that the others do the same. How should we judge this as a model of how people actually behave? Once again we are faced with the same two questions. First, does the assumed behaviour lead to outcomes consistent with empirical or experimental observations? Second, does the empirical or experimental evidence suggest that people actually behave as assumed?

One way to avoid this sort of problem in strategic coordination situations, is simply to find strategies for the individuals that are mutually compatible in the sense of Nash
(1950) that is, where no individual has an incentive to change his strategy if all the other individuals stick to theirs, and to forget about the reasoning necessary to find them. Thus we can prove the existence of such an equilibrium without saying anything about how it is actually achieved or, put alternatively, how agents might manage to coordinate on such a solution. Yet this begs an important question, what sort of plausible reasoning would actually lead people to such an equilibrium? It is worth recalling that the same question applies in much stronger terms to market equilibrium, where nothing is said about who decides what and how the market as a result might move to equilibrium. The saving grace of the market equilibrium is its optimality or efficiency.

No such claim can be made for the Nash equilibrium of a non-cooperative game. Indeed, the equilibrium may be far from a social optimum, and it may be possible as in Hume’s version of the “Prisoners’ Dilemma” to increase all the individuals’ welfare to a level above that achieved at the equilibrium. Now, the question that we have to ask, is twofold, do people, in fact, coordinate on this sort of equilibrium and, if so, what is the behaviour that leads them to do so? What we would like to know is what people will actually do in such situations and how we can appropriately model their behaviour. Of course, by now it should be clear that the answer I will suggest is that the coordination, often observed, is more an emergent phenomenon than the result of sophisticated individual reasoning. To illustrate this, I will present a particularly well known example of this sort of problem, the problem of contributing to a public good and illustrate it with the results of experiments done with Walid Hichri (Hichri and Kirman (2007)).

5.2 The basic public goods problem

The problem of public goods and how to finance them is an old and much debated question in economics. It is a good example of a situation in which people have to act, knowing that what they do will have an impact on the welfare of others, and that what the others do will influence their own welfare. The definition of a pure public good will be familiar to the readers of this book. A pure public good is characterised by two features. Firstly, it is consumed by all the participants in the economy or market in question and the consumption of one individual does not diminish that of another. The fact that somebody else is watching the same TV programme as me does not diminish my consumption of the programme, unlike more standard goods. Secondly, no individual can be excluded from the consumption. There are, of course, many intermediate cases where some specific facility is open to all the residents of some organisation or neighbourhood, for example

The fundamental conflict in this sort of situation is that there are two forces at work. On the one hand, individuals, by contributing more can increase both their own welfare and that of the others. On the other hand, there is a temptation to free ride, that is, to let the others contribute and not to contribute oneself. A situation like this arises when several individuals can use a common resource such as an open piece of land or “common” as it was typically called in England. By limiting the number of sheep that graze on the common, the resource was preserved for the good of all concerned. However, each sheep owner may be tempted to overgraze the land, counting
on the others to be more socially conscious. What corresponds to the contribution by an individual here, is the act of refraining from grazing and it is this that benefits everybody. The non-cooperative solution is for everybody to free ride and it is this that led Hardin (1968) to dub this problem “The Tragedy of the Commons”. In the usual game theoretic models of this situation the Nash equilibrium is for everybody to graze to the maximum. This would translate in the public goods setting to nobody contributing anything to the public good.

As I have said, I will deal with two aspects of this problem here. Firstly how do people actually behave in such situations and secondly, is there a model which gives a good description of that behaviour and which takes account of conscious interaction? To examine this we conducted a series of experiments which complement the huge literature on this subject. I will give more details below but the basic idea is that each individual is in a group and has an initial allotment of money. They can split this money between a contribution to the public good from which everyone benefits and they keep the rest for their private consumption. Once individuals have contributed the total production of public good and the pay-offs to each individual are determined. This is repeated for several periods.

A first remark is in order. Suppose that all the individuals reason as good game theorists, then right from the very first round of the experiment they should contribute the amount corresponding to the Nash Equilibrium. In the most basic version of the model this simply means nobody would contribute anything and nobody would be tempted to move from that position. If we do not observe this then we can rule out the sort of abstract reasoning which would have led people to implement the equilibrium straight away.

Some readers will recall that the situation in repeated games is not the same as that in “one shot” games. In a game played repeatedly with the same players, each player can have a strategy that might “punish” those who contribute little and therefore more cooperation would be observed than in the one shot game. However, in our experiments we are dealing with a finite number of rounds and the previous argument only holds for games with infinite horizons. It is easy to see why. In the last round everyone should play Nash since there is no possibility of subsequent punishment. But, is this so, people should do so in the penultimate round and so forth back to the start. This argument which involves backward induction is standard for such games. So, in such a repeated game people who are good game theorists, or rather act as good game theorists expect them to, should immediately settle to the equilibrium of the “one shot” game. What if they do not do so? Perhaps, since people play repeatedly they “learn how to play Nash”. So, they do not contemplate the common knowledge problem and work everything out from the beginning, but do move towards equilibrium. Thus after a while differences between behaviour will be ironed out and in the end something close to equilibrium would be attained.

As I have said, in public good games individuals decide how much of their money to keep and how much to contribute to a public good which everybody will enjoy. If we look at a one-shot public good game there are two useful benchmarks. On the one hand there is that solution which maximises the total pay-off, the Collective Optimum (CO) and on the other there is the “Nash Equilibrium”, (NE) which I have already mentioned. In the latter, given what the other players contribute, each player chooses
what is in his own best interest, i.e. his “best response” to the others actions. We should, of course, observe, that in this sort of game, both the CO and the NE of the one shot game are defined as an amount which corresponds to the sum of the contributions of the individual players. As soon as the sum corresponding to an equilibrium is positive there are, of course, many different individual contributions which would sum to that amount. As a result of there being so many asymmetric equilibria, the literature has focused on the symmetric equilibria as a test.

Public goods games have turned out to be very useful since we have some well-established stylised facts concerning the experimental results and these can be compared with the theoretical predictions. The first and clearest message from the evidence is that people do not “find” the Nash equilibrium from the outset. Indeed, there is a wealth of information from public goods experiments showing that, with respect to the Nash equilibrium of the one-shot game, people initially perhaps naively, perhaps optimistically, over-contribute to public goods but that, with repetition, they contribute less. Although these experiments have too few rounds, in general, to make meaningful statements about the “convergence” of the total contributions they do decline towards the Nash equilibrium.

As I have mentioned, the players in these experiments are faced with a finite repeated game, which can be solved by backward induction. The solution to this problem in the standard linear model is one of a dominant strategy in which people contribute nothing from the outset. However, it is well established that this is not what actually happens (see e.g. Ledyard (1995)). Therefore, since individuals do not play in this way, we cannot attribute the observed behaviour to that associated with an equilibrium of the finite repeated game. Furthermore players do not establish the cooperative, or socially optimal outcome (CO) and indeed, in general, move collectively away from such a solution. As I have suggested, the way in which this is explained is that it takes some time for people to understand what is happening, and that they “learn to play Nash” or something approaching it. In this chapter I shall examine this idea by analysing average and individual behaviour from the series of public goods experiments that I have mentioned. But, before discussing what people may or may not be learning to do let me present the basic model of a public goods contribution game.

5.3 The model

5.3.1 The basic public goods game:

In the basic game of private contribution to a public good, each subject $i$, $(i = 1, \ldots, N)$ has to split an initial endowment $E$ into two parts: the first part $(E - C_i)$ represents his private share and the other part $C_i$ represents his contribution to the public good. The payoff of each share depends on and varies with the experimental design, but in most experiments is taken to be linear (Andreoni (1995)). The total payoff $\pi_i$ of individual $i$, in that case is given by the following expression:

$$\pi_i = E - C_i + \theta \sum_{j=1}^{N} C_j$$
This linear case gives rise to a corner solution. In fact, assuming that it is common knowledge that players are rational payoff maximisers, such a function gives a Nash equilibrium (NE) for the one shot game at zero and full contribution as social optimum. The dominant strategy for the finite repeated game, is to contribute zero at each step. Nevertheless, experimental studies show that there is generally over-contribution (30 to 70% of the initial endowments) in comparison to the NE.

Attempts to explain this difference between the theoretical and the experimental results are the main subject of the literature on private contribution to public goods. To do so, several pay-off functions with different parameters have been tested in various contexts to try to see the effect of their variation on subjects’ contributions (for surveys, see Davis and Holt (1993), Ledyard (1995) and Keser (2000)).

What about the idea that subjects simply make mistakes? In the linear case, given that the NE is at zero, and giving that subjects could not contribute negative amounts to the public good, error can only be an over-contribution. This suggests modifying the game so that both under and over contributions are possible. To test the error hypothesis experimentally, Keser (1996) performed a new experiment. She proposed a new design in which the payoff function is quadratic and what this does is to make the benefits to all a decreasing function of the total contributions to the public good. With such a design, undercontribution becomes possible and error on average could be expected to be zero. Slightly more technically, the equilibrium is a dominant strategy in the interior of the strategy space. However, the results of Keser’s experiment show that in each period, contributions are above the dominant solution, which leads to the rejection of this error hypothesis.

Yet, the idea of a situation in which, even at the Nash equilibrium, total contributions are positive is appealing. Another way to introduce such an interior solution is to use a linear payoff for the private good and a concave function for the public one and this is what Sefton and Steinberg (1996) did. They still found that “average donations significantly exceed the predicted equilibrium under both treatments, falling roughly midway between the theoretical equilibrium and optimum…”

One of the principle reasons for choosing a model with a positive contribution as an equilibrium is that it allows us to look at situations in which individuals are different, in the sense that they derive more or less benefit from the public good. Those that are perhaps “inequality averse” to use the term coined by Fehr and Schmidt (1999) might wish to contribute more to the public good, while others who are less altruistic might want to contribute less. Examining the data from experiments might enable us to test the hypothesis that such differences exist.

### 5.3.2 The Hichri and Kirman (2007) Model

The theoretical model and design used for the experiments I report in this chapter concern then a public goods game with a concave pay-off from the contributions to the public good. The individual payoff function is

\[ \pi_i = E - C_i + \theta \left( \sum_{j=1}^{N} C_j \right)^{1/2} \]
The Nash equilibrium and the social optimum corresponding to this payoff structure are not trivial solutions but in the interior of the set of the possible choices. The Nash equilibrium for individuals is not a dominant strategy for the finite repeated game. Indeed the solution for that game poses problems for a simple reason. There is a unique Nash equilibrium in the sense that for any Nash equilibrium the group contribution is the same. However, as I have said, that contribution can be obtained by several combinations of individual contributions. Since there are many Nash equilibria for the one-shot game, one might ask precisely what constitutes an equilibrium for the repeated game? The answer is clear, a Nash equilibrium for the game repeated \( n \) times, will be a sequence of \( n \) equilibria for the one shot game. This means that, in such an equilibrium, the same player may make very different contributions at each step. The only requirement is that the total contribution of the group should remain constant. This means that the following sort of arrangement could be an equilibrium. In the first period, one player contributes a lot and the others very little, and then in each subsequent period another takes over the role of leader and makes the large contribution. How, or why, a group would coordinate on such a solution is a different question.

In any event we have two benchmarks for the total contributions. For a group of \( N \) subjects, at the CO the total contribution is given by the following expression:

\[
Y = \sum_{i=1}^{N} y_i = N^2 \theta^2 \frac{\theta^2}{4}
\]

and at the NE is equal to:

\[
Y^* = N \cdot y^*
\]

where \( y^* = \theta^2 \frac{\theta^2}{4} \)

and \( y^* \) is the symmetric individual Nash equilibrium, but again, this is just a reference point and any combination of contributions summing to \( Y^* \) is also an equilibrium.

With such a design, the Nash equilibrium and the social optimum vary with the value of \( \theta \). The idea was to see whether increasing the gain to be had from the public good made any difference to the evolution of individual and aggregate contributions under the different treatments.

In Hichri and Kirman (2007), we gave \( \theta \) four different values, which give four levels for the CO and the NE. The following tables summarize the four treatments (Low, Medium, High and Very High) the different levels of interior solutions for each group of four \((N = 4)\) persons (table 5.1) and for the individual subjects in each group (table 5.2):

<table>
<thead>
<tr>
<th>Value of ( \theta )</th>
<th>Treatment</th>
<th>Endowment</th>
<th>Symmetric NE</th>
<th>CO</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>L</td>
<td>280</td>
<td>4</td>
<td>64</td>
</tr>
<tr>
<td>5.66</td>
<td>M</td>
<td>280</td>
<td>8</td>
<td>128</td>
</tr>
<tr>
<td>6.93</td>
<td>H</td>
<td>280</td>
<td>12</td>
<td>192</td>
</tr>
<tr>
<td>8.94</td>
<td>VH</td>
<td>280</td>
<td>20</td>
<td>280</td>
</tr>
</tbody>
</table>

Table 5.1: The NE and the CO values for the four treatments for one group.
5.3. THE MODEL

<table>
<thead>
<tr>
<th>Value of $\theta$</th>
<th>Treatment</th>
<th>Endowment</th>
<th>Symmetric NE</th>
<th>CO</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>L</td>
<td>70</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>5.66</td>
<td>M</td>
<td>70</td>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>6.93</td>
<td>H</td>
<td>70</td>
<td>3</td>
<td>48</td>
</tr>
<tr>
<td>8.94</td>
<td>VH*</td>
<td>70</td>
<td>5</td>
<td>70</td>
</tr>
</tbody>
</table>

Table 5.2: The symmetric NE and the CO values for the four treatments for one subject.

The set of possible group contributions in our model is very large. In fact, given that each one of the four individuals of a group is endowed with 70 tokens, each group can contribute an amount that varies between zero and 280.

5.3.3 Different levels of information

We also examined situations in which individuals had more or less information about what the members of their group were doing or claimed they were going to do. It might be argued that knowing who had offered what or had given what would lead to more interactive behaviour and change the behaviour of the individuals. For this we added two other features and we again looked not only at the effect of variations in $\theta$ but also at the variability of individual behaviours. In the first variant, we replicated the same experiment as before with the introduction of promises before the decision period relative to the contribution to the public good. The four treatments with promises are called LP, MP, HP and VHP. Note that, from a purely theoretical point of view, announcing how much one intends to contribute should have no impact on the outcome, since this is merely “cheap talk” and there is nothing to oblige a player to do what he says he is going to do. Obviously, in the last period, he has no incentive at all to do what he announces and therefore by the usual backward reasoning he has no interest in doing so at any point in the session.

In the last series of experiments, we introduced, as a parameter, the information about the contribution of some members of one group to the public good. Three treatments were compared to the treatment H. This treatment is used as a benchmark (no-information treatment) and is chosen because it allowed us to keep the CO in the interior of the strategy space but, at a high level. The three new treatments with information are 1N, 1P and 2N and have the same theoretical benchmarks as treatment H (same CO and NE). In theory, the information which is provided after the end of the one-shot game should have no effect on the outcome.

While in treatment H individuals are informed only of the sum of contributions of their group, in treatment 1N each one of the four individuals in a group knows at the end of each period the contribution of his right neighbour, in addition to the sum of contributions of the group. In treatment 1P, each individual in a four persons group has a partner with whom he exchanges information about their own contributions. These partners are the same for all the periods of the game. Finally, in the last treatment (2N),

---

1These are approximate values. The exact values are respectively: $4, 5.6568542, 6.9282032$ and $8.9442719$. We choose these values such that the CO corresponds respectively to 64, 128, 192 and 280.
for each individual information concerns two Neighbours, the right one and the left one. The following figure (figure 5.1) depicts these four treatments:

![Diagram of treatments](image)

**Figure 5.1:** Information about neighbours contributions with an H treatment Design

In all treatments, and for every period, information given to players always concerns the same individuals.

Why did we do all this? Simply because we found as everybody else that people do not play Nash straight away and then we had to try to find out what they were doing. One idea, as I have said, is that they learn from what they observe and modify their behaviour accordingly. In this case it is interesting to see whether having different levels of information will affect their learning. It may be easier to anticipate what the others will do if I have more information about what they did in the past. So we used
the data from these three new treatments as well as the earlier treatments to test the learning models at the population, group and individual level. This allows us to see whether the results in our reference framework are robust to different “institutional frameworks”.

If we are to abandon the idea that people reason game theoretically in the sort of interactive situation we are discussing here then we have to have to be more specific about what they might be doing. Abandoning the “eductive” approach is necessary since all the many experiments that have been discussed in the literature reveal that individuals do not coordinate straight away on a Nash Equilibrium. Hence we need an alternative. The most intuitively appealing is that already mentioned, which is that individuals “learn to play Nash”. To see what is meant by this needs a little discussion of the learning in games literature. But before embarking on this it is worth bearing in mind that most of the learning models in the literature are individual learning models and the fact that a group manages to coordinate on an equilibrium does not mean necessarily that they are all “learning” according to the same model. In fact learning in economics is peculiarly complicated. What one is learning about is often what the others are doing. But the others are also learning, so the object is a moving target and it is not at all obvious that such a process will converge. You may recall that the problem of learning about an environment which is also learning was nicely summed up in figure 3.3.

5.3.4 Learning in games

There are two very different views of learning in games. Population learning suggests that the configuration of strategies in a population game will converge towards some limit, which may or, may not be, a solution of the one-shot game. This, it is argued, is because more successful strategies take over from those that perform less well. This simple evolutionary argument does not explain how the strategies are replaced. One argument of a purely biological type is that the users of the less successful strategies “die” and are replaced by the children of those that use the more successful ones. In the sort of context that we are looking at here this does not make much sense and a more plausible interpretation would be that people switch to using the more successful strategies. What precise reasoning enables them to do this is not specified in general but it is this that lies behind much of the literature on evolutionary game theory. Indeed, in the context of public goods games, this is the sort of idea invoked by Saijo (2008), for example, who classified certain people, from their behaviour, as Nash and found that at the beginning of their public goods experiments 50% of players were “Nash” and, at the end 69% fell into this category. This makes it tempting to believe that the population was evolving towards Nash. But this leaves open an important question. Is it true that the people who switched had “learned to play Nash”, and if so, how and why?

The alternative approach is to actually model the individual learning process and to see if observed behaviour, particularly that in experiments, corresponds to such a model. The usual approach is to assume that all individuals learn in the same way and then to test the learning model on the average observed data. (See Erev and Roth (1998)). In a certain sense, this assumes that the whole group acts like a “representa-
This is very common practice and often gives rise to rather convincing results. However, as Ho et al. (2008), point out, the estimated parameters for the representative individual may not coincide with the average parameters of the population. Furthermore, this approach is fundamentally flawed. To assume that the average player behaves in a certain way is to give way to the same temptation as that offered by the “representative agent” in macroeconomics. In other words we can forget about the aggregation problem and just assume that aggregates behave like individuals or, at least, the average individual. But, the major theme of this book is that it is not, in general logically consistent to attribute the characteristics of an individual to average behaviour. To repeat Summers (1994) objection, if for example, we reject the model of individual behaviour at the aggregate level, how do we know whether we are rejecting the model itself, or the hypothesis that the aggregate can be thought of as learning according to the same model as the individuals?

One way out is to assume that individuals behave according to the same learning model but differ in their parameters. This is the approach adopted, for example, by Ho et al. (2008). Two basic classes of rules have been used. The first of these are the “reinforcement” models in which strategies are updated on the basis of their results in the past, (an approach based on the work of Bush and Mosteller, see e.g. Erev and Roth (1998) and Mookerjee and Sopher (1997)). The second are the so-called “belief” models in which agents update their anticipation of their opponents’ behaviour on the basis of their previous behaviour, fictitious play being a good example (see e.g. Fudenberg and Levine (1998)). A more general model, (experience weighted attraction learning EWA), which incorporates both type of rule has been introduced by Camerer and Ho (1999).

Another possibility is not to find a rule, which encompasses others as special cases, but to allow for different rules and simply to try, on the basis of observed behaviour, to assign agents to rules. This is the procedure followed by Cheung and Friedman (1997), Stahl (1999) and Broseta (2000). There are at least two problems with this sort of approach. Firstly, the rules specified are necessarily arbitrarily chosen, and secondly, the tests are not very powerful since, in such situations, the number of observations is, in general, not very large.

The last and most important point for this chapter has already been mentioned. It is that the Nash Equilibrium for the finitely repeated game is not unique. What is defined is the total sum that individuals should contribute at the equilibrium. However who should contribute what is not determined. If all agents use the same rule one might expect a symmetric result, but, to anticipate a little, this is not what we observe. If each agent learns to contribute a certain amount and the total corresponds to the Nash Equilibrium (NE), then we have to explain how agents come to self organise in this way. More interestingly, if individuals contribute different amounts in different periods but the total still corresponds to the NE then this coordination has also to be explained. One possibility would be that each of the agents is playing a mixed strategy in which he has a certain probability of making a contribution of a given amount. However, a cursory examination of our data reveals that individuals are not playing “mixed strategies” with fixed probabilities of contributing each of the possible sums. Thus the coordination
5.4. THE EXPERIMENTAL RESULTS

5.4. The experimental results

The following section presents the experimental results for the four treatments of both experiments with (treatments LP, MP, HP and VHP) and without (treatments L, M, H and VH) promises and those of the three new treatments with information (treatments 1N, 1P and 2N).

The initial analysis will be at the aggregate level where we have for each treatment the average contribution of the six groups compared to the aggregate NE and to the aggregate CO. These results are reported in figures 5.2 for the four treatments without promises, in figures 5.3 for the four treatments with promises and in figure 5.4 for treatments with information.

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mechanism is unexplained\(^2\). Learning from individual experience would not seem to explain the collective achievement of something near the Nash Equilibrium and we will see how much more complicated the situation becomes once we look at the data at the individual level.

I will proceed as follows in the rest of this chapter. First I present an analysis of the data from a series of games of contributions to public goods and firstly to see what happens, if we follow the standard approach and first determine the trend of contributions and then test a learning model on the average data. The next step is to look at group and then individual data, to examine how this changes the results and then see if some general model such as the EWA with varying parameters can account for individual behaviour. Our case is rather favourable for this sort of test since, by telling agents how much was contributed to the public good in total, at each step, we allow them to know how much they would have obtained from foregone strategies. This avoids a fundamental problem raised by Vriend (1997), as to how agents can update the weight they put on strategies which they have not played if they do not know how much these would have paid. We find that, nevertheless, behaviour differs across groups and individual behaviour is not easily categorized.

It is worth recalling that, in this type of experiment, individuals are divided into groups who play the game for a fixed number of periods. Thus the groups are unaffected by each other’s behaviour. Yet the population, taken as a whole, seems to learn in a simple way. However, the separate groups do not learn as supposed, and their behaviour differs markedly from one group to another. Furthermore, behaviour of the individuals who make up the groups also varies within those groups.

The usual explanation for some of the discrepancies in strategies in the early rounds of public goods games, as Ledyard (1995) explains, is that confusion and inexperience play a role. Indeed, this is one of the basic reasons why repetition has become standard practice in these experiments. Yet, as we will see, this would not be enough to explain some of the individual behaviour observed in our data.

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\(^2\)Efforts have been made to aid the coordination by introducing artificial labels which the players can choose between each round (see Page et al. (2005)) In this situation the players sort themselves into groups by the amounts contributed. Sol et al. (2006) allow players to sign binding agreements as to the contributing group they will form and find that the players do not coordinate on the Nash equilibrium of this game but paradoxically move in the direction of the collective optimum over time.
5.4.1 Basic results without promises:

The first thing to observe when analysing the experimental results is that they reproduce the well known stylised fact that the average group contribution ($\bar{Y}$) decreases over time. In fact, the Very High treatment ($\theta = 8.94$), has 133.33 and 91.5 as values of the first and the last periods (see figure 5.2). In the case of the H and the M treatments, the average group contribution ($\bar{Y}$) decreases during the 10 first periods and stays at a steady level during the rest of the periods of the game. In the last treatment L, this average group contribution starts at 67.67 and decreases steadily during the 25 periods of the game until finishing at 7.16. The decrease in contributions is however less evident in the VH treatment. In general, if we overlook the first five periods that could be assimilated to “learning periods,” contributions are almost steady over the twenty last periods for the M, H and VH treatments.

The results show that contributions vary with the CO level. There is overcontribution in comparison to the NE. As the CO level increases so does overcontribution. Nevertheless, average contributions as a proportion of the CO do not increase. Thus, computing these contributions in relative values by calculating an overcontribution index that takes into account the NE and the CO, shows that, except in the VH treatment, this ratio is constant.

In the L treatment, the average group contribution seems to decrease and to tend steadily to the NE value (which is 4). For this treatment, where the CO level is very low, subjects seem to tend to the theoretical predicted value for the one-shot game. The difference between the theoretical prediction and the experimental results is less evident in this framework as the game proceeds and indeed, such a close approximation to the NE is rarely observed in the experimental literature relative to public goods. It seems somewhat paradoxical that subjects contribute less and learn to play the NE value when the CO is low. For this is precisely the case in which the CO is easy to reach in the sense that it does not require a large contribution. For high levels of the CO, it is, in fact, risky for one subject to cooperate and to try to reach the social optimum by contributing a large amount to the public good. Taking such a risk can lead one subject to share his or her contribution with other subjects that choose not to contribute, and, in so doing, to lose, most of his or her private payoff. Thus the risk when faced with “free riding” behaviour is higher as the CO increases.

The other side of the coin is that the gains to be had from contributing more collectively are higher when the CO is higher. One natural idea is that individuals make generous contributions initially to induce others to do the same. This is, of course, not consistent with optimising non-cooperative behaviour but has been evoked in considerations of non-equilibrium behaviour. This would suggest that individuals indulge in some sort of signalling and this might be their way of getting around the fact that direct communication is ruled out in the game.

5.4.2 Results with promises:

The communication problem is important and one of the main arguments against the way in which experiments are run in economics is that individuals are not allowed, in general, to discuss what they should do. The reason given is that such discussions
5.4. THE EXPERIMENTAL RESULTS

Figure 5.2: Average total contribution in treatments VH, H, M and L without Promises
are difficult to control and that, as a result, meaningful comparisons between different experiments cannot be made. Yet, the focus of much of this book is precisely on how people do interact and coordinate. So, in a sense, the experiments I am discussing here are a step away from this. Without violating the code of economic experiments it is possible to introduce some communication by allowing people to announce what they intend to do. Promises can be introduced in the public goods game as a step preceding real contributions but one which does not involve direct communication between the participants. In each group, and in each period, individuals are asked to announce their intentions as how much they will effectively contribute in the considered period. The sum of these intentions is then revealed to the members of the group. Thus this information become common knowledge before the beginning of the real game in which individuals announce their effective contributions that will be considered when calculating their gains. Recall that strictly speaking, intentions or promises that are not binding are considered in game theory as "cheap talk," since the gains from different actions are not directly affected by their introduction. Also, the NE and the CO of the game are the same as in the game without promises.

As might be intuitively expected, and in concordance with findings in experimental literature, the introduction of promises does increase contributions to the public good, although this increase is not very marked. Consequently, the average group contributions are further from the Nash equilibrium than in treatments without promises. However, this difference diminishes over time, as if the players learn to distrust the announcements of the others.

What is interesting from our point of view is the heterogeneity of groups and individuals behaviour when promises are allowed. In fact, there is at least some evidence that in treatments with promises, behaviour is initially modified in treatments without communication. Since the various levels of the treatment with promises provide essentially similar results I will show only one as an illustration.

![Total contribution in the HP treatment with Promises](image.png)

Figure 5.3: Average Total Contribution in treatments HP with Promises
5.4.3 Results with Differing Information:

While the introduction of promises increases contributions to the public good, information about the contribution of the other members of one group seems to have no effect on the decision of contribution of individuals. In fact, as shown in the following figure (figure 5.4), which shows average total contribution for the three treatments with information and for treatment H (already given in figure 5.3), the experimental results reveal no difference between contributions with information and contributions without. The aggregate behaviour is very similar in the four treatment: overcontribution is evident during all the experiment but decreases over time, as contributions become closer to the NE. This undermines the idea that individuals will change their behaviour once they know who has contributed what. So any coordination that emerges cannot be attributed to the processing of more or less information by the individuals concerned. This is disappointing for those who would like to believe that individuals are, in reality, highly sophisticated and were just being prevented by informational constraints from exercising their sophistication. However it comforts the idea that the participants may be working with simpler rules and not processing all the information potentially available to them.

![Figure 5.4: Average Total Contribution in treatments H, 1N, 1P and 2N](image)

One interesting feature is that in all treatments, contributions start by increasing and are very close to the CO in the first periods of the game. One might be led to believe that individuals try to encourage each other by contributing at a high level initially, though there is no concrete evidence for this. Also, the average total contribution is almost the same for three of the four treatments in the last period and is very close to the
Nash solution. By the end of the experiment the participants have come to coordinate essentially on the non-cooperative equilibrium. Table 5.3 summarises the results for these four treatments

<table>
<thead>
<tr>
<th>Treatment</th>
<th>First period</th>
<th>Average over 25 periods</th>
<th>Last period</th>
<th>Max.</th>
<th>Min.</th>
<th>St. Dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>145.33</td>
<td>93.87</td>
<td>58</td>
<td>145.33</td>
<td>58</td>
<td>21.98</td>
</tr>
<tr>
<td>1N</td>
<td><strong>167.83</strong></td>
<td>94.12</td>
<td>31.83</td>
<td><strong>187.67</strong></td>
<td>31.83</td>
<td>42.97</td>
</tr>
<tr>
<td>1P</td>
<td>129.33</td>
<td>93.51</td>
<td>33.33</td>
<td>150.5</td>
<td>33.33</td>
<td>32.75</td>
</tr>
<tr>
<td>2N</td>
<td>149.17</td>
<td><strong>97.74</strong></td>
<td>32.83</td>
<td>178.17</td>
<td>31.83</td>
<td><strong>43.64</strong></td>
</tr>
</tbody>
</table>

Table 5.3: Average Total Contribution in treatments H, 1N, 1P and 2N

These results are in concordance with other experiments where information is introduced in some way in a public goods game. See, for example Cason and Khan (1999), and Ledyard (1995). All of this reinforces the idea that coordination does not depend essentially on the level of information available.

5.5 Testing a Learning Model

Up to this point we have seen that the individuals in our experiments manage over time to coordinate on something close to Nash equilibrium. As I have said, the standard assertion is that "individuals learn to play Nash". If we are to accept that this is what is going on then we are faced with two problems. The first we have dealt with, which is that the whole group seems to manage to coordinate on the Nash equilibrium. The second is to specify and test a reasonable learning process. The first step is to follow what is done in the literature and examine the validity of a slightly different idea. This is to examine the claim that the population “learns to play Nash”. To do this, we tested a simple learning model on the aggregate level to see whether this model fits the experimental data. The next step was then to see whether this is still the case when we examine individual and group data. Again, the reason for doing this is so as not to be led into the trap I have already discussed. The trap obviously being that it is legitimate to conclude, from the aggregate results that the individuals are behaving in a particular way.

The data that was used for these tests were from treatment H without promises, treatment H with promises and treatments 1N, 1P and 2N with information. This choice is based on the fact that all these treatments have the same theoretical payoff function (high level of social optimum). Since all these cases have the same theoretical solutions we have, in comparable situations, 24 persons playing 25 periods, and we have then 3000 observations or individual decisions as to how much to contribute. To start with, however we just analysed the total contributions.

First I will first present the Reinforcement Learning model, and explain what happened when we tested this simple model at the aggregate level using the experimental data.
5.5. TESTING A LEARNING MODEL

5.5.1 The Reinforcement Learning model:

Two properties of human behaviour in the set of situations we analyse are mentioned in the psychology literature. The first one, known as the "Law of Effect" reflects the fact that choices that have led to good outcomes in the past are more likely to be repeated. The second one is called the "Power Law of Practice" and announces that learning curves tend to be steep initially, and then flatter. Another property is also observed, which is “recency,” according to which recent experience may play a larger role than past experience in determining behaviour.

As already mentioned there exists a wide variety of learning models in the game theory literature. We will use a simple learning model and apply it to our experiments. The model used is the basic reinforcement learning model used for example by Erev and Roth (1998). There are several variations of this model. In the one parameter reinforcement model, each player $i$, at time $t = 0$, before the beginning of the game, has an initial propensity to play his $k$th pure strategy. Let $A_i^k(0)$ be this initial propensity. When a player receives a payoff $x$ after playing his $k$th pure strategy at time $t$, his propensity to play strategy $k$ is updated. The rule of updating these propensities from a period to another is given by the following relation:

\[ A_i^k(t + 1) = A_i^k(t) + x \]

The propensities to play the other pure strategies $j$ are

\[ A_i^j(t + 1) = A_i^j(t) \]

These propensities will allow player $i$ to compute the probability that he plays his $k$th strategies at time $t$. Let this probability be

\[ p_i^k(t) = \frac{\exp(\lambda A_i^k(t))}{\sum_{j=1}^{m_i} \exp(\lambda A_i^j(t))} \]

where the sum is over all of player $i$’s pure strategies $j$.3

5.5.2 The simple test of reinforcement learning at the aggregate level:

We applied the reinforcement learning model to the aggregate level of the 5 treatments. We divide the set of possible contributions ([0; 280]) into ten equal intervals ([0; 28]; [29; 56]; [57; 84]; ...; [252; 280]). At time $t=0$, all the possible levels of contribution have the same attraction and the same probability to be chosen. In period 1, the strategy chosen in the data at the aggregate level receives the payoff $x$ as explained above. At each of the 25 periods, the aggregate contribution of each treatment of one given period played in the real experiment is updated. This model is applied to each of the 5 treatments and the average of these results is calculated and presented in figure 5.5.

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3This well known rule is also referred to as the “Quantal response” rule or the “logit” rule and can be justified as optimizing the trade-off between “exploration” and “exploitation”
figure shows that the strategy that is played the most according to the reinforcement learning model is to contribute an amount corresponding to the fourth interval, that is $[85;112]$. This result obviously corresponds to the experimental data given that we are using this data to update the observed chosen strategies.

**Figure 5.5:** The average probability of playing each of the 10 set of strategies when applying the Reinforcement Learning model to the aggregate level of the six treatments.

### 5.5.3 The EWA Learning model:

I will now present a more sophisticated learning model, the EWA (Experience-Weighted Attraction) model and will show what happened when it was applied it at the aggregate level using the same data as those used in the previous section.

The basic idea of this model is to combine two approaches to learning. That which suggests that people learn from their own experience and that which suggests that people try to respond in the best manner to what they anticipate that others will do. The details of the Experience-Weighted Attraction (EWA) model can be found in Camerer and Ho (1998) where the model was first introduced.

Consider $N$ players indexed by $i$, where $i = \{1, 2, \ldots, N\}$. Let $A_i^j(t)$ denote the attraction of strategy $j$ for player $i$ once period $t$ is played, where $j=1,2,\ldots,m_i$. The attraction of the different strategies in the EWA learning model are updated differently. The chosen strategies receive an attraction equal to the payoff $\pi_i$ player $i$ receives as a result of his choice, while the attractions of the unchosen strategies are updated by adding only a part $\delta$ of the foregone payoff. Thus, the parameter $\delta$ is the weight a player
5.5. TESTING A LEARNING MODEL

puts on the unchosen strategies. Let $s^j_i$ denote the strategy $j$ of player $i$, and $s^t_{-i}$ be the strategies chosen by all the players, except for player $I$, in period $t$. Attractions of strategies are updated according to the payoffs these strategies provide, but also according to the payoff that unchosen strategies would have provided. Recall that it is not always obvious that this information will actually be available to the players. In our case the fact that the individuals are apprised of the total contributions allows them to work out what the foregone payoffs of unchosen strategies would have been. The rule for updating attractions in period $t$ is:

$$A^j_i(t) = \frac{\phi.N(t-1).A^j_i(t-1) + [\delta + (1 - \delta).I(s^j_i, s^t_{-i}(t))]\pi_i(s^j_i, s_{-i}(t))}{N(t)}$$

where $I(x, y)$ is an indicator function that is equal to 1 if $x = y$ and to 0 if not. $\pi_i(s^j_i, s_{-i}(t))$ is the payoff of player $i$ when he plays strategy $j$ while the other players play the combination of strategies $s_{-i}(t)$.

The parameter $\phi$ is a discount factor used to depreciate the previous attractions so that strategies become less attractive over time.

$N(t)$ is the second variable updated in the EWA Learning model. It is the experience weight used to weight lagged attractions when they are updated. $N(t)$ is updated according to the rule:

$$N(t) = \phi(1 - \kappa).N(t - 1) + 1$$

where $t \geq 1$.

The parameter $\kappa$ controls whether the experience weight depreciates more rapidly than the attractions.

At $t = 0$, before the game starts, the two variables $A^j_i(t)$ and $N(t)$ have initial values $A^j_i(0)$ and $N(0)$.

The probability of choosing a strategy $j$ by player $i$ in period $(t + 1)$ is calculated by using a logit form:

$$p^j_i(t + 1) = \frac{\exp(\lambda.A^j_i(t))}{\sum_{k=1}^{m} \exp(\lambda.A^k_i(t))}$$

where $\lambda$ controls the reaction of players to the difference between strategies attractions. A low value of $\lambda$ implies an equal probability for choosing strategies, while a high value supposes that players are more likely to choose strategies with higher attractions.

Note that the reinforcement learning model is a special case of the EWA model. In fact, when $\phi = 1, \kappa = 1, \lambda = 0$ and $N(0) = 1$, attractions are updated in the same way as in the cumulative reinforcement learning model. The EWA learning model includes the reinforcement learning model and other learning models such as belief learning as special cases (Camerer and Ho (1998)).

5.5.4 Simulating the EWA learning model at the aggregate level:

We ran simulations using the same theoretical function as that used in our 5 treatments. We calculate the probability of playing each possible contribution by updating the initial propensities of each strategy using the EWA learning model. The parameters used are those which best fit the model.
First, we played the public goods game for 25 periods where the simulation program chooses randomly in the first period one of the 281 possible levels of contribution ([0; 280]). This means that in the first period, all the strategies have the same probability of being chosen. The propensity of the strategy chosen in the first period is updated according to the EWA learning model. Next, the simulation program calculates for each strategy the probability of its being played in period 2. Obviously, the strategy chosen in period 1 has a greater probability of being chosen in period 2. In the second period, the simulation program chooses a new strategy using the new probabilities that each strategy will be played.

This program is run for 25 periods. Figure 5.6 is an example of 25 strategies belonging to the interval [0; 280] and chosen by the simulation program:

![Figure 5.6: The probability of the 25 strategies actually chosen using the EWA learning model.](image)

We repeat this simulation 1000 times and we calculate the average of the probabilities of all the chosen strategies. The result is presented in figure 5.7:

What is worth noting here? Firstly we see that the probabilities of contributing larger amounts at the end of the experiment are much higher than those observed in the
Figure 5.7: The average of the probabilities of 1000 repeated simulation of the 25 periods public goods game using the EWA learning model.
data. In particular, the probability of choosing a collective contribution between 200 and 280 is high. This suggests that even as an explanation of aggregate behaviour the learning model is not convincing.

This should not be of much concern since my aim here is to suggest that individuals are not all learning in this way and that there is not much reason to suppose that the aggregate does so. Now, let me come to the crucial point of this chapter and look at the group and individual behaviour. Recall that people are playing in groups that have nothing to do with each other so, if aggregation over groups produces more regular behaviour this is a purely statistical phenomenon and has nothing to do with any interaction between groups. Given this, I shall now show the behaviour at the lower levels, that of the groups and the individuals and it will come as no surprise that there is a great deal of volatility.

5.5.5 The group and the individual level:

Although at the aggregate level behaviour seems rather consistent, in all treatments, everything moving towards the Nash equilibrium, at the group level things are very different. The behaviours within the same group and between groups can vary widely. These variations seem to suggest that we observe very different attitudes to contribution. Figure 5.8 shows the contribution of the six groups in treatment H without promises and gives us an idea about this variation between groups.

![Figure 5.8: Contributions of the six groups in treatment H (without promises).](image)

Some groups simply contribute more than others and others have high variability in their contributions. The cynical might believe that it is the economics students who are less willing to contribute whilst those from other disciplines are more altruistic. Sadly
there is some evidence for this.

What is more, even within groups, at the individual level, there is also a difference between the individual and the aggregate behaviour. In fact, individuals simply behave differently. Moreover, the individual behaviour is more difficult to classify because of the considerable volatility of contributions of individual subjects during the 25 periods of the game.

To illustrate the differences in individual behaviour I show the individual contributions in six groups in figure 5.9. It is easy to see that some individuals contribute at a fairly constant level, which may be high or low whilst others vary their contributions considerably.

Jut to have a preliminary idea about the different levels of contributions of individ-

Figure 5.9: Individual contributions in six groups VH treatment. Source Hichri and Kirman (2007)
uals, we classify contributions in 8 intervals of ten each and we present in figure 5.10 the number of times individuals make a contribution belonging to each interval. As in each of the 5 treatments there are 24 persons playing 25 periods, we have then 3000 observations or decision of contribution. As we can see in figure 5.10, none of the intervals has zero weight:

![Figure 5.10: Number of individual contributions for each interval for the 5 treatments (3000 decisions).](image)

Perhaps nevertheless there is some consistency in individual behaviour. If we were to believe that the aggregate behaviour has something to tell us about the individuals then we might, at least, have expected individual contributions to have a higher probability of declining rather than increasing over time. To isolate the different strategies that could explain the differences between the behaviour of different individuals, we
compared, for the 5 treatments and for the 25 decisions of each individual, her contribution in period \( t \) to her contribution in \( t - 1 \). This allows us to know whether individuals react in response to past contributions. We classified this comparison into three possibilities: contribution in \( t \) increases, decreases and remains unchanged in comparison to contributions in \( t - 1 \). Figure 5.11 shows that all three changes were made almost by equal numbers at each point:

![Figure 5.11: Percentage of increasing, decreasing and unchanged individual contributions in \( t \) compared to \( t-1 \) for the 5 treatments (3000 observations)](image)

An alternative way to examine how consistent individual and aggregate behavior are is to test econometrically for treatment H the following model against both individual and aggregate data:

\[
C_t = a + \beta C_{t-1} + \varepsilon_t
\]

We find that \( \beta \) is significantly different from 0 for the aggregate and for the 6 groups while it is not for 13 subjects out of 24. Furthermore, the assumption \( \beta = \beta_i \quad \forall i = 1, \ldots, 4 \) is rejected for all the individuals of the 6 groups. This is clear evidence that individuals were behaving heterogeneously.

### 5.5.6 The test of the EWA learning model at the individual level:

Next to see how individuals were “learning” we applied the same simulation program (run 1000 times) for a set of possible strategies that corresponds to the set available to one person playing the public goods game. This set of strategies is [0; 70]. The player can choose any amount up to 70 to the public good. The payoff of one player depends, of course, not only on his strategy but also on the choice of the three other players of
the same group who are playing the game. The results for a particular individual are presented in figure 5.12:

Figure 5.12: The average of the probabilities of 1000 repeated simulation of the 25 periods public goods game for one player using the EWA learning model.

5.5.7 Comparison with the simple learning model

Simulating the EWA model for single players playing the public goods game shows that contributions belonging to the interval [40; 60] have the highest probabilities of being played (figure 5.7). This is not in accordance with the observed strategies. Furthermore, the EWA model would suggest an essentially monotone evolution. Whilst this is observed at the aggregate level for all treatments, this is far from being the case for the groups and the individuals. The behaviour of players seems to differ widely across individuals and what is more there is no convergence to any common behaviour. Individual players seem to contribute in a way that does not correspond to any simple learning model or at least any of the models discussed here.
5.6 Conclusion

The basic premise in this book is that coherent aggregate behaviour may be the result of the aggregation of very different individuals who interact with each other. Having made this point in the context of markets the idea here was to look at a situation in which individuals are faced with an economic coordination problem and see how they behave and how we might explain their behaviour. In particular, does game theory, the basic approach to understanding strategic interaction, provide us with a good explanation of individual behaviour? The example I chose was that of experiments on contributions to a public good.

The points made in this chapter are rather simple. Firstly, as in all the literature on public goods experiments, players do not find the equilibrium from the outset. They are not the perfectly eductive game theorists sometimes described in the game theoretic literature. Secondly, what seems to be rather systematic behaviour at the aggregate level in a well defined situation in which individuals interact and which might be thought of as corresponding to the behaviour of those individuals, turns out to be a phenomenon due to aggregation. The clear movement towards the Nash equilibrium which persists in various treatments of the public goods game, does not reflect any systematic behaviour at the group or individual level. People within groups interact and react to the contributions of the other members of the group. This may lead to very different levels of total contributions across groups and over time. Furthermore, different groups do not necessarily exhibit the uniform decline in total contributions which is observed at the aggregate level across all treatments.

Different versions of our basic experiment in which we varied the amount of information available and allowed individuals to make promises about their contributions, yield similar results. Hence, the difference between the individual and aggregate level cannot be attributed to some specific institutional feature of our experiments.

What it is that causes the variation across individuals remains an open question. Simple explanations such as “degrees of altruism” or differences in “aversion to inequality” as suggested by Fehr and Schmidt (1999) do not seem to be satisfactory. The idea that people simply vary, in their willingness to give as a result of an intrinsic preference, has some intuitive appeal. This would fit nicely with the idea of an asymmetric equilibrium in which those more willing to give do give more and those less inclined to do so contribute less. The advantage of a model, such as the one which we study, in which the Nash equilibrium corresponds to a positive amount is that it would allow us to observe this. We could then classify individuals accordingly. Over time those who are basically more generous would turn out to be those who contribute most at the Nash equilibrium. However, this is not what we found. Individuals did not move towards a constant level of payment. They switched with others, sometimes contributing larger amounts and then contributing less. There was no sorting of the individuals into the more and less generous. So our results constitute a rejection of the hypothesis that individuals can be simply classified in this way. Had they been doing so we would no longer have had the switching from increasing to decreasing contributions that we observed and which was depicted in figure 5.11.

What seems to be going on is that there are continual actions and interactions between the members of the various groups and somehow this leads to a coordination on
something close to the Nash equilibrium but while the total is stable the contributions vary. One possible interpretation of what is going on is that some individuals try to signal to others by means of their contribution. They may hope, in so doing, to induce higher payments from their colleagues. If this is what they are doing then they violate some of the simple canons of game theory.

What is clear, consistent with all the literature on public good games, is that the players do not, in general manage to coordinate on cooperative behaviour. Thus, in the set-up here, the optimistic conclusion that cooperation will emerge, found in the “Prisoner’s Dilemma” literature does not seem to be justified. Free riding is something which on average increases over time but many individuals do not follow this pattern systematically. To sum up, the behaviour of individuals varies considerably, but the complexity of the interaction does not prevent the individuals from arriving close to an equilibrium. The most mysterious part of this story remains that, despite the failure of all the simple explanations of how individuals behave, they do manage to coordinate on an equilibrium. How they do this is an open problem. That they do so fits nicely with the idea of emergent aggregate behaviour. Nevertheless, the fact that the population manages to coordinate in this way, should not, as we have seen, lead us into the trap of attributing individual behaviour to the aggregate nor, and worse, of concluding from the apparent aggregate learning process, that individuals are learning in this way.
Chapter 6

Segregation: Schelling’s Model

6.1 Introduction

In this chapter I will look at an important social problem, that of racial segregation. This illustrates yet again the basic argument of the book. What happens at the macro level may not reflect individual wishes. However, I would like to introduce another consideration here. It should be clear by now that I believe that economists should be open to lessons from other disciplines. Here I want to argue that by making a physical analogy we can better understand one of the major problems faced by our societies. In the third chapter I already used a simple model from statistical physics to explain the emergence of loyalty. In this chapter I will use another model to analyse the emergence of segregated clusters of individuals. Once again there is a certain reticence on the part of social scientists when faced with physical models but I will come back to this.

But let me turn first to the problem itself. Racial segregation is a persistent phenomena in many cities in many countries. In the U.S, for example, whilst more than one-half of Black Americans now live in middle or upper income households, segregation in housing has persisted in major cities. The natural explanation for this phenomena would be that individuals are racist and prefer to avoid living with people of another race. Thus segregation is no more than a macro phenomenon that reflects, in a consistent way, peoples’ individual sentiments. But Tom Schelling, the 2005 Nobel Prize winner in economics whose influence permeates the whole of this book, showed that, once again the relation between micro and macro behaviour is not so simple. He argued that the degree of segregation that we observe is far from reflecting individual views. At the end of the 60’s he introduced a model of segregation (a good summary of the variants of his model is given in Schelling (1978)), which showed essentially that even if people only have a very mild preference for living with neighbours of their own colour, as they move to satisfy their preferences, complete segregation will occur.

This result was greeted with surprise and has provoked a large number of reactions. The reason for the surprise is that which has figured throughout this book, individuals with a certain motivation, by their interaction, produce an aggregate phenomenon
which was in no way intended by them. To see what is at work here it is worth looking in some detail at the model that Schelling introduced. It can be explained simply and intuitively and this is one of the features of Schelling’s contributions that makes them so appealing. The summary here is based on a recent paper by Pancs and Vriend (2006) which reinforces the surprising relation between micro features and aggregate phenomena in the segregation model, or to use Schelling’s original phrase, the relation between, “Micro motives and macro behavior”.

The basic idea is this. Take a large chess board, and place a certain number of black and white counters on the board, leaving some free places. Then take a counter at random. The counter prefers to be on a square where at most four of his eight neighbours are of a different colour than his own. This “utility function” is illustrated in figure 6.1.

![Schelling's individual preferences](source Pancs and Vriend (2006))

This utility function which depends only on the fraction of neighbours of a different colour can be simply expressed as

$$U(x) = u_R \text{ if } x \leq \rho_R \text{ and } 0 \text{ if } x > \rho_R$$  \hspace{1cm} (6.1)

where $u_R > 0$ and $u_R \in [0, 1]$.

At each step an individual is drawn at random and if he has utility 0 he moves to the nearest unoccupied space where his utility is higher. A sequence of moves illustrating this is shown in Figure 6.2 and it is clear that segregation develops rapidly.

In a recent paper, Kirman and Vinkovic (2006), we have developed an argument to suggest that this result is not, in fact, surprising and that some simple physical theory
Figure 6.2: (source Pancs and Vriend (2006)
can explain the segregation phenomenon. Numerous papers have been written using Schelling’s original model as a starting point and they have used many variants. Typically, the form of the utility function used by Schelling has been questioned as have the number of neighbours, the rules for moving, the amount of unoccupied space and all of these have been claimed to be relevant for the results.

Using simulations in general, a number of authors have argued that modifying the parameters just mentioned yields different patterns of segregation. What has been lacking so far is a theoretical structure which can incorporate all of these considerations and then can produce analytic results for each variation. It is precisely this that we propose in the recent paper mentioned above.

Perhaps the nearest approach to ours is that adopted by Pollicot and Weiss (2005). They however, examine the limit of a Laplacian process in which individuals’ preferences are strictly increasing in the number of like neighbours. In this situation it is intuitively clear that there is a strong tendency to segregation. Yet, Schelling’s result has become famous precisely because the preferences of individuals for segregatin were not particularly strong whereas in the paper by Pollicot and Weiss there is a strong preference for neighbours of one’s own colour.

Indeed this observation leads us directly to the interesting question as to why people have paid so much attention to this model. Pancs and Vriend (2006) give three explanations. Firstly, it is the “micro motives” of the agents that lead to the aggregate result, in other words a macro phenomenon “emerges” from individual behaviour. Furthermore, as we have said, the result at the aggregate level is more extreme than what would have been necessary to make the individuals happy. Secondly, the model is remarkably simple and its workings can be understood immediately. Lastly the model concerns a topic of great concern in the developed world, that of segregation whether it be on racial or any other grounds.

Three things are of particular interest to economists. The first is the organisation of the system into “regions” or clusters, each containing individuals of only one colour and second the shape of the frontier between the regions.

A second question concerns the importance of the number of empty spaces. How does this affect the final configuration? In some analyses, agents are simply swapped with each other if this makes them both happier and there are no free spaces. The question here is as to the role of the free space and will it inevitably wind up as a “no-man’s land” between two clusters?

A third question concerns the distance which an unhappy individual may traverse in order to reach a position in which he is happier. If only local movement is allowed, as opposed to movement over any distance, does this affect the final outcome? A natural economic interpretation would be that there is a cost to moving and that this is an increasing function of distance.

In Vinkovic and Kirman (2006), we develop a physical analogy to the Schelling model. The agent’s happiness or satisfaction in this interpretation corresponds to the energy stored in the agent. An increase in utility would correspond to a decrease in an individual’s internal energy. An agent, therefore, wants to minimize his energy, and this may be achieved, either by taking some action or through the interaction with his environment. In the Schelling model the utility of an agent depends on her local environment and the agent moves if the utility declines below a certain value. The
interpretation of this situation as that of a particle system immediately poses problem for philosophers such as Schabas (2005) who insist on the importance of intention and reason in economic models and these attributes cannot be given to particles. She also argues that a clear analogy between energy and utility has yet to be found, even though many references to such an analogy have been made in the literature. As to the latter, it seems that the case here provides a counter-example. As to the argument about reason and intent, this argument is not relevant here. If agents act in a certain way which corresponds to the behaviour of particles in a system, then the reason for their behaviour does not prevent us using models of inanimate particles that act in the same way. To analyse the behaviour of a stone thrown by a human does not require one to know why he threw it. In the particle analogue the internal energy depends on the local concentration (number density) of like or unlike particles. This analogue is a typical model description of microphysical interactions in dynamical physical systems of gases, liquids, solids, colloids, solutions, etc. Interactions between particles are described with potential energies, which result in inter-particle forces driving particles’ dynamics.

The goal of such models is to study the collective behavior of a large number of particles. In the Schelling model the number of particles is conserved and the total volume in which they move around is constant (that is, the underlying lattice is fixed). The pressure can be also considered constant. The system is not closed, however, because the energy lost by a particle is not transferred to other particles, but transmitted out of the system. (for an economist this is not a zero sum game). Similarly, a particle can gain energy from outside the system when an unlike particle moves into the neighborhood and lowers the particle’s utility. (once again one would say in economics terms that agents generate externalities by their actions). Hence, in physical terms, the system can change its energy only by emitting or absorbing radiation and not by changing its volume or pressure or number of particles.

Since the basic tendency of such a physical system is to minimize its total energy, it can do that only by arranging particles into structures (clusters) that reduce the individual internal energy of as many particles as possible. In other words, interparticle forces attract particles into clustering and the stability of a cluster is determined by this force. Hence, all we need to do is to look at the behavior of this force on the surface of a cluster to see if the surface will be stable or if it will undergo deformations and ripping. If we think of the Schelling model and the utility function that he chose it is clear that the only individuals who do not have high utility are those on the boundary and it is for this reason that the evolution and final form of the boundary are of particular interest.

The initial configuration of the system may well be out of equilibrium. A typical initial state of the Schelling model involves randomized positions of particles in the system. After that we let the system evolve and reach its equilibrium condition. Since we can consider different types of utility functions, the equivalent physical systems can also differ in their underlying definition of interparticle forces. This, in turn will lead to different types of “equilibria”. Three types of equilibria can be immediately predicted based on physical analogues:

i) those which are dynamically unstable: particles cannot create any stable structure and the whole system is constantly changing, with clusters forming and breaking apart all the time. Water clouds are an example of this equilibrium, where water droplets are constantly forming and reevaporating. In a society this would correspond to constantly
changing neighbourhoods with no propensity to settle to any clear spatial structure.

ii) those which are dynamically stable: well defined clusters are formed. They maintain their size despite constant change of its surface due to rearrangement of the surface particles. A wobbling droplet is such an example. A special case of this equilibrium is a cluster that seemingly does not change its shape because moving particles do not accumulate at any specific point on the surface. A small raindrop maintaining its spherical shape is an example of this. This, in the context of spatial arrangements of society, corresponds to the idea that areas on the border between two segregated zones will see movement since it is precisely in these areas that individuals wish to change. The boundary will shift slightly all the time but will not be fundamentally retraced.

iii) those which “freeze”: all particles stop their movement because all of them reach the energy minimum or do not have any available space to move into. The frozen cluster structure can be either completely irregular (amorphous) or well structured. Glass is an example of the former and crystals of the latter. This sort of frozen structure can happen when there is relatively little free space available and when particles can only move locally.

As we have seen, we can study the formation and stability of a cluster by considering the energy of particles on its surface. The interparticle forces derived from the utility function will determine the optimal change of the cluster’s surface as it tries to minimise the energy of surface particles. The first step toward this analysis is a transition from the discretized lattice of the Schelling model into a continuous medium by refining the lattice and taking the limit. Hence instead of counting the number of agents \( DN \) in a discrete area \( DA \) we can write \( dA/dN \). Next we transform the utility function from counting the individuals in a neighborhood around an agent into the measurement of the total solid angle \( q \) covered by different particles around the differential area \( dA \) (Figure 6.3). Utility is replaced with energy \( \varepsilon(\theta) \) with high utility corresponding to low energy and vice versa. This gives the total energy \( dE = ne(q)dA \) stored in \( dA \) or the energy per unit area, \( dE/dA = ne(q) \). Since we are interested in the cluster surface, we take a differential length \( dL \) of the surface and write its energy per unit length as \( dS = ne(q)dL \).

The total surface energy of a cluster is an integral of \( dS \) over the whole cluster surface. It is clear that the energy at a contact point depends on the local curvature at that point. We thus have an expression for the surface tension which is given by taking a local gradient along \( dL \) and the tension force at a point \( r \) is given by

\[
\tilde{F}(\tilde{r}) = -\hat{L} \cdot \nabla_r E(\tilde{r}) = -n\varepsilon(\theta(\tilde{r})) \hat{L}
\]

where \( \hat{L} \) is a unit vector along \( dL \). The energy thus depends on the utility function chosen. In Schelling’s model with the basic step function, energy is a constant if \( q > 180 \) degrees and 0 otherwise. This corresponds to the discrete model in which, the utility of an individual is 1 if at least half of the neighbours are similar and 0 otherwise.

A cluster tries to minimize this surface energy and this is what drives its dynamics. In physics, this force is usually called surface tension. By analysing this we can predict the behavior of a cluster for a given utility function for the individuals. In the continuous version we show that any convex cluster surface experiences a tangential force pushing the surface toward flattening. The outcome of this surface reshaping are
clusters that “prefer” flat surfaces, except for parts covered by a boundary layer made of empty space. In Figure 6.3 we can see the approximation of the discrete model by a continuum.

In Figure 6.3 we can see the forces at work on the frontier trying to “flatten” the surface. When clusters start to grow they compete for particles they can collect. A bigger cluster has a bigger surface for collecting particles, thus also a higher probability of collecting them. Eventually, one cluster grows bigger than others and grows until it collects all available particles. Smaller clusters can exist if they manage to “freeze” their particles, not letting them “evaporate” and travel to the big cluster. If the volume of the big cluster is not much smaller than the total available volume then particles segregate into two distinct clusters separated by a flat surface. Alternatively, a cluster at the center of the available volume forms. Since particles have the same probability to reach the cluster’s surface from any direction, such a cluster will stay close to a spherical shape. In Vinkovic and Kirman (2006) we give detailed illustrations of the sort of cluster patterns that can form under different parameter values.

The growth of one big cluster out of a mixture of particles is a common natural phenomenon and is well known in the physics of metal surfaces, where the evolution of fewer but larger nano-structures out of many smaller nano-structures is an actively studied phenomenon generally known as “coarsening”. The Schelling model is, therefore, just a discretized model of such physical phenomena. This analysis, using a physical analogy, allows us not only to obtain the well-known “surprising” result of Schelling but can also be used to show that other configurations
Figure 6.4: (source Vinkovic and Kirman (2006)
can appear depending on the distance that individuals can move and how much space is available and the particular utility function that is attributed to them. Thus, a rather simple physical analysis allows us to consider a number of interesting phenomena which provide a richer panoply of possibilities than the pure segregation found in the original model.

A first question that one can ask is whether it is inevitable that one large cluster will appear, or in the social context, whether all the people of one race will eventually occupy the same zone? The answer is given by the physical analogy which says that smaller clusters can only exist if they “freeze” their particles, preventing them from “evaporating” and travelling to the big cluster. But what does this mean in our context?

In particular, the most important rule is that which determines whether a particle or individual moves only when the move decreases its energy or increases its utility, or whether it can move even when its energy remains constant.

The former rule leads to the formation of a solid material, where the whole system freezes after all particles reach their minimum possible energy or stay stranded at higher energies or low utilities when there is no available location that would decrease their energy. This can be seen in Figure 6.5 where the role of two factors is illustrated. On the one hand there is the level of tolerance of the individuals of the other race, and on the other, there is the number of free spaces. When the tolerance of others is low, this corresponds to a small value of \( x \), clusters form more easily. The arrows in the figure indicate increased clustering. When there are more free spaces, clusters can be surrounded by unoccupied spaces which means that they do not need to attract other members to improve their utility. However, it is worth noting that when there are few free spaces and the level of intolerance is high the clustering is not strong since too many dissatisfied people are stranded with no space to which they can move.

Suppose, on the other hand we allow agents to move even though such a move will leave them indifferent. Such a rule corresponds to a liquid system where all particles move all the time. In the economic context this corresponds to the case where individuals may change locations for reasons exogenous to the model provided the new location is not worse in terms of the simple utility function. In the physics context these rules determine particle mobility and hence the associated physical structure. In the physical case, solids and glasses have very little or no particle mobility, while liquids have particle mobility even though it may be limited.

An important lesson that we can learn from looking at this sort of model is that the macroeconomic picture may be apparently the same in two cases while what is going on at the micro level may be very different. In the case where particles or individuals are constantly moving, as in the liquid case, the same degree of segregation may be present as when all the individuals of one colour “freeze” into a big cluster, for example. One way in which this can happen is by modifying the choice of utility function. Pancs and Vriend (2006) give a nice example of this. They look at different possibilities for utility functions and these are illustrated in Figure 6.6

Consider the peaked utility function which says that people are happiest when they live in a fully integrated neighbourhood. It would seem obvious that if anything should

\[ x=0.375 \text{ means, for example, that an individual is unhappy if more than 3 of his 8 neighbours are of a different race.} \]
CHAPTER 6. SEGREGATION: SCHELLING’S MODEL

Figure 6.5: (source Vinkovic and Kirman (2006))

Figure 6.6: (source Pancs and Vriend (2006)).
prevent segregation this would be that case. However, what happens is very odd. Individuals are very rarely happy since they are unlikely to be in the ideal situation with four neighbours of each colour. As a result they are constantly moving trying to improve their utility. Thinking about our previous discussion the happy people now are those on the frontier! But the best situation is still a flat frontier, since as soon as it is not flat nobody is happy on either side of it. This means that people will always be on the move, that the frontier will move but the overall pattern at any point in time will be very similar to that found in the original Schelling model. The real difference is that now the picture at the micro level is totally different with most people trying to move to increase their satisfaction.

The situation is shown in Figure 6.7.

![Figure 6.7](source Pancs and Vriend (2006))

The case illustrated is one with a 100x100 grid and 4000 agents of each colour. In the first series of panels one can see the segregation into large clusters in the original Schelling model but what is most interesting is the second series of panels. Here the utility corresponds to the peaked case in Figure 6. We see again the same pattern of segregation but, in fact the individuals are continually moving. At the micro level things are radically different whilst at the macro level they seem very similar.

Another important rule determines how far a particle can move. One rule could allow a particle to move to the closest acceptable location. Another could allow particles to randomly choose between all acceptable locations regardless of their distance. Economically the distinction between the two would correspond to the cost of moving. In the physical context this corresponds to slow and fast diffusion rates, respectively. Sim-
ilarly, if we allow particles to swap places, that is, move even to non-empty locations, then there are two cases and these are illustrated in Figure 6.8.

![Figure 6.8:](image)

If only short jumps are allowed, then we slow down the evolution of the whole system because particles will move mostly to those occupied by their nearest neighbors. If jumps to any distance are allowed, the opposite happens and the diffusion rate becomes high and the whole system will evolve relatively quickly.

### 6.2 The income dimension

One objection to the Schelling model is that it only focuses attention on the racial motives for segregation. In fact, it is clear that other factors play an important role. In particular, individuals may have a preference for the income level of their neighbours. A number of papers have looked at the spatial distribution of incomes but rather few have studied income level as a preferred characteristic. With regard to income distribution there are two standard models in urban economics which predict that incomes in
urban neighborhoods will be quite homogeneous. Tiebout’s (1956) well known analysis implies that households sort themselves into communities with similar tastes and incomes. The monocentric city model which looks at the sorting of people by their distance from the centre of a city, predicts that households who differ only in terms of income will occupy successive (concentric) zones in a monocentric city. Individuals, it was argued by Mills (1967), and Muth (1969), for example, will end up at different locations in space depending on the income elasticity of demand for housing and the cost of commuting. This, of course, necessitates studying the price of housing and I will come back to this. In any event, a standard argument is that spatial differences in the price of land contribute to the formation of new neighborhoods of homogeneous units. All of this suggests that there should be a strong spatial bias toward intra-neighborhood homogeneity and this is what Vandell (1995) suggested. Yet as Hardman and Ioannides (2004) indicate, as soon as one introduces more realistic features into the models these simplistic arguments do not seem to hold. This was already observed by Wheaton (1977) and the detailed analysis by Hardman and Ioannides (2004) reveals that homogeneous income neighborhoods are the exception rather than the rule.

Here the aspect of this problem which I want to emphasise is the role of peoples’ preferences for the level of income in a neighborhood. There is a great deal of evidence that people judge the desirability of neighborhoods by their income level. There are simplistic arguments for this. As an article in the Washington Times put it,

“Most people prefer to live in rich neighborhoods rather than poor ones – why is that? Perhaps they notice rich neighbors tend to keep the environment cleaner, respect property rights of others and do not engage in criminal activities threatening lives of their neighbors”. Rahn (2003).

Once again, although there are elements of the truth in this quote, things are not so simple. Susan Mayer (2001) made a careful study of the statistics and found that whilst overall economic inequality in the U.S. had increased in the period that she studied, within neighborhood inequality had remained essentially constant. The question then becomes, is this consistent with the idea that people sort themselves on the basis of the income level of their neighborhood? To start with, if neighbours’ income is the sole criterion for sorting then there are arguments to suggest that people will want to live in a rich neighbourhood. As Mayer (2001) points out

“… economic segregation follows from the idea that affluent residents generate benefits for their neighbors (Wilson 1987, Durlauf 1996, Jencks and Mayer 1990). As a result, families will pay more to have affluent neighbors, independent of the level of publicly provided goods.”

She goes on to explain that the benefits of affluent neighbors could derive from a higher tax base, which provides any given level of public amenities at a lower tax rate, from better role models (Wilson 1987), or from more effective neighborhood monitoring (Sampson and Laub 1994). She also observes that if everyone saw advantaged neighbors as an advantage and cared only about having advantaged neighbors, neighborhoods would be perfectly sorted by income, because the only way everyone can
avoid having neighbors poorer than themselves is for everyone to have neighbors exactly like themselves.

There are however, other theories that suggest that some families may see wealthier neighbors as a disadvantage. When disadvantaged children must compete with advantaged children for good grades, good jobs, or social status, they are more likely to lose out (Davis 1966, Jencks and Mayer 1990). If there is positive discrimination in favour of children from poorer areas, moving to a richer one may diminish a child’s chance of success. In addition, from a psychological point of view, relative deprivation theory predicts that when the poor compare themselves to the rich, this can lead to unhappiness, stress, and alienation (Merton and Kitt 1950, Davis 1959, Runciman 1966, Williams 1975). If one then focuses on either relative deprivation or competition suggest that if neighbors are a relevant reference group or relevant competitors, families will avoid having richer neighbors. Logically as Mayer (2001) indicates everyone chose neighbors exclusively in this way, we would again observe perfect sorting by income, because the only way everyone can avoid having neighbors richer than themselves is for everyone to have neighbors exactly like themselves. However, note that this argument, like the previous one, is based on the idea that income is the only factor which sorts people into neighbourhoods. Here my main interest is in how income interacts with racial preferences and this immediately complicates things.

Sethi and Somanathan (2004) are among the rare authors who have examined the interaction of these two sorts of preferences. They look at sorting equilibria as a function of peoples’ preferences and as a result of the differences in income between the racial groups together with the relative size of the groups. They explain that the effects of the two criteria, race and income, can produce surprising results.

“…racial disparities in the distribution of income play a subtle and important role in determining patterns of segregation. Even when preferences are strongly pro-integrationist and the ideal neighborhood for all individuals is close to perfectly mixed, complete segregation can result if racial income disparities are negligible or extreme.” Sethi and Somanathan (2004)

Thus, there is no simple monotonic relation between income disparities and segregation. They also observe that a city may remain trapped in the basin of attraction of the segregated equilibrium because of historical patterns of segregation. Furthermore greater integration (and correspondingly greater stratification) lowers neighborhood quality in the poorest neighborhood, which consists disproportionately of black households. As they say,

“The movement of upper-income black households to more affluent communities worsens the conditions for those left behind, a point that has been emphasized by Wilson (1987)” Sethi and Somanathan (2004)

Finally, they remark that from a cross-sectional perspective, cities with lower levels of racial inequality need not be the least segregated.

“And from a historical perspective, the march toward greater integration may be halted and reversed in some cities as racial inequality declines”. Sethi and Somanathan (2004)
6.3. AN EXTENSION OF THE PREVIOUS MODEL

The answer to the question as to which type of segregation will occur when both race and income are preference criteria is therefore far from clear and I will now offer an approach to answering this sort of question.

6.3 An extension of the previous model

In Vinkovic and Kirman (2009) we extended our previous model and tried to see if our physical analogy would help us to examine the problems I have just discussed. To do this, this we had, therefore to specify an individual’s utility function, not only in terms of the racial characteristics of his neighbours, but also in terms of their income.

For the income preference we first make the assumption that there are two levels of income “rich” and “poor” and that the income of a neighbourhood is defined by taking the proportion of poor in it. Furthermore, we make the assumption that everybody wants to live in a rich neighborhood. This seems to be consistent with the observed evidence although, as I have said, there are theories which suggest why people might want to be richer than their neighbours. Hence, in Vinkovic and Kirman (2009), the utility from neighbours income depends exclusively on the fraction of poor neighbors, \( \gamma \in [0, 1] \) and is defined as:

\[
U_I = u_I, \text{ if } \gamma \leq \gamma_I \text{ and } = 0 \text{ otherwise.} \tag{6.3}
\]

The variables \( u_I \) and \( \gamma_I \) are parameters the values of which have to be chosen. Now we can look at the same physical analogy as before and see the corresponding forces exerted on the surface of clusters. Recall that here the situation is somewhat different because of the asymmetry in peoples’ preferences, everybody wants to live with the rich. In the current case there are two types of clusters and accompanying surfaces that can coexist: racial clusters, (blacks and whites) and income clusters (poor and rich). We can derive, as in the racial case which I described before, the surface tension force from its definition as a gradient of energy along the surface. In the case of income the gradient of energy based on the utilities in equation is a step function: the force is non-zero and equal to \( \vec{F}_I = u_I \) only on cluster surfaces with curvatures larger than \( \theta_I = 2\pi\gamma_I \). This means that \( \gamma_I \) determines the cluster evolution. Figure 6.9 illustrates the point.

The rich cluster (red color) shown in the left panel behaves similarly to racial clusters (figure 6.4), except that the surface curvature is defined by \( \theta_I \). The poor cluster (blue color) shown in the right panel, on the other hand, is unstable because the income preference defined in equation 6.3 depends exclusively on the number of poor neighbors. This leads to surface forces that expand surface perturbations and prevent the cluster from growing. Any attempt to form a poor cluster is, therefore, quickly stopped and the cluster disintegrates due to its diffusion into the rich cluster. This, of course, corresponds to the natural economic intuition. Since the surface of a poor cluster in unstable to perturbations; any surface bump is going to grow and push into the rich cluster, destroying the homogeneity of both clusters. In other words, poor clusters quickly evaporate and deposit their particles into the rich cluster. This would lead to an
Figure 6.9:

equilibrium of perfectly mixed poor and rich agents, unless some other force prevents the poor diffusing into the rich clusters.

Now suppose that we combine racial and income utilities and, for simplicity, we do this additively. This will give a utility function of the form

$$U_{RI} = U_R + U_I$$  \hspace{1cm} (6.4)

Since in equation 6.4, racial and income utilities are additive components of utility, this means that racial and income components of the system evolve independently according to their own separate forces. Hence, racial segregation will proceed unaffected by the income preference unless it is a negligible component of the total utility. The only instance where these two utilities interact is when a racial cluster and an income cluster share a surface. Assume that a rich cluster has formed and it shares a part of its surface with a racial cluster, then its surface force is everywhere either zero or $\vec{F}_I = u_I$, except on the shared surface where it is $\vec{F}_I + \vec{F}_R = u_I + u_R$. This net force on the shared surface is, therefore, the largest surface tension force and it deforms the surface faster than the rest of the rich cluster. There are two ways in which this force could be reduced: either by extending the rich cluster over the racial barrier or by contracting it away from the barrier and this is illustrated in Figure 6.10.

This yields an important conclusion: extremes avoid each other, that is, a rich-white cluster will avoid sharing a part of its surface with poor-black. The same is true for rich-black and poor-white.

Now let me complete the picture by taking the last element, house prices into account. For this it is possible to write the utility of an individual directly and to incorporate the effect of the price of housing. First, I have to specify the utility of living at
6.3. AN EXTENSION OF THE PREVIOUS MODEL

Figure 6.10:

location \((x, y)\) at time \(t\) when the price is \(P(x, y, t)\). This is done in equation 6.5.

\[
U_P = (x, y, t) = -u_P \sigma_I \sigma_R P(x, y, t) \tag{6.5}
\]

where \(\sigma_I\) is a factor which translates the fact that for poorer individuals the disutility of housing prices is higher. Thus we set \(\sigma_I \in [0, 1)\) if the individual is rich and \(\sigma_I = 1\) if the individual is poor. Since empirically blacks are poorer than whites we also introduce a second factor \(\sigma_R\) with \(\sigma_R \in [0, 1)\) if the individual is white and \(\sigma_R = 0\) if black.

Next we have to specify how house prices are determined and this will be done in the standard way, that is, as a function of excess demand. The price will thus be modified in time according to changes in demand \(D(x; y; t)\) for this location. The basic unit of time for this process is one iteration step, equivalent to moving one agent. Prices are then updated after one “price cycle” of time length \(t_c\).

Thus the adjustment process is given by,

\[
P(x, y, t) = \frac{|D(x, y, t) - D(x, y, t - t_c)|}{N_{tot}} \text{ if } \frac{t}{t_c} \text{ is an integer and } P(x, y, t-1) \text{ if not.}
\]

Demand \(D(x; y; t)\) is determined by counting all agents in the model that would prefer moving to \((x; y)\). The theoretical minimum demand is zero and the maximum is equal to the total number of people \(N_{tot}\) in the model.

The force exerted by the housing price utility behaves differently than the racial and income forces. The utility \(U_P\) depends only on its local value and it is equivalent.
to a local potential energy $\varepsilon_P(\vec{r}, t)$. The housing price force is a gradient of this scalar field $\vec{F}_P(\vec{r}, t) = -\nabla \varepsilon_P(\vec{r}, t)$ and it behaves like a pressure trying to either expand or compress the clusters. This pressure is forcing particles to follow the price gradient and flow toward regions of lower prices. In other words, other things being equal, individuals will tend to move towards cheaper housing. Unlike racial and income forces, which are zero unless the direction is tangential to the cluster surface, the price pressure force acts in all directions, although with different amplitudes (see figure 6.11).

Now we can express total utility as the sum of the three components which of course is a simplification but provides at least a basis for our analysis. Thus we have,

$$U_{\text{tot}}(x, y, t) = U_R(x, y) + U_I(x, y) - U_P(x, y, t)$$

(6.6)

Given all this we can reach some preliminary conclusions. Firstly, the additive property of utilities in equation 6.6 still implies that \textit{racial segregation will appear whenever racial utility provides a non-negligible contribution to the total utility.}

### 6.3.1 Results of simulations

There is obviously a plethora of possibilities to explore with such a large set of free parameters in our model in. Instead of exhausting all possibilities, we decided to focus on one set of parameters that captures some basic observed characteristics of racial segregation in the United States and then explore what happens when we individually vary these parameters. Here I will just describe what happens in the the “baseline model” which involved the following parameter values, for the numbers of rich and poor of each colour: $N_{wr} = 2250, N_{wp} = 2250, N_{br} = 2250, N_{bp} = 2250$ and the values of the other parameters were given by: $u_R = 1, \rho_R = 0.5, u_I = 1, \gamma_I = 0.5, u_P = 5, \sigma_R = 0.7, \sigma_I = 0.1, t_c = N_{\text{empty}}$.
6.3. AN EXTENSION OF THE PREVIOUS MODEL

The model and its variants were calculated on a 100x100 non-periodic grid, thus, the number of empty space is $N_{\text{empty}} = 1000$. We used two different price update cycles one of which was short of length $N_{\text{empty}}$ and one long of length $N_{\text{tot}}$.

During a short cycle each vacant location becomes occupied once on average, while during a long cycle each agent makes at least one move on average. We used a short cycle for the baseline model because it is more close to a realistic price dynamics in cities, although we also explored what happens when the cycle is long. The numerical experiments start with the initial price of zero everywhere. The system evolves quickly within the first 20,000 iterations when the total system utility, measured by the sum of the individual utilities, gets very close to its “equilibrium” value. After that the total utility (averaged over a price cycle) grows very slowly, while the system evolves further by rearranging particles in order to optimize the clusters’ size and shape. This change in the rate of system evolution happens because initially agents form the smallest clusters possible that would give them the largest increases in their utilities. After that changes in demand are relatively small, which results in small changes of utility for the majority of agents. The evolution becomes dominated by changes in the clusters’ shape and size, with only a small number of agents experiencing large utility changes during a price cycle by gaining on racial or income utility. But this slow evolution due to the constantly varying scalar field of housing prices is an important new feature in the Schelling model. Without this time dependent field the system would quickly approach some equilibrium and freeze. Therefore, in our previous study (Vinkovic & Kirman 2006) we had to introduce an additional rule of letting agents move when their utility remains constant. This produced a “liquid” behavior of the system. Here we do not need this rule because small variations in individual utilities due to changes in the housing price drive the system to behave like a liquid.

In the figures that follow we show the model properties after 1,000,000 iterations. In addition to two-dimensional spatial distribution of agents, we also follow two-dimensional spatial distribution of housing prices, distribution of agents of different type over the full range of housing prices (from 0 to $u_p = 5$), the distribution of agents of different types over a range of individual utility values (from -3 to the maximum of $u_P + u_I = 2$) and the measure of segregation in income and race for different sizes of “city blocks”. Distributions over housing prices and individual utilities are calculated by dividing the prices and utilities into 30 bins. Statistical fluctuations are somewhat reduced by averaging distributions over the last 10% of iterations within a price cycle. The measure of segregation for a city block of $k \times k$ in size is calculated as a sum of deviations from the overall average fraction of blacks or whites:

$$S (k) = \frac{1}{N_k} \sum_{i=1}^{N_k} \left| \frac{N_b}{N_{\text{tot}}} - \frac{N_{b,i}}{N_{b,i} + N_{w,i}} \right| = \frac{1}{N_k} \sum_{i=1}^{N_k} \left| \frac{N_w}{N_{\text{tot}}} - \frac{N_{w,i}}{N_{b,i} + N_{w,i}} \right|$$

where $N_k = (N_x - k) (N_y - k)$ is the total number of possible positions of the city block within the city and $N_{b,i}$ and $N_{w,i}$ are the number of blacks and whites within a city block $i$. The same function can be written for income segregation if indexes for black and white are replaced with poor and rich. Initial random positioning of agents introduces some unintended segregation. This constitutes the minimum segregation and it is marked by a dotted line in segregation plots. Double-counting prevents an
easy estimate of the maximum segregation value, but what is important to observe is how strongly it deviates from the minimum value associated with a random distribution. Notice that the level of segregation differs for different sizes of city blocks. In general, segregation is stronger on smaller scale, while the city block equal to the whole city in size has zero segregation by definition. The results for the baseline model are shown in figure 6.5. Racial segregation reached the level of “complete” segregation where racial clusters eventually merge into one big cluster (for details see Vinkovic and Kirman (2006). Spatial segregation by income is almost negligible. It exists only on small scale and it is not capable of evolving into larger clusters as I suggested in the theoretical considerations. This is also visible on the spatial distribution of prices, which does not indicate any large price cluster forming. However, the number of people in various price groups shows that richer agents occupy more expensive housing. They can afford this because the impact of prices ($\sigma_I = 0.1$) on their utility is much smaller than for poor people. If only the poor side of the income distribution is considered (housing prices below $\sim 2$), whites on average occupy slightly higher housing prices as expected from $\sigma_R = 0.7$. Notice how different segregation properties coexist in this system: spatial racial segregation, but no spatial income segregation, while there is income segregation in the housing price distribution. This is important to understand because any partial study that would look only at one property (for example, distribution over housing prices) might yield misleading conclusions. The results for the baseline model are shown in figure 6.12.

![Figure 6.12](image-url)

Figure 6.12:

It is easy to see that the baseline model shows spatial racial segregation, but no significant spatial income segregation. The far left panel is a spatial distribution of agents of different types. The middle panel is a spatial distribution of housing prices.
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The lack of noticeable clustering of prices is an indication of negligible spatial income segregation. The far right panels are distributions over housing prices, individual utility values and measures of segregation (see equation 6.7; dotted line is segregation due to random positioning of agents). The system segregates in income over housing prices, even though it does not show spatial income segregation.

6.3.2 Other cases

In Vinkovic and Kirman (2009) we studied a number of variants on the baseline model and this produced some interesting changes in the spatial configurations. I will give one example here and it concerns limiting the amount of free space in the area. This might be thought of as an arrival of new inhabitants who put more pressure on the existing housing stock and eliminate what were previously empty spaces. The empty spaces play a very important role in the Schelling model. In Vinkovic & Kirman (2006) we explained how empty space takes the role of a boundary layer that can stabilize a cluster surface that would undergo deformations if in direct contact with the unlike cluster. This is why wide avenues or city parks often become boundaries of racial and income clusters in urban areas. When we reduced the number of empty cells in our baseline model from 1000 to 400, an extensive spatial income segregation emerged (see figure6.6). In addition there were some interesting features of this spatial income sorting. Poor clusters are identified with uniformly low housing prices, while rich clusters experience dramatic house price changes from very high to very low. This price oscillations happen because in one price cycle the price of certain units is so high that only rich people can afford them. This drastically reduces the demand for these locations and deflates their prices to very low levels in the next cycle. These very low prices generate a high demand and the prices inflate again to high values in the next cycle, which perpetuates the price oscillations.

It is easy to see why income clusters emerge under these conditions. If a poor agent manages to find a very low price vacant spot in a rich cluster it will occupy it. However, in the next price cycle the housing price of this location is going to increase above what the poor agent can afford and it will be forced to move out. On the other hand, a rich agent is far more resilient to price changes because housing prices do not affect its utility as much. A smaller number of empty cells in the system also reduced the probability for poor agents to find a low price housing in rich clusters. This, of course, reinforced the income clustering. In the particular example we chose the number of free cells was reduced to 400. The model with reduced number of empty cells (400 in this example: \( N_{wr} = N_{wp} = N_{br} = N_{bp} = 2400 \)) shows an additional spatial income segregation in comparison with the baseline model as can be seen in figure 6.13. This is also visible in the middle panel where poor clusters have low housing prices, while rich clusters experience very volatile prices. Notice also that the highest prices are essentially concentrated on the boundaries of racial clusters and as I have mentioned before, how extremes try to avoid each other (rich white clusters avoid poor black, and rich black avoid poor white).

This resulted in increased stability of rich clusters and gave them ability to grow. As I already predicted in , emergence of rich clusters also leads to the phenomenon of extremes avoiding each other. Rich white clusters tend to be separated from poor black
clusters by a zone of either poor whites or rich blacks. Similarly, rich blacks avoid poor whites. An additional feature is noticeable in the case of simultaneous emergence of rich and low income clusters: the highest prices are found at the boundaries of racial clusters within rich clusters. This is due to an extra demand for these locations. Not only is it true that all the poor would like to be in these rich locations, but also a big fraction of black and white rich agents would benefit in their racial utility from moving to this boundary.

An obvious question about our model is as to the volatility of the price of houses occupied by the rich. This is, in large part, due to the price adjustment mechanism that we used. An alternative would be to have an auction for each vacant cell and to establish prices in this way. Another feature which is of importance is the speed of price adjustment. How many people can move at existing prices before the next price adjustment step is made? If this is large, i.e., we slow down the adjustment process by having a long period between price changes, then the system tends to “freeze” as the matching between agents and cells improves. This is still not enough, however, to eliminate price volatility. However, the overall utility of the population, measured by the sum of the utilities increases.

We also examined the cases where there were essentially poor people and, in this case, segregation essentially disappears. We then considered the case where the percentage of poor blacks was larger as was the case in 1960 when the proportion of blacks living in lower income households was 80% as opposed to 50% today, (U.S. Census Bureau (2000). Racial clustering in this version of the model leads to income segregation in whites, with the additional property that white clusters are mostly rich. This is a direct consequence of black clusters being also poor clusters by their nature.
6.3. AN EXTENSION OF THE PREVIOUS MODEL

Since rich white clusters avoid direct contact with poor black clusters, they are forced to move into the interior of white clusters, while poor whites occupy the boundary between white and black clusters. In addition, a large fraction of poor whites scatters into the poor black neighborhoods in search for lower housing prices, which leaves white clusters richer on average. Rich blacks, on the other hand, do not cluster because of their small number (any attempt to cluster is quickly disturbed by a large influx of poor blacks). Hence, the apparently different behaviour of rich whites and rich blacks is not the cause of segregation in this model, but a consequence of it.

Although we considered other cases such as differences in the quality of housing which can have an important influence on segregation, I will just mention one last case. This is where people of one race are more racially tolerant than those of the other race. Such asymmetries have been observed, and Cutler et al. (1999) showed theoretically that relative differences between housing prices of blacks and whites can indicate which race is exercising larger racial preference and causing segregation. If segregation is caused by white preferences for white neighborhoods, then whites will pay relatively more for housing than blacks as segregation rises. This will reduce the relative housing costs of blacks compared to those of whites. If discrimination and/or black preferences for black neighborhoods are the causes of segregation, then blacks will pay relatively more for housing than whites in more segregated cities. This will increase housing costs for blacks relative to whites.

We introduced asymmetric racial preferences between blacks and whites in order to test the ability of our model to reproduce these trends. In one variant of our model blacks were more tolerant than whites ($\sigma_R = 0.625$ for blacks and $\sigma_R = 0.375$ for whites), while in the other model the opposite was the case. We found that, indeed the distribution of housing prices shows the trend predicted by Cutler et al. (1999). However we also found that an additional pattern emerged: the less tolerant racial group creates mostly rich racial clusters. This is a direct consequence of the increased demand for racially intolerant clusters, while the demand for racially tolerant clusters decreased (that is, the need for clustering is low in the case of racially tolerant agents). The rich can afford the inflated housing price of racially intolerant clusters, while the race intolerant poor have difficulty clustering because as soon as they start forming a cluster the housing price increases.

6.3.3 A little empirical evidence

There is, by now, a huge empirical literature on racial and economic segregation, it not my purpose to make even a perfunctory survey of that literature but I will cite two examples of studies of U.S. cities, New York, (Mobius (2007)) and Chicago, (Mobius and Roseblatt, (2001). Both of these illustrate the expansion of ghettoes and examine the mechanism underlying this expansion and both inherit characteristics of Schelling’s model.

To take New York first, the idea here is to look at the development of Harlem over time. As Mobius (2007) explain, this figure shows the evolution of Harlem which was an all white upper class area in the 19th century, then received an inflow of Jews from Eastern Europe in the 1890’s and developed an Italian section in the North East. There
were a few blacks in blocks on the periphery of central Harlem and these are marked in black in the figure. Then West Harlem became a black ghetto and after 1920 the ghetto expanded to the East and to the South. By the end of the decade there were blacks living as far south as 110th street. Although Mobius’s analysis is based essentially on the racial criterion, what is particularly interesting, is that prices were highest, as predicted by our model on the frontier of the ghetto as it expanded.

Other cities have seen a considerable expansion in terms of ghettos. As Mobius (2007) explains,

“In the wake of the Great Migration, northern US cities, in particular, be-
6.4. CONCLUSION

Came indeed much more segregated. In 1940 the average African American lived in a residential area that was 37.6 percent black and by 1970 that share had increased to almost 70 percent. Cutler, Glaeser and Vigdor (1997) found in a sample of 313 US cities that only 5 cities had ghettos in 1910 but more than a third had one by 1970. Most of these almost exclusively black neighborhoods formed around the principal black cluster of concentration that happened to exist before the Great Migration.” Mobius (2007) p.33

In figure 6.15 we see the evolution of the black areas in Chicago, and from left to right the panels correspond to the years 1940, 1950 and 1960. What is interesting here is how exactly this expansion took place. What Mobius and Rosenblatt (2001) argue is that the expansion of black neighbourhoods was essentially one dimensional in that people in a street are worried about who their immediate right and left neighbours are, much more than being concerned about the general racial composition of the block. Mobius (2007) also describes what he calls “avenue waves” of segregation in Chicago due to propagation along streets.

What neither of these authors provide, however, is an empirical analysis of the dynamics of housing prices over the period of 20 years and this would be an interesting topic for future research to see whether our analysis can explain this aspect.

6.4 Conclusion

Schelling’s model is a wonderfully simple vehicle for examining the emergence of segregation. The patterns that emerge are dependent on the underlying assumptions in a rather subtle way and with Dejan Vinkovic we used a physical analogy to make sense of the various patterns that emerge. We complicated the model in various ways and found that the sort of patterns that we observed in the model were consistent with the empirical evidence. The reader will hardly be convinced by the very sketchy outline of the model and the results that I have given here but may be persuaded to go to the papers in question where full details are given. The introduction of preferences over more than characteristic such as race or income complicates the problem and the results of the two influences may be counter-intuitive. As Sethi and Somonathan (2004) say,

“The conventional view is rooted in the intuition that if households sort themselves across neighborhoods on the basis of income, then racial income disparities will be mirrored in segregated residential patterns. This intuition fails when households make location decisions based on multiple neighborhood characteristics. When individuals care also about the racial composition of their communities, the relationship between inequality and segregation is more complex and depends in subtle ways on both intraracial and interracial disparities in income.”

Sethi and Somonathan (2004) p. 1298
The sort of analysis I have sketched in this chapter shows ways in which we can try to understand one of the major empirical puzzles. Why despite increasing racial tolerance and the narrowing of racial income disparities has segregation persisted. Of course, one could make two arguments, firstly if this is what people choose why should we be concerned? Yet this is counter to the views that people claim to have. Secondly maybe we have just got locked in to an inferior equilibrium and it persists despite the emergence of other superior potential equilibria. In any event we cannot afford to simply ignore the problem since as Cutler and Glaeser (1997) show segregation continues to penalise blacks. They compared the outcomes of blacks between cities and found, in particular that blacks in racially more segregated cities earn less income and are more likely to become single mothers or drop out of high school.

In conclusion then, the message from this chapter is that Schelling’s original insight as to the fact that macro phenomena are not necessarily a reflection of underlying individual preferences is both confirmed and reinforced by our analysis. Furthermore, the social and economic consequences of this interaction may be important.
Chapter 7

Conclusion

This book has been something of a walk through the foothills of economics. It reached no lofty heights nor, I hope, was there so much formalism that the reader felt as if he lacked oxygen. It is possible that someone who had not passed along the path of theoretical economics sometimes felt a little out of breath but surely no more than that. My idea and aim has been to suggest that we should recentre economics on different themes than the traditional ones of efficiency and equilibrium and that we should place interaction and coordination at the centre of our interests. In particular, we should focus on interaction and its consequences for aggregate behaviour. To do this requires, I believe, rethinking, in a fundamental way, how we model economic activity. Doing this does not simply mean taking on board each criticism or suggestion from psychology, biology, physics or sociology and then putting our usual tools to work. This, can and has, been done by those who are past masters at using principal agent, game theoretic or other standard economic models. However, I would like to see a much more fundamental revamping of our discipline. We should accept that individuals are suspended in a web of relations and linked directly and indirectly to others. Our preferences cannot be defined only in terms of an ordering over material opportunities but are rather, at least in part, determined by the preferences and actions of others. Since this is true of all the individuals one is faced with a system which is in evolution with no special reason as to why it might converge to some stationary state.

This sort of idea as I have said before, is far from new, and it has strong echoes of the Austrian School which fell from favour because of its “lack of rigour” and also because of its ideological associations. Yet reading Schumpeter on “creative destruction” and Hayek on “self organisation” one is struck by the insights which preceded, by far, the appearance of what we are pleased to call complexity theory. The Austrians were not so preoccupied with efficiency or the progress towards efficiency as are modern economists, indeed Von Mises argued that it is impossible to to determine and meaningless to suggest that the real economy is closer to the final state of rest and therefore manifests a superior coordination of plans and greater allocative efficiency, at one instant of time than it was at a previous instant (Mises Human Action pp.245-246). He had a vision of a market in continuous restless movement drifting back towards a state of rest but constantly perturbed by the arrival of fresh information or by the discovery
of opportunities by the actors.

Thus Mises has the idea that individuals look for opportunities but often do not have the information necessary to be able to profit from them. He does not view individuals as optimising but rather, as opportunistic. Individuals never possess the information necessary to fully optimise but this does not prevent them from trying to seize opportunities as they arise. Our of this constant search and adjustment some sort of consistency of actions arises. This is what Hayek argued, since he had the view that markets self organise but, in so doing, make internal adjustments as the participants learn new modes of behaviour or, simply how to get around the existing rules. Once again, as I have said, learning is important but individuals may be attempting to learn about moving targets, in particular other individuals who are themselves learning. There is no good reason to believe that such a process will converge, as we have seen. But once we recognise this we can put paid to a standard myth. The idea that markets do self-organise has been used to justify the injunction to leave them to their work. The less interference there is the better. Yet since the self organisation may not be stable such a view is not justified. Whilst, there is a constant restructuring of the economy and the relations within it, there is the always the possibility that this process will lead to a sudden and possibly catastrophic change at the aggregate level. In the simplified models of financial markets that I presented we saw how the fact that people herd on successful strategies can generate bubbles and crashes. This does not proclaim the existence of long periods of apparent stability.

As Sornette (2003) argues, a stock market crash is not the result of short-term exogenous events, but rather involves a long-term endogenous buildup, with exogenous events acting merely as triggers. In particular, Sornette shows that financial crashes are the result of the “spontaneous emergence of extreme events in self-organizing systems,” and observes that “extreme events are characteristic of many complex systems.” This echoes Minsky’s (1982) reflection on the “disruptive internal processes” in the economy.

Each of the chapters of this book has tried to show what can happen at the aggregate level when individuals interact with each other. When we consider this interaction, as we have seen, we have also to look at the structure of that interaction. In other words, the network that links people together can have an important impact on the outcome of the whole system. Take the following observation of Andrew Haldane of the Bank of England. He compares the SARS epidemic and the current financial crisis and says:

“Both events were manifestations of the behaviour under stress of a complex, adaptive network. Complex because these networks were a cat’s cradle of interconnections, financial and non-financial. Adaptive because behaviour in these networks was driven by interactions between optimising, but confused, agents. Seizures in the electricity grid, degradation of eco-systems, the spread of epidemics and the disintegration of the financial system? each is essentially a different branch of the same network family tree.” Haldane (2009) p.3

While I would prefer to replace the word “optimising” by “opportunistic”, what is striking here is the recognition of the importance of the view of the economy as
a system of interacting agents and the emphasis on the structure of the relationships between those individuals. In the chapter on fish markets, we saw how patterns of links emerge and their influence on market outcomes. The difficulty for economics, of absorbing the lessons from graph theory and networks is precisely that the individuals are subject to the direct influence of others and while this idea is commonplace in sociology, it is far from being so in economics. To return to markets for a minute, it is interesting to hear what a sociologist who has studied economic activity intensively has to say,

“Modern markets are social structures that consist of roles, conventions, and power struggles. The telecommunications market is thus analogous to the Lutheran Church, or to the Detroit school system. Sociologists have approached explaining the social structures and conventions found in markets much as they approach explaining structures and conventions in a church or a school system. Common sense tells us that that markets and economic conventions are shaped by economic laws. Sociologists find that concrete social processes matter too”. Dobbin (2004)

Again, we have seen how aggregate phenomena such as racial segregation can emerge from the interactive behaviour of relatively tolerant individuals. Here, I argue, there are lessons to be learned from physics where the study of self organising systems is widely undertaken. The rejection of the idea that we can ignore these lessons because the theory in question was designed for particles without purpose or intent is, I claim, mistaken. Once the rules whatever their origins, which govern the behaviour and the interaction are well defined, we can legitimately apply such models. Indeed, economists have not hesitated over the last century to do this as Mirowski (1989) has shown. But now it seems to be a question of “better the physics you know than that which you do not know”. Yet we have much to learn, I believe, from statistical physics. This point is forcefully argued by Mark Buchanan (2005) in his defence of what he calls “social physics”.

Nevertheless, it is always tempting for economists to avoid this route, to revert to individualistic explanations of aggregate phenomena and to put the role of interaction to one side. When we looked at how individuals coordinate their contributions to public goods, it is simple to argue that some individuals are intrinsically more generous than others and to leave it at that. Yet, as we have seen, careful examination of the experimental evidence shows that things are much more complicated than this and that individuals switch between generosity and free riding, no doubt as a result of their interaction with and reaction to the other contributors.

Perhaps the most convincing argument of all is that in a host of other disciplines, from statistical physics to the neurosciences, to biology and to the social sciences other than economics have come to the conclusion that systems of individuals behave in a fundamentally different way from that of the individuals who make them up. A group of biologists argued as follows:

“Understanding the mechanisms governing and regulating the emergence of structure and heterogeneity within cellular systems, such as the developing embryo, represents a multiscale challenge typifying current integra-
tive biology research, namely, explaining the macroscale behaviour of a system from microscale dynamics.” Baker et al. (2008), p.251.

But here again the emphasis is on dynamics and not on stationary states and economists should perhaps be more willing to focus on the dynamics of our systems and, in particular, the out of equilibrium dynamics. Again what should have been evident from almost every page in this book is the radical difference between individual and collective behaviour. This, as an earlier quote made clear, is of primary concern to neuroscientists but the problem is present in many if not most disciplines.

To cite Robert Laughlin a Nobel Laureate in physics, quoted by Mark Buchanan, (2007),

“I am increasingly persuaded that all physical law that we know has collective origins, not just some of it. In other words, the distinction between fundamental laws and the laws descending from them is a myth…Physical law cannot generally be anticipated by pure thought, but must be discovered experimentally, because control of nature is achieved only when nature allows this through a principle of organisation… What physical science has to tell us is that the whole being more than the sum of its parts is not merely a concept but a physical phenomenon. Nature is regulated not only by a microscopic rule base, but by powerful and general principles of organisation” Robert Laughlin, (2005), “A Dufferebt Universe”

Once again the emphasis is on the organisation and coordination of activity and not on the precise details of the individuals or particles that engender that activity. Yet also there is here the clear idea that our analysis should not be based on hypotheses derived from pure introspection. Werner Hildenbrand has argued for some years now that, for precisely this reason, we should focus on statistical properties of the economy and not on individual behaviour. This is the opposite position from those who still wish to reduce the behaviour of the whole to that of an individual, and yet seems far more reasonable. Looking at the distribution of choices and seeing what characteristics that distribution has is more informative than worrying about the individuals’ reasons for making those choices. In particular, if we add the feature that these choices will be correlated for various reasons such as those so nicely explained in “Animal Spirits”, (Akerlof and Shiller (2009)), we should worry much more about the evolution of the aggregate and less about the strict individualistic rationality of the economic agents.

Having got as far as this the reader may well ask “but how does this help us?” If we accept the general idea that is behind the rather simple and limited examples that I have discussed and apply it to the economy as a whole can we say anything constructive? As Akerlof and Shiller say :

“…people seem to be rethinking their views of the economy. The recent economic turmoil has brought back to the table many questions that had been considered settled. Now people are seeking new answers urgently. We see it in the newspapers, We see it in the think tanks, and at conferences and in the corridors of economics departments”. Akerlof and Shiller (2009) Animal Spirits, p.174
What they argue for is a much more active role for the government in the economy, even though this is anathema for those whose ideological positions have been comforted by the last 20 years economic experience. The view of the economy that I have tried to convey is that it is indeed a self-organising system which is not stable and is always in danger of sliding like the slime in “Emergence” over the edge into a recession or even a depression. But given this we should not hope to be able to design a system of constraints which, when imposed now, will prevent it ever doing so in the future. Given the obvious problems that have been posed by the recent evolution in the world’s economy it would not be very intelligent just to sit back until the ship gets back on course. So we have seen massive interventions by governments, but these were short term measures, no doubt measures to restrict leverage and to restrain the granting of credit will help to prevent an exact reproduction of what happened this time. But, at least it seems to me that even if such measures are put in place and the financial system is revamped at some point the slime will find a few cracks through which to slide. So the government’s role is not simply to retrace the lines of tennis courts or to adjust the height of the baskets, but rather to be ready to anticipate and identify those changes that may make what should be a gentlemanly, and even fair, sport turn into a rowdy free for all.

But the point of this book was not to make policy recommendations but rather to persuade people to rethink the way in which we look at the economy. To do so requires a mental effort and a willingness to put aside our comfortable basic conventions. But if doing this helps us to understand a little better how the economy works this will surely be worthwhile. I am not particularly optimistic about transforming the way in which most of my colleagues think and it would be pretentious to think otherwise. For as Aristotle said,

“In some ways, the effect of achieving understanding is to reverse completely our initial attitude of mind. For everyone starts (as we have said) by being perplexed by some fact or other: for instance... the fact that the diagonal of a square is incommensurable with the side. Anyone who has not yet seen why the side and the diagonal have no common unit regards this as quite extraordinary. But one ends up in the opposite frame of mind... for nothing would so much flabbergast a mathematician as if the diagonal and side of a square were to become commensurable.”

Aristotle

However there will be many young economists who may be tempted to explore the route that I have mapped out and all that I can say is that the journey may be more stimulating than the arduous and rather barren treks that economists have undertaken in recent years. As somebody said, we have spent the twentieth century in economics developing and perfecting a model based on nineteenth century physics. Perhaps now in the twenty first century we can move on to physics from the twentieth century. The hope that I see is that there are some signs that economic theory like the economy itself is also a complex adaptive system and it just might be self-organising so that it slides collectively over the edge into a new and possibly more promising phase.
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